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Brief paper

PD control with on-line gravity compensation for robots with elastic joints: Theory and experiments☆

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Abstract

A proportional-derivative (PD) control with on-line gravity compensation is proposed for regulation tasks of robot manipulators with elastic joints. The work extends a previous PD control with constant gravity compensation at the desired configuration. The control law requires measuring only position and velocity on the motor side of the elastic joints, while the on-line gravity compensation torque uses a biased measure of the motor position. It is proved via a Lyapunov argument that the control law globally asymptotically stabilizes the desired robot configuration. A simulation study on a two-joint arm reveals the better performance that can be obtained with the new scheme as compared to the case of constant gravity compensation. Moreover, the proposed controller is experimentally tested on an eight-joint cable-driven robot manipulator, in combination with a point-to-point interpolating trajectory, showing the practical advantages of the on-line compensation.

Keywords: Regulation; Elastic joint robots; Gravity compensation; Lyapunov stability

1. Introduction

Control algorithms conceived for completely rigid robots may guarantee a stable behavior even if a certain degree of elasticity in the actuation system and motor transmission elements, or in the link structure, is present (Sweet & Good, 1985). The price to pay, however, is a typical degradation of robot performance. In fact, elasticity of mechanical transmissions induces position errors at the robot end effector because of static deformation under gravity. In addition, it may become a source of instability in case of interaction between the robot and the environment (Spong, 1989) or when feedback is based on the measure of link variables only (De Luca & Tomei, 1996).

When the effects of transmission flexibility are non-negligible, the control design has to be revisited in order to account for the elastic phenomena. Elasticity is assumed to be concentrated at the n robot joints and the number of Lagrangian configuration variables in the robot dynamics is doubled with respect to the rigid case, leading to a set of n motor and n link second-order nonlinear equations.

For robot manipulators with elastic joints, different control solutions are available for trajectory tracking as well as for regulation tasks (De Luca & Tomei, 1996). For trajectory tracking tasks, one can resort to high-performing but complex control strategies, such as the linearizing and decoupling nonlinear feedback (Spong, 1987; De Luca & Lucibello, 1998), an integral manifold approach based on a singular perturbation model of the robot dynamics (Spong, Khorasani, & Kokotovic, 1987), or an adaptive neural network control (Kobayashi & Ozawa, 2003). For

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regulation tasks, instead, Tomei (1991) has proved that a simple proportional-derivative (PD) controller suffices to globally stabilize a robot with elastic joints about any desired configuration. The control law includes a constant gravity compensation term, which is evaluated at the desired reference position, and needs to feed back only position and velocity of the motors.

In the case of rigid robots, it is well known that global regulation to a desired configuration q_d can be achieved by a PD control law, either with a constant gravity compensation term $g(q_d)$ and sufficiently high positional gains (Arimoto & Miyazaki, 1984), or with a nonlinear gravity compensation term $g(q)$ evaluated on line at the current configuration, see Sciavicco and Siciliano (2000). The evaluation of the gravity term may be avoided only by resorting to an additional saturated integral term (Alvarez-Ramirez, Kelly, & Cervantes, 2003). In the presence of joint elasticity, utilizing an on-line gravity compensation is more complex than in the rigid case. On one hand, the gravity torque depends on the robot link positions q whereas quite often only the motor positions θ are measurable. On the other hand, a PD control with on-line gravity compensation based on the motor positions $g(\theta)$ does not lead to the desired final equilibrium configuration. In addition, the stability analysis is complicated by the non-collocation between the available control torque (on the motor side) and the gravity torque to be compensated (acting on the link side of joint elasticity).

The contribution of this paper is a PD control law with *on-line* gravity compensation for robot manipulators with elastic joints, which requires only motor measurements and has guaranteed global stabilization properties. The main idea is to use a new variable, named *gravity-biased* motor position, for evaluating the gravity torque at each configuration. One feature of this controller is to allow more flexibility in the tuning of the proportional gains, as compared to the original controller proposed by Tomei (1991), e.g., for obtaining better transients. In addition, this regulation scheme can be effectively combined with a point-to-point interpolating trajectory, so as to prevent motor saturation effects typically occurring during the first instants of motion when a step change is commanded on the robot joint positions.

The paper is organized as follows. Section 2 recalls dynamic modelling of robot manipulators with elastic joints, and its notable properties. The PD control law with on-line gravity compensation is introduced in Section 3. The analysis of the closed-loop equilibria and the proof of asymptotic stability via a Lyapunov argument are presented in Section 4. In Section 5 simulation results on a simple two-joint planar arm are utilized to compare the on-line vs. the constant gravity compensation schemes. Experimental results on the *Dexter* robot, an 8-joint cable-driven articulated manipulator, are reported in Section 6. Concluding remarks are enlightened in the final section.

2. Dynamic model of robots with elastic joints

The following two assumptions are made in describing the dynamics of robots with elastic joints:

(A1) The robot manipulator is an open kinematic chain of rigid bodies, driven by electrical actuators through elastic joints undergoing small deformations in the domain of linear elasticity.

(A2) Rotors of motors are uniform bodies balanced around their rotation axes.

The robot dynamic model can be written as follows (Tomei, 1991):

$$B(q)\ddot{q}_c + C(q_c, \dot{q}_c)\dot{q}_c + e(q) + K_e q_c = m, \tag{1}$$

where $q_c = [q^T \;\; \theta^T]^T$ is the $(2n \times 1)$ vector of configuration variables, being q and θ the $(n \times 1)$ vectors of link positions and motor positions (reflected through the gears), respectively. In view of Assumptions (A1) and (A2), the $(2n \times 2n)$ robot inertia matrix $B(q)$ and the $(2n \times 1)$ gravitational torque vector $e(q)$ are independent of θ. Moreover,

$$C(q_c, \dot{q}_c)\dot{q}_c = \dot{B}(q)\dot{q}_c - \frac{1}{2}\left(\frac{\partial}{\partial q_c}(\dot{q}_c^T B(q)\dot{q}_c)\right)^T$$

is the $(2n \times 1)$ vector of centrifugal and Coriolis torques, $K_e q_c$ represents the $(2n \times 1)$ vector of elastic torques and, on the right-hand side of (1), m is the $(2n \times 1)$ vector of external torques producing work on q_c.

Eq. (1) can be rearranged into two equations, one for the link side and the other for the motor side, if the contributions to the robot dynamics are decomposed as follows. The $(2n \times 2n)$ robot inertia matrix $B(q)$ can be partitioned in four $(n \times n)$ block matrices

$$B(q) = \begin{bmatrix} B_1(q) & B_2(q) \\ B_2^T(q) & B_3 \end{bmatrix}, \tag{2}$$

where B_1 takes into account the inertial properties of rigid links, B_2 considers the coupling between each spinning actuator and the previous links, and B_3 is a constant diagonal matrix including the motor inertia (scaled through the squared gear ratios).

