

LEARNING GRAVITY COMPENSATION IN ROBOTS: RIGID ARMS, ELASTIC JOINTS, FLEXIBLE LINKS

ALESSANDRO DE LUCA AND STEFANO PANZIERI

*Dipartimento di Informatica e Sistemistica, Università degli Studi di Roma 'La Sapienza', Via Eudossiana 18,
I-00184 Roma, Italy*

SUMMARY

The setpoint regulation problem for robotic manipulators is a basic task that can be solved either by PID control or by model-based gravity compensation. These approaches are commonly applied both to rigid arms and to robots with flexible links and/or elastic joints. However, PID control requires fine and lengthy tuning of gains in order to achieve good performance over the whole workspace. Moreover, no global convergence proof has yet been given for this control law in the case of flexible links or elastic joints. On the other hand, a constant or even a configuration-dependent gravity compensation is only an approximate solution when an unknown payload is present or when model parameters are poorly estimated. In this paper a simple iterative scheme is proposed for generating exact gravity compensation at the desired setpoint without the knowledge of dynamic model terms. The resulting control law is shown to be global asymptotically stable for rigid arms as well as for manipulators with elastic joints or flexible links. Starting with a PD action on the error at the joint (i.e. motor) level, an additional feedforward term is built and updated at discrete instants. Convergence of the scheme is proved under a mild condition on the proportional gain, related to a bound on the gravity terms. In the presence of concentrated or distributed flexibility a structural property of the joint or of the link stiffness is further required, largely satisfied in practice. Simulation results are given for a three-link rigid arm and experimental results are also presented for a two-link robot with a flexible forearm.

KEY WORDS Robot control Regulation Elastic joints Flexible manipulators Iterative learning

1. INTRODUCTION

Regulation of multilink manipulators is often performed using linear feedback laws that exploit inherent physical properties of the mechanical system. In the absence of gravity it can be shown that a simple proportional derivative (PD) feedback at the joint (motor) error level is sufficient to asymptotically stabilize any arm configuration. This result is formally proved in Reference 1 for rigid arms, in Reference 2 for robots with joint elasticity and in Reference 3 when dealing with distributed link flexibility. In this last case a full state feedback including the deflection variables may improve the transient characteristics.⁴

For rigid manipulators under gravity the most direct approach to setpoint regulation is to globally cancel the gravity terms and still apply PD control with positive definite gains. This leads to a *non-linear feedback control* law which may be difficult to implement but yields exponential stability. Under a mild condition on the proportional gain this scheme can be simplified to *constant gravity compensation* as evaluated at the desired configuration;^{1,5} a purely linear feedback law with a feedforward action is then obtained (denoted PD+). In this

case the proportional gain should be chosen so as to dominate the gradient of the gravity force over the whole robot workspace.

When flexible components are present in the robotic structure, a similar strategy based on PD plus a constant gravity feedforward has also been shown to asymptotically stabilize robots with elastic joints² and with flexible links.⁶ For the elastic joint case feedback is closed only around the motor variables, while for the flexible link case only the joint (rigid) variables are used for control. This control strategy works under a further structural assumption on the joint and on the link stiffness respectively.

In all cases an exact knowledge of the gravity vector is assumed. This is difficult to achieve, e.g. for a robot picking up multiple unknown payloads, and therefore an on-line identification or an adaptive procedure would be required. In particular, the latter asks for a proper factorization of the non-linear gravity term⁵ and is quite sensitive to unmodelled dynamics.

For robots with flexible components it is well known that controlling the end-effector is quite a different problem from controlling the motion of the arm joints. In this case the source of steady state end-effector error under *pure* joint PD control is twofold: firstly, a displacement is present at the motor level just as in the fully rigid case; secondly, a further displacement is introduced by the arm (joint and/or link) deflection. High-gain feedback will reduce but not eliminate the total error, exciting in fact higher-order deformation modes and leading to longer transition times owing to the poorly damped oscillations.

On the other hand, to compensate for gravity effects, a standard model-independent remedy is the addition of an *integral term* to the linear PD law. However, several problems arise with the design of an efficient PID control, partly owing to the non-linear nature of the robot dynamics. Typically, saturation will occur during large-transient phases and reset or anti-wind-up procedures have to be devised when starting far from the final position.⁷

From a theoretical point of view, asymptotic stability of robot PID control has been proved *only* for rigid arms. This result holds *locally* around the desired configuration and requires complex inequalities among the proportional, derivative and integral gains to be satisfied.^{8,9} In practice, some of these drawbacks may be overcome by adding the integral action only near the final point, so that gross motion is performed with PD control, while fine positioning is achieved with PID. However, no formal proof of convergence has been given for this modified law.

In this paper we consider the *setpoint regulation problem under gravity for general manipulators*, i.e. with or without joint elasticity and with or without link flexibility. A fast iterative scheme is proposed that builds up the required compensation at the final configuration from a very limited knowledge of the robot gravity terms. A PD-based control law is applied iteratively *at the motor level*, while a constant gravity feedforward is learned at discrete instants, without the explicit introduction of an integral error term nor the use of high-gain feedback.

An easy-to-check sufficient condition is derived that guarantees the *global asymptotic convergence* of the scheme to zero steady state error, taking into account the robot non-linearities in the analysis. This condition is obtained within a general modelling set-up and then particularized for the two cases of fully rigid arms and of robots with elastic joints but rigid links. Whenever arm deflections are present, it is further assumed that *arm stiffness dominates gravity effects*, in analogy with References 2 and 6. This mild assumption is always satisfied in real robots.

Simulation results showing the performance of the overall scheme are presented for a three-link rigid arm moving in the vertical plane. Experimental results are also reported for the two-

link lightweight manipulator with a flexible forearm available in the Robotics Laboratory of our Department.¹⁰ This arm has been tilted from the horizontal plane so as to include gravity effects.

2. ROBOT MODEL PROPERTIES

The dynamic model properties relevant to our control approach are reviewed here for rigid robots, for robots with rigid links but flexible joints and for robots with flexible links.

Consider first a robot arm composed of a serial chain of n rigid links, actuated at the joints by motors connected through rigid transmissions. Denoting by θ and u the $n \times 1$ vectors of motor co-ordinates and motor torques respectively (both already scaled by the reduction ratio), the standard dynamic equations of motion are written as

$$B(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) + g(\theta) = u \quad (1)$$

with the $n \times n$ positive definite symmetric inertia matrix B and the $n \times 1$ vectors h and $g = (\partial U / \partial \theta)^T$ of Coriolis and centrifugal forces and of gravity forces respectively. It can be shown that the Hessian of the gravitational potential energy U is bounded as

$$\left\| \frac{\partial g}{\partial \theta} \right\| = \left\| \frac{\partial^2 U}{\partial \theta^2} \right\| \leq \alpha_1 \quad (2)$$

for both rotational and prismatic joints.⁵ In the following we will use as norm of a matrix A the spectral one, i.e. $\|A\| = \sqrt{[\lambda_{\max}(A^T A)]}$ ($\|A\| = \lambda_{\max}(A)$ for a symmetric matrix).

