## Robotics II

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## Exercise 1

Consider a planar 3R robot with unitary link lengths as in Fig. 1. Taking into account the robot redundancy, a velocity control scheme is active so as to track desired end-effector position trajectories while trying to locally maximize the minimum Cartesian distance of the robot body from the obstacles.

- In the shown configuration $\boldsymbol{q}=\left(30^{\circ},-30^{\circ}, 30^{\circ}\right)$, the end-effector is assigned a unitary velocity $\boldsymbol{v}$ in the positive $\boldsymbol{x}_{0}$ direction. Specify the velocity control scheme and provide the associated numerical value of the command vector $\dot{\boldsymbol{q}} \in \mathbb{R}^{3}$.
- Compare with a minimum velocity norm solution that neglects the presence of the obstacle.


Figure 1: A planar 3R robot moving in the presence of an obstacle

## Exercise 2

The two-mass flexible system in Fig. 2 moves under gravity and is subject to a control force $F$. The position coordinates $q_{1}$ and $q_{2}$ have their zero in a position where the elastic spring is undeformed.

- Derive the dynamic model assuming that all friction effects can be neglected.
- Determine all forced equilibrium configurations of the system.
- Design a feedforward plus linear feedback control scheme using only measurements of the position $q_{1}$ and velocity $\dot{q}_{1}$ of the first mass, which is able to regulate the position of the second mass to a constant desired position $q_{2, d}$.
- Prove the global asymptotic (actually, exponential) stability of the desired closed-loop equilibrium. Hint: The closed-loop system dynamics is affine, and a simple analysis can be made by linearizing the system around the desired equilibrium, which removes constant terms.


Figure 2: A system of two masses $m_{1}$ and $m_{2}$, coupled by an elastic transmission of stiffness $k$

