Robotics II

January 7, 2020

Exercise 1

Consider the robot in Fig. 1 with $N = 3$ joints, one prismatic and two revolute. Each link of the robot has uniformly distributed mass, center of mass on its physical link axis, and a diagonal barycentric link inertia matrix. We assume that friction at the joints can be neglected. The robot is commanded at the joint level by a generalized vector of forces/torques $\boldsymbol{\tau} \in \mathbb{R}^3$.

a) Derive the dynamic model of the robot in the Lagrangian form $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}.\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}$.

b) Find a linear parametrization $\boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\,\boldsymbol{a} = \boldsymbol{\tau}$ of the robot dynamics in terms of a vector $\boldsymbol{a} \in \mathbb{R}^p$ of dynamic coefficients and of a $3 \times p$ regressor matrix $\boldsymbol{Y}$. Discuss the minimality of p.

c) Determine which of the $10N = 30$ dynamic parameters of the links are irrelevant for the describing the motion of the robot.

d) Given a desired smooth trajectory $\boldsymbol{q}_d(t) \in C^2$ in the joint space, design for this robot an adaptive control law that globally asymptotically stabilizes the tracking error $\boldsymbol{e}(t) = \boldsymbol{q}_d(t) - \boldsymbol{q}(t)$ to zero, without any a priori knowledge of the robot dynamic parameters.

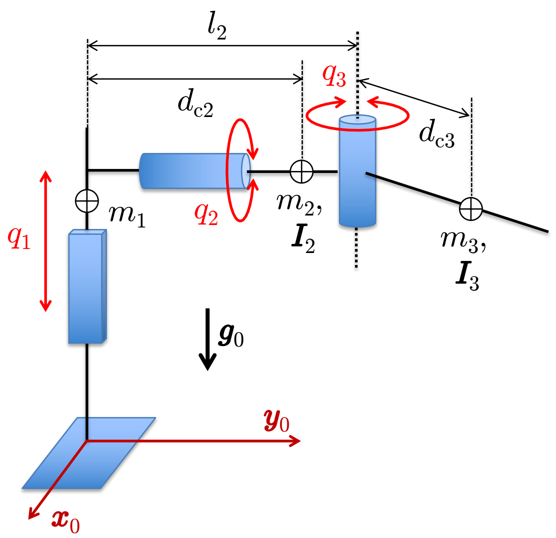

Figure 1: A PRR robot with coordinates $\boldsymbol{q} = (q_1\ q_2\ q_3)^T$ and relevant kinematic/dynamic parameters.

Exercise 2

A number of questions and statements are reported on the Questionnaire Sheet. Fill in your answers and/or comments on the same sheet, providing also a short motivation/explanation for each item. Add your name on the sheet and return it.

[210 minutes, open books]

Robotics II - Questionnaire Sheet

January 7, 2020

Name: ____

Answer to the questions or comment the statements, providing also a *short* motivation/explanation for each of the following 4 items.

1. Write down the calibration equation for a planar 2R robot in which the only inaccurate values are the link lengths l_1 and l_2. Describe briefly how to set up a kinematic calibration procedure in this caee.

2. At a given $\boldsymbol{q} \in \mathbb{R}^N$, we have to choose the velocity command $\dot{\boldsymbol{q}} \in \mathbb{R}^N$ that minimizes the objective function $H = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{A}^{-1}\dot{\boldsymbol{q}}$, with $\boldsymbol{A} > 0$, while satisfying the task $\boldsymbol{J}\dot{\boldsymbol{q}} = \dot{\boldsymbol{x}} \in \mathbb{R}^M$, with $M < N$ and $\text{rank}\{\boldsymbol{J}\} = M$. Two commands have been computed as
$$\dot{\boldsymbol{q}}' = \boldsymbol{A}^{-1}\boldsymbol{J}^T\left(\boldsymbol{J}\boldsymbol{A}^{-1}\boldsymbol{J}^T\right)^{-1}\dot{\boldsymbol{x}} \quad \text{and} \quad \dot{\boldsymbol{q}}'' = \boldsymbol{J}^{\#}\dot{\boldsymbol{x}} - \left(\boldsymbol{I} - \boldsymbol{J}^{\#}\boldsymbol{J}\right)\nabla_{\dot{\boldsymbol{q}}}H, \quad \text{with } \nabla_{\dot{\boldsymbol{q}}}H = \boldsymbol{A}^{-1}\dot{\boldsymbol{q}}.$$
Which command is better? Why?

3. Consider an assembly task, in which a peg having an equilateral triangle as section is to be inserted at a slow but constant speed V in a similar hole with reduced clearance. Define a suitable task frame and the natural and artificial constraints for this task.

4. For a 2-dof RP robot in the horizontal plane, write the explicit expression of an energy-based scalar residual, able to detect collisions when all the robot joints are in motion. Determine also which type of contact forces in the plane $\boldsymbol{F}_c \in \mathbb{R}^2$ (i.e., where they are applied on the robot, and in which direction) cannot be detected by this method, even if the robot is not at rest.