The $(2n \times 2n)$ matrix $C(q_c, \dot{q}_c)$, by resorting to the so-called decomposition in Christoffel symbols, can be expressed as

$$C(q_c, \dot{q}_c) = C_A(q, \dot{\theta}) + C_B(q, \dot{q}), \tag{3}$$

where

$$C_A(q, \dot{\theta}) = \begin{bmatrix} C_{A1}(q, \dot{\theta}) & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_B(q, \dot{q}) = \begin{bmatrix} C_{B1}(q, \dot{q}) & C_{B2}(q, \dot{q}) \\ C_{B3}(q, \dot{q}) & 0 \end{bmatrix},$$

being C_{A1}, C_{B1}, C_{B2}, C_{B3} suitable $(n \times n)$ matrices.

The gravitational torque takes on the form

$$e(q) = \begin{bmatrix} g(q) \\ 0 \end{bmatrix}, \tag{4}$$

where $g(q) = (\partial U_g(q)/\partial q)^T$, being $U_g(q)$ the potential energy due to gravity.

The $(2n \times 2n)$ matrix K_e in the elastic torque can be written in terms of the $(n \times n)$ diagonal and positive definite matrix K of joint stiffness coefficients as follows:

$$K_e = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}$$

and, finally, the vector of generalized forces acting on q_c can be expressed as

$$m = \begin{bmatrix} 0 \\ u \end{bmatrix},$$

where u is the torque vector produced by the n motors.

For model (1), the following four properties hold (De Luca & Tomei, 1996):

(P1) The inertia matrix $B(q)$ is symmetric and positive definite for all q.
(P2) The matrix $B_2(q)$ is strictly upper triangular.
(P3) If a representation in Christoffel symbols is chosen for the elements of $C(q_c, \dot{q}_c)$, the matrix $\dot{B} - 2C$ is skew-symmetric.
(P4) A positive constant α exists such that

$$\left\| \frac{\partial g(q)}{\partial q} \right\| = \left\| \frac{\partial^2 U_g(q)}{\partial q^2} \right\| \leqslant \alpha \quad \forall q, \tag{5}$$

where the matrix norm of a symmetric matrix $A(q)$ is given by $\lambda_{max}(A(q))$, i.e., its largest (real) eigenvalue at q.[1] Inequality (5) implies

$$\|g(q_1) - g(q_2)\| \leqslant \alpha \|q_1 - q_2\| \tag{6}$$

for any q_1, q_2. It should be explicitly remarked that this inequality holds whatever argument is used for evaluating the gravity vector.

3. PD control with on-line gravity compensation

In this section, a control law is proposed which is aimed at regulating the robot link positions to a desired constant configuration q_d. The assumption is made that only the motor variables θ and $\dot{\theta}$ are measurable or, at least, θ is measurable and $\dot{\theta}$ is obtained by accurate numerical differentiation. The control law includes a proportional-derivative action in the space of motor variables, combined with an on-line gravity compensation in lieu of the constant gravity compensation as done by Tomei (1991).

The PD control with *constant* gravity compensation in (Tomei, 1991) is expressed as

$$u = K_P(\theta_d - \theta) - K_D\dot{\theta} + g(q_d), \tag{7}$$

where $K_P > 0$ and $K_D > 0$ are both symmetric (and typically diagonal) matrices, and

$$\theta_d = q_d + K^{-1} g(q_d). \tag{8}$$

Under the assumption that the stiffness matrix K and the proportional gain matrix K_P comply with the following condition

$$\lambda_{min}(\bar{K}) := \lambda_{min}\left(\begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} \right) > \alpha, \tag{9}$$

the control law (7) yields global asymptotic stability of the (unique) closed-loop equilibrium state $(q, \theta, \dot{q}, \dot{\theta}) = (q_d, \theta_d, 0, 0)$.

Similarly to the rigid joint case, a better transient behavior is to be expected if some kind of gravity compensation is performed at any configuration during motion. However, note that the gravity vector in (4) depends on the link variables q, which are assumed not to be measurable. In addition, it is easy to show that using $g(\theta)$, with the measured motor positions in place of the link positions, leads to an incorrect closed-loop equilibrium.

Therefore, the PD control with *on-line* gravity compensation is introduced as

$$u = K_P(\theta_d - \theta) - K_D\dot{\theta} + g(\tilde{\theta}), \tag{10}$$

where $K_P > 0$ and $K_D > 0$ are both symmetric (and typically diagonal) matrices, and

$$\tilde{\theta} = \theta - K^{-1} g(q_d). \tag{11}$$

The variable $\tilde{\theta}$ is a *gravity-biased* modification of the measured motor position θ. While the term $g(\tilde{\theta})$ provides only an *approximate* cancellation of gravity at any robot configuration during motion, it leads to the *correct* gravity compensation at steady state, even without a direct measure of q. As a matter of fact, the control law (10) can be implemented using only motor variables.

4. Theoretical result

Sufficient conditions for the global asymptotic stability of system (1) under control (10) are hereafter derived.

Theorem. *If the condition* $\lambda_{min}(\bar{K}) > \alpha$ *holds true, then the closed-loop system* (1), (10) *has the unique equilibrium configuration* $(q, \theta, \dot{q}, \dot{\theta}) = (q_d, \theta_d, 0, 0)$. *Moreover, this equilibrium is globally asymptotically stable.*

[1] This is the matrix norm naturally induced by the Euclidean norm on vectors, e.g., $\|q\| = \sqrt{\sum_{i=1}^{n} q_i^2}$.

Proof. The equilibrium configurations of the closed-loop system (1), (10) are computed by setting $\dot q=\dot\theta=0$ and $\ddot q=\ddot\theta=0$. This yields

$$g(q)+K(q-\theta)=0, \tag{12}$$

$$K(\theta-q)=K_{\mathrm P}(\theta_{\mathrm d}-\theta)+g(\tilde\theta). \tag{13}$$

From (12) it follows that, at any equilibrium, $\theta=q+K^{-1}g(q)$. Taking this into account and adding (12) to (13) leads to

$$K_{\mathrm P}(\theta_{\mathrm d}-\theta)+g(\tilde\theta)-g(q)=0.$$

Indeed $(q_{\mathrm d},\theta_{\mathrm d})$ is a closed-loop equilibrium configuration, since $\tilde\theta_{\mathrm d}:=\theta_{\mathrm d}-K^{-1}g(q_{\mathrm d})=q_{\mathrm d}$ from (8) and (11) so that $g(\tilde\theta_{\mathrm d})=g(q_{\mathrm d})$.

