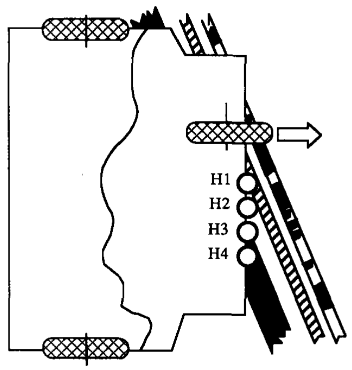

Fig. 7. AGV intersects the guide-path after avoiding an obstacle.

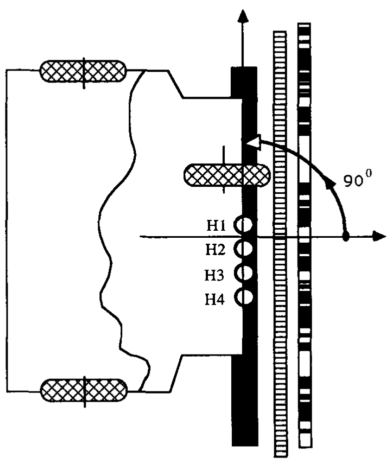

Fig. 8. AGV aligns its four reading heads on the guide-path and then turns 90° to reposition itself correctly on the guide-path.

Once the AGV is correctly repositioned on the guide track, the "navigator" authorizes the "guide-path tracking" module to take over the vehicle motion control. Despite the inherent positional errors occurring during the odometric obstacle avoidance, the "absolute position measurement" function allows the AGV to immediately recover its position on the guide path. The "navigator" monitors the AGV current position P and stops the vehicle when it reaches the destination position D, previously specified on the robot's keyboard (shown in Fig. 5) by the user.

IV. Conclusion

This paper presents a pseudorandom encoding method that allows an AGV to recover its absolute position at any point on its guide-path. This is especially significant in situations where the AGV has to avoid obstacles that may appear on its path. Beside the obvious guide-path, the proposed method requires two additional 1-bit-wide tracks, one for the pseudorandom code and one for the synchronization of code readings.

An experimental AGV system was built to test the described absolute position measurement method and its application for AGV navigation when unexpected obstacles are encountered on the guide-path. The results recommend this technique for implementation as a stand-alone function to cost-effectively upgrade existent optically guided industrial AGV's.

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Adaptive PD Controller for Robot Manipulators

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***Abstract*—Referring to the point-to-point control problem, this work presents a PD control algorithm that is adaptive with respect to the gravity parameters of robot manipulators. The proposed controller is shown to be globally convergent. Following the same approach, an application to the tracking problem is also presented. Simulation tests are included, with reference to a robot having three degrees of freedom.**

I. Introduction

In this paper, we refer to the so-called *point-to-point* control of robot manipulators. As known [1], control laws based on feedback from the positions and velocities of the joints have been shown to be globally asymptotically stable, provided that the gravity terms are compensated. It also has been shown that PD controllers may be used for trajectory tracking, with accuracy related to the velocity feedback gains [2]. Moreover, such control algorithms are robust with respect to uncertainties on the inertia parameters; namely, even if the inertia parameters are not known, the global asymptotic stability is ensured. Conversely, uncertainties on the gravity parameters (such as the payload) may lead to undesired steady-state errors.

To circumvent this problem, Arimoto and Miyazaki [1] proposed a PID control algorithm that, however, guarantees only local asymptotic stability [3, pp. 385-388]. Moreover, to ensure the stability, the gain matrices must satisfy complicated inequalities, which depend on the initial conditions.

The purpose of this work is to show how an adaptive PD controller can be designed. The proposed controller yields the global asymptotic stability of the whole system even if the inertia and gravity parameters are unknown, provided that upper and lower bounds of the inertia matrix are available. The convergence is

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ensured for any value of the proportional and derivative gain matrices, assumed to be symmetric positive definite. The only constraint is in the adaptation gain, which has to be greater than a lower bound. In the common case in which only the robot payload is unknown, one integrator is sufficient to implement this controller, while a PID algorithm requires as many integrators as the number of the links.

This paper is organized as follows. In Section II we illustrate the robot model and some related useful properties. Section III is devoted to the derivation of some interesting results on the stability of the PD controller. In particular, we show that a PD controller with imperfect gravity compensation is still stable, but the equilibrium point is, in general, different from the desired one. The derivation of the adaptive PD control law is presented in Section IV. The stability proof makes use of a Lyapunov function, similar to that introduced in [4], that reveals itself useful also for the design of adaptive tracking algorithms, as is shown in Section V. The performances of the proposed control law have been compared with those of the PID controller by means of simulation tests referred to a three-link robot. The results are reported in Section VI and some conclusions are drawn in Section VII.