When the motion transmission elements are elastic, a further set of n generalized co-ordinates has to be considered in order to fully describe the robot arm configuration. In Reference 2 the complete modelling is carried out using the link position variables θ_i in addition to the motor variables θ . The potential energy associated with the joint elasticity is then written as $U_e = \frac{1}{2}(\theta_i - \theta)^T K(\theta_i - \theta)$, where $K > 0$ is the joint stiffness diagonal matrix. The $2n$ second-order differential equations of motion become

$$\begin{bmatrix} B_1(\theta_i) & B_2(\theta_i) \\ B_2^T(\theta_i) & B_3 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_i \\ \ddot{\theta} \end{bmatrix} + C(\theta, \theta_i, \dot{\theta}, \dot{\theta}_i) \begin{bmatrix} \dot{\theta}_i \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} g_1(\theta_i) \\ 0 \end{bmatrix} + \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{bmatrix} \theta_i \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (3)$$

In the above partitioning the motor torques perform work only on the associated motor variables, while the gravity forces act only on the link side. Similarly to (2), one has

$$\left\| \frac{\partial g_1}{\partial \theta_i} \right\| \leq \alpha_1 \quad (4)$$

being structurally $g_1 = g$ of the rigid case.²

In order to preserve a unified notation, we introduce the $n \times 1$ deflection variables $\delta = \theta_i - \theta$ to be used in place of θ_i . Applying this linear transformation leads to a model of robots with elastic joints in the form

$$\begin{bmatrix} B_1(\theta + \delta) + (B_2 + B_2^T)(\theta + \delta) + B_3 & B_1(\theta + \delta) + B_2^T(\theta + \delta) \\ B_1(\theta + \theta) + B_2(\theta + \delta) & B_1(\theta + \delta) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} h_1(\theta, \delta, \dot{\theta}, \dot{\delta}) \\ h_2(\theta, \delta, \dot{\theta}, \dot{\delta}) \end{bmatrix} + \begin{bmatrix} g_1(\theta + \delta) \\ g_1(\theta + \delta) \end{bmatrix} + \begin{bmatrix} 0 \\ K\delta \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (5)$$

When considering the presence of flexible links, the Lagrangian technique can still be employed to derive the dynamic equations of motion,¹¹ using a finite set of space functions

to describe link deformation shapes. A linear model is in general sufficient to capture the dynamics of each flexible link, with the slender links seen as Euler–Bernoulli beams with proper boundary conditions. By extending δ to be the $m \times 1$ co-ordinate vector of generalized deformations (possibly including both joint and link ones), the closed-form dynamic equations can be written as $n + m$ second-order non-linear differential equations in the given form¹²

$$\begin{bmatrix} B_{\theta\theta}(\theta, \delta) & B_{\theta\delta}(\delta, \delta) \\ B_{\delta\theta}^T(\theta, \delta) & B_{\delta\delta}(\theta, \delta) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} h_{\theta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \\ h_{\delta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \end{bmatrix} + \begin{bmatrix} g_{\theta}(\theta, \delta) \\ g_{\delta}(\theta, \delta) \end{bmatrix} + \begin{bmatrix} 0 \\ K\delta \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (6)$$

Note that the link deformation is implicitly described in a frame *clamped* at the joint side, implying that the control does not enter directly in the equations of the flexible part.

Comparing (6) with (5), we immediately see that the joint elastic model is a particularization of the general flexible case, though with different terms. For our purposes the most relevant difference is in the structure of the gravity vector $g = (g_{\theta}, g_{\delta})$. In the elastic joint case the two subvectors are equal ($g_{\theta} = g_{\delta} = g_1$) and have a functional dependence limited to the *sum* $\theta + \delta$. This allows us to bound the gradient of the whole gravity vector in (5) solely on the basis of inequality (4).

On the other hand, the more general dependence of the gravity components in (6) requires further analysis. Under the hypothesis of *small link deformation* the model validity extends up to a limited amount of stored elastic potential energy $U_e = \frac{1}{2} \delta^T K \delta \leq U_{e,\max}$, where the diagonal positive definite matrix K is the one appearing in (6). As a result, denoting by $q = (\theta, \delta)$ the $(n + m) \times 1$ vector characterizing the arm configuration, we have the inequality

$$\left\| \frac{\partial g}{\partial q} \right\| \leq \gamma_0 + \gamma_1 \|\delta\| \leq \gamma_0 + \gamma_1 \sqrt{\left(\frac{2U_{e,\max}}{\lambda_{\max}(K)} \right)} =: \alpha \quad (7)$$

where $\gamma_0, \gamma_1, \alpha > 0$. Similarly,

$$\left\| \frac{\partial g_{\theta}}{\partial q} \right\| \leq \alpha_{\theta}, \quad \left\| \frac{\partial g_{\delta}}{\partial q} \right\| \leq \alpha_{\delta} \quad (8)$$

These can be easily proved by observing that the gravity terms contain only trigonometric functions of θ and linear/trigonometric functions of δ . As a direct consequence of (7) we have

$$\|g(q_1) - g(q_2)\| \leq \alpha \|q_1 - q_2\|, \quad \forall q_1, q_2 \in \mathbb{R}^{n+m} \quad (9)$$

while similar inequalities are obtained from (8).

In the following we will consider the general flexible model (6), but it is intended that the obtained results fully apply also to robots with *rigid links* via proper simplifications. In particular, δ vanishes (together with g_{δ}) in the *rigid joint* case so that $g = g_{\theta}$ and

$$\alpha = \alpha_{\theta} = \alpha_1, \quad \alpha_{\delta} = 0 \quad (10)$$

In contrast, in the *elastic joint* case $g_{\theta} = g_{\delta} = g_1$ and

$$\alpha = 2\alpha_1, \quad \alpha_{\theta} = \alpha_{\delta} = \alpha_1 \quad (11)$$

where the properties of the spectral norm of a matrix¹³ have been taken into account.

3. PD+ CONTROL

For a desired constant motor position θ_d under perfect knowledge of the gravity term the input torque u can be chosen as a *linear PD+ control*, i.e.

$$u = K_P(\theta_d - \theta) - K_D\dot{\theta} + g_{\theta}(\theta_d, \delta_d), \quad K_P > 0, \quad K_D > 0 \quad (12)$$

with the associated δ_d implicitly defined as the solution of

$$g_\delta(\theta_d, \delta) + K\delta = 0 \quad (13)$$

This includes the case of fully rigid arms,¹ where $\delta_d = 0$, and that of elastic joints,² where $\delta_d \neq 0$.

