



Robotics 2

Robot Interaction with the Environment

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DIPARTIMENTO DI INGEGNERIA INFORMATICA
AUTOMATICA E GESTIONALE ANTONIO RUBERTI



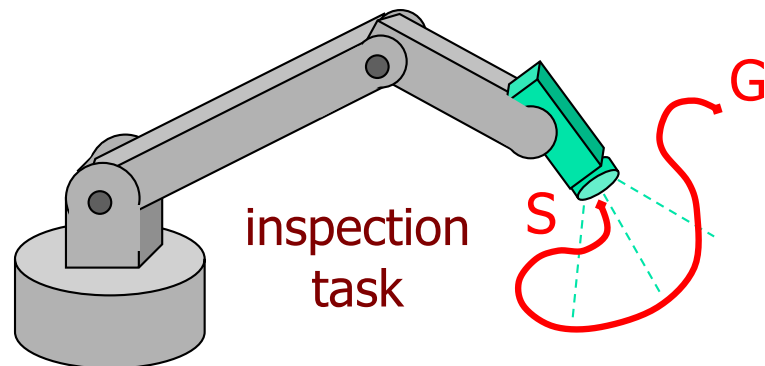
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Robot-environment interaction

a robot (end-effector) may interact with the **environment**

- **modifying the state** of the environment (e.g., pick-and-place operations)
- **exchanging forces** (e.g., assembly or surface finishing tasks)

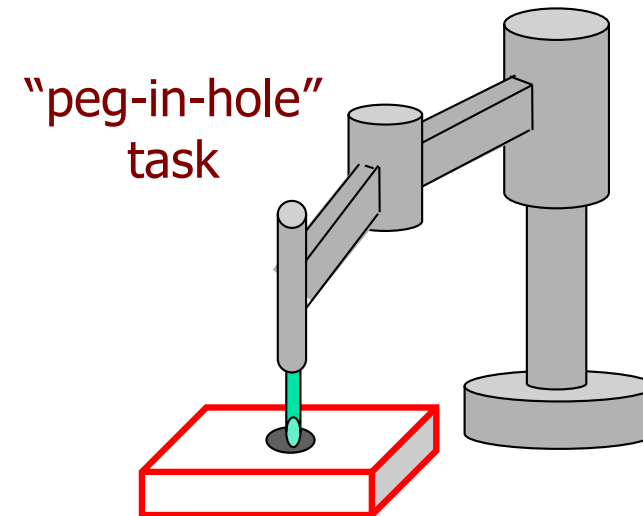
control of free motion



sensors: position (encoders)
at the joints* or
vision at the Cartesian level

*and velocity (by numerical differentiation
or, more rarely, with tachos)

control of compliant motion



sensors: as before +
6D force/torque
(at the robot wrist)



Robot compliance

PASSIVE

robot end-effector equipped with **mechatronic devices** that “comply” with the **generalized forces** applied at the TCP = Tool Center Point

RCC = Remote Center of Compliance device

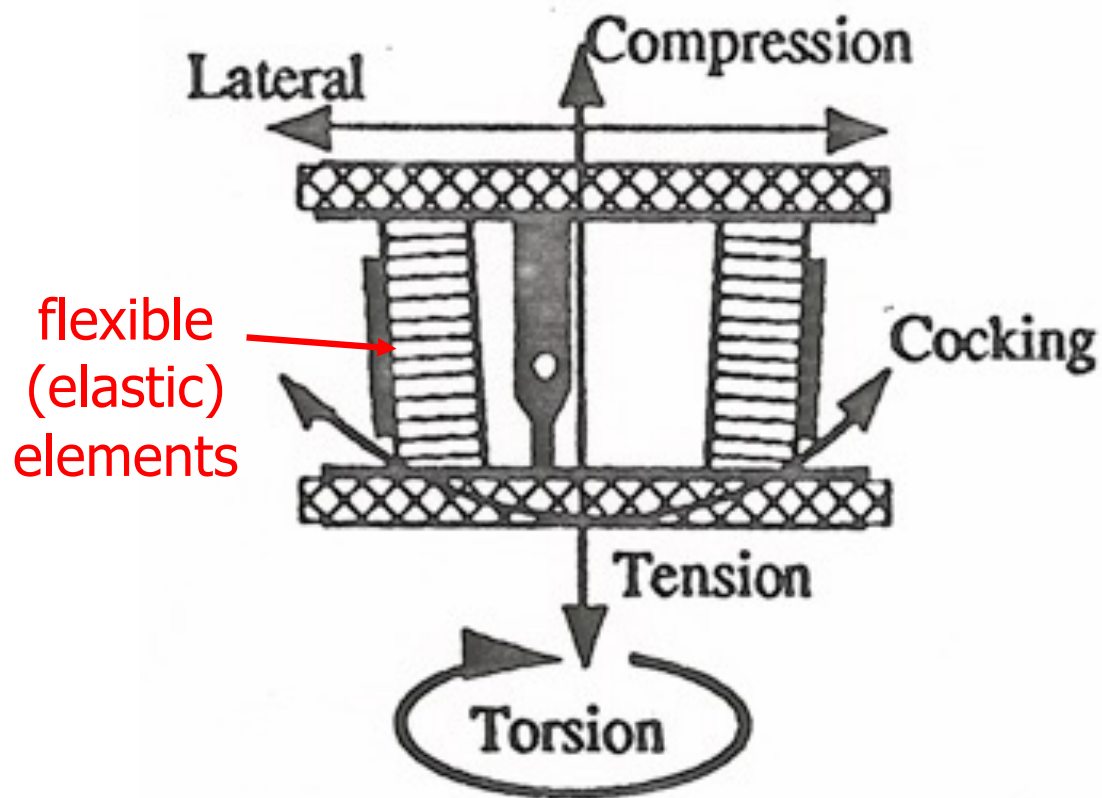


ACTIVE

robot is moved by a **control law** so as to react in a desired way to **generalized forces** applied at the TCP (typically measured by a F/T sensor)

- **admittance** control
contact forces \Rightarrow velocity commands
- **stiffness/compliance** control
contact displacements \Rightarrow force commands
- **impedance** control
contact displacements \Leftrightarrow contact forces

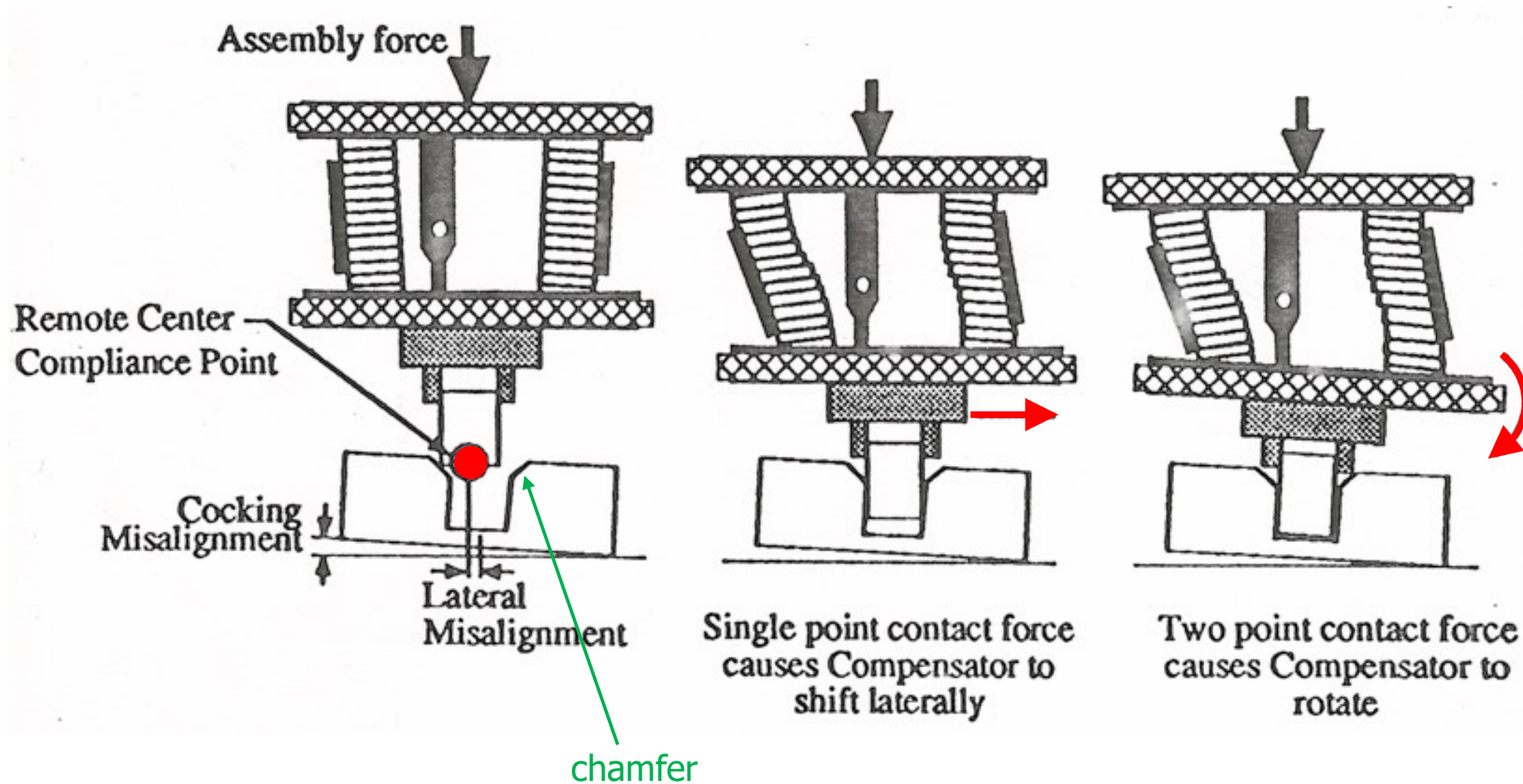
RCC device



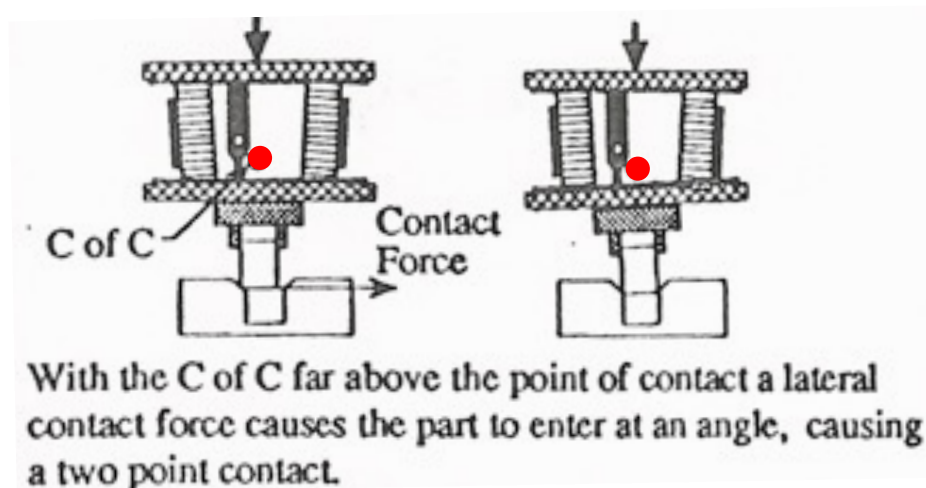
RCC models of different size by ATI

RCC behavior

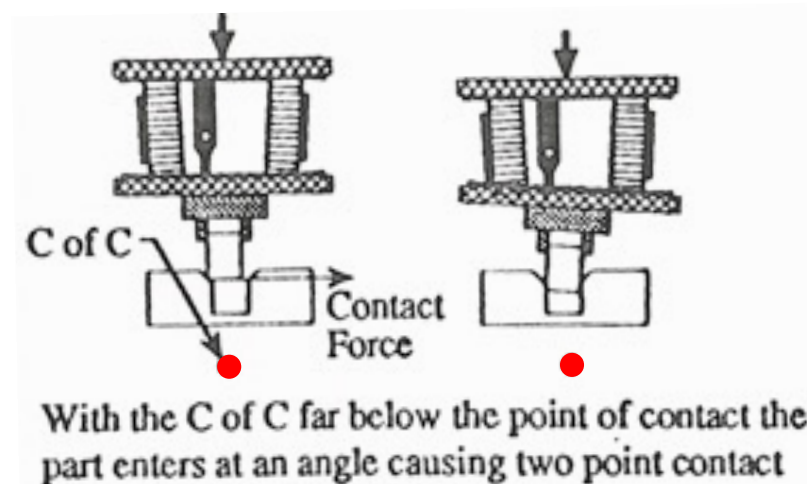
in case of misalignment errors in assembly tasks