II. Robot Model

Consider the dynamics of an n-link rigid robot as described by

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + e(q) + F\dot{q} = u \tag{1}$$

where q is the $n \times 1$ vector of the joint coordinates; $B(q)$ is the inertia matrix, which is symmetric positive definite and bounded for any q; $C(q, \dot{q})$ takes into account the Coriolis and centrifugal forces and is linear with respect to $\dot{q}$ and bounded with respect to q; F is the diagonal positive semidefinite matrix of the viscous friction coefficients; u is the vector of the applied torques; and $e(q)$ is the vector of the gravity forces given by

$$e(q) = \frac{\partial U(q)}{\partial q} \tag{2}$$

where $U(q)$ is the gravitational energy of the robot that is bounded for any q. The vector $e(q)$ and its partial derivative with respect to q are also bounded. The dynamic model (1) has the following important properties.

Property 1: Given a proper definition of C that is not unequivocally defined by the form $C(q, \dot{q})\dot{q}$, the matrix $\dot{B} - 2C$ is skew-symmetric [1], [5], [6]. One possible definition for the elements of C which leads to the skew-symmetry of $\dot{B} - 2C$ is [5]

$$C_{ij}(q, \dot{q}) = \frac{1}{2}\left[\dot{q}^T \frac{\partial B_{ij}}{\partial q} + \sum_{k=1}^{n}\left(\frac{\partial B_{ik}}{\partial q_j} - \frac{\partial B_{jk}}{\partial q_i}\right)\dot{q}_k\right], \quad i, j = 1, \ldots, n.$$

This definition implies that

$$\dot{B}(q) = C(q, \dot{q}) + C^T(q, \dot{q}). \tag{3}$$

Property 2: The matrices B and C and the vector e are linear in terms of robot and load parameters [7].

Property 3: Since $C(q, \dot{q})$ is bounded in q and linear in $\dot{q}$, a positive constant k_C exists such that

$$\|C(q, \dot{q})\| \le k_C \|\dot{q}\|. \tag{4}$$

III. Stability of the PD Controller

As is known, with reference to the point-to-point control of manipulators, a controller consisting of independent local PD feedback at each joint ensures the global asymptotic stability of the whole system, provided that the gravity terms are exactly compensated. This result is stated by the following theorem.

Theorem 1 [1]: Consider the control law

$$u = e(q) - K_P(q - q_0) - K_D\dot{q} \tag{5}$$

where K_P and K_D are symmetric positive definite constant matrices and q_0 is the desired position. The equilibrium point $q = q_0$, $\dot{q} = 0$ of (1), (5) is globally asymptotically stable. ■

In the following, we show that it is not necessary to compensate $e(q)$ for all values of q. Indeed, the simpler control law

$$u = e(q_0) - K_P(q - q_0) - K_D\dot{q} \tag{6}$$

suffices, under suitable assumptions, to achieve global asymptotic stability. Throughout this paper, we use the notations $\lambda_M(A)$ and $\lambda_m(A)$ to indicate the largest and the smallest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(x)$, for any $x \in R^n$. The norm of vector x is defined as $\|x\| = (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$ and that of matrix A is defined as the corresponding induced norm $\|A\| = (\max_{\text{eigenvalue}} A^T A)^{\frac{1}{2}}$. This definition implies that if A is symmetric positive definite, we have $\|A\| = \lambda_M(A)$.

The partial derivatives of $e(q)$ being bounded, a positive constant M_1 exists such that

$$\left\|\frac{\partial e(q)}{\partial q}\right\| \le M_1, \qquad \forall q \in R^n \tag{7}$$

which implies

$$\|e(q_1) - e(q_2)\| \le M_1\|q_1 - q_2\|, \qquad \forall q_1, q_2 \in R^n. \tag{8}$$

We now prove a theorem that states that the control law (6) globally asymptotically stabilizes the closed-loop system (1), (6) if the proportional action is sufficiently strong. For instance, if we take K_P as a diagonal matrix $K_P = \text{diag}[k_{p1}, \ldots, k_{pn}]$, it suffices that $k_{pi} > M_1$, $1 \le i \le n$.

Theorem 2: Consider the system (1), (6). If $\lambda_m(K_P) > M_1$, then the equilibrium point $q = q_0$, $\dot{q} = 0$ is globally asymptotically stable.

