Theorem 1: For any bounded measurement vector sequence $\{\phi_k\}$, the algorithm has the following properties:

3) $\|\tilde{\theta}_k\|_{\mathrm{WC}}$ is bounded and nonincreasing;

4) $\|\tilde{\theta}_k\|$ is bounded by

$$\|\tilde{\theta}_k\| \le \left[\sigma_k^2 \lambda_{\max}(P_k)\right]^{\frac{1}{2}} \le \left[\sigma_0^2 \lambda_{\max}(P_0)\right]^{\frac{1}{2}}. \tag{2}$$

Proof: It can be readily proved from formulas (6), (10), and (11) of the original paper[1] that $\sigma_k^2 P_k \le \sigma_{k-1}^2 P_{k-1}$. Hence, from Definition 1, $\|\tilde{\theta}_k\|_{\mathrm{WC}}$ is bounded and nonincreasing. Since $\|\tilde{\theta}_k\| \le \|\tilde{\theta}_k\|_{\mathrm{WC}}$, $\|\tilde{\theta}_k\|$ is bounded, and (2) is satisfied. This completes the Proof of Theorem 1. □

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Comments on "Adaptive Variable Structure Set-Point Control of Underactuated Robots"

Alessandro De Luca and Giuseppe Oriolo

***Abstract*—In this note, we point out the fallacies of a recent paper concerning the stabilization of underactuated robots.**

***Index Terms*—Underactuated robots, variable structure control.**

I. INTRODUCTION

In a recent paper,[1] a set-point control method for generic underactuated robots is proposed. Under the only hypothesis (hereafter denoted as H) that the constant desired configuration q_d satisfies the dynamic constraint (3) of the paper, i.e., that the gravitational torque G_u acting on the passive joints is zero at q_d, the authors claim to achieve global convergence of the (active and passive) joint errors to zero using a variable structure controller.

This result, which would be striking with respect to the state of the art in the field, is unfortunately false, even if the dynamic model is exactly known. We show this first through intuitive counterexamples and then pointing out the inconsistency in the given proof of Theorem 1, the main result of the paper.

II. COUNTEREXAMPLES

An examination of the proof of Theorem 1 reveals that neither the controllability of the underactuated robot nor the fact that q_d is a closed-loop equilibrium are required. This suggests that the proof is wrong, as will be demonstrated in Section III. First, we present some intuitive counterexamples to the claimed convergence of the proposed controller.

3) Consider the same underactuated $2R$ robot of the paper (the so-called Pendubot). If the center of mass of the second link lies on the second joint axis ($l_{c2} = 0$), then $G_u(q) \equiv 0$ and *any* constant configuration $q_d = (q_{1d}, q_{2d})$ satisfies the working hypothesis H, i.e., the dynamic constraint (3) [which takes the form (29) for this robot]. Thus, the proposed variable structure controller should be able to achieve set-point regulation regardless of the choice of q_d.

However, being in this case[2] $m_{au} = m_{uu} = I_2$ and $h = 0$, the passive joint equation (29) can be integrated as

$$q_1(t) + q_2(t) = q_1(0) + q_2(0) = \text{const} \qquad \forall t$$

having assumed zero initial velocities. This means that the second link orientation w.r.t. the ground will not change. Hence, although hypothesis H is satisfied, the proposed controller (in fact, any controller) cannot drive the robot to a configuration q_d which does not satisfy the above integral constraint.

4) Assume now that the underactuated robot is controllable ($l_{c2} \neq 0$), and that $G_a(q_d) \neq 0$, i.e., a constant torque is required on the active joints to sustain the robot at the desired equilibrium. For the Pendubot, this occurs at all configurations with $q_{1d} + q_{2d} = \pm 90^\circ$ (second link vertical) and $q_{1d} \neq \pm 90^\circ$ (first link not vertical). The discontinuous controller (12–14) cannot provide the correct steady-state torque. In fact, when $s = 0$ and $\dot{s} = 0$ the control torque becomes

$$u_a = -\rho_r \Phi_r \operatorname{sign}(0).$$

It is easy to verify that, no matter how the value of the vector sign function is defined at zero, u_a will never provide $G_a(q_d)$. For example, letting sign $(0) = [-1 \cdots -1]^T$, one obtains through (31)

$$u_a = 2\rho_r g \cos q_{1d}$$

while

$$G_a(q_d) = (m_1 l_{c1} + m_2 l_1) g \cos q_{1d}.$$

Noting that

$$2\rho_r > \rho_r \ge \|\alpha_r\| > m_1 l_{c1} + m_2 l_1$$

where (11) and (30) of the paper have been used, it can be concluded that $|u_a| > G_a(q_d)$. Therefore, q_d is not even a closed-loop equilibrium—asymptotic convergence is out of question. A similar conclusion is reached for the smoothed controller (24)–(26), since the latter directly gives $u_a = 0$ when $s = 0$ and $\dot{s} = 0$.