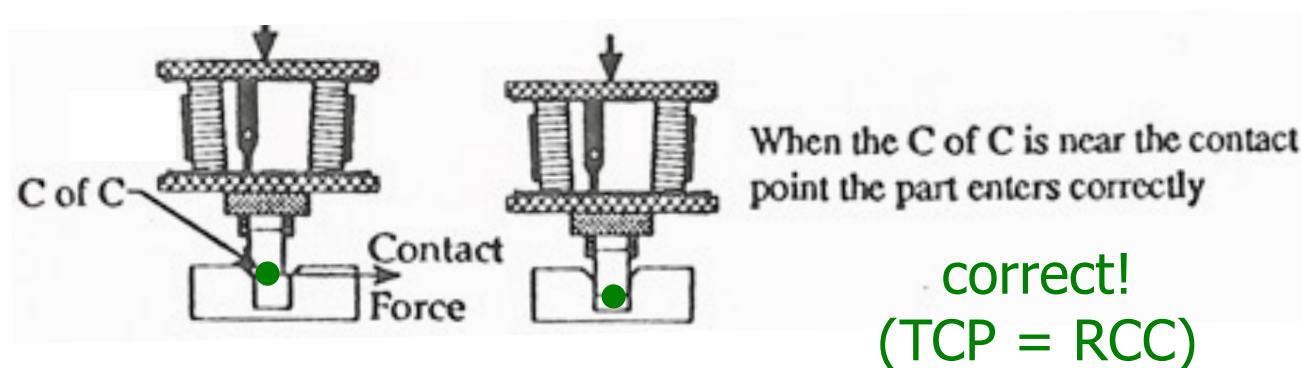
Effects of RCC positioning



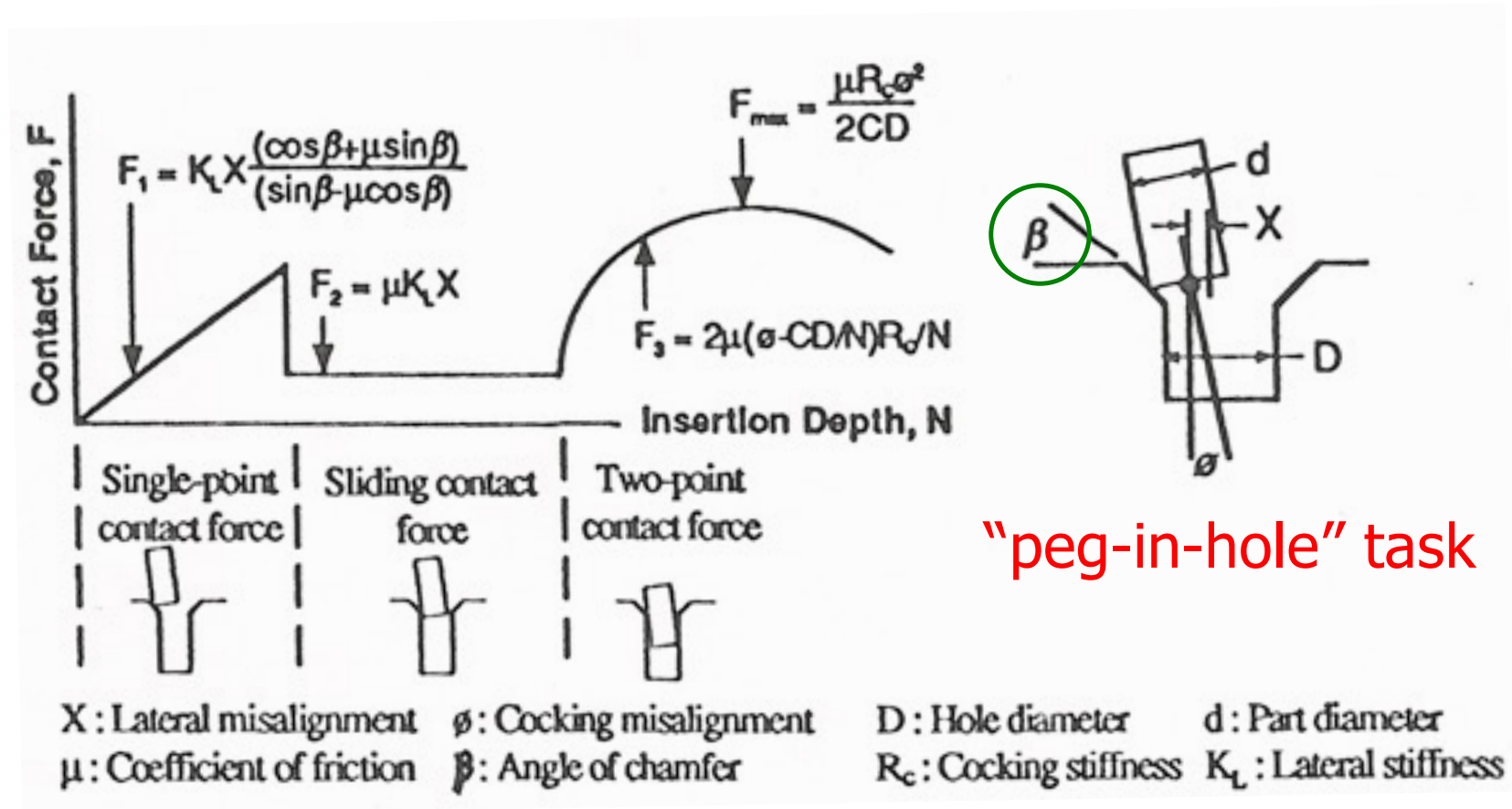
too high...



too low...

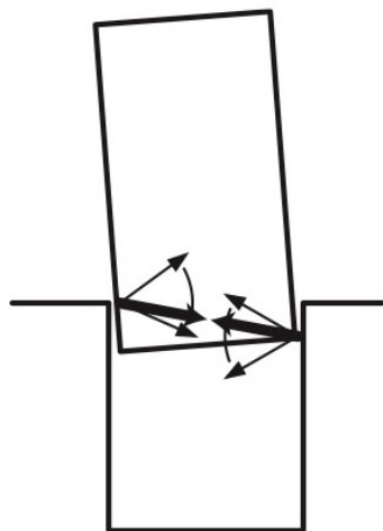


Typical evolution of assembly forces

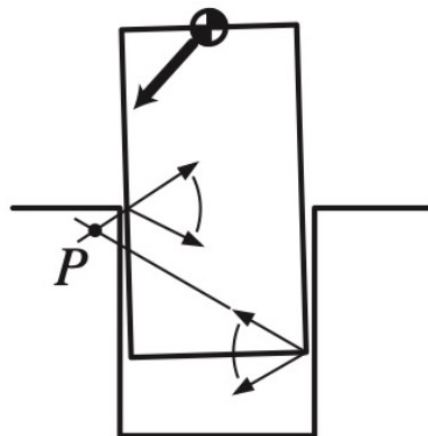


chamfer angle β = to ease the insertion,
related also to the tolerances of the hole

Wedging and jamming at contacts



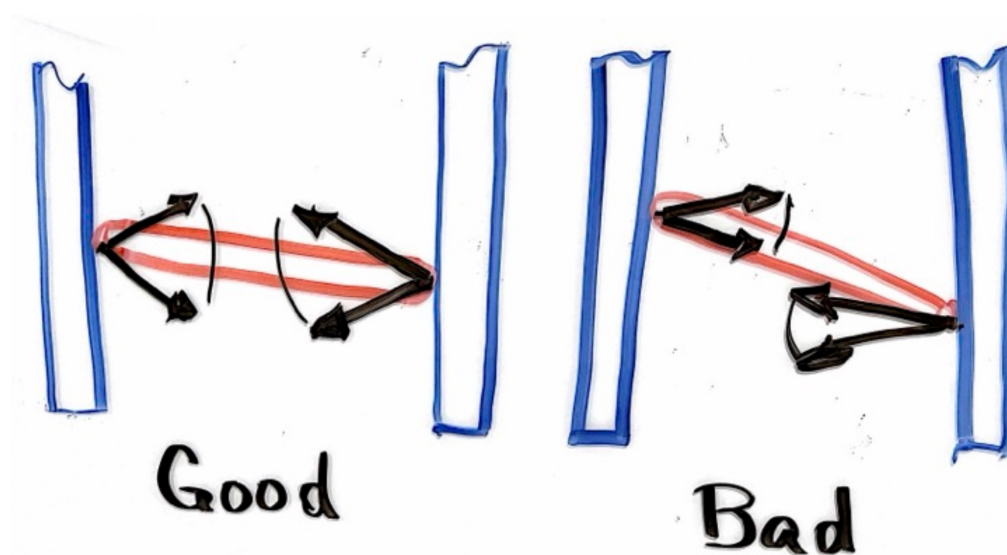
Wedged



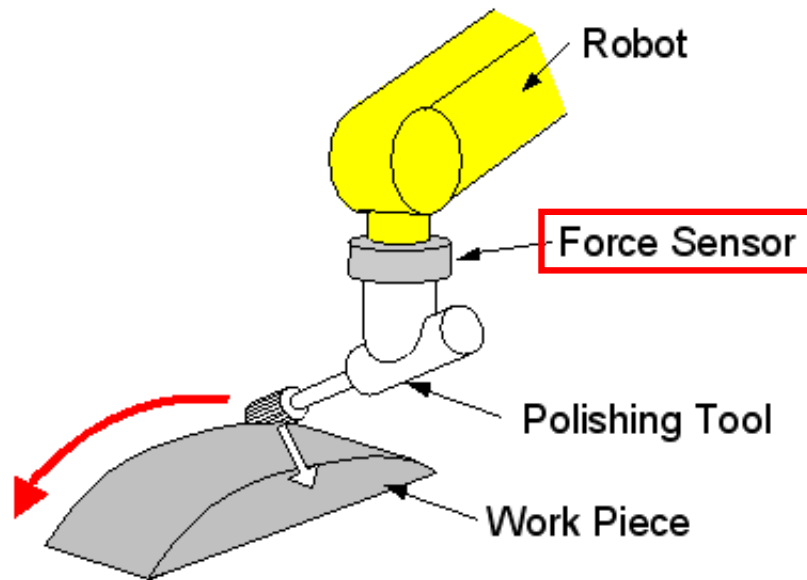
Jammed

friction problems
in two-point contact
for peg-in-hole **assembly**

(no) slippage of a coin
in **manipulation**
by a parallel gripper



Active compliance for contour following



Following with constant pushing force



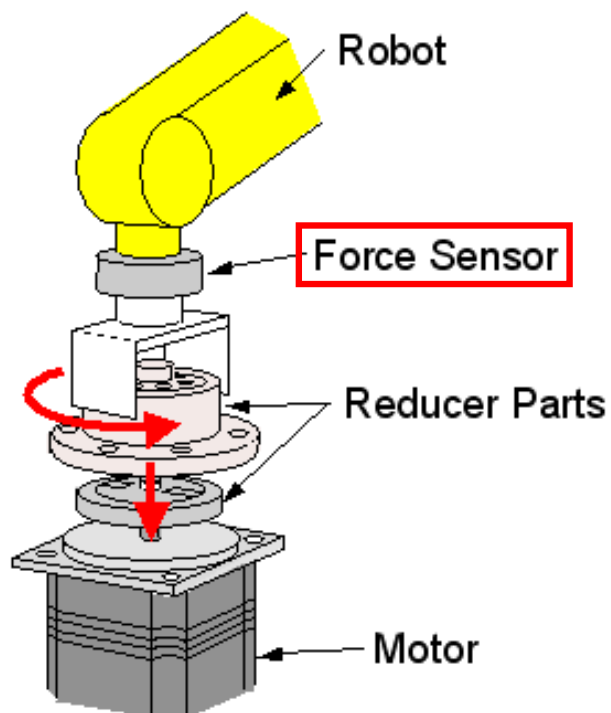
Washstand



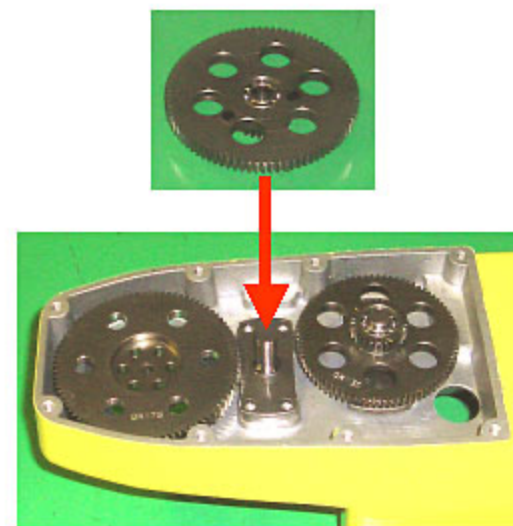
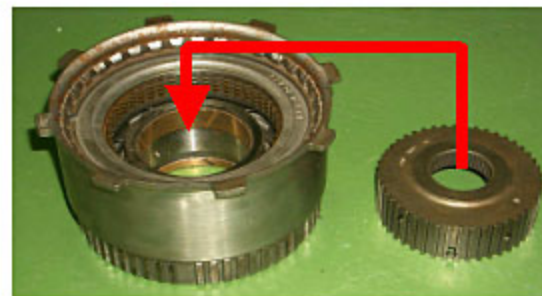
Metal Cabinet

Active compliance

"matching" of mechanical parts



Phase matching by force sensing



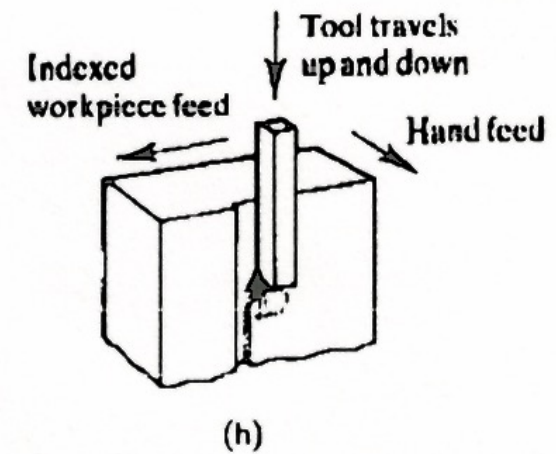
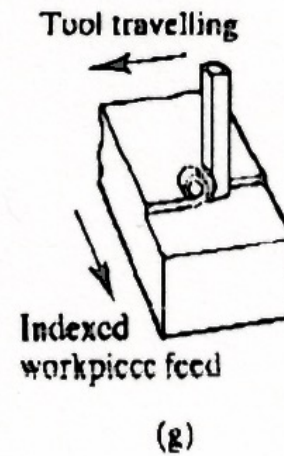
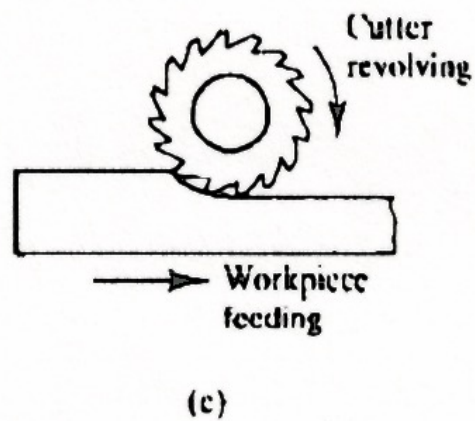
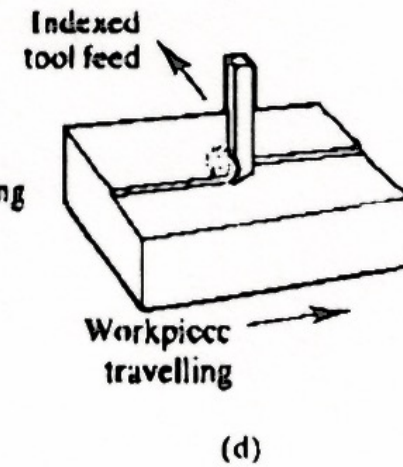
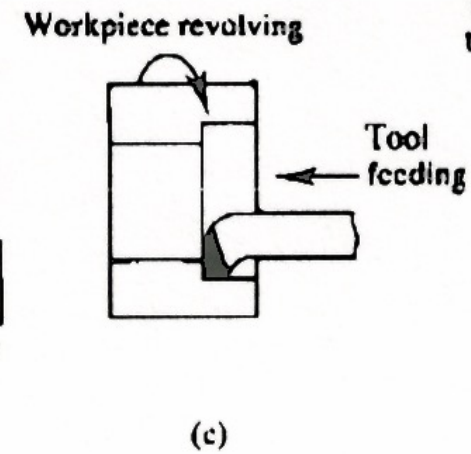
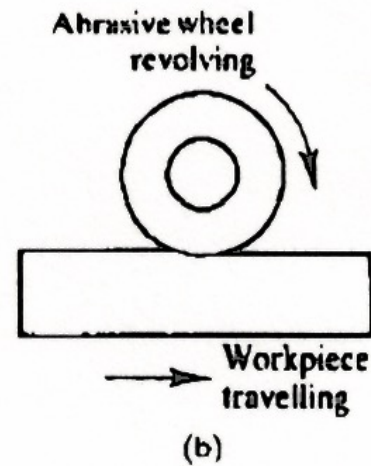
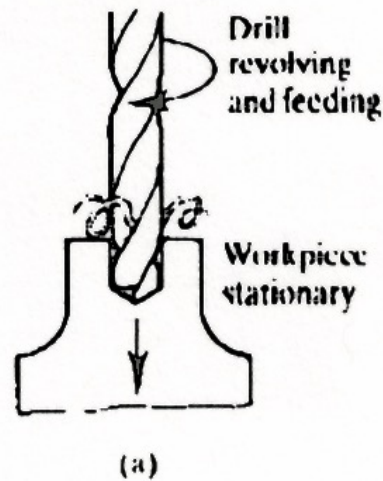
Gear Parts