Proof: The system (1), (6) has the unique equilibrium point $q = q_0$, $\dot{q} = 0$. Indeed, the equilibrium positions of (1), (6) are the solutions of

$$K_P(q - q_0) = e(q_0) - e(q). \tag{9}$$

The hypothesis $\lambda_m(K_P) > M_1$ and (8) enable us to write

$$\|K_P(q - q_0)\| \ge \lambda_m(K_P)\|q - q_0\| > M_1\|q - q_0\| \ge \|e(q_0) - e(q)\|, \qquad \forall q \ne q_0.$$

The previous chain of inequalities implies that the left-hand side of (9) is different from the right-hand side for every $q \ne q_0$. Therefore, $q = q_0$ is the unique solution of (9). Consider the function

$$P_1(q) = U(q) - q^T e(q_0) + \frac{1}{2}q^T K_P q - q^T K_P q_0. \tag{10}$$

The stationary values of (10) are given by the solutions of $\frac{\partial P_1(q)}{\partial q} = 0$, which coincides with (9) and, consequently, has the unique solution $q = q_0$. Moreover,

$$\frac{\partial^2 P_1(q)}{\partial q^2} = K_P + \frac{\partial e(q)}{\partial q}. \tag{11}$$

Owing to (7) and to the assumption $\lambda_m(K_P) > M_1$, the matrix (11) is positive definite, and we conclude that $P_1(q)$ has an absolute minimum at $q = q_0$. Let us introduce the candidate Lyapunov function

$$v(q, \dot{q}) = \frac{1}{2}\dot{q}^T B(q)\dot{q} + P_1(q) - P_1(q_0) \tag{12}$$

which is positive definite with respect to $q = q_0$, $\dot{q} = 0$. Differentiating (12), we get

$$\dot{v}(q, \dot{q}) = \tfrac{1}{2}\dot{q}^T\dot{B}(q)\dot{q} + \dot{q}^T[-C(q, \dot{q})\dot{q} - e(q) - F\dot{q} - K_P(q - q_0) - K_D\dot{q} + e(q_0)] + \dot{q}^T\frac{\partial U(q)}{\partial q} - \dot{q}^Te(q_0) + \dot{q}^TK_P(q - q_0). \quad (13)$$

Substituting (2) and (3) into (13), we have $\dot{v}(q, \dot{q}) = -\dot{q}^T(K_D + F)\dot{q}$ and, therefore, $\dot{v}$ is negative semidefinite. A direct application of the Lasalle theorem [8, p. 108] gives the thesis. ∎

Remark: The hypothesis $\lambda_m(K_P) > M_1$ guarantees that $q = q_0$, $\dot{q} = 0$ is the unique equilibrium point of (1), (6) for every value of q_0, which is a necessary condition for global asymptotic stability. ∎

If the gravity vector $e(q)$ is not perfectly known, it cannot be exactly compensated in (5). Suppose we have an estimate $\hat{e}(q)$ obtained by $\hat{e}(q) = \frac{\partial \hat{U}(q)}{\partial q}$, where $\hat{U}(q)$ is the available estimate of the gravitational energy that is assumed bounded for any q. The PD control law (5) becomes

$$u = \hat{e}(q) - K_P(q - q_0) - K_D\dot{q}. \quad (14)$$

The equilibrium positions of (1), (14) are solutions of

$$K_P(q - q_0) + e(q) - \hat{e}(q) = 0. \quad (15)$$

If $e(q_0) - \hat{e}(q_0) \neq 0$, the point $q = q_0$, $\dot{q} = 0$ is no longer an equilibrium point of (1), (14). However, by increasing the proportional gain matrix K_P we find that (15) has a unique solution $\hat{q}_0$, arbitrarily close to q_0, and that the associated equilibrium point $q = \hat{q}_0$, $\dot{q} = 0$ is globally asymptotically stable. Analogous to (8), we assume that a positive constant M_2 exists such that

$$\|\hat{e}(q_1) - \hat{e}(q_2)\| \leq M_2\|q_1 - q_2\|, \quad \forall q_1, q_2 \in R^n. \quad (16)$$

Theorem 3: Consider the system (1), (14). If $\lambda_m(K_P) > M_1 + M_2$, then (15) has only one solution $q = \hat{q}_0$, and the associated equilibrium point $q = \hat{q}_0$, $\dot{q} = 0$ is globally asymptotically stable.

Proof: Consider the function

$$P_2(q) = U(q) - \hat{U}(q) + \tfrac{1}{2}q^TK_Pq - q^TK_Pq_0.$$

Since $U(q)$, $\hat{U}(q)$ are bounded and $\frac{1}{2}q^TK_Pq - q^TK_Pq_0$ is a convex function (having minimum equal to $-\frac{1}{2}q_0^TK_Pq_0$), $P_2(q)$ has an absolute minimum for a finite value of q. This fact implies that the equation $\frac{\partial P_2(q)}{\partial q} = 0$, which coincides with (15), has at least one solution, corresponding to the absolute minimum of $P_2(q)$. Let $q = \hat{q}_0$ be this solution. We can write

$$K_P(\hat{q}_0 - q_0) + e(\hat{q}_0) - \hat{e}(\hat{q}_0) = 0. \quad (17)$$

Subtracting (17) from (15), we get

$$K_P(q - \hat{q}_0) + [e(q) - e(\hat{q}_0)] - [\hat{e}(q) - \hat{e}(\hat{q}_0)] = 0. \quad (18)$$