Further analysis allows showing that such equilibrium is unique. Adding $K(\theta_{\mathrm d}-q_{\mathrm d})-g(q_{\mathrm d})=0$ to both (12) and (13) yields

$$K(q-q_{\mathrm d})-K(\theta-\theta_{\mathrm d})=g(q_{\mathrm d})-g(q),$$

$$-K(q-q_{\mathrm d})+(K+K_{\mathrm P})(\theta-\theta_{\mathrm d})=g(\tilde\theta)-g(q_{\mathrm d}),$$

or, using the matrix $\bar K$ defined in (9),

$$\bar K\begin{bmatrix}q-q_{\mathrm d}\\ \theta-\theta_{\mathrm d}\end{bmatrix}=\begin{bmatrix}g(q_{\mathrm d})-g(q)\\ g(\tilde\theta)-g(q_{\mathrm d})\end{bmatrix}. \tag{14}$$

Under the hypothesis of the Theorem, i.e. condition (9), it is

$$\left\|\bar K\begin{bmatrix}q-q_{\mathrm d}\\ \theta-\theta_{\mathrm d}\end{bmatrix}\right\|^2\geqslant\lambda^2_{\min}(\bar K)\left\|\begin{bmatrix}q-q_{\mathrm d}\\ \theta-\theta_{\mathrm d}\end{bmatrix}\right\|^2 =\lambda^2_{\min}(\bar K)(\|q-q_{\mathrm d}\|^2+\|\theta-\theta_{\mathrm d}\|^2), \tag{15}$$

while using inequality (6) and the identity $\tilde\theta-q_{\mathrm d}=\theta-\theta_{\mathrm d}$ gives

$$\left\|\begin{matrix}g(q_{\mathrm d})-g(q)\\ g(\tilde\theta)-g(q_{\mathrm d})\end{matrix}\right\|^2=\|g(q_{\mathrm d})-g(q)\|^2+\|g(\tilde\theta)-g(q_{\mathrm d})\|^2 \leqslant\alpha^2(\|q-q_{\mathrm d}\|^2+\|\theta-\theta_{\mathrm d}\|^2). \tag{16}$$

By comparing (15) with (16) it follows that, when $\lambda_{\min}(\bar K)>\alpha$, the equality in (14) holds only for $(q,\theta)=(q_{\mathrm d},\theta_{\mathrm d})$, which is thus the unique equilibrium configuration of the closed-loop system (1), (10).

To demonstrate asymptotic stability of the closed-loop system, a candidate Lyapunov function is defined in terms of an auxiliary configuration-dependent function $P(q,\theta)$. This is expressed as

$$P(q,\theta)=\tfrac12(q-\theta)^{\mathrm T}K(q-\theta)+\tfrac12(\theta_{\mathrm d}-\theta)^{\mathrm T} \times K_{\mathrm P}(\theta_{\mathrm d}-\theta)+U_{\mathrm g}(q)-U_{\mathrm g}(\tilde\theta) \tag{17}$$

and differs from the similar function proposed by Tomei (1991). Under condition (9), this function has a unique minimum in $(q_{\mathrm d},\theta_{\mathrm d})$. In fact, the necessary condition for a minimum of $P(q,\theta)$ is

$$\nabla P(q,\theta)=\begin{bmatrix}\nabla_q P\\ \nabla_\theta P\end{bmatrix}=\begin{bmatrix}K & -K\\ -K & K\end{bmatrix}\begin{bmatrix}q\\ \theta\end{bmatrix} +\begin{bmatrix}g(q)\\ K_{\mathrm P}(\theta-\theta_{\mathrm d})-g(\tilde\theta)\end{bmatrix}=0. \tag{18}$$

Eq. (18) is exactly in the form (12), (13), which in turn is equivalent to (14). Using the same arguments as above, it can be demonstrated that $\nabla P(q,\theta)=0$ only at $(q_{\mathrm d},\theta_{\mathrm d})$. Moreover, the sufficient condition for a minimum

$$\nabla^2 P(q_{\mathrm d},\theta_{\mathrm d})=\begin{bmatrix}K & -K\\ -K & K+K_{\mathrm P}\end{bmatrix} +\left.\begin{bmatrix}\dfrac{\partial g(q)}{\partial q} & 0\\ 0 & -\dfrac{\partial g(\tilde\theta)}{\partial\theta}\end{bmatrix}\right|_{q=q_{\mathrm d},\theta=\theta_{\mathrm d}}>0$$

is satisfied, using again assumption (9). By setting $P_{\mathrm d}:=P(q_{\mathrm d},\theta_{\mathrm d})=g^{\mathrm T}(q_{\mathrm d})K^{-1}g(q_{\mathrm d})$, the candidate Lyapunov function can be written as

$$V(q,\theta,\dot q,\dot\theta)=\tfrac12\dot q_{\mathrm c}^{\mathrm T}B(q)\dot q_{\mathrm c}+P(q,\theta)-P_{\mathrm d}\geqslant 0. \tag{19}$$

Indeed, V is zero only at the desired equilibrium state $q=q_{\mathrm d}$, $\theta=\theta_{\mathrm d}$, $\dot q=\dot\theta=0$. Along the trajectories of the closed-loop system (1), (10), the time derivative of V becomes

$$\begin{aligned}\dot V&=\dot q_{\mathrm c}^{\mathrm T}B(q)\ddot q_{\mathrm c}+\frac12\dot q_{\mathrm c}^{\mathrm T}\dot B(q)\dot q_{\mathrm c}+(\dot q-\dot\theta)^{\mathrm T}K(q-\theta)\\ &\quad-\dot\theta^{\mathrm T}K_{\mathrm P}(\theta_{\mathrm d}-\theta)+\dot q^{\mathrm T}\left(\frac{\partial U_{\mathrm g}(q)}{\partial q}\right)^{\mathrm T}-\dot\theta^{\mathrm T}\left(\frac{\partial U_{\mathrm g}(\tilde\theta)}{\partial\theta}\right)^{\mathrm T}\\ &=\dot q_{\mathrm c}^{\mathrm T}\left(-C(q_{\mathrm c},\dot q_{\mathrm c})\dot q_{\mathrm c}-e(q)-K_{\mathrm e}q_{\mathrm c}+m+\frac12\dot B(q)\dot q_{\mathrm c}\right)\\ &\quad+\dot q^{\mathrm T}(K(q-\theta)+g(q))-\dot\theta^{\mathrm T}K(q-\theta)\\ &\quad-\dot\theta^{\mathrm T}(K_{\mathrm P}(\theta_{\mathrm d}-\theta)+g(\tilde\theta))\\ &=\dot q^{\mathrm T}(-K(q-\theta)-g(q)+K(q-\theta)+g(q))\\ &\quad+\dot\theta^{\mathrm T}(K(q-\theta)-K(q-\theta)+K_{\mathrm P}(\theta_{\mathrm d}-\theta))\\ &\quad+\dot\theta^{\mathrm T}(-K_{\mathrm D}\dot\theta+g(\tilde\theta)-K_{\mathrm P}(\theta_{\mathrm d}-\theta)-g(\tilde\theta))\\ &=-\dot\theta^{\mathrm T}K_{\mathrm D}\dot\theta\leqslant 0,\end{aligned} \tag{20}$$

where the identity $\dot{\tilde\theta}=\dot\theta$ and the skew-symmetry of matrix $\dot B-2C$ have been used. Since $\dot V=0$ if and only if $\dot\theta=0$, substituting $\dot\theta(t)\equiv 0$ into the closed-loop equations yields

$$B_1(q)\ddot q+C_{B1}(q,\dot q)\dot q+g(q)+Kq=K\theta=\text{constant}, \tag{21}$$

$$B_2^{\mathrm T}(q)\ddot q+C_{B3}(q,\dot q)\dot q-Kq =-K\theta+K_{\mathrm P}(\theta_{\mathrm d}-\theta)+g(\tilde\theta)=\text{constant}. \tag{22}$$