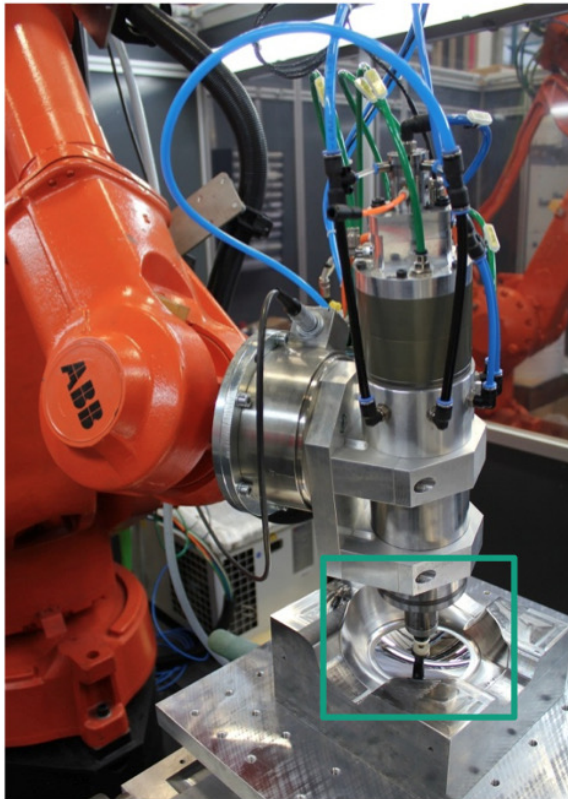
Tasks with environment interaction

- **mechanical machining**
 - deburring, surface finishing, polishing, assembly,...
- **tele-manipulation**
 - force feedback improves performance of human operators in master-slave (leader-follower) systems
- **contact exploration for shape identification**
 - force and velocity/vision sensor fusion allow 2D/3D geometric identification of unknown objects and their contour following
- **dexterous robot hands**
 - power grasp and fine in-hand manipulation require force/motion cooperation and coordinated control of the multiple fingers
- **cooperation of multi-manipulator systems**
 - the environment includes one or more other robots with their own dynamic behaviors
- **physical human-robot interaction**
 - humans as active, dynamic environments that need to be handled under full safety premises ...

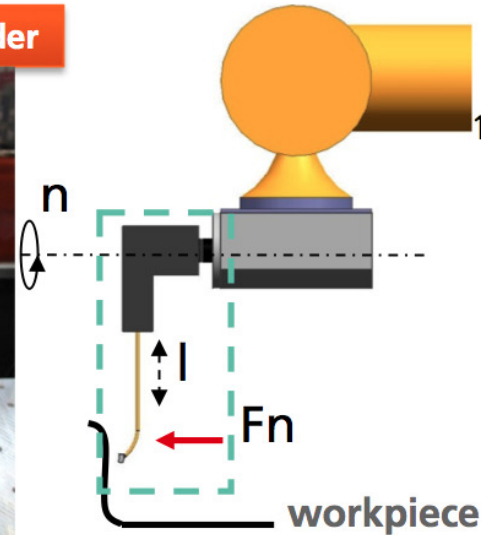
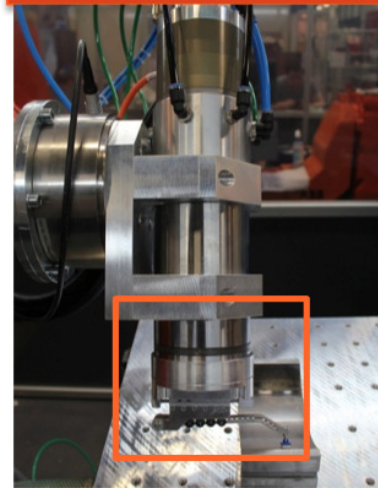
Examples of mechanical machining



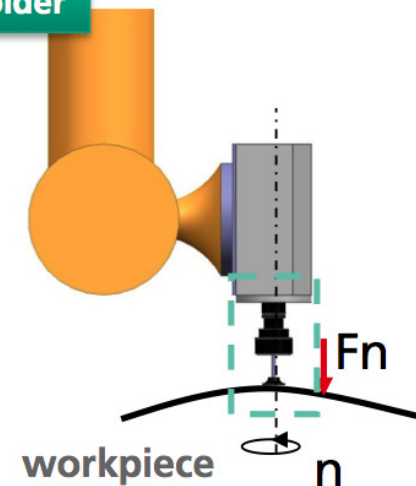
Abrasive finishing of surfaces



Translational tool holder



Rotational tool holder



Main properties:

- synchronous motor
- rotation : 100 - 36.000 rpm
- power : 6 kW
- mass : 16 kg
- automated tool exchanger
- pneumatic channels for force control (x3)

Abrasive finishing of surfaces

video



technological processes: cold forging of surfaces
and hammer peening by pneumatic machine

Non-contact surface finishing

video

Fluid Jet technology

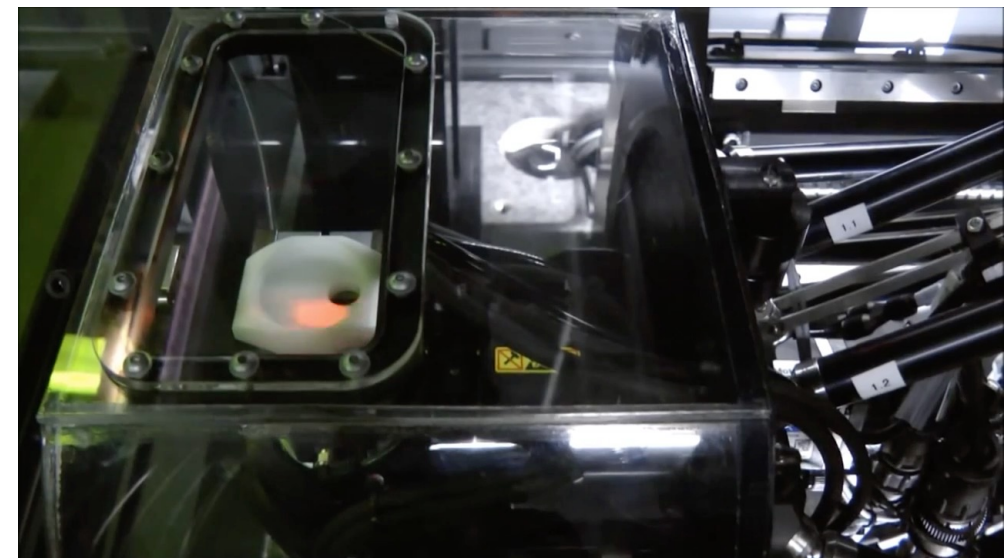


H2020 EU project for the
Factory of the Future (FoF)



Pulsed Laser technology

video





In all cases ...

- for physical interaction tasks, the **desired motion** specification and execution should be integrated with complementary data for the **desired force**
 - ➔ **hybrid force/motion** planning and control objectives
- the exchanged forces/torques at the contact(s) with the environment can be explicitly **set under control** or simply **kept limited** in an indirect way

Evolution of control approaches

a bit of history from the late 70's-mid '80s ...



- **explicit control of forces/torques only** [Whitney]
 - used in quasi-static operations (assembly) in order to avoid deadlocks during part insertion
- **active admittance and compliance control** [Paul, Shimano, Salisbury]
 - contact forces handled through position (**stiffness**) or velocity (**damping**) control of the robot end-effector
 - robot reacts as a compressed **spring** (with **damper**) in selected/all directions
- **impedance control** [Hogan]
 - a desired dynamic behavior is imposed to the robot-environment interaction, e.g., a "model" with forces acting on a **mass-spring-damper**
 - mimics the human arm behavior moving in an unknown environment
- **hybrid force-motion control** [Mason]
 - decomposes the **task space** in complementary sets of directions where **either** force **or** motion is controlled, based on
 - a **purely kinematic** robot model [Raibert, Craig]
 - the actual **dynamic model** of the robot [Khatib]



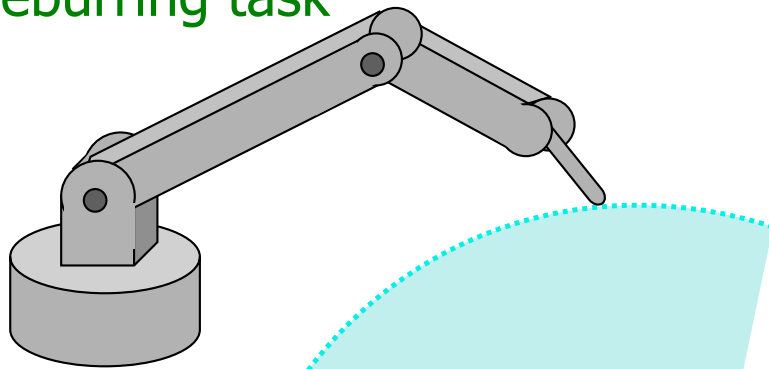
appropriate for fast and accurate motion in dynamic interaction...

Interaction tasks of interest

interaction tasks with the environment that require

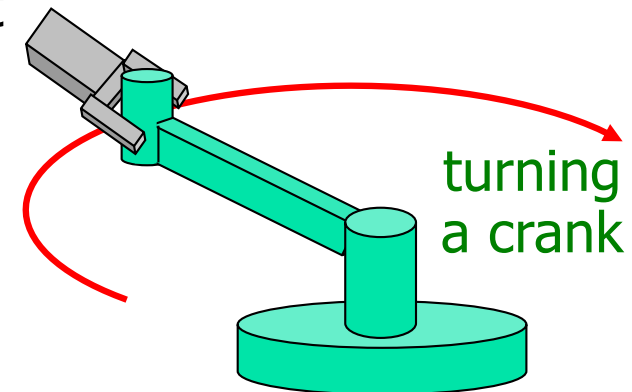
- **accurate following/reproduction** by the robot end-effector of desired trajectories (even at **high speed**) defined on the surface of objects
- **control of forces/torques** applied at the contact with environments having low (**soft**) or high (**rigid**) stiffness

deburring task



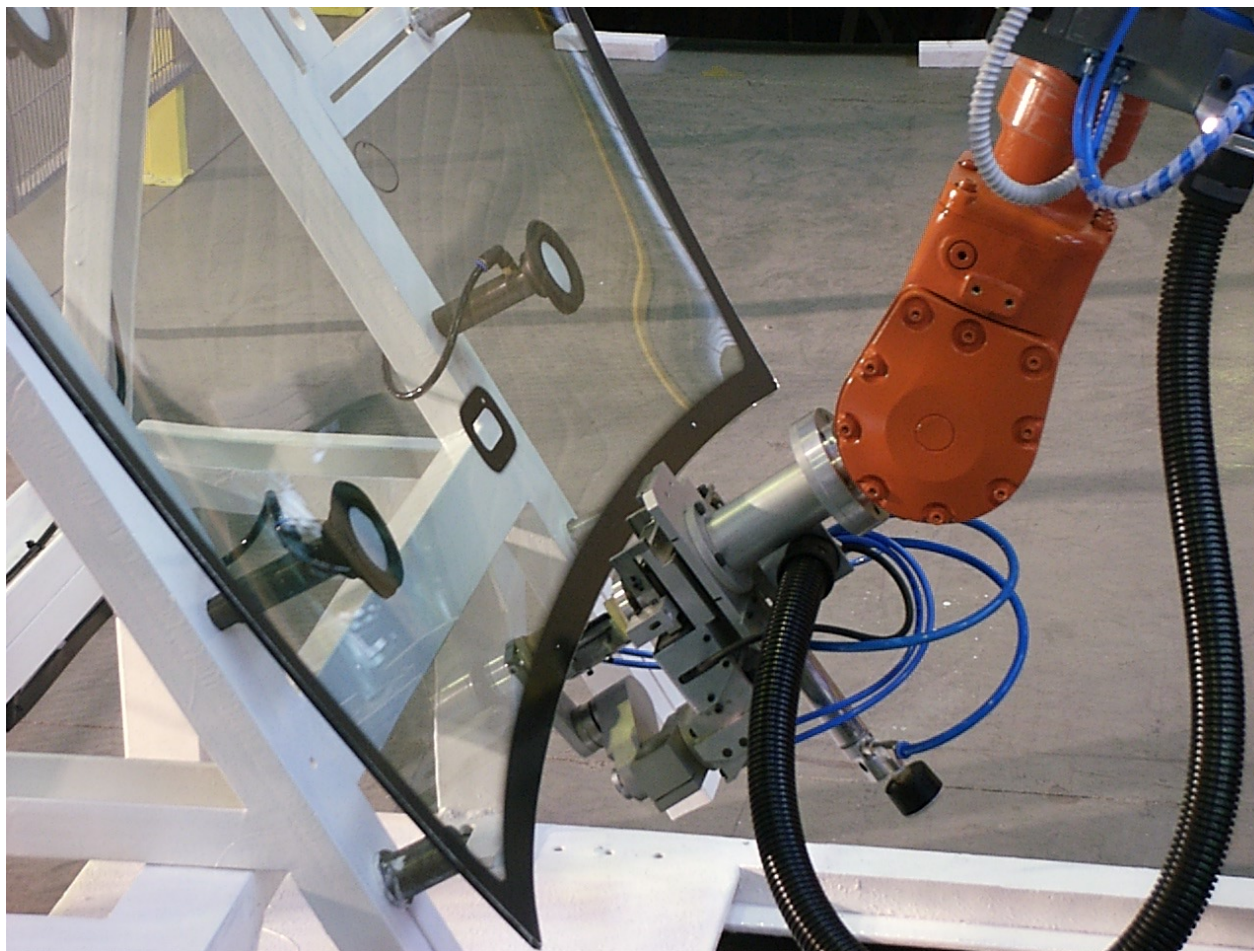
e.g., removing extra glue from the border of a car windshield

robot



e.g., opening a door

Robotized deburring of windshields



c/o ABB Excellence Center in Cecchina (Roma), 2002



Impedance vs. Hybrid control

environment model (↔ domain of control application)

impedance control

- environment = mechanical system undergoing **small but finite deformations**
- contact forces arise as the result of a balance of two **coupled dynamic systems** (robot+environment)
- ➔ desired dynamic characteristics are assigned to the force/motion interaction