Since by hypothesis $\lambda_m(K_P) > M_1 + M_2$, (8) and (16) imply that $q = \hat{q}_0$ is the only solution of (18). The global asymptotic stability of the associated equilibrium point $q = \hat{q}_0$, $\dot{q} = 0$ can then be proved by considering the Lyapunov function $v(q, \dot{q}) = P_2(q) - P_2(\hat{q}_0) + \frac{1}{2}\dot{q}^TB(q)\dot{q}$, and proceeding as in the proof of Theorem 2. ∎

The previous theorem ensures the stability of the PD controller, even with imperfect gravity compensation. Unfortunately, if $e(q_0) - \hat{e}(q_0) \neq 0$ this controller leads to a steady-state error equal to $q_0 - \hat{q}_0$. One way to overcome this difficulty has been proposed in [1] and consists of modifying the control law (14) by adding integral actions. The resulting PID controller is given by

$$u = -K_P(q - q_0) - K_D\dot{q} - K_I\int_0^t (q - q_0)\, d\tau.$$

However, the stability is ensured if the gain matrices K_P, K_D, and K_I are positive definite and are chosen so that complex conditions, some of which depend on the initial conditions $q(0)$, are satisfied. For a given choice of K_P, K_D, and K_I, the asymptotic stability is ensured if the initial condition $q(0)$ belongs to a suitable region. In that sense, the PID controller guarantees only local stability.

An alternative way to eliminate the steady-state errors is proposed in the following section. The approach is that of incorporating suitable parameter adaptation dynamics into the PD controller. This approach allows us to obtain a globally convergent controller. The choice of the related gain matrices is greatly simplified with respect to the PID control law. Moreover, the order of the adaptation dynamics is equal to the number of unknown gravitational parameters. Hence, in the frequent case in which only the mass of the payload is unknown, one integrator is needed against the n integrators required by the PID controller. At last, the adaptive PD controller exhibits better performance, as will be shown in Section VI.

IV. Adaptive PD Controller

Since the gravity vector $e(q)$ is linear in terms of robot parameters, it can be expressed as $e(q) = E(q)p$, where p is the $m \times 1$ unknown parameter vector, which is assumed constant, and $E(q)$ is a known matrix. Even if the inertia matrix is supposed unknown, we assume known upper and lower bounds on the magnitude of its eigenvalues and known constant k_C that appears in (4). We also assume a known upper bound on the coefficients of friction matrix F. Consider the control law

$$u = -K_P\tilde{q} - K_D\dot{q} + E(q)\hat{p} \quad (19)$$

with the parameter adaptation dynamics

$$\dot{\hat{p}} = -\beta E^T(q)\left[\gamma\dot{q} + \frac{2\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}}\right] \quad (20)$$

in which $\tilde{q} = q - q_0$ is the position error, K_P and K_D are symmetric positive definite matrices, β is a positive constant, and γ is such that

$$\gamma > \max\left\{\frac{2\lambda_M(B)}{\sqrt{\lambda_m(B)\lambda_m(K_P)}}, \frac{1}{\lambda_m(K_D)} \cdot \left[\frac{(\lambda_M(K_D) + \lambda_M(F))^2}{2\lambda_m(K_P)} + 4\lambda_M(B) + \frac{k_C}{\sqrt{2}}\right]\right\}. \quad (21)$$

Theorem 4: Consider the system (1), (19), and (20). If γ satisfies (21), then $\tilde{q}(t)$, $\dot{q}(t)$ and $\hat{p}$ are bounded for any $t \geq 0$. Moreover,

$$\lim_{t\to\infty}\left\|\begin{bmatrix}\tilde{q}\\ \dot{q}\end{bmatrix}\right\| = 0.$$

Proof: As suggested by a work of Koditschek on the adaptive control of a rigid body for attitude tracking [4], we select as candidate Lyapunov function ($\tilde{p} = p - \hat{p}$)

$$v(\tilde{q}, \dot{q}, \tilde{p}) = \gamma\left(\tfrac{1}{2}\dot{q}^TB(q)\dot{q} + \tfrac{1}{2}\tilde{q}^TK_P\tilde{q}\right) + \frac{2\dot{q}^TB(q)\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} + \frac{1}{2\beta}\tilde{p}^T\tilde{p} \quad (22)$$

which is positive definite since by hypothesis $\gamma >$

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$\frac{2\lambda_M(B)}{\sqrt{\lambda_m(B)\lambda_m(K_P)}}$. The time derivative of (22) is given by

$$\dot{v}(\tilde{q}, \dot{q}, \tilde{p}) = -\gamma \dot{q}^T(K_D + F)\dot{q} - 2\frac{\tilde{q}^T K_P \tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} + 2\frac{\dot{q}^T B\dot{q}}{1 + 2\tilde{q}^T\tilde{q}} + 2\frac{\dot{q}^T C\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} - 2\frac{\dot{q}^T(K_D + F)\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} - 8\frac{\dot{q}^T B\tilde{q}\dot{q}^T\tilde{q}}{(1 + 2\tilde{q}^T\tilde{q})^2}.$$

