Knowledge representation and semantic technologies

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Exercise on Reasoning about Actions

(a) Axiomatize the following scenario, appropriately with action precondition and effect axioms, and obtain successor state axioms.

Fluents:

- `doorOpen(s)` - The door is open in situation s.
- `insideRoom(s)` - The robot is inside the room in situation s.

Actions:

- `openDoor` - The robot opens the door. This can be done if the robot is not inside the room and the door is closed (that is, not open), and has the effect that the door will be open.
- `closeDoor` - The robot closes the door. This can be done if the robot is inside the room and the door is open, and has the effect that the door will be closed.
- `enter` - The robot enters the room. This requires that the door is open and the robot is not inside the room, and has the effect that the robot will be inside the room.

Initial situation description: Initially the robot is not inside the room and the door is closed (that is, not open).

(b) Show, by applying regression, that the sequence of actions `openDoor, enter, closeDoor` is executable in the initial situation.

(c) Show, by applying regression, that the robot will be inside the room after the sequence of actions `openDoor, enter, closeDoor`.

Solution (a)

Action precondition axioms:

$$\begin{array}{l} Poss(\texttt{openDoor}, s) \equiv \neg\texttt{insideRoom}(s) \wedge \neg\texttt{doorOpen}(s) \\ Poss(\texttt{closeDoor}, s) \equiv \texttt{insideRoom}(s) \wedge \texttt{doorOpen}(s) \\ Poss(\texttt{enter}, s) \equiv \texttt{doorOpen}(s) \wedge \neg\texttt{insideRoom}(s) \end{array}$$

Effect axioms:

$$\begin{array}{l} \texttt{doorOpen}(do(\texttt{openDoor}, s)) \\ \neg\texttt{doorOpen}(do(\texttt{closeDoor}, s)) \\ \texttt{insideRoom}(do(\texttt{enter}, s)) \end{array}$$

Effect axioms in normal form:

$$
\begin{array}{l}
(a = \texttt{openDoor}) \rightarrow \texttt{doorOpen}(do(a, s)) \\
(a = \texttt{closeDoor}) \rightarrow \neg\texttt{doorOpen}(do(a, s)) \\
(a = \texttt{enter}) \rightarrow \texttt{insideRoom}(do(a, s)) \\
\mathit{false} \rightarrow \neg\texttt{insideRoom}(do(a, s))
\end{array}
$$

Successor state axioms:

$$
\begin{array}{l}
\texttt{doorOpen}(do(a, s)) \equiv (a = \texttt{openDoor}) \vee (\texttt{doorOpen}(s) \wedge (a \neq \texttt{closeDoor})) \\
\texttt{insideRoom}(do(a, s)) \equiv (a = \texttt{enter}) \vee (\texttt{insideRoom}(s) \wedge \mathit{true})
\end{array}
$$

Initial situation description:

$$
\begin{array}{l}
\neg\texttt{insideRoom}(S_0) \\
\neg\texttt{doorOpen}(S_0)
\end{array}
$$

Solution (b)

To check the executability of the sequence of actions `openDoor, enter, closeDoor` (legality task), we have to establish whether the following formula is a logical consequence of the action specification and the initial situation description:

$$
\begin{array}{l}
Poss(\texttt{openDoor}, S_0) \wedge \\
Poss(\texttt{enter}, do(\texttt{openDoor}, S_0)) \wedge \\
Poss(\texttt{closeDoor}, do(\texttt{enter}, do(\texttt{openDoor}, S_0)))
\end{array}
$$

We now apply the regression operator to the above formula: from

$$
\begin{array}{l}
\mathcal{R}[Poss(\texttt{openDoor}, S_0) \wedge \\
Poss(\texttt{enter}, do(\texttt{openDoor}, S_0)) \wedge \\
Poss(\texttt{closeDoor}, do(\texttt{enter}, do(\texttt{openDoor}, S_0)))]
\end{array}
$$

we obtain

$$
\mathcal{R}[Poss(\texttt{openDoor}, S_0)] \tag{1}
$$

$$
\wedge\ \mathcal{R}[Poss(\texttt{enter}, do(\texttt{openDoor}, S_0))] \tag{2}
$$

$$
\wedge\ \mathcal{R}[Poss(\texttt{closeDoor}, do(\texttt{enter}, do(\texttt{openDoor}, S_0)))] \tag{3}
$$

First, observe that:

$$
\mathcal{R}[Poss(\texttt{openDoor}, S_0)] = \neg\texttt{insideRoom}(S_0) \wedge \neg\texttt{doorOpen}(S_0)
$$

and the above formula holds in the initial situation. So, formula (1) holds in the initial situation.

