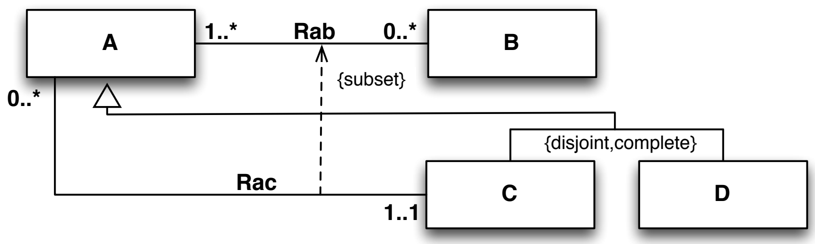
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods – Jan. 22, 2014

(Time to complete the test: 2 hours)

Exercise 1. Consider the following UML class diagram.

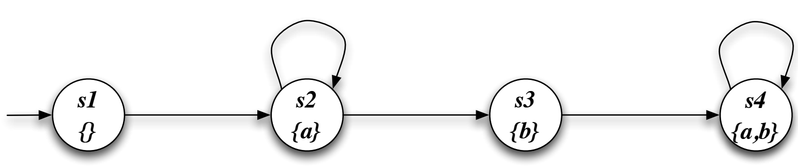


1. Express it in FOL.
2. Express it in *DL-Lite*$_{\mathcal{A}}$, highlighting parts that are not expressible.
3. Given the ABox $A = \{C(c)\}$ and the boolean conjunctive query $q(x) \leftarrow Rab(x,y), Rab(y,z), A(z)$, return the certain answer by exploiting the *DL-Lite*$_{\mathcal{A}}$ rewriting algorithm.

Solution

$$
\begin{array}{ll}
q(x) \leftarrow \underline{Rab(x,y)}, Rab(y,z), A(z) & Rac \sqsubseteq Rab \\
q(x) \leftarrow Rac(x,y), \underline{Rab(y,z)}, A(z) & Rac \sqsubseteq Rab \\
q(x) \leftarrow Rac(x,y), Rac(y,z), \underline{A(z)} & C \sqsubseteq A \\
q(x) \leftarrow Rac(x,y), Rac(y,z), \underline{C(z)} & \exists Rac^- \sqsubseteq C \\
q(x) \leftarrow Rac(x,y), \underline{Rac(y,z)}, \underline{Rac(w,z)} & w = y, z = z \\
q(x) \leftarrow Rac(x,y), \underline{Rac(y,z)} & A \sqsubseteq \exists Rac \\
q(x) \leftarrow Rac(x,y), \underline{A(y)} & C \sqsubseteq A \\
q(x) \leftarrow Rac(x,y), \underline{C(y)} & \exists Rac^- \sqsubseteq C \\
q(x) \leftarrow \underline{Rac(x,y)}, \underline{Rac(v,y)} & x = v, y = y \\
q(x) \leftarrow \underline{Rac(x,y)} & A \sqsubseteq \exists Rac \\
q(x) \leftarrow \underline{A(x)}, & C \sqsubseteq A \\
q(x) \leftarrow C(x) & \Longrightarrow x = c
\end{array}
$$

Exercise 2. Model check the Mu-Calculus formula $\mu X.\nu Y.(a \vee [next]X) \wedge [next]Y$ and the CTL formula $AFAGa$ against the following transition system:



Exercise 3. Check whether CQ q_1 is contained in CQ q_2, reporting canonical DBs and homomorphism:

$$
\begin{array}{lcl}
q_1(x_r) & \leftarrow & e(x_r, x_g), e(x_g, x_b), e(x_b, x_r). \\
q_2(x) & \leftarrow & e(x, y), e(y, z), e(z, x), e(z, v)e(v, w), e(w, z).
\end{array}
$$

Solution

- Freeze:

$$
\begin{array}{lcl}
q_1(r) & \leftarrow & e(r, x_g), e(x_g, x_b), e(x_b, r). \\
q_2(r) & \leftarrow & e(r, y), e(y, z), e(z, r), e(z, v)e(v, w), e(w, z).
\end{array}
$$

- Build canonica DB of q_1

e	
r	x_g
x_g	x_b
x_b	r

- Guess assignment of (existential) variables in q_2 that makes the atoms in q_2 true in the canonical DB of q_1.

$$y = x_g$$
$$z = x_b$$
$$v = r$$
$$w = x_g$$

- Build canonical DB for the query q_2.
- Check that the assignment extended to constants (interpreted as themselves) is an homomorphism.

Exercise 4. Compute the certain answers to the CQ $q(x) \leftarrow M(x, y), E(y)$ over the incomplete database (naive tables):

$E(mployee)$

$name$
Smith
$null_1$
Brown

$M(anager)$

mgr	mgd
Smith	$null_1$
$null_1$	Brown
Brown	$null_2$

Solution

- Evaluate q over the database as it was a complete database
- Filter out all answers where null appears (certain answers are constituted by tuples of constants in $Cons$)

Answer: $\{$*Smith*$\}$

Exercise 5. Compute the weakest precondition for getting $\{x = 100\}$ by executing the following program:

```
x := y + 50;
if (y > 0) then
  x := y + 100
else x := y + 200;
x := x + y;
```

Solution

```
{y = -50}

x := y + 50;


{(y > 0 & y=0) | (y =< 0 & y=-50)} = {y = -50}

if (y > 0) then

      {y + y + 100 = 100} = {y=0}
```

```
      {x+y = 100}

      {y + 200 + y = 100} = {y = -50}
else x := y + 200;
      {x+y = 100}

{x+y = 100}
x := x + y;
{x=100}
```