For the dynamic model (6) it has been shown⁶ that under the assumption

$$\lambda_{\min} \begin{pmatrix} K_P & O \\ O & K \end{pmatrix} > \alpha \quad (14)$$

the state $q = q_d = (\theta_d, \delta_d)$, $\dot{q} = 0$ is the *unique equilibrium* of the closed-loop system (6), (12), i.e. satisfying

$$g_\theta(\theta, \delta) = K_P(\theta_d - \theta) + g_\theta(\theta_d, \delta_d) \quad (15)$$

$$g_\delta(\theta, \delta) = -K\delta \quad (16)$$

Condition (14) can always be satisfied provided that an assumption on the structural flexibility holds:

$$\lambda_{\min}(K) = \min_{i=1, \dots, n} \{k_i\} > \alpha \quad (17)$$

with K diagonal. This is not restrictive in general and depends on the relative magnitude of link and/or joint stiffness versus gravity. As a result, by choosing the proportional control gain so that $\lambda_{\min}(K_P) > \alpha$, the equilibrium state $q = q_d$, $\dot{q} = 0$ of system (6) under control (12) is asymptotically stable.

We finally remark that the same condition (14) guarantees a unique equilibrium point also when an approximate constant gravity compensation \hat{g}_θ is used in (12) in place of $g_\theta(\theta_d, \delta_d)$. In this case the equilibrium point will indeed be a $\bar{q} = (\bar{\theta}, \bar{\delta}) \neq q_d$.

4. ITERATIVE CONTROL SCHEME

An iterative compensation scheme that achieves setpoint regulation in the general (flexible) robot *without knowledge of gravity* is introduced as follows. In particular, our objective will be to drive the vector of robot motor variables, θ , to a specified value θ_d .

Let $q_0 = (\theta_0, \delta_0)$ be the initial arm configuration. The control law during iteration i is defined as

$$u = \frac{1}{\beta} K_P(\theta_d - \theta) - K_D\dot{\theta} + u_{i-1}, \quad \beta > 0 \quad (18)$$

for $i = 1, 2, \dots$, where the term u_{i-1} is a constant feedforward. If $u_0 = 0$, which is the common initialization, the first iteration is performed with a simple joint PD control. Indeed, one may collect the best available information on the required gravity term by setting $u_0 = \hat{g}_\theta(q_d)$, where the 'hat' denotes an estimate.

At the end of the i th iteration, system (6) under control (18) reaches an equilibrium state $q = q_i = (\theta_i, \delta_i)$, $\dot{q} = 0$ satisfying

$$g_\theta(\theta_i, \delta_i) = \frac{1}{\beta} K_P(\theta_d - \theta_i) + u_{i-1} \quad (19)$$

$$g_\delta(\theta_i, \delta_i) = -K\delta_i \quad (20)$$

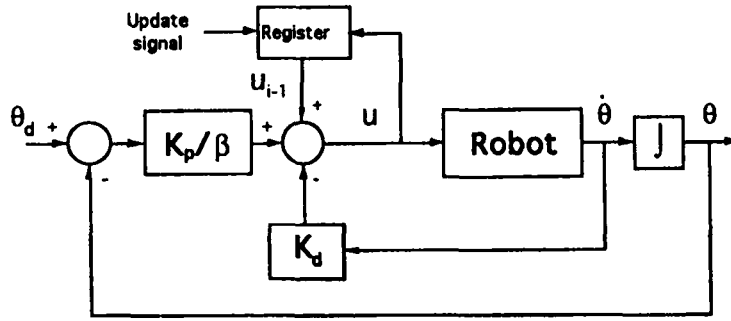


Figure 1. The iterative learning controller

Note that the unknown gravity term $g_\theta(q_i)$ is determined through the *reading* of the control effort at steady state. For the *next* iteration the feedforward is instantaneously updated as

$$u_i = \frac{1}{\beta} K_P(\theta_d - \theta_i) + u_{i-1} \quad (21)$$

and control (18) is applied again, starting from the current configuration q_i . A block diagram of the overall controller is shown in Figure 1.

Our main result is the following.

Theorem

The sequence $\{\theta_0, \theta_1, \dots\}$ converges to θ_d , starting from any initial q_0 , provided that

- (a) $\lambda_{\min}(K) > \alpha$
- (b) $\lambda_{\min}(K_P) > \alpha$
- (c) $0 < \beta \leq \frac{1}{2}(\alpha - \alpha_\delta)/\alpha_\theta$. ■

Proof. Let $e_i = \theta_d - \theta_i$. At the end of the i th iteration, equations (19) and (21) imply that $u_i = g_\theta(q_i)$ at the steady state q_i and so

$$\|u_i - u_{i-1}\| = \|g_\theta(q_i) - g_\theta(q_{i-1})\| \leq \alpha_\theta \|q_i - q_{i-1}\| \leq \alpha_\theta (\|\theta_i - \theta_{i-1}\| + \|\delta_i - \delta_{i-1}\|) \quad (22)$$

where the first inequality in (8) was used. From equation (20), using the second inequality in (8) and hypothesis (a), we have

$$\begin{aligned} \|\delta_i - \delta_{i-1}\| &\leq \|K^{-1}\| \|g_\delta(q_i) - g_\delta(q_{i-1})\| < \frac{1}{\alpha} \alpha_\delta \|q_i - q_{i-1}\| \\ &\leq \frac{\alpha_\delta}{\alpha} (\|\theta_i - \theta_{i-1}\| + \|\delta_i - \delta_{i-1}\|) \end{aligned} \quad (23)$$

or, since $\alpha > \alpha_\delta$,

$$\|\delta_i - \delta_{i-1}\| < \frac{\alpha_\delta}{\alpha - \alpha_\delta} \|\theta_i - \theta_{i-1}\| \quad (24)$$

Combining (22) and (24) gives

$$\|u_i - u_{i-1}\| < \frac{\alpha\alpha_\theta}{\alpha - \alpha_\delta} \|\theta_i - \theta_{i-1}\| \leq \frac{\alpha\alpha_\theta}{\alpha - \alpha_\delta} (\|e_i\| + \|e_{i-1}\|) \quad (25)$$