An even more general counterexample follows.

5) Consider any underactuated robot moving in the absence of gravity, and assume that the robot is initially at rest with the active joints already at their desired value, so that $s_a = 0$. An easy calculation based on expressions (8) and (9) shows that the discontinuous controller (12)–(14) gives $u_a = 0$, i.e., the

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[1] C.-Y. Su and Y. Stepanenko, *IEEE Trans. Automat. Contr.*, vol. 44, pp. 2090–2093, Nov. 1999

[2] Note that the expressions of m_{aa} and m_{uu} in (28) of the paper should be reversed.

robot does not move and the error on the passive joints cannot be recovered. Therefore, no reconfiguration maneuvers can be performed if the active joints are already placed. The same is trivially true for the smoothed controller (24)–(26), even in the presence of gravity.

III. Error in the Proof

Theorem 1 of the paper provides the theoretical basis for the authors' claim that the proposed variable structure control achieves set-point regulation. Yet, there is a serious inconsistency in the proof, as shown below.

First of all, note that the proof relies on (19), i.e., the fact that $\dot{V} \leq -s^T K_s s$. In turn, this is established through (18), where the authors have used the expression of $\dot{k}$ as given by (14) for the case $k \neq 0$. However, when $k = 0$, (18) [and, thus, (19)] does not hold. In fact, in this case one obtains

$$\dot{V} \leq -s^T K_s s + \frac{\delta}{\|s_a\|^2 + \delta}(\|\alpha_f s_u^T \Phi_f\| + s_u^T K_u s_u)$$

with the second term in the right-hand side clearly positive. Hence, the proof of Theorem 1 is correct unless $k(t) = 0$ at some instant t. Unfortunately, this is exactly what happens in general: k *goes to zero in finite time*, invalidating the proof.[3] Below, we illustrate this fact analytically in a particular case and through a typical simulation result.

As in counterexample $3)$ above, consider an underactuated robot in the absence of gravity, and assume that the robot is at rest with the active joints already placed. Being $s_a = 0$, both the discontinuous controller (12)–(14) and the smoothed controller (24)–(26) yield $u_a = 0$, i.e., the robot will *not* move. The evolution of k is then given by (14) as

$$\dot{k} = -\left[\frac{\delta_1 \eta}{\delta}(\rho_f \|s_u^T \Phi_f\| + s_u^T K_u s_u)\right]\frac{1}{k}.$$

Denoting the constant quantity in square brackets as $\beta > 0$, the solution is

$$k^2(t) = k^2(0) - 2\beta t$$

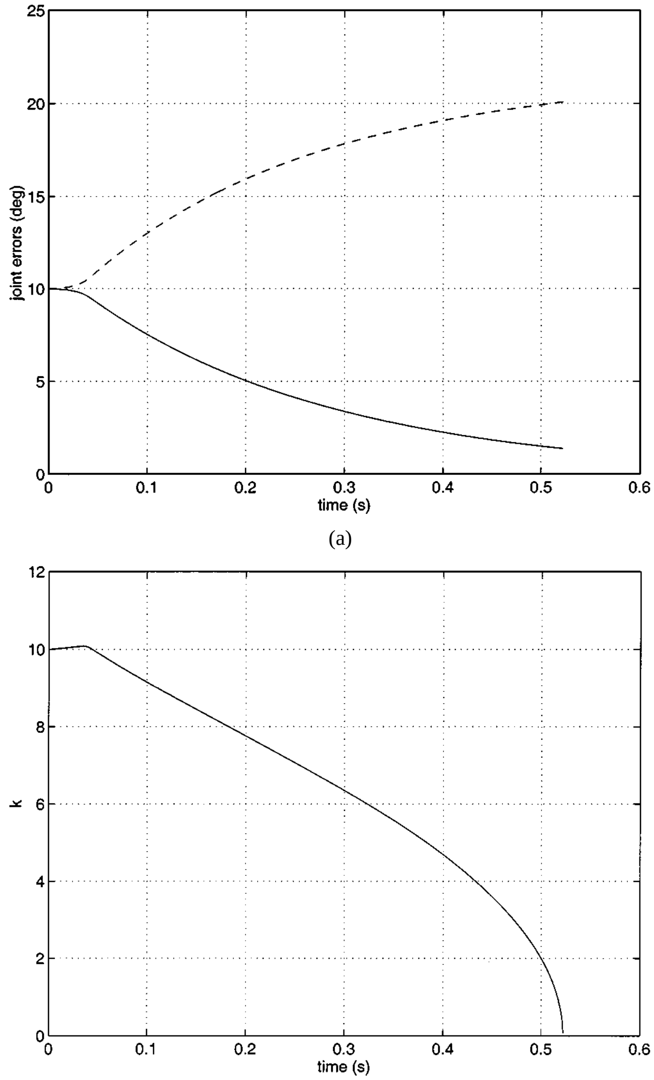
showing that k goes to zero at $\bar{t} = k^2(0)/2\beta$.