Now note that

$$\frac{2\dot{q}^T B\dot{q}}{1 + 2\tilde{q}^T\tilde{q}} \le 2\lambda_M(B)\|\dot{q}\|^2 \tag{23}$$

$$2\frac{\dot{q}^T C\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} \le k_C\|\dot{q}\|^2 \frac{2\|\tilde{q}\|}{1 + 2\|\tilde{q}\|^2} \le \frac{k_C}{\sqrt{2}}\|\dot{q}\|^2 \tag{24}$$

$$\frac{8\dot{q}^T B\tilde{q}\dot{q}^T\tilde{q}}{(1 + 2\tilde{q}^T\tilde{q})^2} \le \lambda_M(B)\|\dot{q}\|^2 \frac{8\|\tilde{q}\|^2}{1 + 4\|\tilde{q}\|^4} \le 2\lambda_M(B)\|\dot{q}\|^2 \tag{25}$$

which imply that

$$\dot{v} \le -\gamma\lambda_m(K_D)\|\dot{q}\|^2 - 2\lambda_m(K_P)\frac{\|\tilde{q}\|^2}{1 + 2\|\tilde{q}\|^2} + \left(4\lambda_M(B) + \frac{k_C}{\sqrt{2}}\right)\|\dot{q}\|^2 + 2\frac{(\lambda_M(K_D) + \lambda_M(F))\|\dot{q}\|\|\tilde{q}\|}{1 + 2\|\tilde{q}\|^2}.$$

Since

$$\gamma > \frac{1}{\lambda_m(K_D)}\left[\frac{(\lambda_M(K_D) + \lambda_M(F))^2}{2\lambda_m(K_P)} + 4\lambda_M(B) + \frac{k_C}{\sqrt{2}}\right]$$

the function $\dot{v}$ is negative semidefinite and vanishes if and only if $\tilde{q} = 0$, $\dot{q} = 0$. By applying the Lasalle theorem [8, p. 108] the thesis is proved. ■

Remark: Since the constants on the right-hand side of (21) are bounded, we can always choose γ so that (21) is satisfied.

V. An Application to the Tracking Problem

The Lyapunov function introduced in the proof of Theorem 4 can be also useful to derive adaptive tracking algorithms. In the sequel, we show how the adaptive control law proposed in [5] can be modified to avoid the introduction of the *virtual reference trajectory*. Consider the control law

$$u = \hat{B}(q)\ddot{q}_d + \hat{C}(q, \dot{q})\dot{q}_d + \hat{e}(q) + \hat{F}\dot{q}_d - K_P\tilde{q} - K_D\dot{\tilde{q}} \tag{26}$$

where K_P and K_D are symmetric positive definite matrices; $\tilde{q} = q - q_d$ is the error between the actual and the desired trajectory; and $\hat{B}$, $\hat{C}$, $\hat{F}$, and $\hat{e}$ are estimates of the robot matrices whose parameters are updated according to a certain parameter adaptation law. Since the robot dynamics is linear in terms of robot parameters, we can write

$$(B - \hat{B})\ddot{q}_d + (C - \hat{C})\dot{q}_d + (F - \hat{F})\dot{q}_d + e - \hat{e} = Y(q, \dot{q}, \dot{q}_d, \ddot{q}_d)\tilde{p} \tag{27}$$

in which $\tilde{p} = p - \hat{p}$ is the difference between the true parameter vector p and its estimate $\hat{p}$. It is shown in [5] that the parameter adaptation law

$$\dot{\hat{p}} = -Y^T(q, \dot{q}, \dot{q}_d, \ddot{q}_d)\dot{\tilde{q}} \tag{28}$$

along with the control law (26) yield global stability. However, to ensure the asymptotic stability, the following modified control and adaptation algorithms were proposed [5]

$$u = \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{e}(q) + \hat{F}\dot{q}_r - K_D(\dot{q} - \dot{q}_r)$$
$$\dot{\hat{p}} = -Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\dot{q} - \dot{q}_r)$$