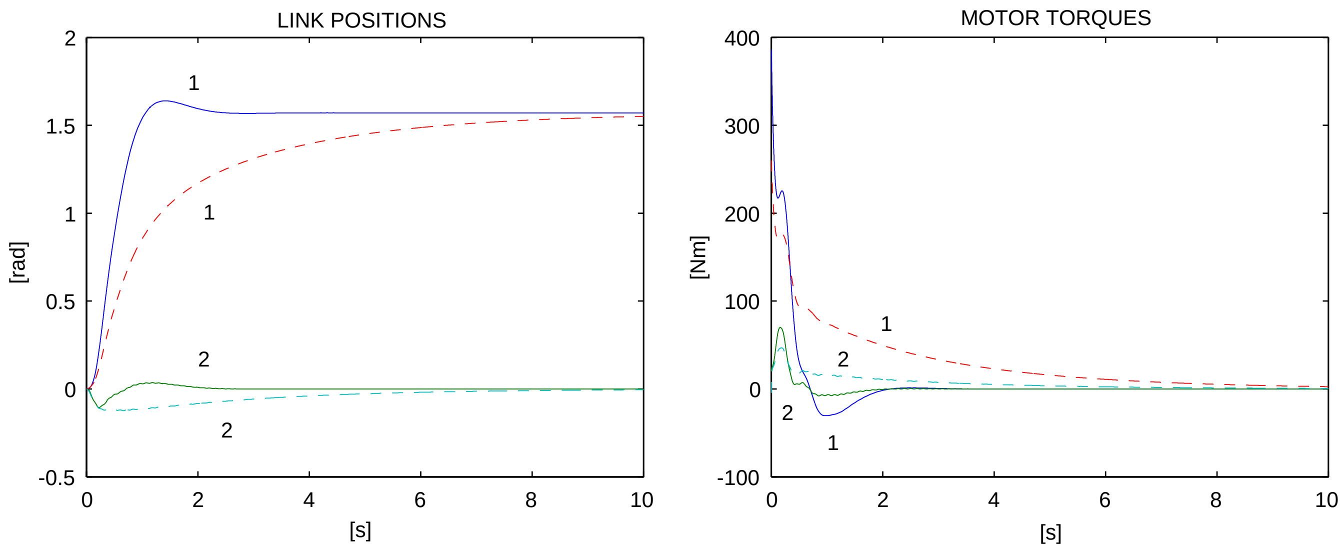

Fig. 1. Link positions and motor torques with on-line (solid) and constant (dashed) gravity compensation.

By virtue of Property (P2) and the expression of $C_{B3}(q, \dot{q})$, from (22) it follows that $\dot{q}(t) \equiv 0$. This in turn simplifies (21) to

$$g(q) + K(q - \theta) = 0. \tag{23}$$

It has already been shown that system (22), (23) has the unique solution $(q, \theta) = (q_d, \theta_d)$, provided that condition (9) holds true. Therefore, $q = q_d$, $\theta = \theta_d$, $\dot{q} = \dot{\theta} = 0$ is the largest invariant subset contained in the set of states such that $\dot{V} = 0$. By La Salle's Theorem, global asymptotic stability of the desired set point can be concluded. □

Remark 1. The sufficient condition (9) is the same as in (Tomei, 1991). It can be always satisfied by increasing the smallest eigenvalue of K_P, provided that $\lambda_{min}(K) > \alpha$. This latter assumption is by no means restrictive and is always satisfied in practice, since it requires the joints to be stiff enough to prevent the manipulator from falling down under the action of its weight.

Remark 2. If q_d is an open-loop equilibrium configuration (i.e. $g(q_d) = 0$), then $\tilde{\theta} = \theta$ and a gravity-biased modification of the measured motor position is not required.

Remark 3. Differently from the rigid joint case, the sufficient condition involving the position gain K_P cannot be relaxed when passing from constant to on-line gravity compensation. This is because the controller performs only an approximate gravity compensation during transients and employs measured variables at the motor side, rather than at the link side. The latter would eliminate the need of a positive lower bound on K_P in order to guarantee convergence. In principle, it should be possible to estimate the current value of q from motor measurements and then use this estimate $\bar{q}$ for on-line gravity compensation (i.e., with $g(\bar{q})$). However, by including a dynamic observer, the resulting control law would be more complex than the proposed static feedback law (10). A preliminary result along this direction has been recently presented by Ott, Albu-Schäffer, Kugi, Stramigioli, and Hirzinger (2004), where an iterative (discrete-time) scheme determines, for each motor measurement θ, the value $\bar{q}$ to be used for correct gravity compensation.

Remark 4. In the presence of uncertainties on K and g, the controller (10) would not yield convergence to the desired q_d. Nonetheless, some robustness features are present when using approximate estimates $\hat{K}$ and $\hat{g}$ in place of the exact terms in (10). Following a similar analysis as in (Tomei, 1991), it can be shown that the closed-loop equilibrium configuration $(q, \theta) = (q^*, \theta^*)$ satisfying

$$g(q) + K(q - \theta) = 0 \quad K(\theta - q) = K_P(\hat{\theta}_d - \theta) + \hat{g}(\hat{\theta}),$$

with $\hat{\theta}_d = q_d + \hat{K}^{-1}\hat{g}(q_d)$ and $\hat{\theta} = \theta - \hat{K}^{-1}\hat{g}(q_d)$, is still unique and globally asymptotically stable under the same assumption of the Theorem. Indeed, the higher is K_P the closer will be q^* to q_d.

5. Simulation results

Simulation tests have been carried out in order to measure dynamic performance of the PD controller with on-line vs. constant gravity compensation.

A simple planar arm with two revolute joints and uniform cylindric links moving in the vertical plane is considered. Its relevant dynamic parameters are: $l_1 = l_2 = 0.5$ m (link lengths); $m_1 = 20$, $m_2 = 10$ kg (link masses); $r_1 = r_2 = 0.1$ m (link section radii). With these values, the upper bound in (5) is $\alpha \simeq 133$. The joint stiffness matrix is