On the other hand, from equation (21)

$$\|u_i - u_{i-1}\| = \frac{1}{\beta} \|K_P e_i\| \quad (26)$$

From equations (25) and (26), using hypothesis (b), it follows immediately that

$$\frac{1}{\beta} \alpha \|e_i\| < \frac{1}{\beta} \lambda_{\min}(K_P) \|e_i\| \leq \frac{1}{\beta} \|K_P e_i\| < \frac{\alpha \alpha_\theta}{\alpha - \alpha_\delta} (\|e_i\| + \|e_{i-1}\|) \quad (27)$$

Reorganizing terms, since hypothesis (c) implies $\alpha - \alpha_\delta - \beta \alpha_\theta > 0$, we obtain

$$\|e_i\| < \frac{\beta \alpha_\theta}{\alpha - \alpha_\delta - \beta \alpha_\theta} \|e_{i-1}\| \quad (28)$$

Therefore the error norm in (28) satisfies a contraction-mapping condition if

$$\frac{\beta \alpha_\theta}{\alpha - \alpha_\delta - \beta \alpha_\theta} \leq 1 \quad (29)$$

which is again guaranteed by hypothesis (c). As a result, $\lim_{i \rightarrow \infty} \|e_i\| = 0$ and asymptotic convergence of $\{\theta_i\}$ to θ_d is proved for any initial arm configuration q_0 . ■

Hypothesis (b) for rigid robots and hypotheses (a) and (b) for flexible robots are those needed to show that the motor PD control law with constant gravity compensation is globally asymptotically stable when the gravity vector is *known*. In the present case they are needed to ensure that the robot arm under control (18) has a unique steady state solution at every iteration, thus enforcing safe operation against drifting motions outside 2π -rotations.¹ The new hypothesis (c) guarantees convergence of the iterative scheme (21) and in particular that $\lim_{i \rightarrow \infty} u_i = g_\theta(q_d)$, with *a priori* knowledge limited to the bounds (7) and (8) on the gravity terms.

A series of remarks are now in order.

- (i) The same proof can be followed for rigid robot arms. Using (10), it follows from (c) that $\beta \leq \frac{1}{2}$, as first shown in Reference 14. Thus, merging conditions (b) and (c), the overall proportional gain matrix $\hat{K}_P = K_P/\beta$ in (18) has to be chosen so as to satisfy

$$\lambda_{\min}(\hat{K}_P) > 2\alpha = 2\alpha_1 \quad (30)$$

- (ii) In the case of elasticity concentrated only at the joint, using (11), it follows that $\beta \leq (2\alpha_1 - \alpha_1)/2\alpha_1 = \frac{1}{2}$, as in the rigid case.
- (iii) For robots with flexible links it is shown in Reference 15 that when the lower part of the gravity vector can be assumed to be independent of δ , i.e. $g_\delta = g_\delta(\theta)$, then a different bound can be derived for the gain β as

$$\beta \leq \frac{1}{2} \frac{\alpha}{\alpha_\theta(1 + \alpha_\delta/\alpha)} \quad (31)$$

which is strictly larger than the one given by condition (c), so that a smaller \hat{K}_P can be used.

- (iv) The iterative scheme can be interpreted as a discrete-time PID control in which the integral term is updated only at fixed instants. Moreover, this approach combines in an automatic way the benefits of a PD control far from the destination and of an integral action close to the goal, avoiding wind-up effects. As a further merit of the

scheme, one should consider that gains with guaranteed *global convergence* properties are easily selected.

- (v) The iterative scheme (18), (21) is also reminiscent of learning control algorithms that achieve reproduction of repetitive trajectories for rigid^{16,17} or flexible^{18,19} robot arms. However, *no repositioning* of the arm to the initial configuration is performed (nor required) here at any iteration.
- (vi) The bounds (7) and (8) on the gravity terms may be evaluated by taking into account the maximum admissible payload so as to ensure exact setpoint regulation under all operating conditions. Moreover, they can be obtained directly through experimental trials.
- (vii) As a drawback, since each update of the feedforward term should be performed at steady state, in principle the control scheme converges to the desired position in a doubly infinite time. However, ultimate boundedness of the error in finite time is obtained by updating the feedforward term as soon as error variations definitely drop below a given threshold, even before a complete stop. Moreover, it should be emphasized that a fixed PD control with gravity compensation guarantees not just asymptotic but rather *exponential convergence* to the unique equilibrium, as follows from the results in Reference 20. Therefore for the present application a time bound can be derived in order to predict convenient updating instants.
- (viii) An interesting aspect is to estimate the distance from necessity of the sufficient conditions (a)–(c). This point can be investigated through simulations and experiments. In our experience the above criteria are rather stringent, as will be shown in the following sections.

5. SIMULATION RESULTS FOR A RIGID ROBOT

The proposed control scheme has been simulated on a 3R rigid robot arm moving in the vertical plane (Figure 2). The three robot links have equal length $l_i = 0.5$ m, with uniformly distributed masses of 30, 20 and 10 kg respectively.

In this case the gravity vector is given by

$$\begin{aligned} g_1(\theta) &= g_0 [(m_1 d_1 + m_2 l_1 + m_3 l_1) \sin \theta_1 + (m_2 d_2 + m_3 l_2) \sin(\theta_1 + \theta_2) + m_3 d_3 \sin(\theta_1 + \theta_2 + \theta_3)] \\ g_2(\theta) &= g_0 [(m_2 d_2 + m_3 l_2) \sin(\theta_1 + \theta_2) + m_3 d_3 \sin(\theta_1 + \theta_2 + \theta_3)] \\ g_3(\theta) &= g_0 m_3 d_3 \sin(\theta_1 + \theta_2 + \theta_3) \end{aligned} \quad (32)$$

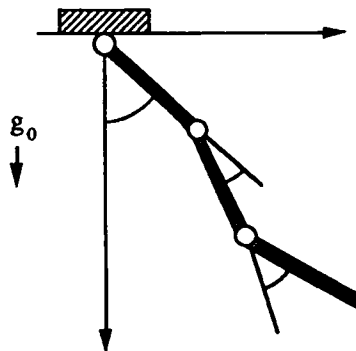


Figure 2. A 3R rigid robot arm in the vertical plane

where $d_i = l_i/2$, $g_0 = 9.8$ is the gravitational acceleration and $\theta = (0, 0, 0)$ corresponds to the straight downward position (of minimum potential energy). With the nominal data of this rigid case the maximum value $\alpha = \alpha_1 \approx 400$ is attained at $\theta = 0$.

A motion from $\theta = 0$ to the straight arm position of maximum gravity force ($\pi/2$ of first-joint rotation) is commanded using as proportional and derivative gains

$$\hat{K}_P = \text{diag}\{1000, 600, 280\}, \quad K_D = \text{diag}\{200, 100, 20\} \quad (33)$$

The update (21) for u_i is made at *fixed* intervals of 3 s. Figures 3–5 and 6–8 show the errors and applied torques respectively over 10 s. In this case two updates are sufficient for regulating the error to zero within 7.5 s. Note that only the first position gain in (33) satisfies the sufficient condition (30), while the gain of the third joint is even smaller than α_1 .