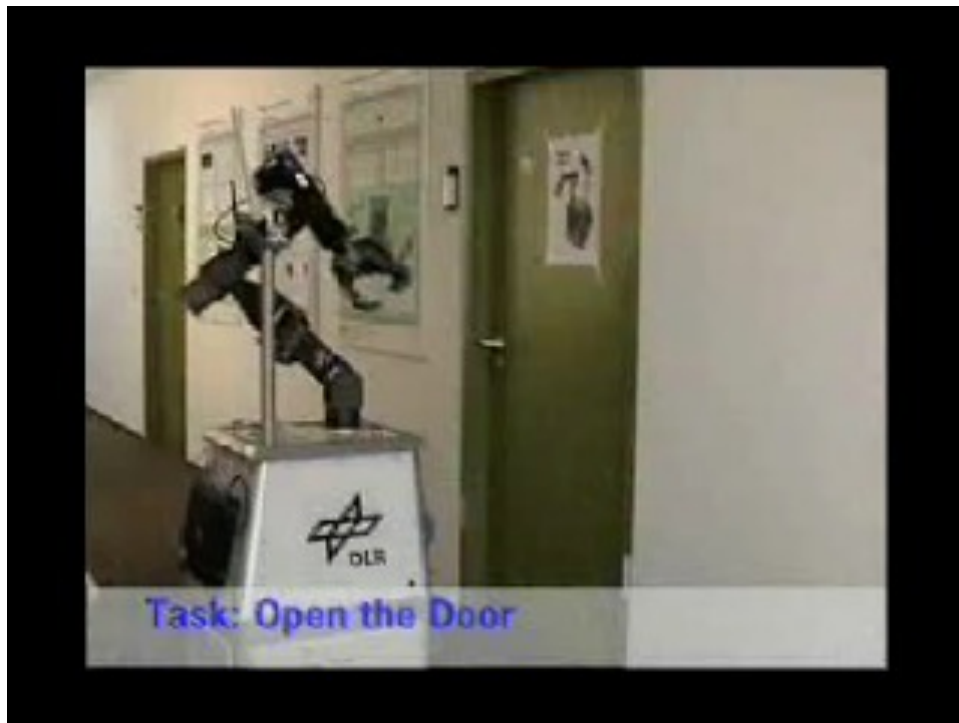
hybrid force/motion control

- a **rigid environment** reduces the degrees of freedom of the robot when in (bi-/uni-lateral) contact
- contact forces result from attempts to violate **geometric constraints** imposed by the environment
- ➔ task space is decomposed in sets of directions where **only motion** or **only reaction forces** are feasible

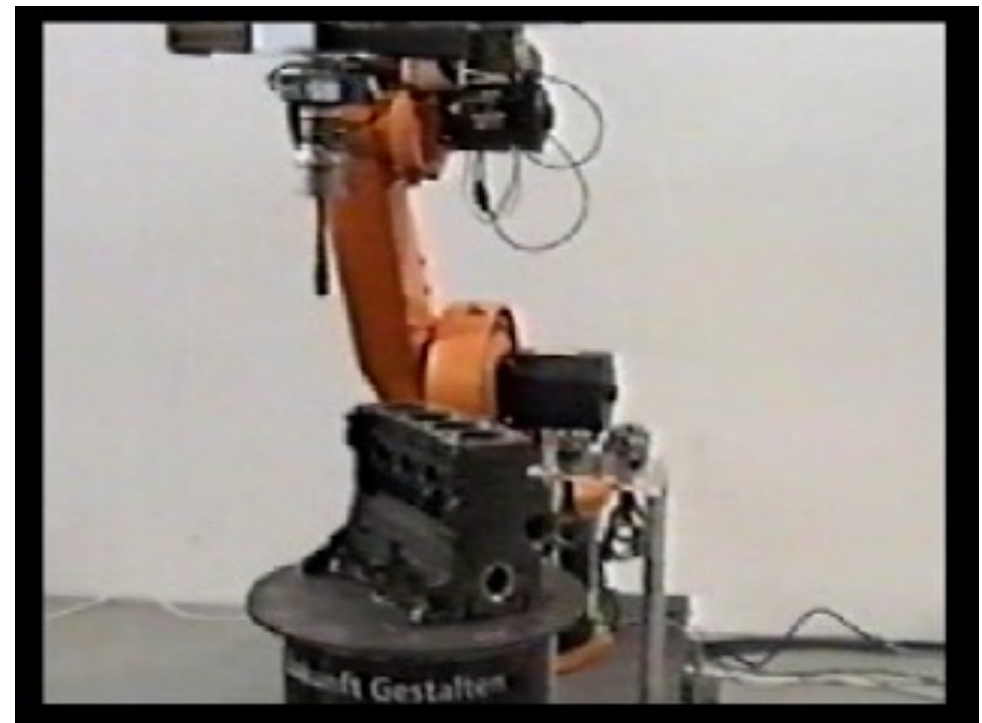
- the required **level of knowledge** about the environment geometry is only **apparently** different between the two control approaches
- however, **measuring contact forces** may not be needed in impedance control, while it always necessary in hybrid force/motion control

Impedance vs. Hybrid control

- opening a door with a mobile manipulator under **impedance control**
- piston insertion in a motor based on **hybrid control** of force-position (visual)

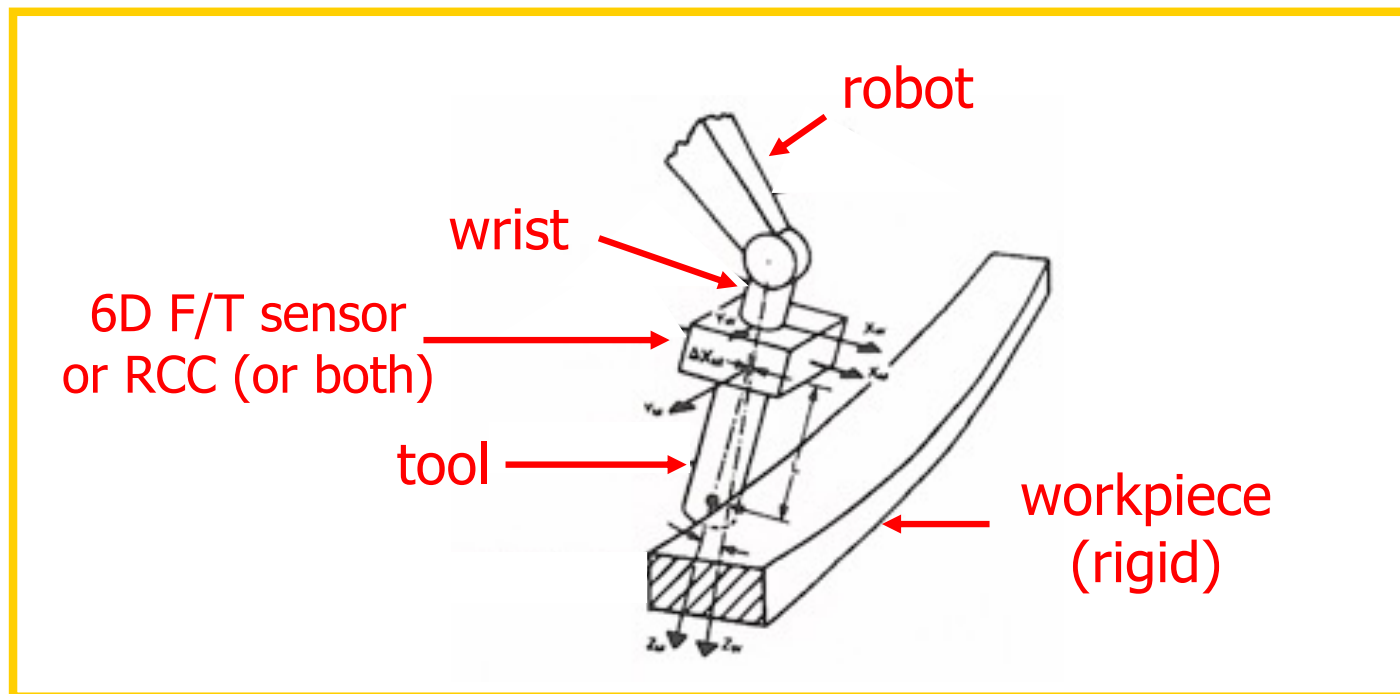


video



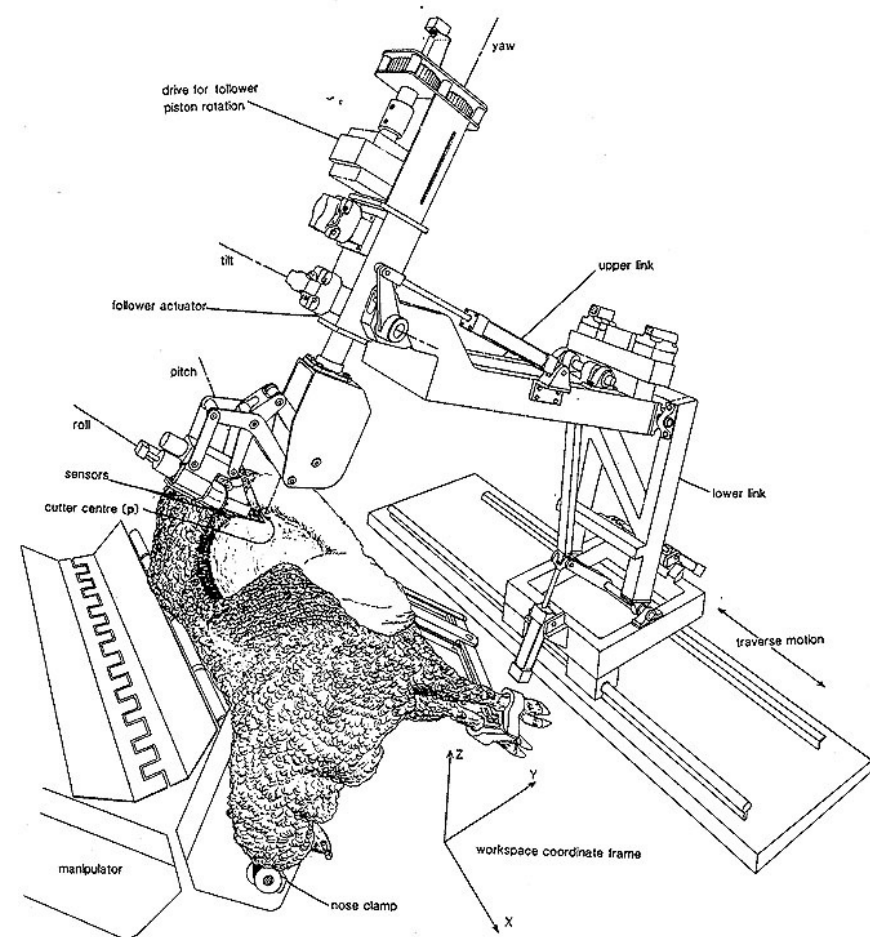
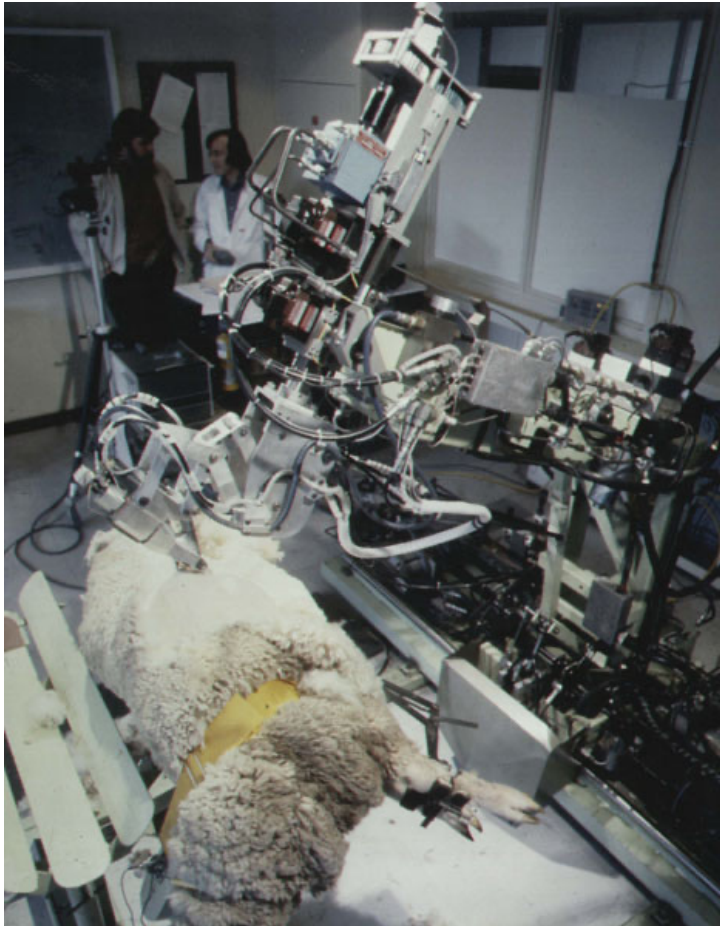
video

A typical constrained situation ...



the robot end-effector follows in a stable and accurate way the geometric profile of a **very stiff** workpiece, while applying a desired contact force