where $q_r = q_d - A\int_0^t \tilde{q}\, d\tau$ (with A being a Hurwitz matrix) is the so-called virtual reference trajectory. The use of the virtual reference trajectory can be avoided by suitably modifying the adaptation algorithm (28), as is shown in the sequel. Assume that $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are bounded and $\|\dot{q}_d(t)\| \le M$. Consider the parameter adaptation algorithm

$$\dot{\hat{p}} = -\beta Y^T(q, \dot{q}, \dot{q}_d, \ddot{q}_d)\left(\gamma\dot{\tilde{q}} + \frac{2\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}}\right) \tag{29}$$

where β is a positive constant and γ is such that

$$\gamma > \max\left\{\frac{2\lambda_M(B)}{\sqrt{\lambda_m(B)\lambda_m(K_P)}}, \frac{1}{\lambda_m(K_D)} \cdot \left[\frac{(\lambda_M(K_D) + \lambda_M(F) + k_C M)^2}{2\lambda_m(K_P)} + 4\lambda_M(B) + \frac{k_C}{\sqrt{2}}\right]\right\}. \tag{30}$$

Theorem 5: Consider the system (1), (26), and (29). If γ satisfies (30), then $\tilde{q}$, $\dot{\tilde{q}}$ and $\tilde{p}$ are bounded for any $t \ge t_0$. Moreover

$$\lim_{t\to\infty}\left\|\begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix}\right\| = 0.$$

Proof: Substituting (26) into (1) and taking (27) into account, we obtain

$$B(q)\ddot{\tilde{q}} + C(q, \dot{q})\dot{\tilde{q}} + F\dot{\tilde{q}} = -K_P\tilde{q} - K_D\dot{\tilde{q}} - Y(q, \dot{q}, \dot{q}_d, \ddot{q}_d)\tilde{p}. \tag{31}$$

Consider the candidate Lyapunov function

$$v(\tilde{q}, \dot{\tilde{q}}, \tilde{p}, t) = \gamma\left[\tfrac{1}{2}\dot{\tilde{q}}^T B(q)\dot{\tilde{q}} + \tfrac{1}{2}\tilde{q}^T K_P\tilde{q}\right] + 2\frac{\dot{\tilde{q}}^T B(q)\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} + \frac{1}{2\beta}\tilde{p}^T\tilde{p} \tag{32}$$

which is positive definite and decrescent [8, p. 195] by virtue of (30). Owing to (3) and (29), the time derivative of (32) along (31) is given by

$$\dot{v}(\tilde{q}, \dot{\tilde{q}}, \tilde{p}, t) = -\gamma\dot{\tilde{q}}^T(K_D + F)\dot{\tilde{q}} - \frac{2\tilde{q}^T K_P\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} + \frac{2}{1 + 2\tilde{q}^T\tilde{q}} \cdot \left[\dot{\tilde{q}}^T C(q, \dot{q})\tilde{q} + \dot{\tilde{q}}^T B(q)\dot{\tilde{q}} - \dot{\tilde{q}}^T(K_D + F)\tilde{q}\right] - \frac{8\dot{\tilde{q}}^T B(q)\tilde{q}\dot{\tilde{q}}^T\tilde{q}}{(1 + 2\tilde{q}^T\tilde{q})^2}.$$

Recalling (23), (25) and observing that

$$\frac{2\dot{\tilde{q}}^T C(q, \dot{q})\tilde{q}}{1 + 2\tilde{q}^T\tilde{q}} \le \frac{2\|\dot{\tilde{q}}\|\|\tilde{q}\|}{1 + 2\|\tilde{q}\|^2}(k_C\|\dot{\tilde{q}}\| + k_C M) \le \frac{k_C}{\sqrt{2}}\|\dot{\tilde{q}}\|^2 + \frac{2k_C M\|\tilde{q}\|\|\dot{\tilde{q}}\|}{1 + 2\|\tilde{q}\|^2}$$

we can write

$$\dot{v} \le -\gamma\lambda_m(K_D)\|\dot{\tilde{q}}\|^2 - \frac{2\lambda_m(K_P)}{1+2\|\tilde{q}\|^2}\|\tilde{q}\|^2 + \left(4\lambda_M(B) + \frac{k_C}{\sqrt{2}}\right)\|\dot{\tilde{q}}\|^2 + \frac{2(\lambda_M(K_D)+\lambda_M(F)+k_C M)}{1+2\|\tilde{q}\|^2}\|\dot{\tilde{q}}\|\|\tilde{q}\|. \quad (33)$$