As a second example, a regulation to $\theta_d = (3\pi/4, 0, 0)$ was attempted with smaller position gains. In particular, $K_P = 500I$ was used, which satisfies condition (b) but not the bound (30). Update intervals for u_i are slowed down to 6 s in order to allow steady-state conditions to be reached.

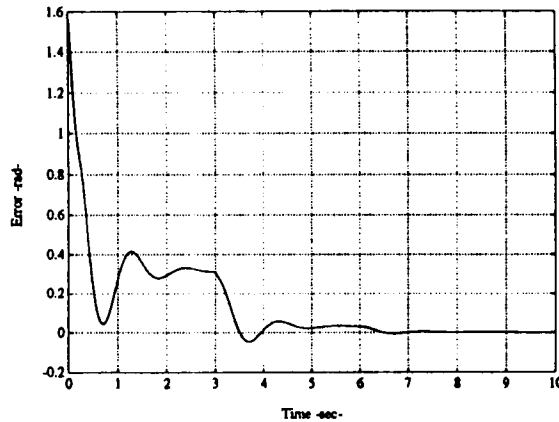


Figure 3. Position error for joint 1 — $q_d = (\pi/2, 0, 0)$

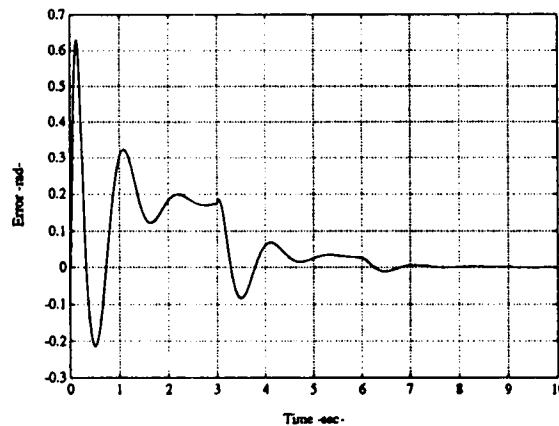


Figure 4. Position error for joint 2

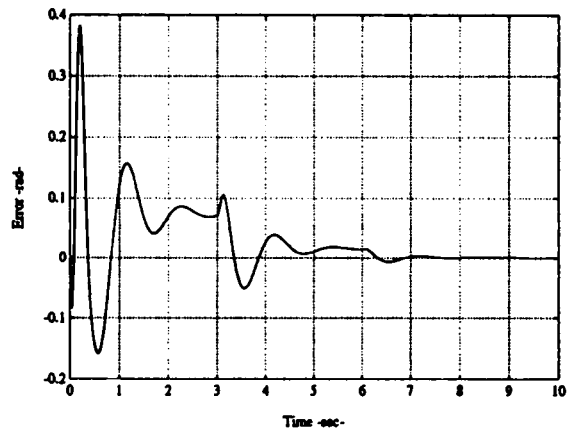


Figure 5. Position error for joint 3

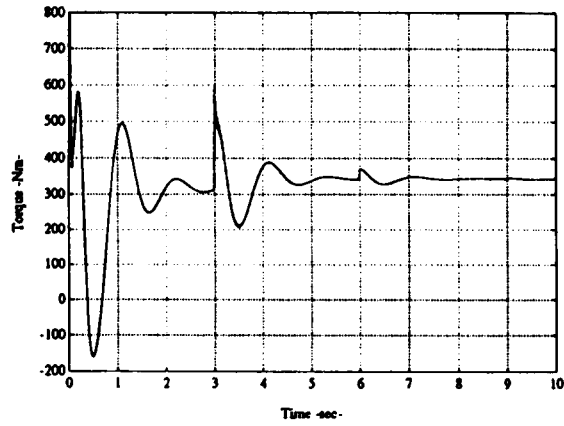
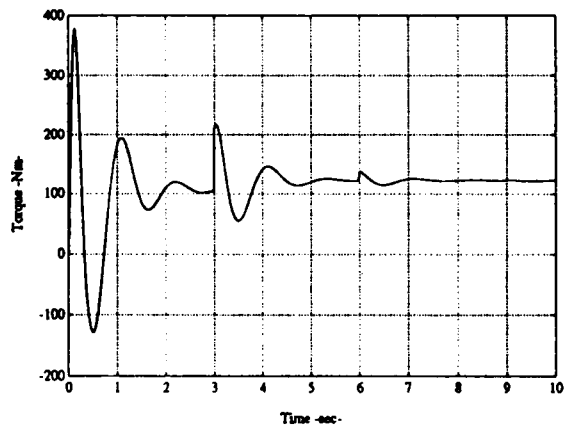
Figure 6. Applied torque for joint 1 — $q_a = (\pi/2, 0, 0)$ 

Figure 7. Applied torque for joint 2

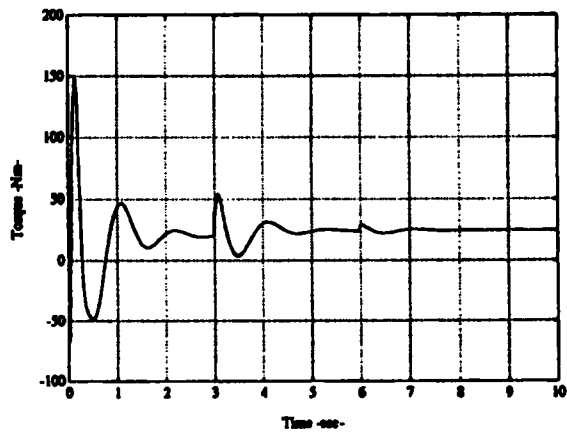


Figure 8. Applied torque for joint 3

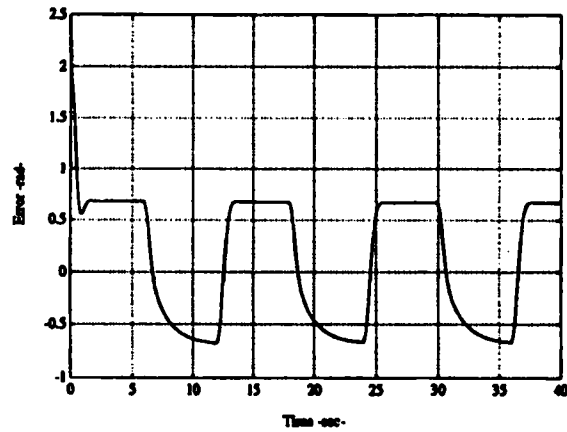
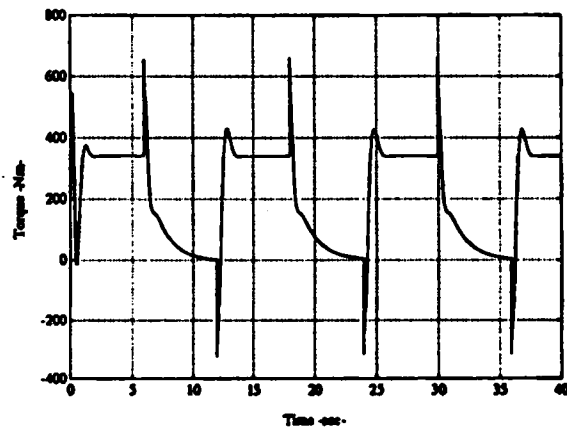
Figure 9. Position error for joint 1 with reduced gains — $q_d = (3\pi/4, 0, 0)$ 