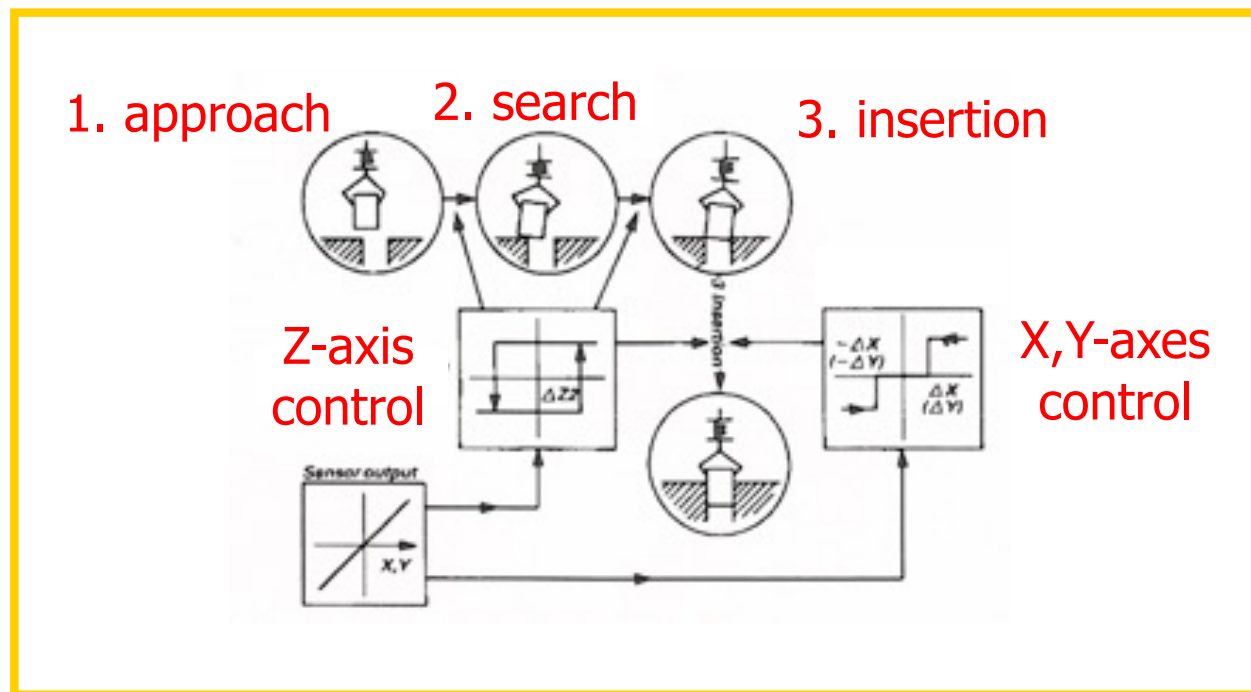
An unusual compliant situation ...



Trevelyan (AUS): **Oracle** robotic system in a test dated 1981

...is the sheep happy?

A mixed interaction situation



processing/reasoning on force measurements
 leads to a sequence of **fine motions**
 ⇒ correct completion of insertion task with
 help of (sufficiently large) passive compliance

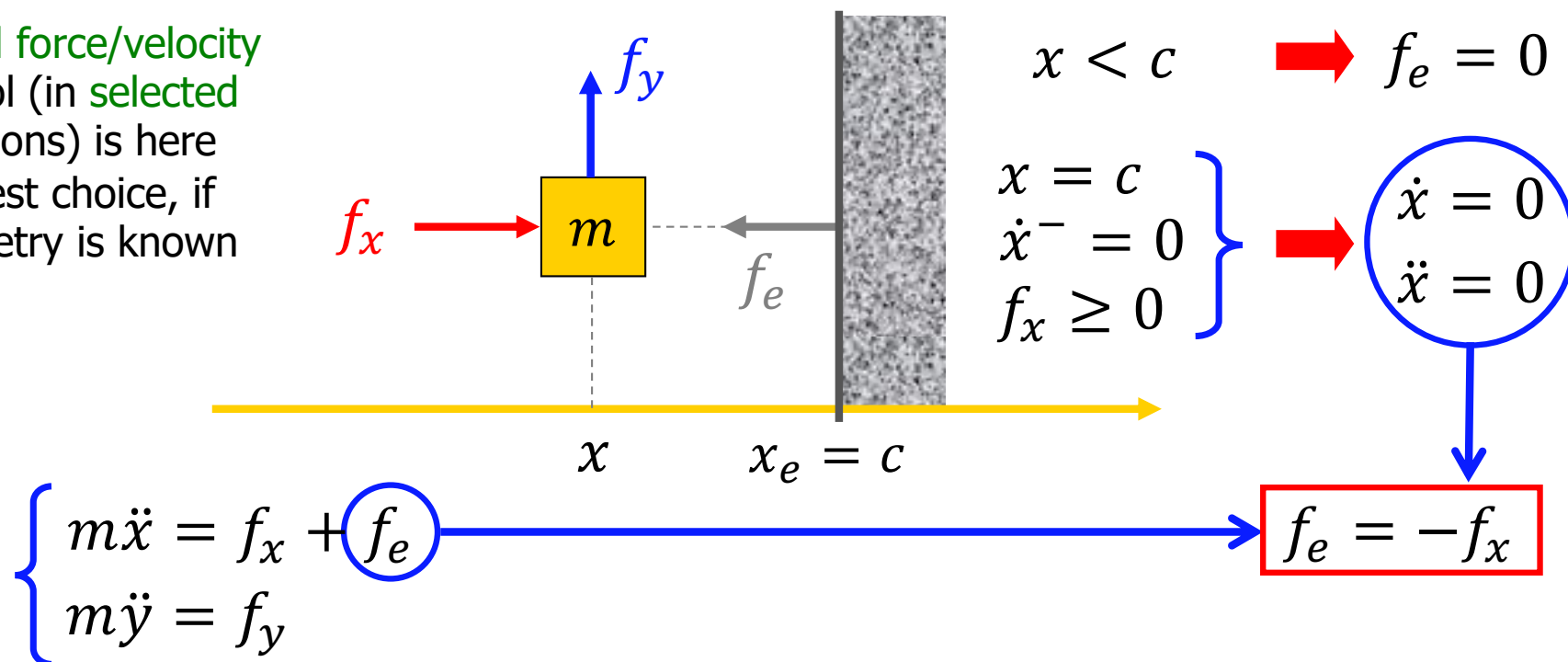


Ideally constrained contact situation

a first possible modeling choice for very stiff environments



hybrid force/velocity control (in selected directions) is here the best choice, if geometry is known



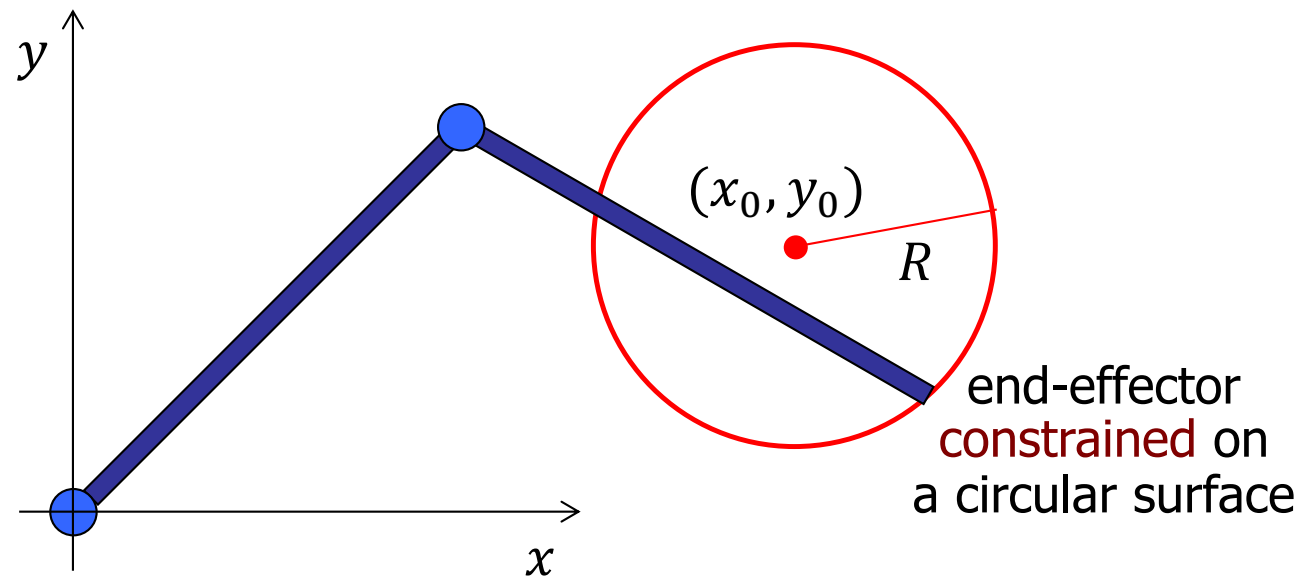
“ideal” = robot (here, a Cartesian mass) + environment are both **infinitely STIFF** (and **no friction** at the contact)

in case of an **impact** ($x = c, \dot{x}^- > 0$) that is purely “elastic” (i.e., with conservation of total momentum and total kinetic energy) $\Rightarrow \dot{x}^+ = -\dot{x}^-$ (f_e is an impulse!)



In more complex situations

- how can we describe **more complex contact situations**, where the **end-effector** of an articulated robot (not yet reduced to a Cartesian mass via feedback linearization control) is **constrained** to move **on an environment surface** with nonlinear geometry?
- example: a planar 2R robot with end-effector moving on a circle





Constrained robot dynamics - 1

- consider a robot in free space described by its Lagrange **dynamic model** and a **task output function** (e.g., the end-effector pose)

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = u$$

$$r = f(q)$$

$$q \in \mathbb{R}^n$$

- suppose that the task variables are subject to $m < n$ (bilateral) **geometric constraints** in the general form $k(r) = 0$ and define

$$h(q) = k(f(q)) = 0$$

- the **constrained robot dynamics** can be derived using again the Lagrange formalism, by defining an **augmented Lagrangian** as

$$L_a(q, \dot{q}, \lambda) = L(q, \dot{q}) + \lambda^T h(q) = T(q, \dot{q}) - U(q) + \lambda^T h(q)$$

where the **Lagrange multipliers** λ (a m -dimensional vector) can be interpreted as the **generalized forces** that arise at the contact when attempting to violate the constraints



Constrained robot dynamics - 2

- applying the **Euler-Lagrange equations** in the extended space of generalized coordinates **q** and multipliers **λ** yields

$$\frac{d}{dt} \left(\frac{\partial L_a}{\partial \dot{q}} \right)^T - \left(\frac{\partial L_a}{\partial q} \right)^T = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right)^T - \left(\frac{\partial L}{\partial q} \right)^T - \left(\frac{\partial}{\partial q} (\lambda^T h(q)) \right)^T = u$$

$$\left(\frac{\partial L_a}{\partial \lambda} \right)^T = h(q) = 0 \quad \leftarrow \text{contact forces do NOT produce work}$$

$$\Rightarrow \left\{ \begin{array}{l} M(q)\ddot{q} + c(q, \dot{q}) + g(q) = u + A^T(q)\lambda \quad (\star) \\ \text{subject to } h(q) = 0 \end{array} \right.$$

where we defined the **Jacobian of the constraints** as the matrix

$$A(q) = \frac{\partial h(q)}{\partial q}$$

that will be assumed of **full row rank** ($= m$)



Constrained robot dynamics - 3

- we can **eliminate the appearance of the multipliers** as follows

- differentiate the constraints twice w.r.t. time

$$h(q) = 0 \Rightarrow \dot{h} = \frac{\partial h(q)}{\partial q} \dot{q} = A(q) \dot{q} = 0 \Rightarrow \ddot{h} = A(q) \ddot{q} + \dot{A}(q) \dot{q} = 0$$

- substitute the joint accelerations from the dynamic model (★)
(dropping dependencies)

$$AM^{-1}(u + A^T \lambda - c - g) + \dot{A} \dot{q} = 0$$

- solve for the multipliers

invertible $m \times m$ matrix,
when A is full rank

$$\begin{aligned} \lambda &= (AM^{-1}A^T)^{-1}(AM^{-1}(c + g - u) - \dot{A}\dot{q}) \\ &= (A_M^\#)^T (c + g - u) - (AM^{-1}A^T)^{-1}\dot{A}\dot{q} \end{aligned}$$

the inertia-weighted
pseudoinverse of the
constraint Jacobian A

to be replaced in the dynamic model...

constraint
forces λ are
uniquely
determined
by the robot
state (q, \dot{q})
and input u !!



Constrained robot dynamics - 4

- the final **constrained dynamic model** can be rewritten as

$$M(q)\ddot{q} = \underbrace{\left[I - A^T(q)(A_M^\#(q))^T \right]}_{\text{dynamically consistent projection matrix}} (u - c(q, \dot{q}) - g(q)) - M(q)A_M^\#(q)\dot{A}(q)\dot{q}$$

dynamically consistent projection matrix

where $A_M^\#(q) = M^{-1}(q)A^T(q)(A(q)M^{-1}(q)A^T(q))^{-1}$ and with

$$\lambda = (A_M^\#(q))^T (c(q, \dot{q}) + g(q) - u) - (A(q)M^{-1}(q)A^T(q))^{-1} \dot{A}(q)\dot{q}$$

- if the robot state $(q(0), \dot{q}(0))$ **at time $t = 0$** satisfies the constraints, i.e.,

$$h(q(0)) = 0, \quad A(q(0))\dot{q}(0) = 0$$

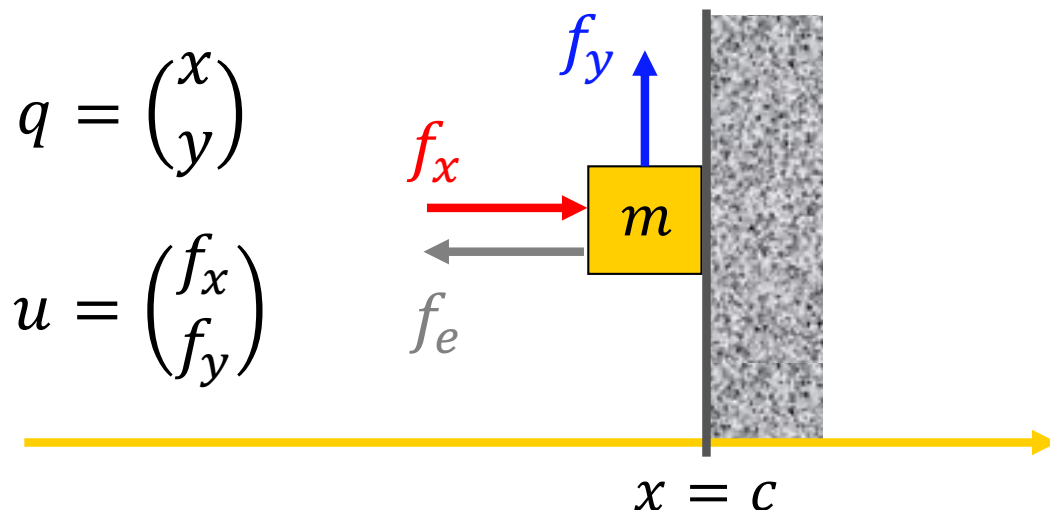
then the robot evolution described by the above dynamics will be consistent with the constraints **for all $t \geq 0$** and **for any $u(t)$**