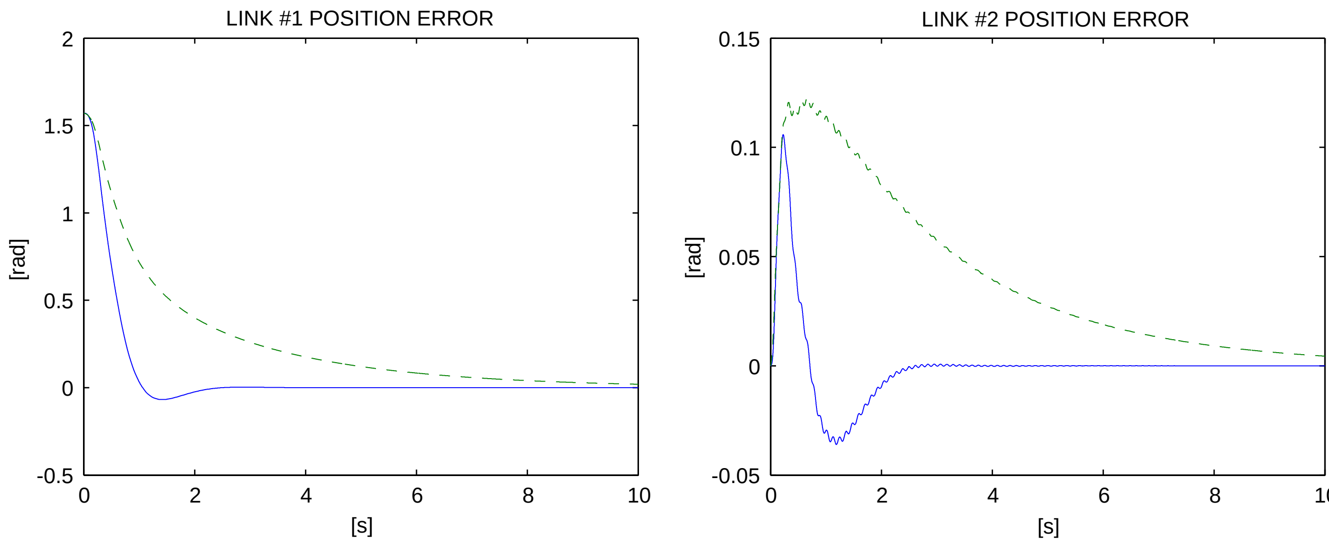

Fig. 2. Position errors with on-line (solid) and constant (dashed) gravity compensation: link 1 (left), link 2 (right).

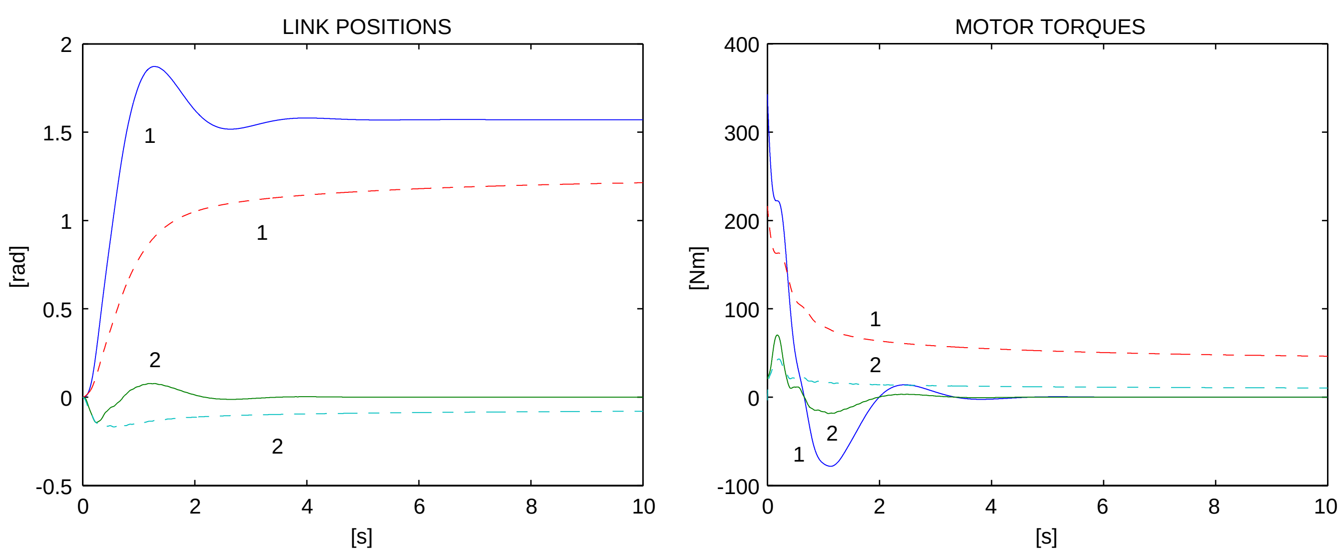

Fig. 3. Link positions and motor torques with on-line (solid) and constant (dashed) gravity compensation—lower gains.

$K = \mathrm{diag}\{1000, 1000\}$ Nm/rad. The motor inertia matrix is $B_3 = \mathrm{diag}\{6.183, 0.858\}$.

The task is a swing up with a step change of the link positions from the initial configuration $q_i = [0\ \ 0]^{\mathrm{T}}$ (the arm is stretched horizontally) to the desired configuration $q_{\mathrm{d}} = [\pi/2\ \ 0]^{\mathrm{T}}$ rad (arm stretched vertically and upward). The robot is initially in an equilibrium state, under the action of the control torque $u = g(q_i)$ and with the two elastic joints slightly deformed. Therefore, the initial values of the motor positions have been set to $\theta_i = [0.1275\ \ 0.0245]^{\mathrm{T}}$ rad.

In the first run, the gains of both controllers in (7) and (10) are set to

$$K_{\mathrm{P}} = \mathrm{diag}\{180, 180\}, \quad K_{\mathrm{D}} = \mathrm{diag}\{80, 80\},$$

and the results obtained are shown in Figs. 1 and 2. Although both controllers achieve the correct steady-state position, the transient with the on-line gravity compensation is faster than that with the constant gravity compensation. This is achieved at the cost of just slightly higher motor torques at the start of the motion. When the requested motion is reversed, i.e., from $q_i = [\pi/2\ \ 0]^{\mathrm{T}}$ to $q_{\mathrm{d}} = [0\ \ 0]^{\mathrm{T}}$, differences in behavior are less evident. After extensive simulations, it has been observed in general that, for a given set of gains, transient performance with the on-line gravity compensation scheme is usually better than with constant gravity compensation.

In order to assess the conservativeness of condition (9) for the on-line compensation scheme, the same task was