Figure 10. Applied torque for joint 1 with reduced gains

Figure 9 shows 40 s of the first-joint position error. A persistent oscillatory behaviour results as a consequence of the poor learning capabilities: the robot arm switches alternatively from an almost horizontal configuration, where the maximum torque effort is stored, to the upward straight configuration, where the error feedback torque balances the learned feedforward term so as to give a rather small net torque (see Figure 10). Note also that since $\hat{\lambda}_{\min}(K_P) > \alpha_1$, there is still a unique equilibrium configuration for each applied feedforward. As a result, this choice of reduced gains gives quantitative information on how much the sufficient condition (30) can be relaxed in rigid robot arms.

6. EXPERIMENTAL RESULTS FOR A FLEXIBLE ROBOT

The design of the iterative control algorithm has also been carried out for the two-link lightweight manipulator with a flexible forearm available in the Robotics Laboratory of our Department. The arm is a planar mechanism constituted by two links, respectively $l_1 = 0.3$ m and $l_2 = 0.7$ m long, connected by revolute joints with direct drive actuators and mounted on a fixed base as shown in Figure 11.

The upper link is rigid, while the second link, weighing $m_2 = 1.8$ kg, is very flexible in the plane of motion. The first two natural frequencies of vibration are

$$f_1 = 4.716 \text{ Hz}, \quad f_2 = 14.395 \text{ Hz} \quad (34)$$

with the stiffness coefficients of the diagonal matrix K being

$$k_1 = 878.02 \text{ N}, \quad k_2 = 8180.56 \text{ N} \quad (35)$$

A Lagrangian dynamic model of this flexible robot can be found in Reference 10. In our experiments the manipulator base has been tilted by $\gamma_t \approx 6^\circ$ from the horizontal plane so as to include gravity effects. The associated model term $g(\theta, \delta)$ is given below, including only the first two modes:

$$g_\theta = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T, \quad g_\delta = \begin{bmatrix} g_3 \\ g_4 \end{bmatrix}^T \quad (36)$$

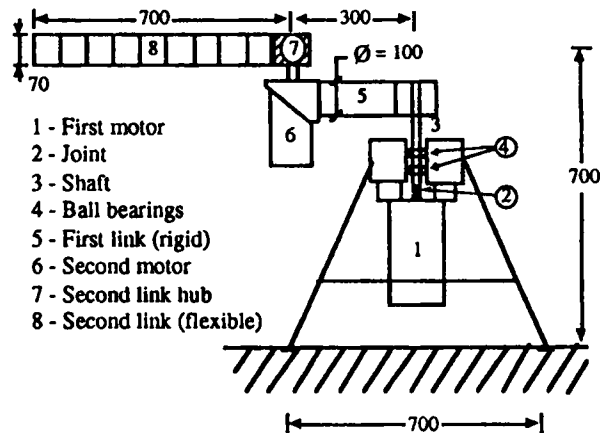


Figure 11. The two-link flexible robot arm at DIS

with

$$\begin{aligned} g_1 &= A_1 \sin \theta_1 + A_2 \sin(\theta_1 + \theta_2) + (A_3 \delta_1 + A_4 \delta_2) \cos(\theta_1 + \theta_2) \\ g_2 &= A_2 \sin(\theta_1 + \theta_2) + (A_3 \delta_1 + A_4 \delta_2) \cos(\theta_1 + \theta_2) \\ g_3 &= A_3 \sin(\theta_1 + \theta_2), \quad g_4 = A_4 \sin(\theta_1 + \theta_2) \end{aligned} \quad (37)$$

The constant coefficients are

$$\begin{aligned} A_1 &= g_0 [m_1 l_{c1} + (m_2 + m_{h2} + m_p) l_1], & A_2 &= g_0 (m_2 l_{c2} + m_p l_2) \\ A_3 &= g_0 (m_p \phi_{1e} + v_1), & A_4 &= g_0 (m_p \phi_{2e} + v_2) \end{aligned} \quad (38)$$

with $g_0 = 9.8 \sin \gamma_t$ the gravitational acceleration, $m_1 l_{c1} = 0.111 \text{ kg m}$, $m_2 l_{c2} = 0.537 \text{ kg m}$, $m_{h2} = 3.1 \text{ kg}$, $m_p = 0 \text{ kg}$, $\phi_{1e} = -1.446 \text{ m}$, $\phi_{2e} = 1.369 \text{ m}$, $v_1 = 0.48 \text{ kg m}$ and $v_2 = 0.18 \text{ kg m}$.

With the convention used, $q = (\theta_1, \theta_2, \delta_1, \delta_2) = (0, 0, 0, 0)$ corresponds to the straight downward position. Note that for this robot g_s is only a function of θ , so that the bound (31) can be used for the gain β . A value $\alpha \approx 2.85$ is obtained for the nominal data, while the same value is used as an upper limit for α_θ and α_δ . From (35) it is also apparent that the structural hypothesis (a) is largely satisfied.

In the first experiment a motion from $q = 0$ (undeformed arm) to the straight position of maximum gravity force ($\pi/2$ of first-joint clockwise rotation) is commanded using as proportional and derivative gains

$$\hat{K}_P = \text{diag}\{10.7, 11.6\}, \quad K_D = \text{diag}\{1.6, 0.85\} \quad (39)$$

The update (21) for u_i is made at *fixed* intervals of 5 s. Figures 12–15 show the joint errors and applied torques over 14 s. Two updates are sufficient for regulating the error to zero within 11 s, since the position gains in (39) satisfy the combined conditions (b) and (c).

The evolution of the tip deflection angle as seen from the second-link base is given in Figure 16, indicating that a maximum deflection of about $0.7 \times 9 \times (\pi/180) = 10 \text{ cm}$ is attained during motion, while the residual tip deformation is about 2 cm.