- this is a useful **simulation model** (constrained **direct** dynamics)



Example – ideal mass

constrained robot dynamics



$$q = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$u = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$M\ddot{q} = u \quad \text{robot dynamics in free motion}$$

$$M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$h(q) = x - c = 0 \Rightarrow A(q) = \begin{pmatrix} 1 & 0 \end{pmatrix} \Rightarrow A_M^\#(q) = \dots = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\left(I - A^T(q)(A_M^\#(q))^T \right) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{dynamically consistent projection matrix}$$

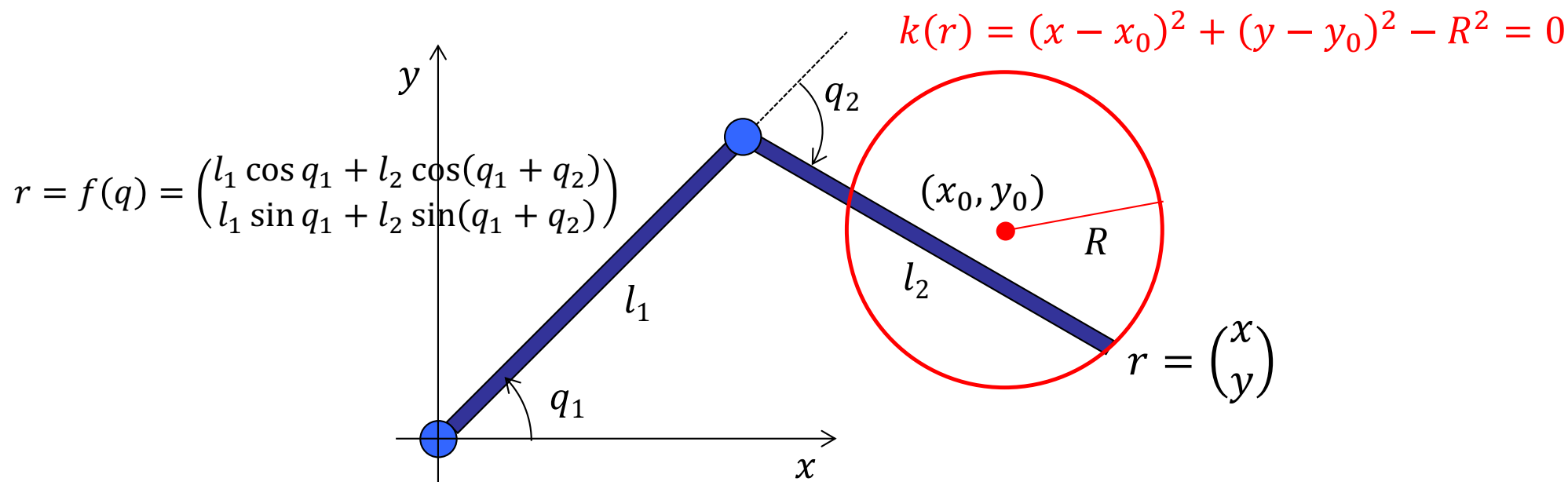
$$\lambda = -(A_M^\#(q))^T u = -\begin{pmatrix} 1 & 0 \end{pmatrix} u = -f_x \quad \text{multiplier (contact force } f_e)$$

$$M \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = M\ddot{q} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} u = \begin{pmatrix} 0 \\ f_y \end{pmatrix} \quad \text{constrained robot dynamics}$$



Example – planar 2R robot

constrained robot dynamics



$$h(q) = k(f(q)) = (l_1 \cos q_1 + l_2 \cos(q_1 + q_2) - x_0)^2 + (l_1 \sin q_1 + l_2 \sin(q_1 + q_2) - y_0)^2 - R^2 = 0$$

$$\begin{aligned} \dot{h} &= \frac{\partial k}{\partial r} \frac{\partial r}{\partial q} \dot{q} = [2(x - x_0) \quad 2(y - y_0)] J_r(q) \dot{q} \\ &= \underbrace{[2(l_1 c_1 + l_2 c_{12} - x_0) \quad 2(l_1 s_1 + l_2 s_{12} - y_0)]}_{A(q)} J_r(q) \dot{q} = A(q) \dot{q} \end{aligned}$$



Reduced robot dynamics - 1

- by imposing m constraints $h(q) = 0$ on the n generalized coordinates q , it is also possible to **reduce** the description of the constrained robot dynamics to a **$n - m$ dimensional** configuration space
- start from constraint matrix $A(q)$ and **select** a matrix $D(q)$ such that

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}_{n \times n \text{ matrix}} \text{ is a } \textbf{nonsingular} \rightarrow \begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = (E(q) \quad F(q))$$

- define the $(n - m)$ -dimensional vector of **pseudo-velocities** v as the linear combination (at a given q) of the robot generalized velocities

$$v = D(q)\dot{q} \rightarrow \dot{v} = D(q)\ddot{q} + \dot{D}(q)\dot{q}$$

- inverse relationships (from “pseudo” to “generalized” velocities and accelerations) are given by

$$\dot{q} = F(q)v \quad \ddot{q} = F(q)\dot{v} - (E(q)\dot{A}(q) + F(q)\dot{D}(q))F(q)v$$

↔ properties of **block products** in inverse matrices have been used for eliminating the appearance of \dot{F} (often F is only known **numerically**)

Reduced robot dynamics – 2

whiteboard ...



$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = \begin{pmatrix} E(q) & F(q) \end{pmatrix} \quad \text{a number of properties from this definition...}$$

two matrix inverse products

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} \begin{pmatrix} E(q) & F(q) \end{pmatrix} = \begin{pmatrix} A(q)E(q) & A(q)F(q) \\ D(q)E(q) & D(q)F(q) \end{pmatrix} = \begin{pmatrix} I_{m \times m} & 0 \\ 0 & I_{(n-m) \times (n-m)} \end{pmatrix}$$

three useful identities!

$$\begin{pmatrix} E(q) & F(q) \end{pmatrix} \begin{pmatrix} A(q) \\ D(q) \end{pmatrix} = E(q)A(q) + F(q)D(q) = I_{n \times n}$$

➔ differentiating w.r.t. time $\dot{E}A + E\dot{A} + \dot{F}D + F\dot{D} = 0 \quad \triangleleft$

from pseudo-velocity $v = D(q)\dot{q}$
 since F is a right inverse of the
 full row rank matrix D ($DF = I$)

➔ $\dot{q} = F(q)v$ (in fact,
 $D\dot{q} = DFv = v$)

➔ differentiating w.r.t. time $\dot{q} = F(q)v$

$$\begin{aligned} \ddot{q} &= F\dot{v} + \dot{F}v = F\dot{v} + (\dot{F}D)\dot{q} \quad (\triangleleft) \\ &= F(q)\dot{v} - (\cancel{\dot{E}A} + E\dot{A} + F\dot{D})Fv \\ &= F(q)\dot{v} - (E(q)\dot{A}(q) + F(q)\dot{D}(q))F(q)v \end{aligned}$$



Reduced robot dynamics - 3

- consider again the dynamic model (★), dropping dependencies

$$M\ddot{q} + c + g = u + A^T \lambda$$

- since $AE = I$, multiplying on the left by E^T isolates the multipliers

$$E^T(M\ddot{q} + c + g - u) = \lambda$$

- since $AF = 0$, multiplying on the left by F^T eliminates the multipliers

$$F^T M \ddot{q} = F^T (u - c - g)$$

- substituting in the latter the generalized accelerations and velocities with the pseudo-accelerations and pseudo-velocities leads finally to

$(n-m) \times (n-m)$ invertible positive definite matrix \rightarrow

$$(F^T M F) \dot{v} = F^T (u - c - g + M(E\dot{A} + F\dot{D})Fv)$$

which is the reduced $(n-m)$ -dimensional dynamic model

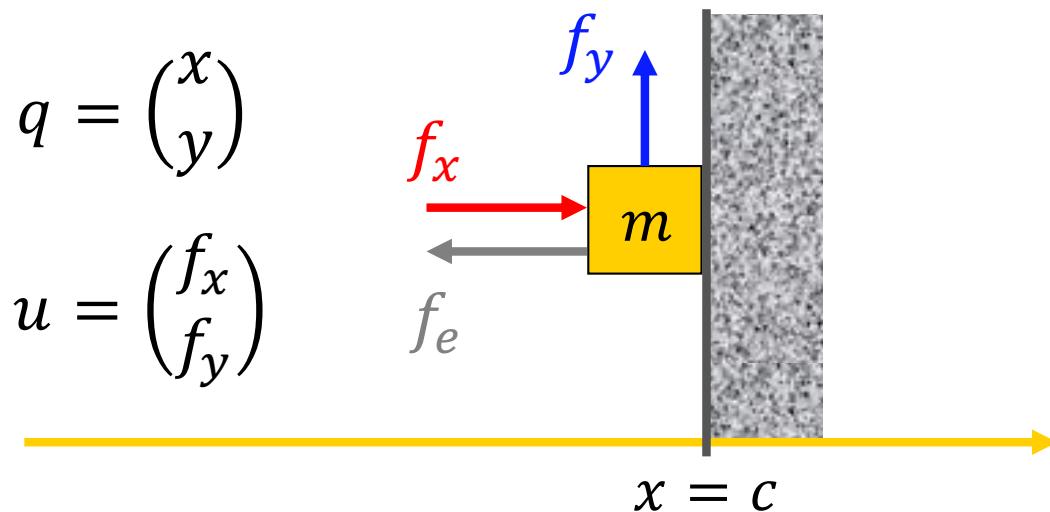
- similarly, the expression of the multipliers becomes

$$\lambda = E^T (MF\dot{v} - M(E\dot{A} + F\dot{D})Fv + c + g - u) \quad (\S)$$



Example – ideal mass

reduced robot dynamics



$$q = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$u = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$M\ddot{q} = u \quad \text{robot dynamics in free motion}$$

$$M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$h(q) = x - c = 0 \Rightarrow A = \begin{pmatrix} 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} A \\ D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} E & F \end{pmatrix}$$

$$\Rightarrow v = D\dot{q} = \dot{y} \quad \text{pseudo-velocity}$$

$$\lambda = E^T(MF\dot{v} - u)$$

multiplier
(contact
force f_e)

$$= \begin{pmatrix} 1 & 0 \end{pmatrix} \left(\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \dot{v} - \begin{pmatrix} f_x \\ f_y \end{pmatrix} \right) = -\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix} = -f_x$$

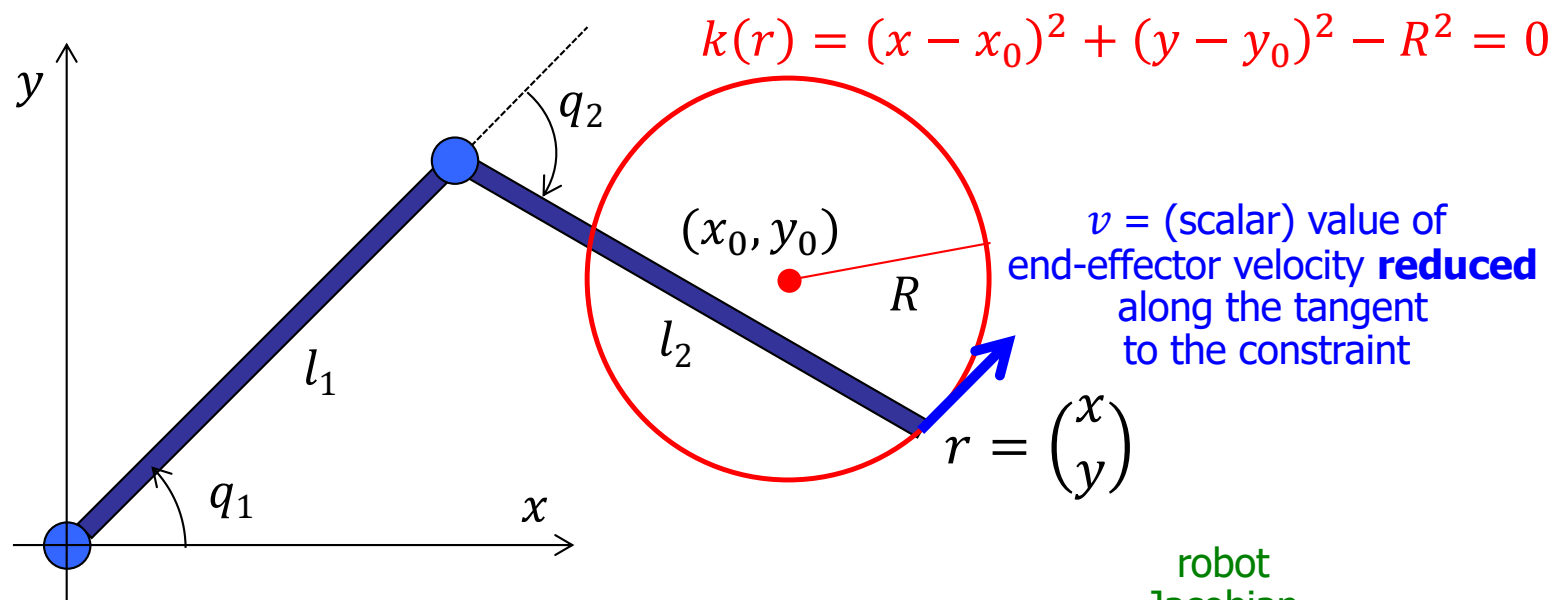
$$(F^T M F)\dot{v} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \dot{v} = m\dot{v} = f_y = F^T u$$

reduced
robot dynamics



Example – planar 2R robot

reduced robot dynamics



$$A(q) = \begin{bmatrix} 2(x - x_0) & 2(y - y_0) \end{bmatrix} J_r(q)$$

$$= \begin{bmatrix} 2(l_1 c_1 + l_2 c_{12} - x_0) & 2(l_1 s_1 + l_2 s_{12} - y_0) \end{bmatrix} J_r(q)$$

robot Jacobian
↓

a feasible **selection** of matrix $D(q)$

$$D(q) = \begin{bmatrix} -\frac{1}{2}(y - y_0) & \frac{1}{2}(x - x_0) \end{bmatrix} J_r(q) \quad \Rightarrow \quad \det \begin{pmatrix} A(q) \\ D(q) \end{pmatrix} = R^2 \cdot \det J_r(q) \neq 0$$

out of robot singularities
↓

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = (E(q) \quad F(q)) \quad \Rightarrow \quad \boxed{v} = D(q) \dot{q} \quad \Rightarrow \quad \dot{q} = F(q) v = J_r^{-1}(q) \begin{pmatrix} -2(y - y_0)/R^2 \\ 2(x - x_0)/R^2 \end{pmatrix} v$$

a scalar

Control based on reduced robot dynamics



- the reduced $n - m$ dynamic expressions are more compact but also more complex and less used for simulation purposes than the n -dimensional constrained dynamics
- however, they are useful for **control design** (reduced **inverse** dynamics)
- in fact, it is straightforward to verify that the **feedback linearizing** control law

$$u = (c + g - M(E\dot{A} + F\dot{D})Fv) + MFu_1 - A^T u_2$$

applied to the **reduced robot dynamics** and to the **expression (8) of the multipliers** leads to the closed-loop system

$$\dot{v} = u_1 \quad \lambda = u_2$$

Note: these are **exactly** in the form of the ideal mass example of **slide #25**, with $v = \dot{y}$, $u_1 = f_y/m$, $\lambda = f_e$, $u_2 = -f_x$ (being $n = 2$, $m = 1$, $n - m = 1$)