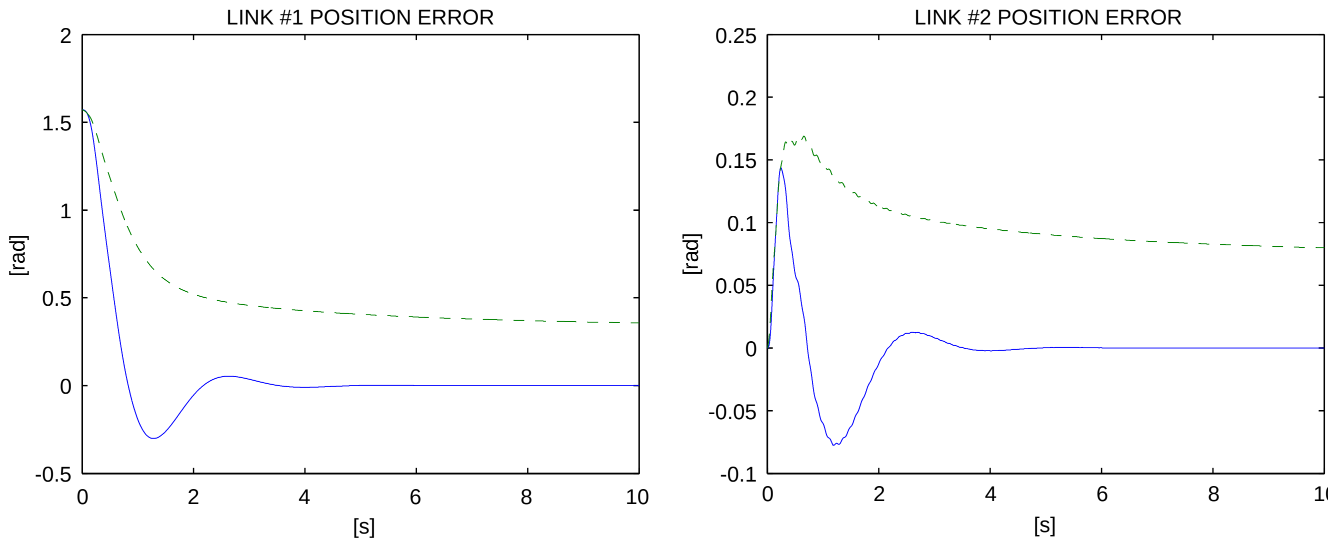

Fig. 4. Position errors with on-line (solid) and constant (dashed) gravity compensation: link 1 (left), link 2 (right)—lower gains.

executed again with lower gains

$$K_P = \text{diag}\{150, 150\}, \quad K_D = \text{diag}\{50, 50\},$$

not satisfying the sufficient condition (9) for global asymptotic stability. Interestingly enough, only the controller with on-line compensation achieves the desired reconfiguration, while a different steady-state position is reached under the controller with constant compensation (see Figs. 3 and 4).

Further simulations have confirmed that the PD control with on-line gravity compensation continues to work with a choice of lower position gains, even when the sufficient condition is violated. Moreover, for increasing values of the joint stiffness (in the limit $K \to \infty$), the range of feasible values of K_P allowing exact regulation with on-line gravity compensation extends down to zero, thus recovering the rigid case, i.e., for any $K_P > 0$. This does not happen, instead, in the case of constant gravity compensation. On the other hand, the differences between the two controllers tend to vanish when increasing arbitrarily the positional gains. However, high gains may provoke motor saturation for a step reference change and, most notably, excite unmodeled dynamics.

6. Experimental results

In this section, an experimental comparison of the PD controllers with constant and with on-line gravity compensation is presented, showing also the practical relevance of motor saturation, static friction, and uncertain dynamics.

The robot used for the experiments is an eight-joint cable-driven robot manipulator, named *Dexter*, manufactured by Scienzia Machinale (Fig. 5). It has a mechanical transmission system realized by pulleys and steel cables. As an example, Fig. 6 shows one of the *Dexter* joint/link pair.

Fig. 5. The *Dexter* robot.

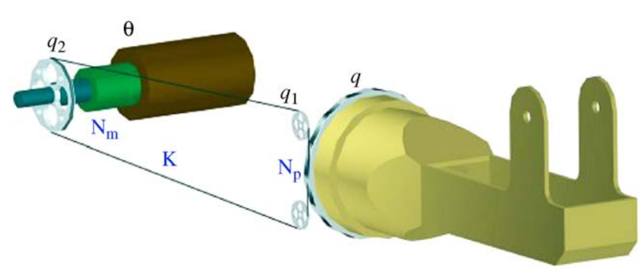

Fig. 6. A cable-driven joint/link pair.

Remote actuation through cables allows decreasing the distribution of the link total masses from the robot base up to the end effector, thus lightening the robot mechanical structure at the cost of introducing joint transmission elasticity. The total mass of the eight links is about 27 kg.

Table 1
Joint stiffness coefficients of the *Dexter* manipulator, expressed in (Nm/rad)

Joint	1	2	3	4	5	6	7	8
Stiffness coefficient	10^5	10^5	6.34×10^3	3.60×10^3	2.69×10^3	1.69×10^3	1.23×10^2	2.06×10^2

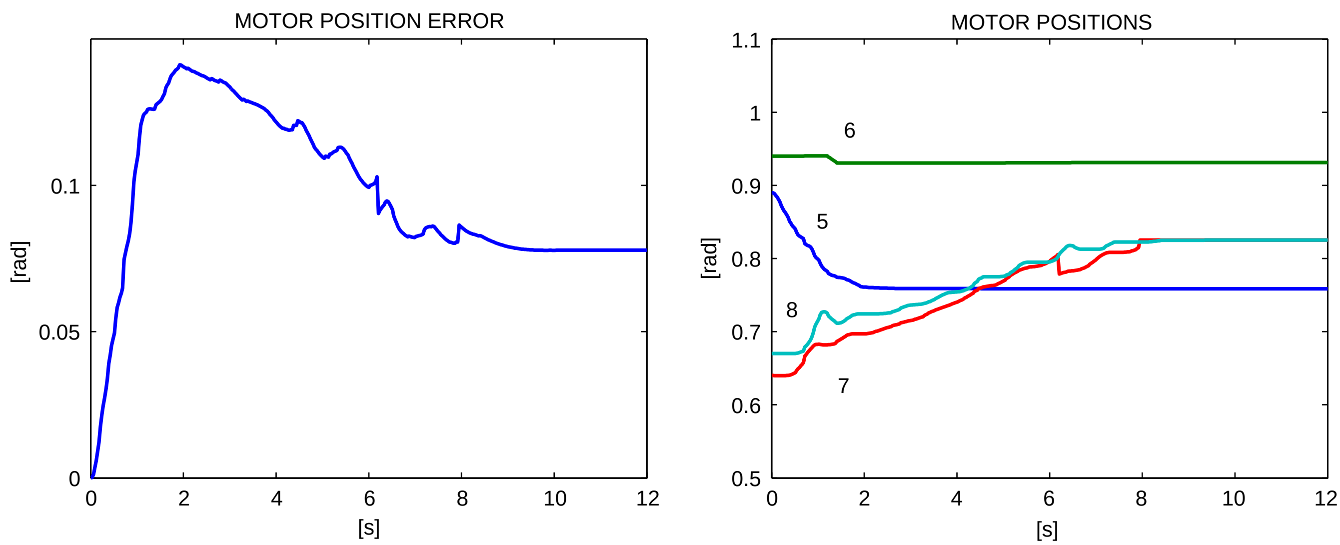

Fig. 7. Norm of all motor position errors (left) and motor variables 5–8 (right) for PD control with constant gravity compensation (desired time-varying joint trajectory).