From (33), owing to (30), we obtain

$$\dot{v} \le -\phi(\|\tilde{q}\|, \|\dot{\tilde{q}}\|) \quad (34)$$

where ϕ is positive definite with respect to $\|\tilde{q}\|$, $\|\dot{\tilde{q}}\|$. From (32) and (34) we obtain that $\tilde{q}$, $\dot{\tilde{q}}$, and $\tilde{p}$ are bounded. By hypothesis, $\dot{q}_d$ and $\ddot{q}_d$ are also bounded. From (31), it follows that $\ddot{\tilde{q}}$ is bounded and, as a consequence, $\tilde{q}$ and $\dot{\tilde{q}}$ are uniformly continuous. The inequality (34) enables us to write

$$\int_{t_0}^{\infty}\phi(\|\tilde{q}\|, \|\dot{\tilde{q}}\|)\,dt \le -\int_{t_0}^{\infty}\dot{v}\,dt = v(t_0) - v(\infty) < \infty. \quad (35)$$

Since $\tilde{q}$ and $\dot{\tilde{q}}$ are uniformly continuous, (35) implies [9, p. 210] $\lim_{t\to\infty}\phi(\|\tilde{q}\|, \|\dot{\tilde{q}}\|) = 0$ which, in turn, implies

$$\lim_{t\to\infty}\left\|\begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix}\right\| = 0.$$

■

VI. Case Study

The adaptive PD controller presented in Section IV has been tested by numerical simulations referred to a three revolute jointed robot whose links are 0.5 m long. Frictional forces have been neglected. The nonzero entries of the inertia matrix B and of the gravity vector e that completely characterize the robot model (1) are given by

$$\begin{aligned} B_{11} &= a_1 + a_2\cos^2 q_2 + a_3\cos^2(q_2+q_3) + a_4\cos q_2\cos(q_2+q_3)\\ B_{22} &= a_5 + a_4\cos q_3, \quad B_{23} = B_{32} = a_6 + a_7\cos q_3 \quad (36)\\ B_{33} &= a_8 \quad e_2 = b_1\cos q_2 + b_2\cos(q_2+q_3),\\ e_3 &= b_2\cos(q_2+q_3). \end{aligned}$$

The values of the parameters a_i and b_i, referred to payloads m_p of 0 and 5 kg, have been reported in Table I. The aim of the simulation tests was to compare the PD and the PID controllers with the proposed adaptive PD control law (denoted by APD), assuming a nominal payload of 5 kg and actual payloads of 0 and 5 kg. As is displayed by (36), vector e is linear in terms of the parameters b_1 and b_2. Since

$$b_1 = 189.1708 + 4.9008\, m_p$$
$$b_2 = 52.9286 + 4.9008\, m_p$$

we can write the gravity vector in the form $e(q) = e_A(q) + e_B(q)m_p$, where e_A and e_B are known vectors. Consequently, the adaptive PD control law becomes

$$u = -K_P(q-q_0) - K_D\dot{q} + e_A(q) + e_B(q)\hat{m}_p$$
$$\dot{\hat{m}}_p = -\beta e_B^T(q)\left[\gamma\dot{q} + \frac{2(q-q_0)}{1+2(q-q_0)^T(q-q_0)}\right].$$

The considered problem is that of regulation about a reference position. The gain matrices of the three controllers were chosen as follows (all values are in SI units)

TABLE I
Robot Parameters

$m_p = 0$ kg	$m_p = 5$ kg
$a_1 = 23.380$	$a_1 = 23.380$
$a_2 = 9.2063$	$a_2 = 10.456$
$a_3 = 2.4515$	$a_3 = 3.7015$
$a_4 = 5.4000$	$a_4 = 7.9000$
$a_5 = 82.399$	$a_5 = 84.899$
$a_6 = 2.6274$	$a_6 = 3.8744$
$a_7 = 2.7000$	$a_7 = 3.9500$
$a_8 = 25.779$	$a_8 = 27.027$
$b_1 = 189.17$	$b_1 = 213.67$
$b_2 = 52.928$	$b_2 = 77.432$

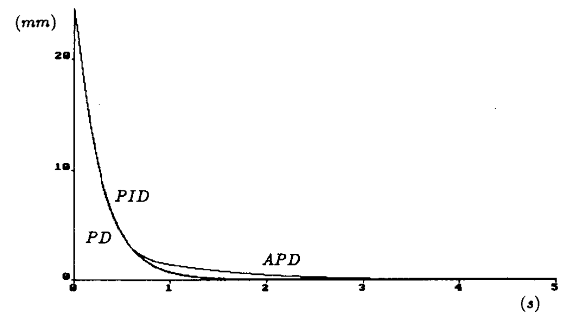

Fig. 1. End effector error: nominal payload = actual payload, initial error = 24 mm.

