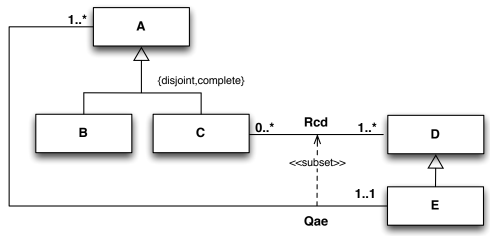
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods – September 12, 2014

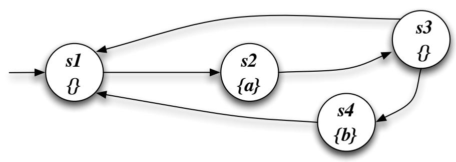
(Time to complete the test: 2 hours)

Exercise 1. Consider the following UML class diagram.



1. Express it in FOL.
2. Express it in *DL-Lite*$_{\mathcal{A}}$, highlighting the parts that are not expressible.
3. Given the ABox $\mathcal{A} = \{A(c)\}$, compute the certain answer to the query $q(x) : -Rcd(x, y), D(y)$, using the rewriting technique for *DL-Lite*$_{\mathcal{A}}$.

Exercise 2. Model check the Mu-Calculus formula $\nu X.\mu Y.((b \wedge \langle next\rangle X) \vee \langle next\rangle Y)$ and the CTL formula $AG(AFa \wedge EFb \wedge EG\neg b)$ (showing its translation in Mu-Calculus) against the following transition system:



Exercise 3. Consider the following predicates: $Supplier(x, y)$, saying that x is a supplier in city y; $Item(x, y)$, saying that item x has color y; and $Sells(x, y, z)$ saying that supplier x sells item y at price z. Express in FOL the following boolean queries, stating which ones are CQs (do not use abbreviations for cardinalities):

1. *There exists a supplier in NY selling a blue item.*
2. *There exists a supplier in NY selling at least two blue items.*
3. *There exists a supplier in NY selling only blue items*
4. *There exists a supplier in NY selling all blue items.*
5. *Return the pairs of suppliers such that the first supplier sells at least one item at a cheaper price than the second one.*
6. *Return the pairs of suppliers such that the first supplier sells all items that the second one sells, and at a cheaper price.*

Exercise 4. Compute the certain answers to the following CQs over the following incomplete database (naive tables), and discuss how you obtained the result:

$$q(x) \leftarrow Sells(x, y), Item(y, z) \qquad q(x, z) \leftarrow Sells(x, y), Item(y, z)$$

$Sells$

$supplier$	$item$
Smith	$null_1$
$null_2$	item1
Brown	$null_3$
Green	item2
White	$null_5$
$null_4$	$null_3$

$Item$

$item$	$color$
item1	blue
$null_1$	red
item2	$null_{10}$
$null_3$	$null_{11}$

Exercise 5. Check the truth of the following Hoare triple, assuming as invariant: $i \leq 64$, explaining in details the technique used:

```
{i=1}  while(i<64) do i:= i*2  {i=64}
```