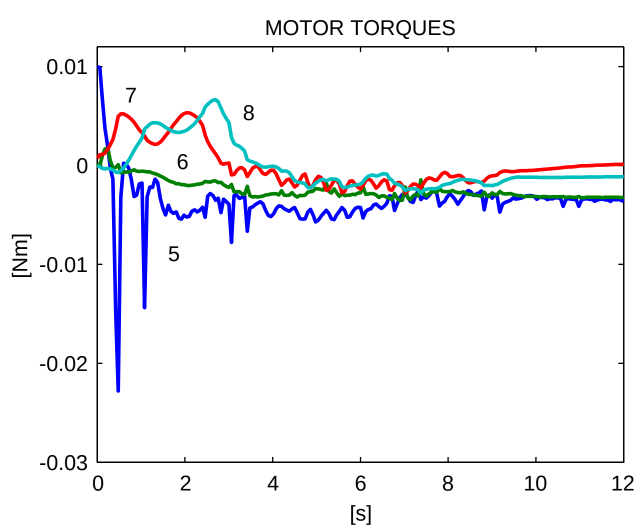

Fig. 8. Torques at motors 5–8 for PD control with constant gravity compensation (desired time-varying joint trajectory).

The robot dynamic model is expressed in terms of the $n = 8$ variables θ defining the motor positions, and the $n = 8$ variables q defining the link positions, for a total of 16 position variables. Incremental encoders allow measuring motor positions θ during motion, while motor velocities $\dot{\theta}$ are reconstructed through numerical differentiation (Euler method, with 10 ms step).

The cable stiffness coefficients for the *Dexter* manipulator are reported in Table 1. As one can observe, joints 1 and 2 have higher stiffness values with respect to the other joints, being the motors directly coupled to the shafts through harmonic drives. Hence, their elasticity can be neglected as compared to that affecting joints 3–8, which is mostly due to cable transmission deformation, see Zollo, De Luca, and Siciliano (2004).

Control laws are written in C^{++} programming language and run on a PC Pentium II under DOS Operating System. The motor commands are sent to the actuation system every 10 ms, by means of two MEI 104/DSP-400 control boards.

The issue of motor saturation becomes evident in the experiments on the *Dexter* manipulator. When a constant gravity compensation is used, even very short regulation tasks to a constant desired configuration may not be accomplished. The large initial error and the addition of the gravity term evaluated at the destination can lead to a high torque demand so that actuators saturate. Conversely, with the use of on-line gravity compensation, the task can be performed in such cases, but only for short distances (nearby 3–4 cm in the Cartesian space) between the initial and the desired configuration.

In order to overcome the critical issue of motor saturation, a point-to-point quintic polynomial trajectory (with zero velocity and acceleration boundary conditions) has been planned, guiding the robot manipulator from an initial

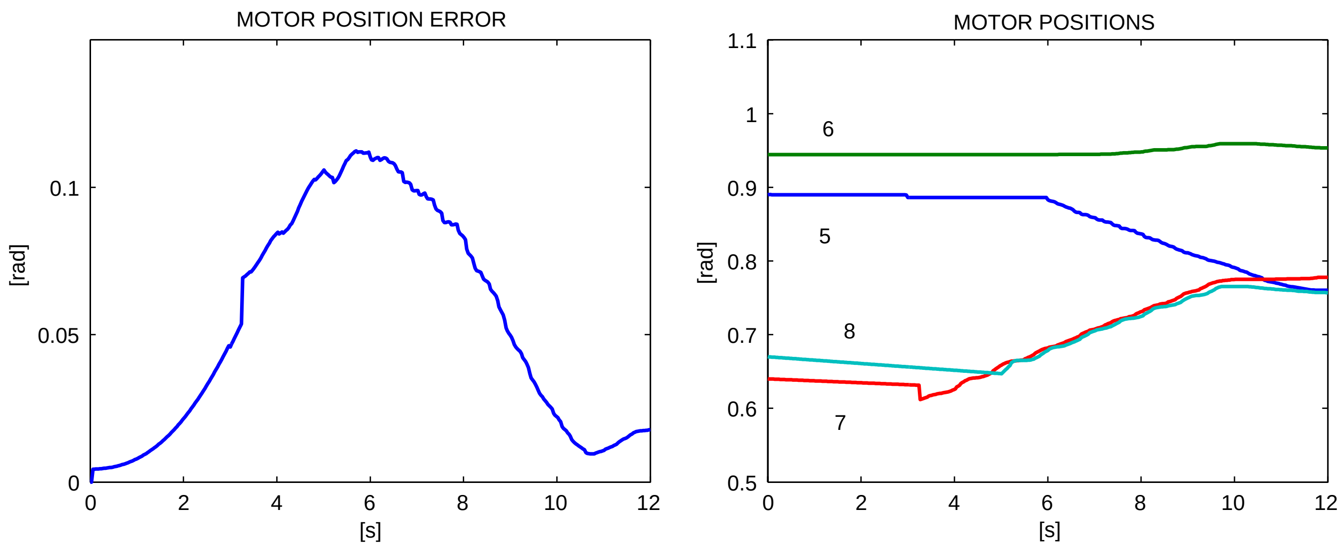

Fig. 9. Norm of all motor position errors (left) and motor variables 5–8 (right) for PD control with on-line gravity compensation (desired time-varying joint trajectory).

joint configuration

$$q_i = [1.57\ \ 0.00\ \ 2.49\ \ 0.83\ \ -0.89\ \ 0.94\ \ 0.64\ \ 0.66]^{\mathrm{T}}\ \mathrm{rad}$$

to the desired configuration

$$q_{\mathrm{d}} = [1.57\ \ 0.30\ \ 2.74\ \ 0.83\ \ -0.77\ \ 0.96\ \ 0.78\ \ 0.77]^{\mathrm{T}}\ \mathrm{rad}$$

in a time interval of 10 s, plus 2 s for the adjustment. In this way, both controllers (7) and (10) can perform the motion with sufficiently high positional gains. Note that q_{d} is not an open-loop equilibrium configuration. For this task, the Cartesian motion is approximately 15 cm, with a reorientation of about $\pi/4$ rads.

The first experiments were run with the following proportional and derivative gains for both controllers:

$$K_{\mathrm{P}} = \mathrm{diag}\{80, 80, 30, 20, 16, 8, 2, 2\},$$

$$K_{\mathrm{D}} = \mathrm{diag}\{10, 10, 9, 3, 2.5, 2, 0.1, 0.1\}. \quad (24)$$

The results are shown in Figs. 7 and 8 for constant gravity compensation and, respectively, in Figs. 9 and 10 for on-line gravity compensation. In particular, Figs. 7 and 9 display the norm of the motor position error of all eight joints and the evolution of motor variables 5–8 (the most involved in the motion), as recorded by the encoders on the motor shafts. For the sake of graph scaling, the evolution of joint 5 is shown with the opposite sign. Comparing Fig. 7 with Fig. 9 indicates a reduction of the overall positional error obtained thanks to on-line gravity compensation. Note that the error norm starts from zero in both cases, as the reference trajectory starts from the initial robot position. On the other hand, a small residual error is present at steady state due to the effects of static friction and/or inaccurate estimate of the gravity term. This steady-state error, however, is about one order of magnitude smaller when gravity is compensated on