The above phenomenon is indeed general and clearly observed in simulations. As an example, consider a $2R$ robot with the first joint actuated, moving in the horizontal plane. The dynamic parameters are the same used in Section II of the paper. The robot is initially in equilibrium at $(q_{10},\, q_{20}) = (10^\circ,\, 80^\circ)$ while the desired configuration is $(q_{1d},\, q_{2d}) = (0^\circ, 70^\circ)$. Fig. 1 shows the results obtained by simulating the discontinuous controller (12)–(14): the first joint error goes to zero, while the second tends to increase to a constant value. Meanwhile, k is decreasing and then rapidly approaching zero, which is reached approximately at $\bar{t} = 0.53$ s. These plots suggest that the decrease of V (which is proved as long as $k \neq 0$) comes indeed from a decrease of s_a and k, while s_u grows uncontrolled. Similar results are obtained in the presence of gravity (motion in the vertical plane), as well as using the smoothed controller (24)–(26).

IV. Conclusion

We have shown through intuitive counterexamples, analytic arguments and numerical simulations that the variable structure controller proposed by Su and Stepanenko does not solve the set-point regulation problem for underactuated robots. Still, it may work in very special cases in which the desired configuration is "naturally" attractive for the underactuated system. The only simulation reported by the authors

[3]In particular, the authors' expedient of setting in (14) $\dot{k} = \delta$ when $k = 0$ is a useless artifice, because V will still increase at that instant.



(a)

(b)

Fig. 1. Simulation results. (a) Evolution of joint errors e_1 (—) and e_2 (- -). (b) Evolution of k.

is one of these fortunate cases, because the objective there was to drive an underactuated $2R$ robot under gravity to the *downward* equilibrium position. However, it is obvious that the latter can be asymptotically stabilized much more easily by injecting viscous damping in the system: a trivial analysis of the tangent linearization at $q_d = (-90^\circ, 0^\circ)$ shows that simply setting $u_a = -20\dot{q}_1$ is sufficient to achieve local asymptotic stability, with remarkably better performance than reported in the paper.

As a final comment, we wish to emphasize that the problem of stabilizing underactuated robots is much more complex than implied in the paper, and still open in the general case. Recent literature indicates that a clear distinction must be made between the cases of presence and absence of gravity. Under gravity, the tangent linearization of the system becomes controllable. Hence, smooth stabilization is possible and can be obtained through classical control techniques such as feedback linearization and passivity [1]. However, in complete contrast with the authors' hint in the Introduction, a greater challenge is to be faced in the absence of gravity. In this case, Brockett's necessary conditions [2] for smooth stabilizability are violated, and nonsmooth feedback laws must be used, exactly as in the case of first-order nonholonomic systems. The controller proposed by Su and Stepanenko is indeed discontinuous, but switching is introduced there only to account for the uncertainty on the dynamic model; lacking a mechanism for overcoming Brockett's obstruction, stabilization will not be achieved, even in the nominal case. A correct solution comes from nonconventional design approaches, e.g., see [3].

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Comments on "Adaptive Variable Structure Set-Point Control of Underactuated Robots"

Tao Zhang

***Abstract*—This note points out several technical errors in the above paper. It is shown that the choice of Lyapunov function is inadequate for the system stability analysis. The asymptotic tracking of the unactuated joints is not achievable under certain conditions.**

I. MAIN POINTS

The paper[1] does not show $k(t) \neq 0, \forall t \geq 0$. Suppose that there exists a time $t = t_1$ such that $k(t_1) = 0$. Then, $k(t_1)\dot{k}(t_1)/\eta = 0$ and

$$u_c(t) = \frac{s_a}{\|s_a\|^2 + \delta}\left[\rho_f\left\|s_u^T\Phi_f\right\| + s_u^T K_u s_u\right], \qquad t = t_1.$$