Then, we have that

$$
\begin{array}{l}
\mathcal{R}[Poss(\texttt{enter}, do(\texttt{openDoor}, S_0))] = \\
\texttt{doorOpen}(do(\texttt{openDoor}, S_0)) \wedge \neg\texttt{insideRoom}(do(\texttt{openDoor}, S_0))
\end{array}
$$

and applying the regression operator to the above formula, we obtain

$$
\begin{array}{l}
\mathcal{R}[\texttt{doorOpen}(do(\texttt{openDoor}, S_0)) \wedge \neg\texttt{insideRoom}(do(\texttt{openDoor}, S_0))] = \\
\mathcal{R}[\texttt{doorOpen}(do(\texttt{openDoor}, S_0))] \wedge \neg\mathcal{R}[\texttt{insideRoom}(do(\texttt{openDoor}, S_0))]
\end{array}
$$

Now:

- $\mathcal{R}[\texttt{doorOpen}(do(\texttt{openDoor}, S_0))] = (\texttt{openDoor} = \texttt{openDoor}) \vee (\texttt{doorOpen}(S_0) \wedge (\texttt{openDoor} \neq \texttt{closeDoor}))$, which is equivalent to *true*;
- $\mathcal{R}[\neg\texttt{insideRoom}(do(\texttt{openDoor}, S_0))] = \neg((\texttt{openDoor} = \texttt{enter}) \vee (\texttt{insideRoom}(S_0) \wedge true))$, which is equivalent to $\neg\texttt{insideRoom}(S_0)$, which holds in the intial situation.

Therefore, formula (2) holds in the initial situation.

Finally, we consider formula (3):

$$\mathcal{R}[Poss(\texttt{closeDoor}, do(\texttt{enter}, do(\texttt{openDoor}, S_0)))] =$$
$$\texttt{insideRoom}(do(\texttt{enter}, do(\texttt{openDoor}, S_0))) \wedge \texttt{doorOpen}(do(\texttt{enter}, do(\texttt{openDoor}, S_0)))$$

We now apply the regression operator to the above formula, obtaining:

$$\mathcal{R}[\texttt{insideRoom}(do(\texttt{enter}, do(\texttt{openDoor}, S_0)))] \wedge \mathcal{R}[\texttt{doorOpen}(do(\texttt{enter}, do(\texttt{openDoor}, S_0)))]$$

Now:

- $\mathcal{R}[\texttt{insideRoom}(do(\texttt{enter}, do(\texttt{openDoor}, S_0)))] =$
  $(\texttt{enter} = \texttt{enter}) \vee (\texttt{insideRoom}(do(\texttt{openDoor}, S_0)) \wedge true)$, which is equivalent to *true*;
- $\mathcal{R}[\texttt{doorOpen}(do(\texttt{enter}, do(\texttt{openDoor}, S_0)))]=$
  $(\texttt{enter} = \texttt{openDoor}) \vee (\texttt{doorOpen}(do(\texttt{openDoor}, S_0)) \wedge (\texttt{enter} \neq \texttt{closeDoor}))$,
  and this formula is equivalent to $\texttt{doorOpen}(do(\texttt{openDoor}, S_0))$.
  We now apply regression to the last formula, obtaining
  $\mathcal{R}[\texttt{doorOpen}(do(\texttt{openDoor}, S_0))] =$
  $(\texttt{openDoor} = \texttt{openDoor}) \vee (\texttt{doorOpen}(S_0) \wedge (\texttt{openDoor} \neq \texttt{closeDoor}))$,
  which is equivalent to *true*.

Therefore, formula (3) holds in the initial situation.

Consequently, the sequence of actions `openDoor, enter, closeDoor` is executable in the initial situation S_0.

Solution (c)

Using the regression theorem, we now check that the robot will be inside the room after the sequence of actions `openDoor, enter, closeDoor` (projection task).

We start from the formula

$$\texttt{insideRoom}((\texttt{closeDoor}, (\texttt{enter}, (\texttt{openDoor}, S_0))))$$

Applying the regression operator, we obtain

$$(\texttt{closeDoor} = \texttt{enter}) \vee (\texttt{insideRoom}((\texttt{enter}, (\texttt{openDoor}, S_0))) \wedge true)$$

which is equivalent to

$$\texttt{insideRoom}((\texttt{enter}, (\texttt{openDoor}, S_0)))$$

Applying the regression operator again, we obtain

$$(\texttt{enter} = \texttt{enter}) \vee (\texttt{insideRoom}((\texttt{openDoor}, S_0)) \wedge true)$$

which is equivalent to *true*.

Therefore, the initial formula holds in the initial situation, i.e., the robot will be inside the room after the execution of the sequence of actions `openDoor, enter, closeDoor` in the initial situation S_0.