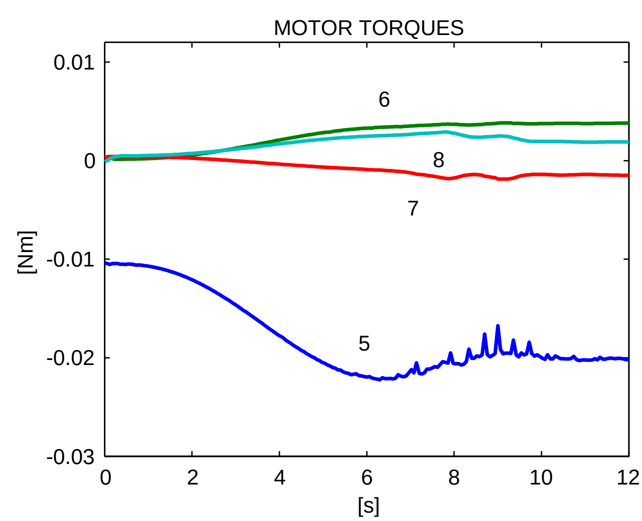

Fig. 10. Torques at motors 5–8 for PD control with on-line gravity compensation (desired time-varying joint trajectory).

line. Finally, this improved performance is obtained at no additional cost in terms of control effort. The torque profiles of motors 5–8, obtained from motor current data and shown in Figs. 8 and 10, have similar peak values but are considerably smoother in the case of on-line gravity compensation.

In order to test for the behavior with higher control gains, the same motion task has been repeated using the proportional gains

$$K_{\mathrm{P}} = \mathrm{diag}\{110, 110, 50, 35, 26, 15, 4, 4\} \quad (25)$$

in place of those in (24). However, the control law (7) could not be applied in this case. In fact, during the first few seconds of motion, the constant gravity torque $g(q_{\mathrm{d}}) \neq 0$ has a larger value than the gravity torque $g(\tilde{\theta})$ used in the control

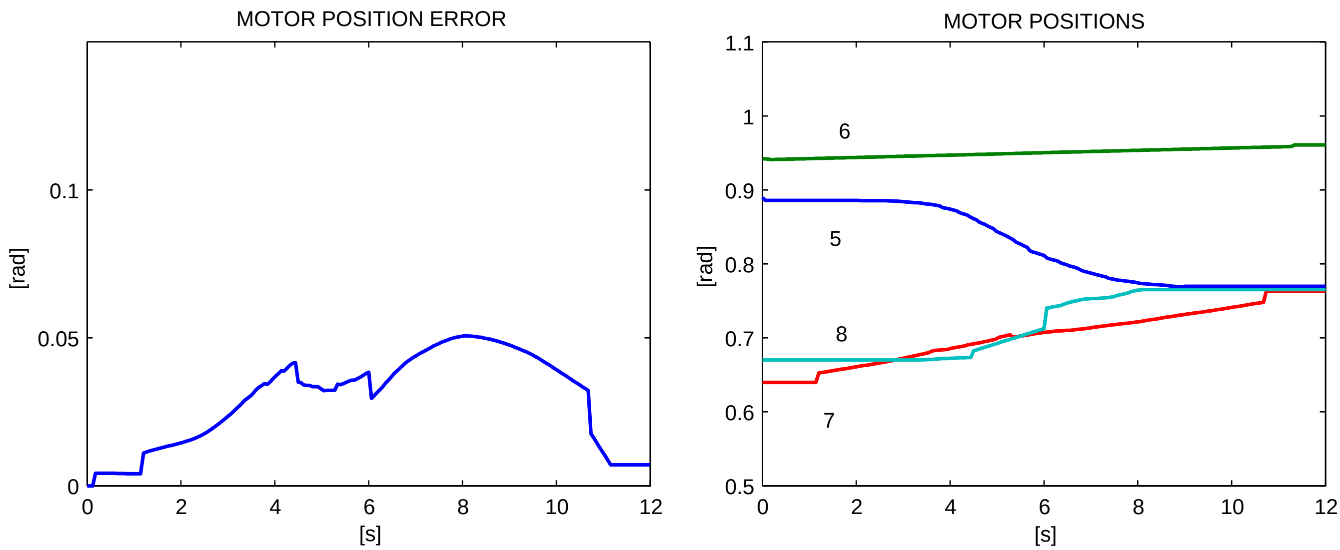

Fig. 11. Norm of all motor position errors (left) and motor variables 5–8 (right) for PD control with on-line gravity compensation—higher gains.

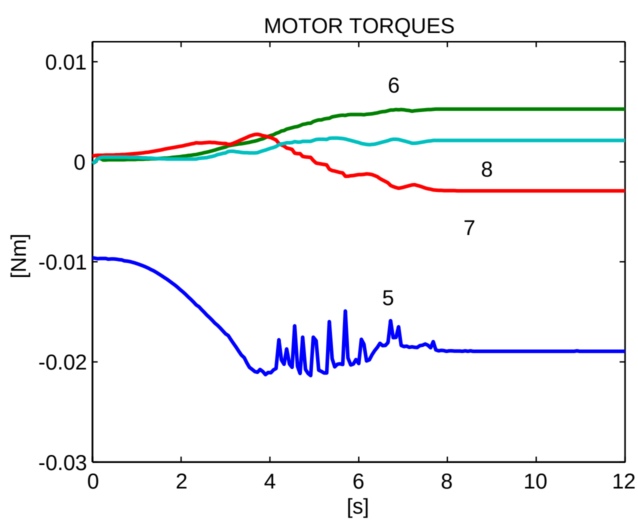

Fig. 12. Torques at motors 5–8 for PD control with on-line gravity compensation—higher gains.

law (10). When added to the error multiplied by the higher proportional gain K_P in (25), this leads again to motor saturation. Instead, motion could be safely completed with the on-line gravity compensation. As expected, the results shown in Figs. 11 and 12 indicate a reduction of both the transient and steady-state errors, with a similar control torque effort.

7. Conclusion

For robots with elastic joints performing regulation tasks, the combination of a PD control action on the motor variables with on-line gravity compensation has been proposed. In this latter term, the adoption of a gravity-biased motor position variable, in place of the actual link position, avoids the use of extra position sensors on the link side of joint elasticity. Global asymptotic stability of this control law has been proved through a Lyapunov argument, and its performance has been evaluated by means of both simulations on a two-link arm with elastic joints and experiments on an eight-joint cable-driven robot manipulator.

The results have shown that the proposed controller with on-line gravity compensation typically outperforms the previous controller with constant gravity compensation in terms of transient behavior and design flexibility. In particular, control torques of similar magnitude are obtained, but with a smoother time course and a sensible reduction of the average positional transient errors. Also, a wider range of proportional gains turns out to be feasible using PD control with on-line gravity compensation. This has been exploited, together with the use of an interpolating trajectory for regulation tasks, in order to avoid the problem of actuator saturation.

Finally, it is worth mentioning that the use of the gravity-biased motor variable has been proved successful also for regulation tasks in the Cartesian space. Zollo, Siciliano, De Luca, Guglielmelli, and Dario (2003) (see also Zollo, Siciliano, De Luca, Guglielmelli, & Dario, 2005) have used such a variable also in the direct and differential kinematic terms to regulate compliance at the robot end effector.

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