$$\begin{aligned} &PD: && K_P = \mathrm{diag}[10000], && K_D = \mathrm{diag}[3000]\\ &PID: && K_P = \mathrm{diag}[10000], && K_D = \mathrm{diag}[3000],\\ & && K_I = \mathrm{diag}[100] & &\\ &APD: && K_P = \mathrm{diag}[10000], && K_D = \mathrm{diag}[3000],\\ & && \gamma = 0.1, \quad \beta = 100. & & \end{aligned}$$

The initial condition $\hat{m}_p(0)$ for the APD controller was set equal to the nominal payload. In the first simulation the actual payload coincided with the nominal payload (5 kg), e.g., the payload was assumed exactly known. In Figs. 1 and 2 are reported the distances between the actual and the desired position of the robot end effector, expressed in a Cartesian frame, starting by an initial distance of 24 and 412 mm, respectively. These figures show that, when the payload is known, the performances of the PD controller are better than those of the PID and APD controllers (the error settles to zero much faster). The PID controller has a very long settling time when the initial error is quite large (see Fig. 2). The APD controller performs worse than the PD does, even though $\hat{m}_p(0)$ is equal to the true payload. This is caused by the fact that the dynamics of $\hat{m}_p(t)$ is driven by the position error $q - q_0$ and, therefore, the estimate of the gravity term is not exact in the transient.

In order to check the performance of the controllers when the payload is not exactly known, the same tests were repeated by adopting an actual payload (0 kg) different from the nominal one. The corresponding errors are drawn in Figs. 3 and 4, which show that the PD controller has a nonzero steady-state error, due to imperfect gravity compensation. With low initial error, the PID controller possesses the best performances (Fig. 3), but its dynamic behavior considerably deteriorates as the initial error increases (Fig. 4). In particular, a lot of time is needed to reach with accurate precision the desired position. Conversely, the APD controller is scarcely influenced by the initial error (as the PD controller), even if

More — on the outside.

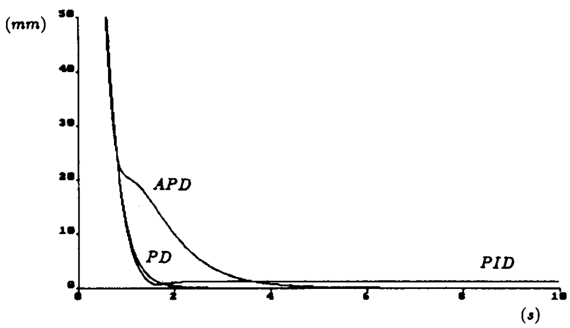

Fig. 2. End effector error: nominal payload = actual payload, initial error = 412 mm.

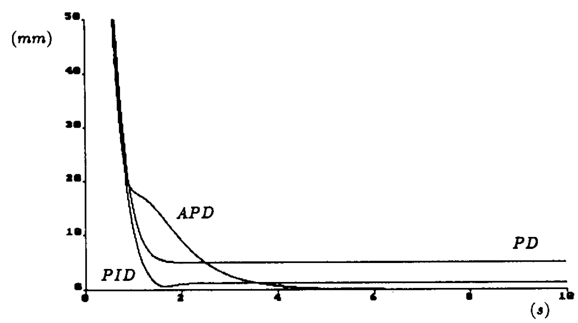

Fig. 4. End effector error: nominal payload ≠ actual payload, initial error = 412 mm.

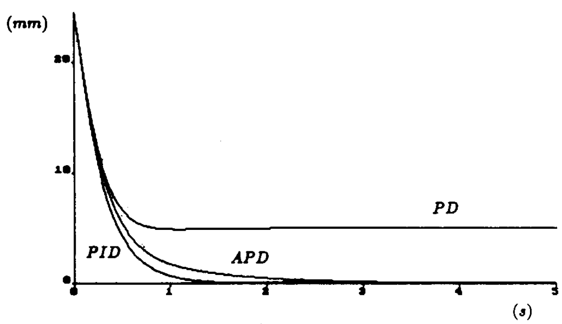

Fig. 3. End effector error: nominal payload ≠ actual payload, initial error = 24 mm.

it guarantees zero steady-state error (as the PID controller). Therefore, the proposed APD controller ensures the best dynamic performances when the payload is not *a priori* known and the initial positioning error has a large range of variation.

VII. Conclusions

In practice, the robot parameters are never exactly known. This is especially true for the payload, which can vary during operations. Therefore, the use of robust control laws is to be preferred. We have shown that a PD controller, which is robust with respect to inertial and frictional parameters, also can be made adaptive with reference to the gravity parameters. The resulting adaptive PD controller has been proved to be globally asymptotically stable. Moreover, the choice of the gain matrices, which ensure the stability, is greatly simplified with respect to the PID control algorithm.

Simulation tests have shown that the performances of the adaptive PD controller are scarcely influenced by the initial error, while the performances of the PID control law deteriorate considerably as this error increases.

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