In the second experiment $\theta_d = (-3\pi/4, 0)$ is the desired joint position to be reached from the same initial configuration using as gains

$$\hat{K}_P = \text{diag}\{5.7, 6.2\}, \quad K_D = \text{diag}\{2.5, 1.34\} \quad (40)$$

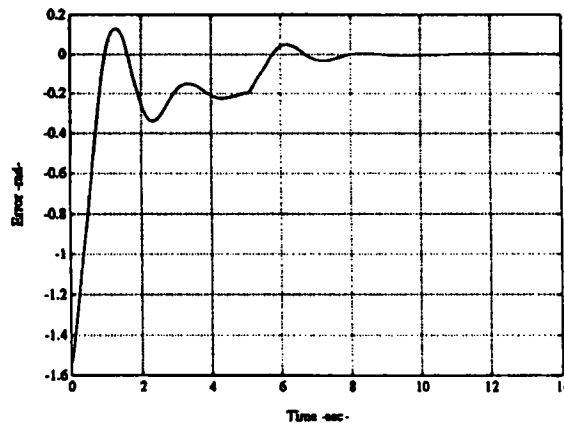


Figure 12. Position error for joint 1 — $\theta_d = (-\pi/2, 0)$

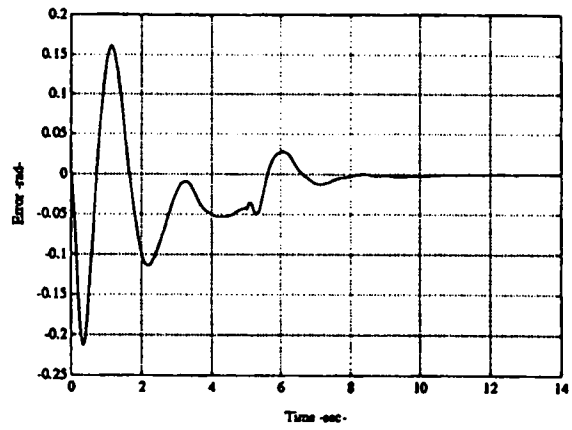


Figure 13. Position error for joint 2

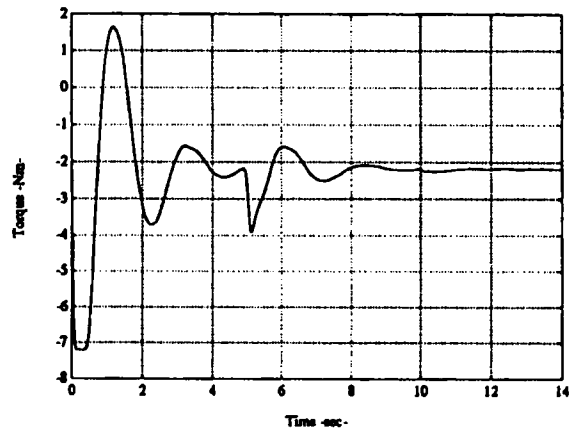


Figure 14. Applied torque for joint 1

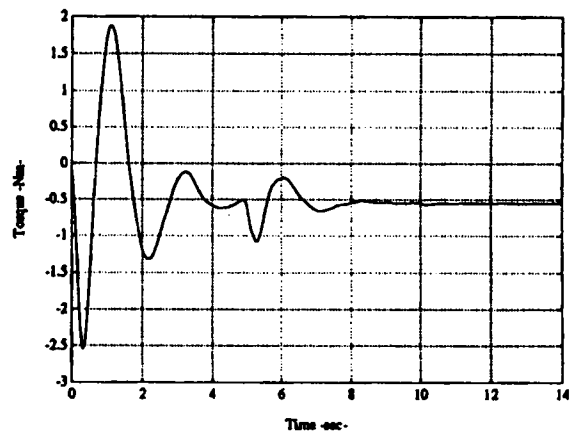


Figure 15. Applied torque for joint 2

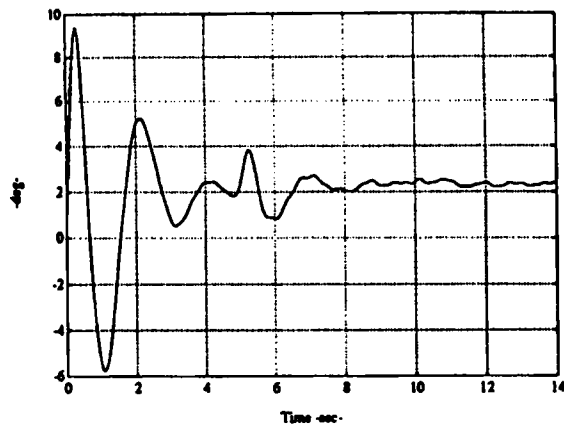


Figure 16. Tip deflection angle

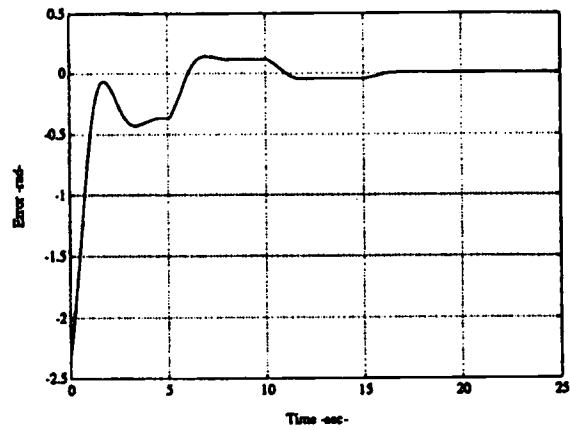
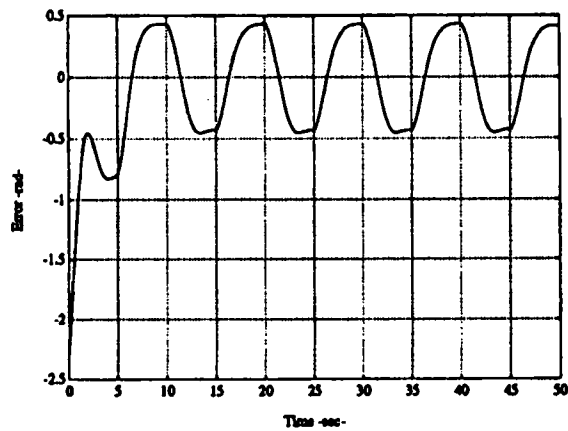
Figure 17. Position error for joint 1 — $\theta_d = (-3\pi/4, 0)$ 

Figure 18. Position error for joint 1 with reduced gains

The evolution of the error at joint 1 over 25 s is displayed in Figure 17, showing that four updates are now necessary for obtaining convergence. No special care was taken to minimize the duration of the motion: a faster global transient could have been obtained by updating sooner the feedforward u_2 , then u_3 and finally u_4 . This example shows the capability of learning the exact gravity compensation also when the 'wall' of maximum gravity force has to be overcome.