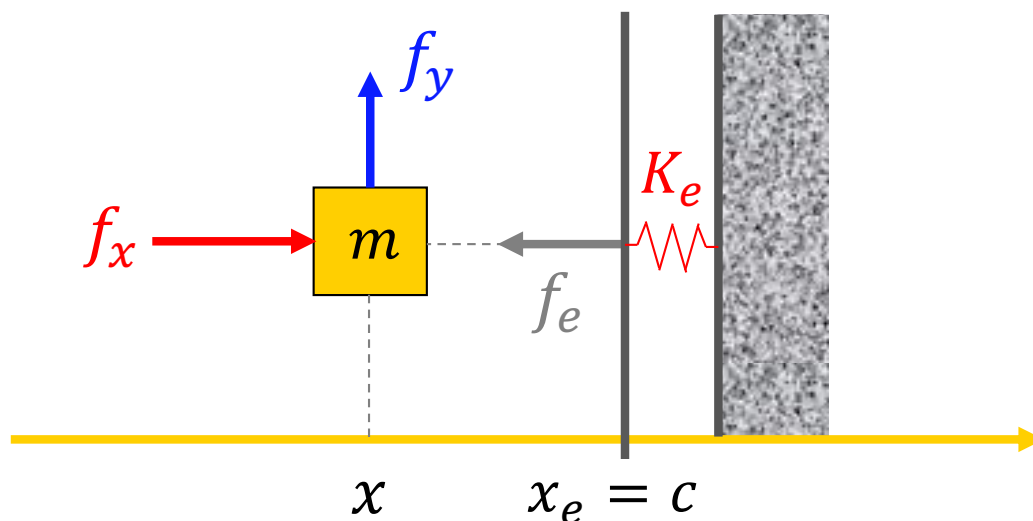
↑
here m is a mass! 😎

↑
here m is a dimension! 😎



Compliant contact situation

a second possible modeling choice for softer environments



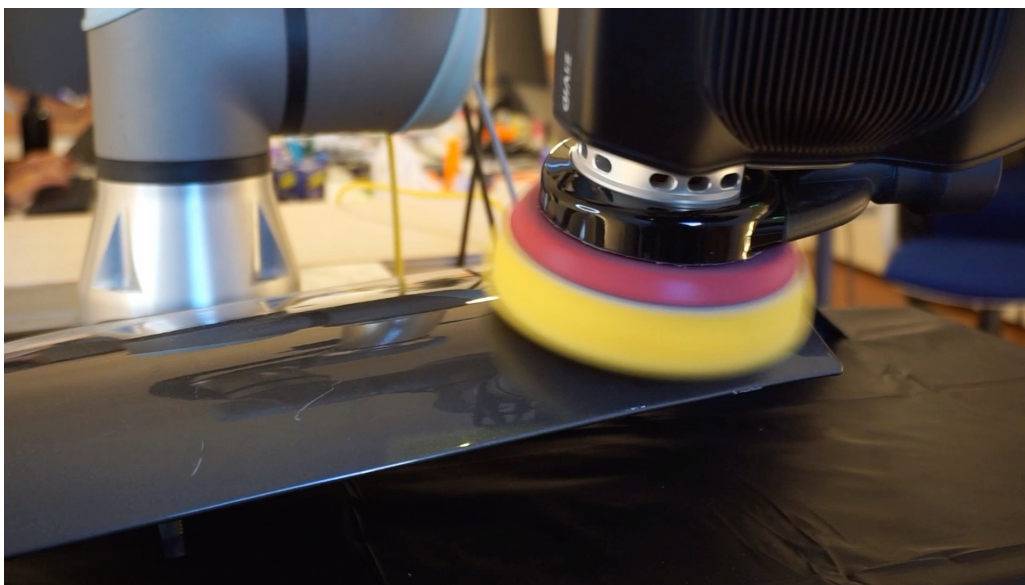
compliance/impedance control (in all directions) is here a good choice that allows to handle

- uncertain position
- uncertain orientation of the wall

$$\begin{cases} m\ddot{x} = f_x + f_e \\ m\ddot{y} = f_y \end{cases} \quad \begin{cases} x < c & \Rightarrow f_e = 0 \\ x \geq c & \Rightarrow f_e = K_e(x - x_e) \end{cases}$$

with $K_e > 0$ being the **stiffness** of the environment

Surface following with force control



video

UR10 6R robot with stereo camera
and F/T sensor at the end-effector
soft tool on hard surface



Flexiv RIZON 7R robot
with joint torque sensors
hard tool on soft surface

video

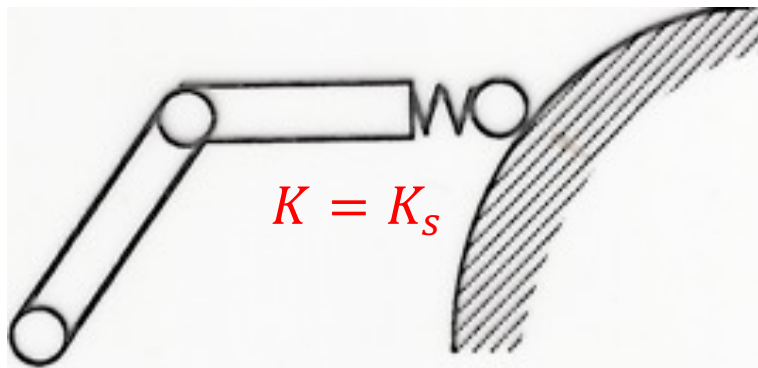


Robot-environment contact types

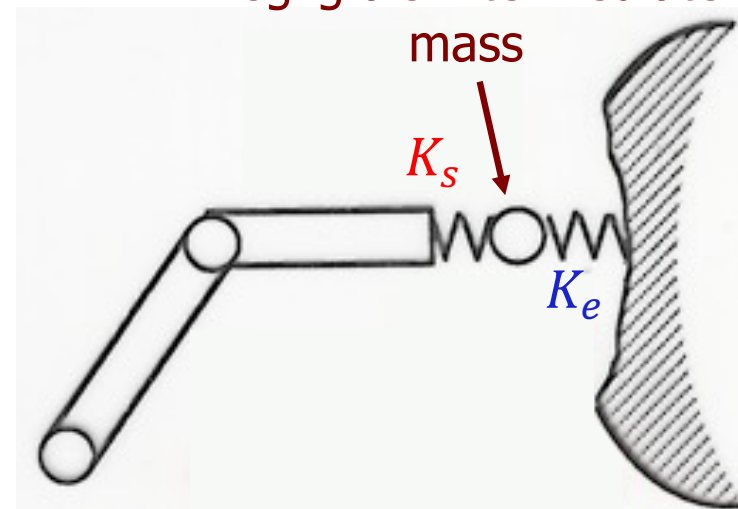
modeled by a single elastic constant

compliant
force sensor

rigid environment

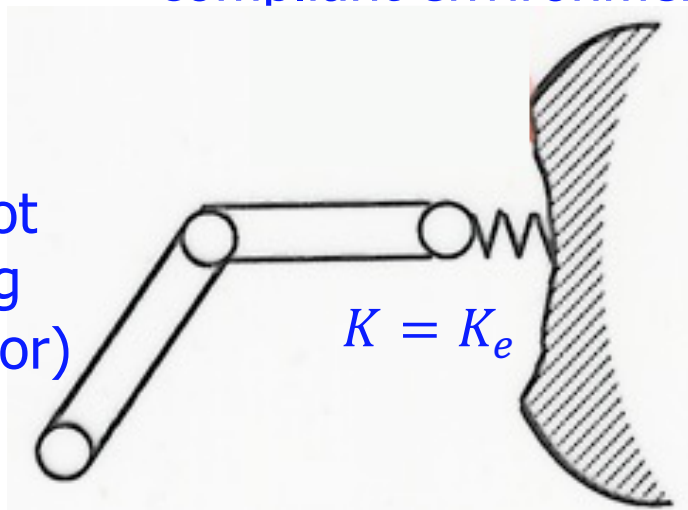


negligible intermediate
mass



compliant environment

rigid robot
(including
force sensor)



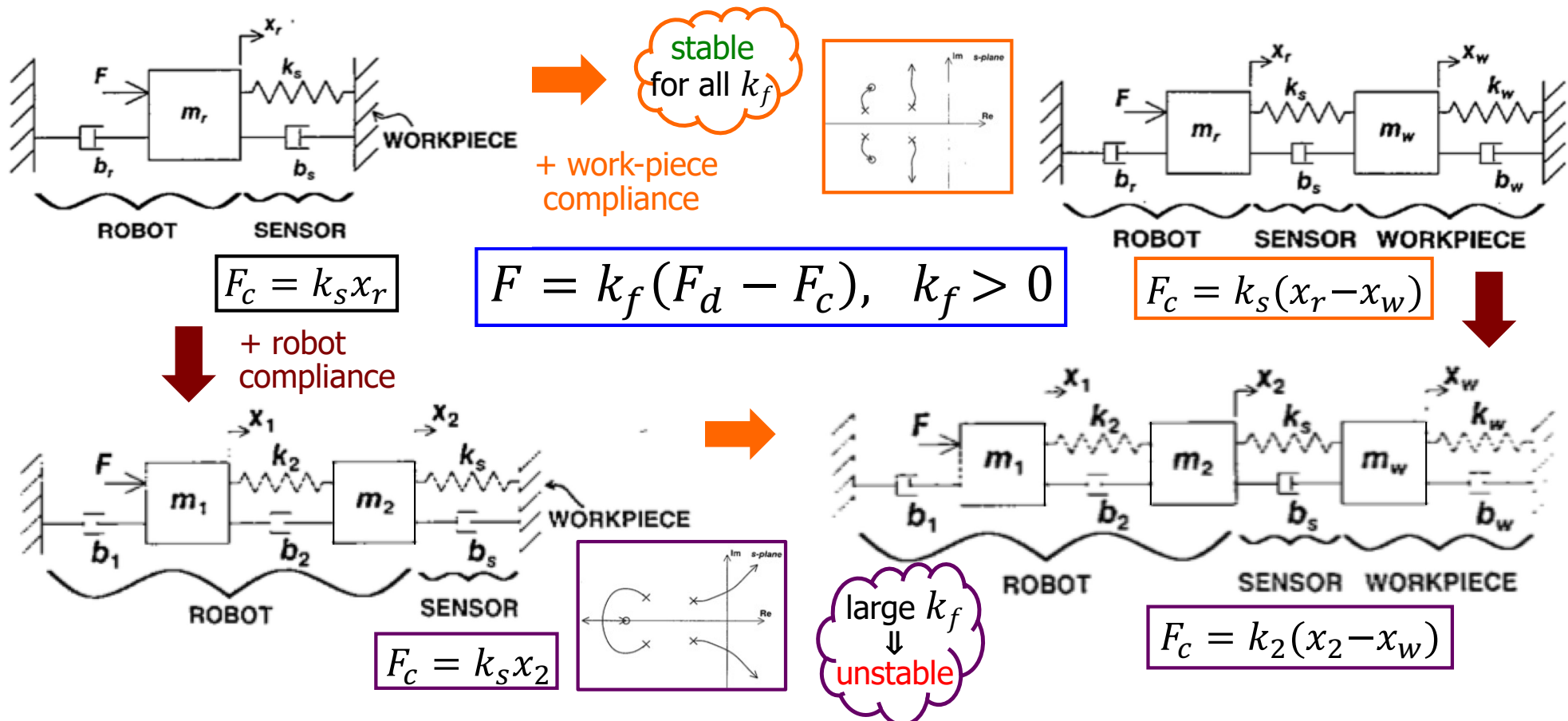
$$\frac{1}{K} = \frac{1}{K_s} + \frac{1}{K_e} \Rightarrow K = \frac{K_s K_e}{K_s + K_e}$$

series of springs =
sum of compliances
(inverse of stiffnesses)