The left-hand side of inequality (18) given in the paper[1] becomes

$$\begin{aligned} &-s_a^T u_c + s_u^T K_u s_u + k\dot{k}/\eta \\ &\quad = -s_a^T u_c + s_u^T K_u s_u \\ &\quad = \frac{-\|s_a\|^2}{\|s_a\|^2+\delta}\left[\rho_f\left\|s_u^T\Phi_f\right\| + s_u^T K_u s_u\right] + s_u^T K_u s_u \\ &\quad = \frac{1}{\|s_a\|^2+\delta}\left[-\rho_f\|s_a\|^2\left\|s_u^T\Phi_f\right\| + \delta s_u^T K_u s_u\right] \\ &\quad > \frac{1}{\|s_a\|^2+\delta}\left[-\rho_f\|s_a\|^2\left\|s_u^T\Phi_f\right\| - \delta_1\rho_f\left\|s_u^T\Phi_f\right\|\right] \\ &\qquad\quad \forall s_u \neq 0. \end{aligned}$$

Therefore, the inequality (18) does not hold when $k = 0$ and $s_u \neq 0$. Once the inequality (18) of the paper[1] is broken down, the proof of Theorem 1 is questionable.

Even if $k(t) \neq 0$ is assumed for all time, there is still an error in the proof of Theorem 1. Considering the case of $k(t) \neq 0$ and $s_a(t) = 0$, it can be seen from (12) and (13) that control input $u_a(t) = 0$, i.e., there is no control action added on the robot system. No matter how you tune the parameter $k(t)$, the changing of $k(t)$ does not affect robot dynamics. In such a case, the choice of Lyapunov function (16) is invalid because signals $s(t)$ and $k(t)$ come from two separated systems [there is no physical relationship between $s(t)$ and $k(t)$]. In fact, once s_a goes to zero, the unactuated joints will not be affected by any control parameters. We can see this from Fig. 1 of the simulation example given in the paper,[1] which indicates that after the tracking error s_a of joint 1 converging to zero for $t > 10s$, the tracking error of unactuated joint 2 does not go to zero.

In the following, we use a special case to show how an incorrect result is derived in the paper.[1] Suppose there exists a time $t_1 > 0$ such that $k(t) \neq 0$ and $s_a(t) = 0, \forall t \geq t_1$. Let

$$V_s = \tfrac{1}{2} s^T M(q) s.$$

By following the similar procedure of the paper, the time derivative of V_s along (15) is

$$\begin{aligned} \dot{V}_s &= s^T\left(\tfrac{1}{2}\dot{M} - B\right)s + s^T\left(-K_s s + \begin{bmatrix} u_a + \Phi_r\alpha_r + K_a s_a \\ \Phi_f\alpha_f + K_u s_u \end{bmatrix}\right) \\ &= s_u^T \Phi_f \alpha_f \qquad \forall t \geq t_1. \end{aligned}$$

Clearly, the tracking error s_u of the unactuated joints may not converge to zero due to the item $s_u^T\Phi_r\alpha_r$ being indefinite. However, by checking the adaptive law (14) under this case, it becomes

$$\frac{k\dot{k}}{\eta} = -\frac{\delta_1}{\delta}\left[\rho_f\left\|s_u^T\Phi_f\right\| + s_u^T K_u s_u\right], \qquad \text{for } k \neq 0,\ \forall t > t_1.$$

Integrating both sides of the above equation over $[t_1, \infty]$ leads to

$$\begin{aligned} \int_{t_1}^{\infty}\left[\rho_f\left\|s_u^T\Phi_f\right\| + s_u^T K_u s_u\right] dt &= -\frac{\delta}{\delta_1\eta}\int_{t_1}^{\infty} k\dot{k}\, dt \\ &= \frac{\delta}{2\delta_1\eta}\left[k^2(t_1) - k^2(\infty)\right] \\ &\leq \frac{\delta}{2\delta_1\eta} k^2(t_1) < \infty. \end{aligned}$$

This implies that $s_u \in L_2^n$. Combining it with $\dot{s}_u \in L_\infty^n$ can result in $s_u \to 0$ as $t \to \infty$. This is an unreasonable result because parameter $k(t)$ does not affect system dynamics when $s_a = 0$, the tuning law (14) itself cannot conclude the L_2^n property of s_u. That is why the incorrect conclusion is obtained in Theorem 1 for the asymptotic stability of the unactuated joints.

In summary, the major problem of the paper lies in the use of the adaptive law (14) and the choice of Lyapunov function (16) for the case $s_a = 0$. In view of the fact that the controller (12)–(14) does not have any control action on the underactuated dynamics when $s_a = 0$, under the present control structure it is impossible to achieve the asymptotic tracking for both joints unless the underactuated dynamics themselves are asymptotically stable.

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