As a final experiment the same motion was performed *halving* the positional gains in (40), i.e. with $\hat{K}_P = \text{diag}\{2 \cdot 85, 3 \cdot 1\}$, satisfying hypothesis (b) but not the additional condition (c). Figure 18 shows 50 s of the first-joint motion with no zeroing of the error (the other two joints display a similar behaviour). Thus, relaxation of the proportional gains by 50% will destroy the convergence guaranteed by our theorem; this gives an upper bound for the distance to necessity of the stated conditions also in the flexible case.

7. CONCLUSIONS

A simple iterative and model-independent control scheme has been presented for setpoint regulation under gravity in a general class of robots, with or without flexibility concentrated at the joints or distributed along the links. The scheme generates the exact gravity compensation required at the setpoint, starting initially with a PD control law and updating at discrete instants an additional feedforward term.

For all types of robot arms the linear feedback is always closed at the motor level, thus using directly available variables. A lower-bound condition on the magnitude of the proportional gain in the PD control part is sufficient to prove global convergence of the scheme. In the rigid case this condition leads to a proportional gain which is twice as large as that needed for ensuring the existence of a unique closed-loop equilibrium point. The same bound is obtained for the elastic joint case, while for robots with flexible links the bound is slightly increased. Both simulation and experimental results have shown the effectiveness of the method, pointing out that the convergence conditions are also close to being necessary.

The approach was developed for the regulation of a desired configuration of the arm as expressed by its motor (i.e. joint) positions; deformation variables, when present, are not needed for feedback nor in the feedforward update. If the tip location of a flexible arm is of interest, a similar learning scheme can be derived, still closing the feedback loop at the motor level but taking into account the value of link (or joint) deformation in the process of updating the feedforward term at intermediate steady states.

ACKNOWLEDGEMENTS

This paper is based on work partly supported by the Consiglio Nazionale delle Ricerche under contract 92.01115.PF67 (Progetto Finalizzato Robotica).

REFERENCES

1. Takegaki, M., and S. Arimoto, 'A new feedback method for dynamic control of manipulators', *Trans. ASME J. Dyn. Syst., Meas., Control*, **102**, 119–125 (1981).
2. Tomei, P., 'A simple PD controller for robots with elastic joints', *IEEE Trans. Automatic Control*, **AC-36**, 1208–1213 (1991).
3. De Luca, A., and B. Siciliano, 'Relevance of dynamic models in analysis and synthesis of control laws for flexible manipulators', in Gentina, J. C., and S. G. Tzafestas (eds), *Robotics and Flexible Manufacturing Systems*, Elsevier, Amsterdam, 1992, pp. 161–168.

4. Lee, H. G., S. Arimoto and F. Miyazaki, 'Liapunov stability analysis for PDS control of flexible multi-link manipulators', *Proc. 27th IEEE Conf. on Decision and Control*, Austin, TX, 1988, pp. 75–80.
5. Tomei, P., 'Adaptive PD controller for robot manipulators', *IEEE Trans. Robot. Automat.*, RA-7, 565–570 (1991).
6. De Luca, A., and B. Siciliano, 'An asymptotically stable joint PD controller for robot arms with flexible links under gravity', *Proc. 31st IEEE Conf. on Decision and Control*, Tucson, AZ, 1992, pp. 325–326.
7. Åström, K. J., and B. Wittenmark, *Computer-controlled Systems: Theory and Design*, 2nd edn, Prentice-Hall International, London, 1990.
8. Arimoto, S., and F. Miyazaki, 'Stability and robustness of PID feedback control for robot manipulators of sensory capability', in Brady, M., and R. P. Paul (eds), *Robotics Research: 1st Int. Symp.*, MIT Press, Boston, MA, 1984, pp. 783–799.
9. Khorrami, F., and Ü. Özgüner, 'Decentralized control of robot manipulators via state and proportional–integral feedback', *Proc. IEEE Int. Conf. on Robotics and Automation*, Philadelphia, PA, 1988, pp. 1198–1203.
10. De Luca, A., L. Lanari, P. Lucibello, S. Panzneri and G. Ulivi, 'Control experiments on a two-link robot with a flexible forearm', *Proc. 29th IEEE Conf. on Decision and Control*, Honolulu, HI, 1990, pp. 520–527.
11. Book, W. J., 'Recursive Lagrangian dynamics of flexible manipulator arms', *Int. J. Robot. Res.*, 3, (3), 87–101 (1984).
12. De Luca, A., and B. Siciliano, 'Closed-form dynamic model of planar multi-link lightweight robots', *IEEE Trans. Syst., Man, Cyber.*, SMC-21, 826–839 (1991).
13. Stewart, G. W., and J. Sun, *Matrix Perturbation Theory*, Academic, San Diego, CA, 1990.
14. De Luca, A., and S. Panzneri, 'A simple iterative scheme for learning gravity compensation in robot arms', *Proc. 36th ANIPLA Ann. Conf. (Automation 1992)*, Genova, 1992, pp. 459–471.
15. De Luca, A., and S. Panzneri, 'An iterative scheme for learning gravity compensation in flexible robot arms', *Proc. 12th IFAC World Congr.*, Sydney, 1993, vol. 8, pp. 191–196.
16. Arimoto, S., S. Kawamura and F. Miyazaki, 'Bettering operation of robots by learning', *J. of Robot. Syst.*, 1, 123–140 (1984).
17. De Luca, A., G. Paesano and G. Ulivi, 'A frequency-domain approach to learning control: implementation for a robot manipulator', *IEEE Trans. Ind. Electron.*, IE-39, 1–10 (1992).
18. Poloni, M., and G. Ulivi, 'Iterative learning control of a one-link flexible manipulator', *Proc. 3rd IFAC Symp. on Robot Control*, Vienna, 1991, pp. 393–398.
19. De Luca, A., and G. Ulivi, 'Iterative learning control of robots with elastic joints', *Proc. IEEE Int. Conf. on Robotics and Automation*, Nice, 1992, pp. 1920–1926.
20. Tomei, P., 'An adaptive PD control algorithm for robots', in Kaashoek, M. A., J. H. van Schuppen and A. C. M. Ran (eds), *Progress in Systems and Control Theory Series*, Vol. 4, *Robust Control of Linear Systems and Nonlinear Control*, Birkhäuser, Boston, MA, 1990, pp. 575–582.