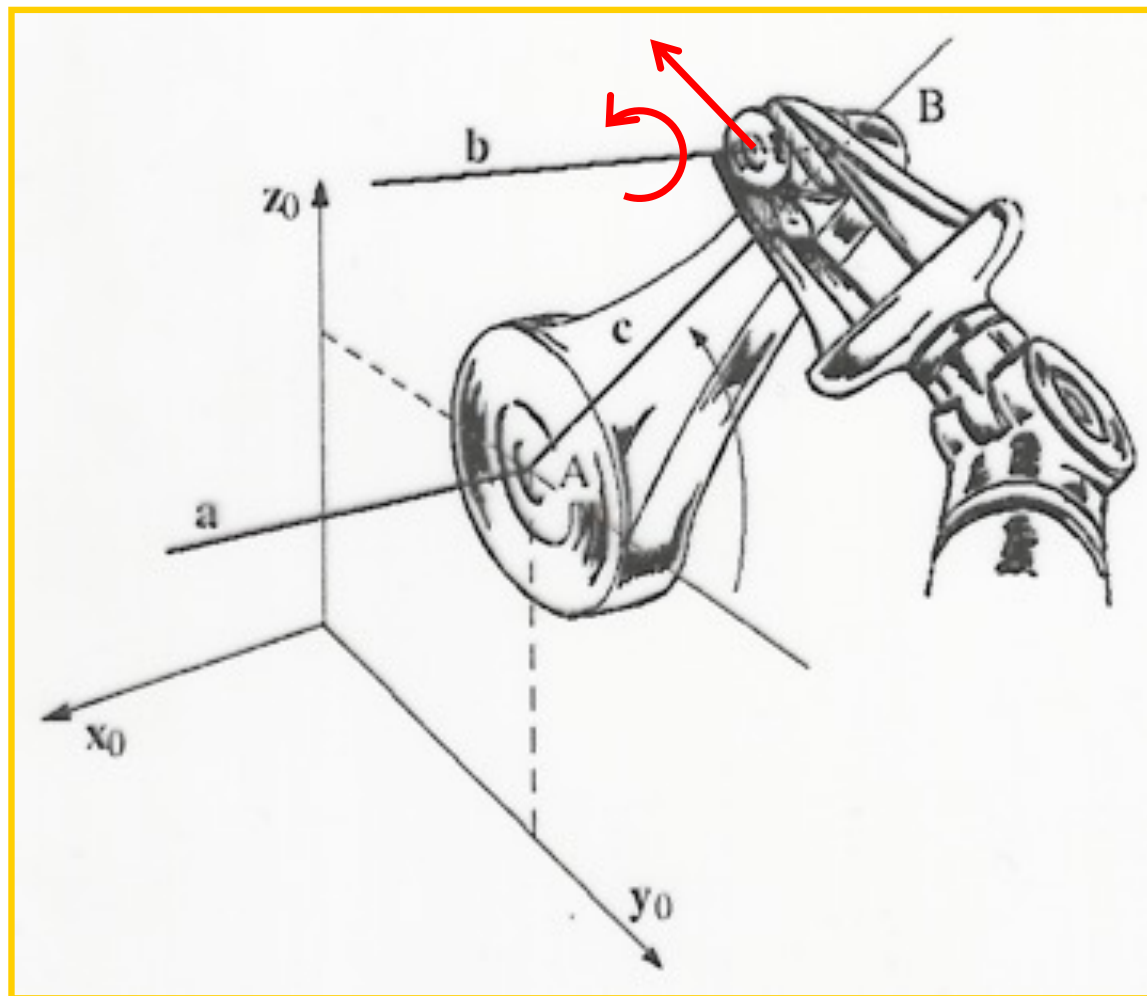
Force control

1-dof robot-environment linear dynamic models

- with a **force sensor** (having stiffness k_s and damping b_s) measuring the contact force F_c
- stability** analysis of a **proportional** control loop for regulation of the contact force (to a desired constant value F_d) can be made using the **root-locus method** (for a varying k_f)
- by including/excluding **work-piece compliance** and/or **robot (transmission) compliance**



Tasks requiring hybrid control



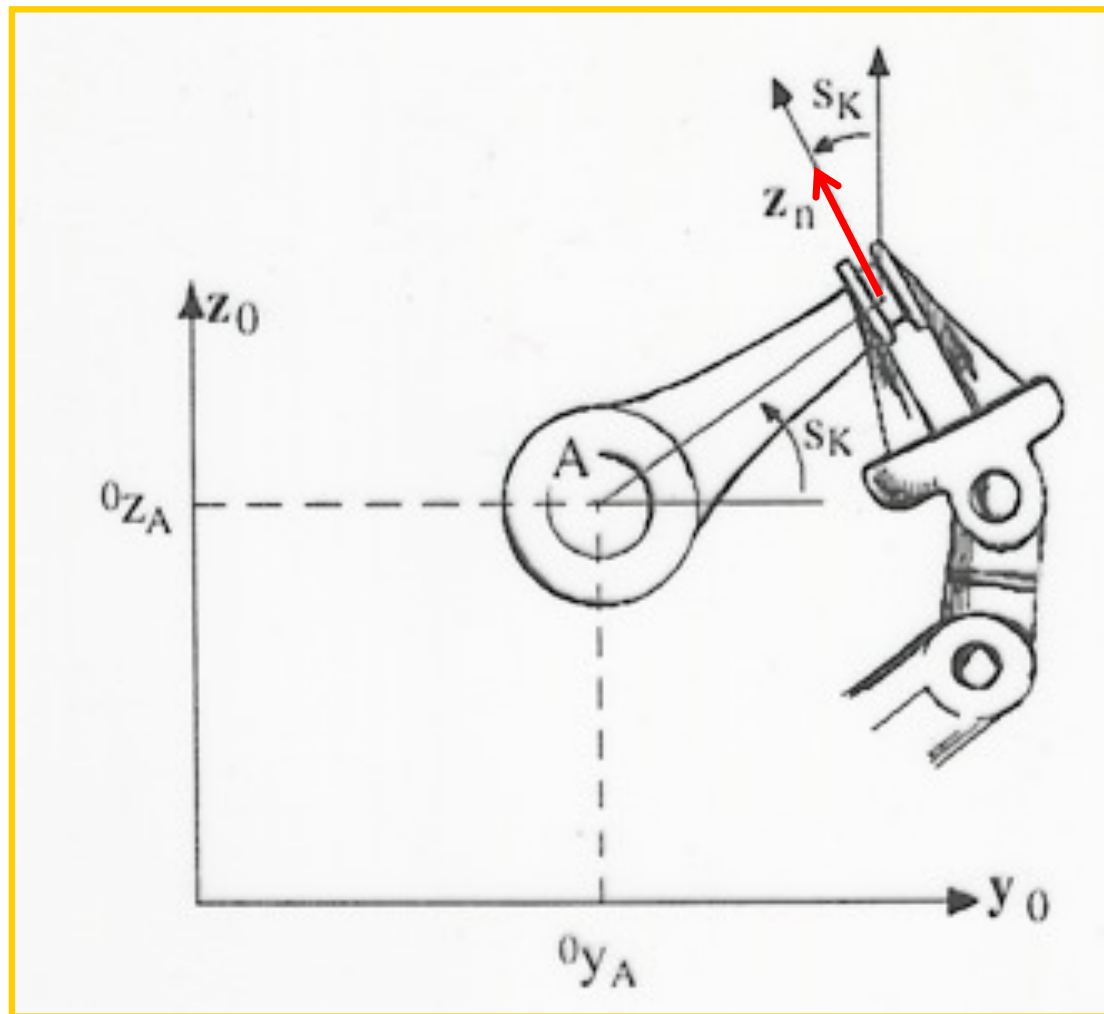
two generalized **directions** of instantaneous free motion at the contact: *tangential velocity & angular velocity around handle axis*



four directions of generalized reaction forces at the contact

the robot should turn a crank having a **free-spinning** handle

Tasks requiring hybrid control



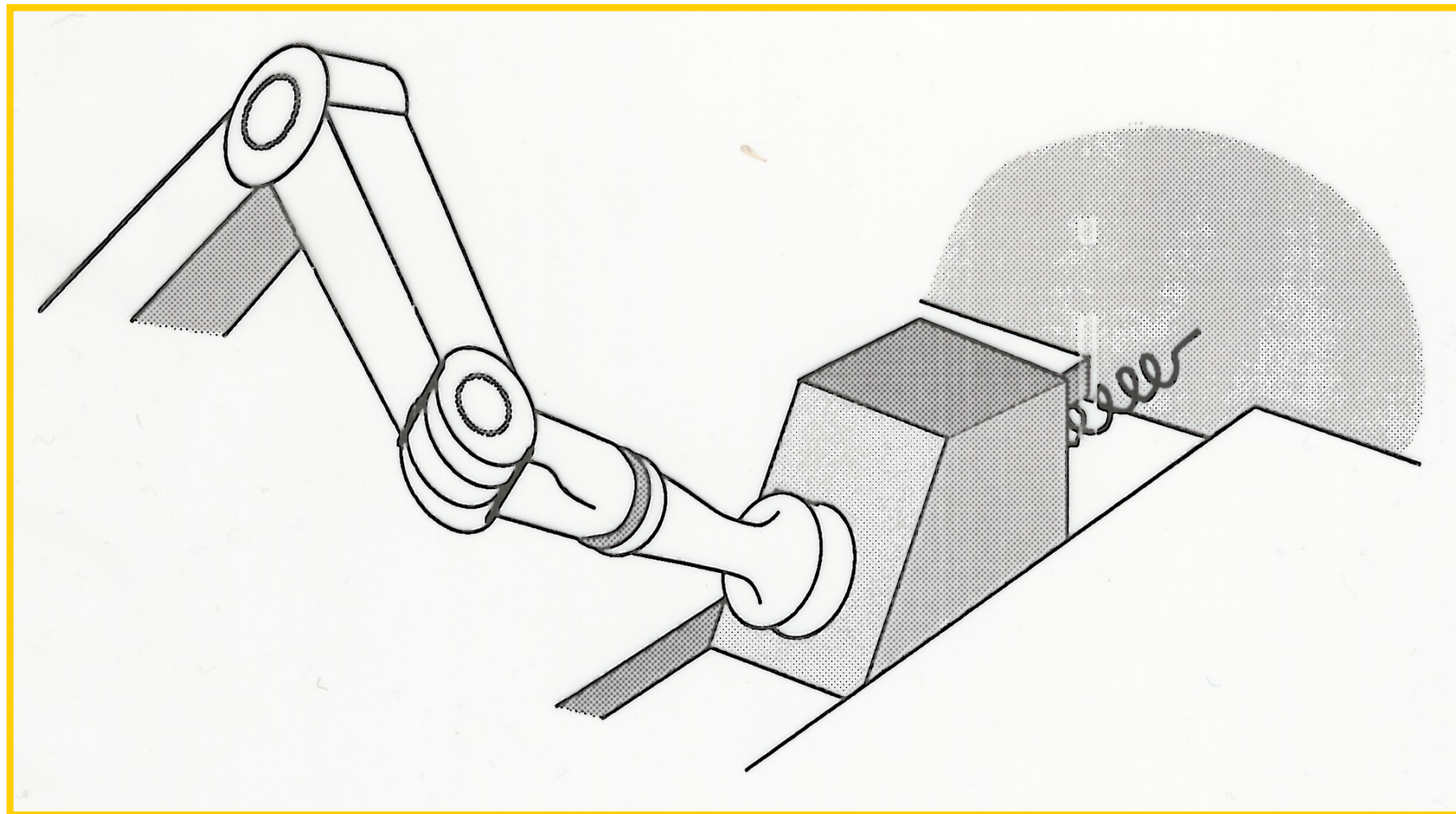
one direction only
of instantaneous
free motion
at the contact:
tangential velocity



five directions
of generalized
reaction forces
at the contact

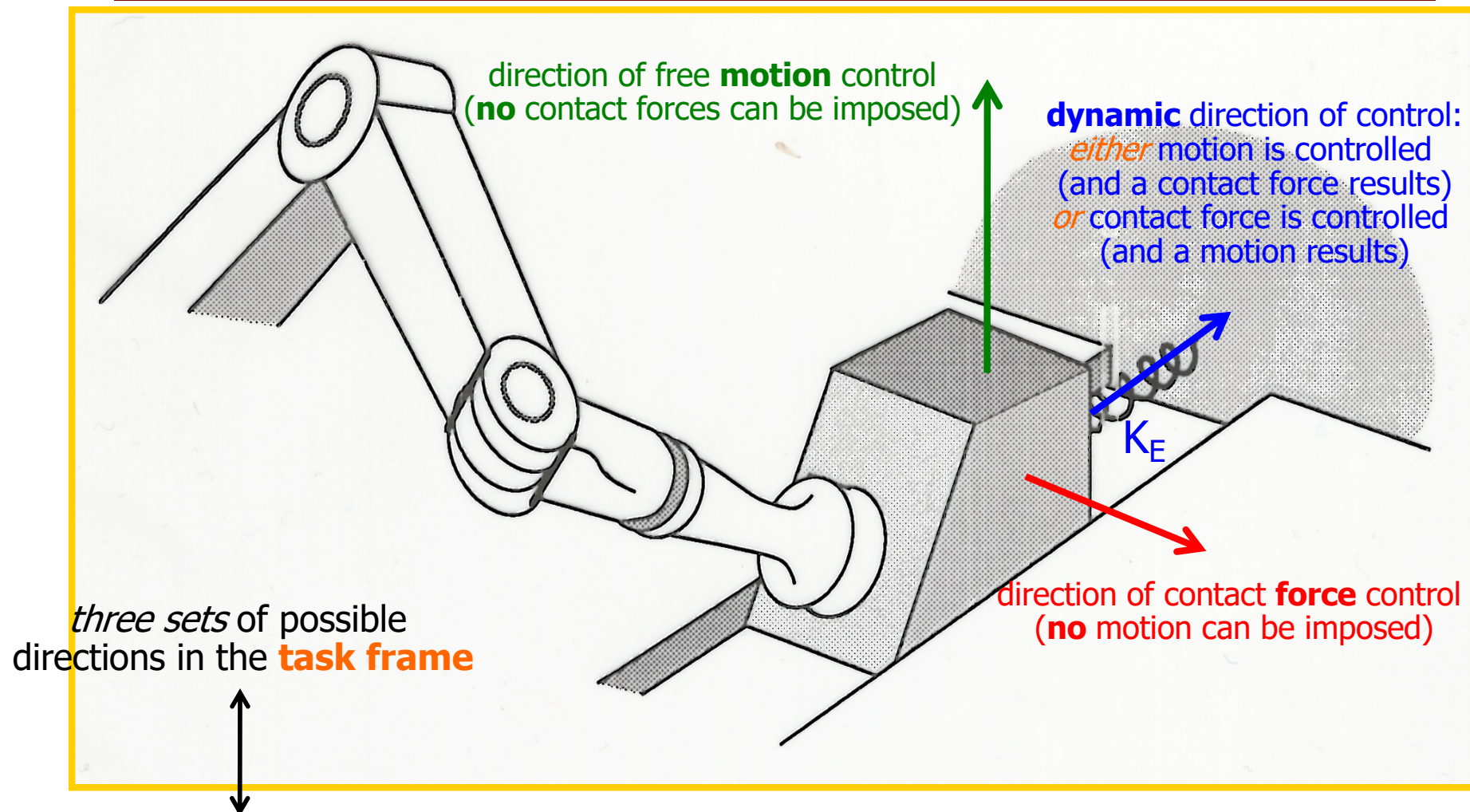
the robot should turn a crank
having a **fixed** handle

Tasks requiring hybrid control



the robot should push a mass
elastically coupled to a wall and constrained in a guide

Tasks requiring hybrid control



generalized **hybrid** modeling and control for **dynamic** environments

A. De Luca, C. Manes: IEEE Trans. Robotics and Automation, vol. 10, no. 4, 1994