## Robotics II

September 22, 2014

## Exercise 1

The actuation device shown in Fig. 1 is a servomechanism with a DC motor coupled to a rotating load through a flexible transmission.


Figure 1: A 1-dof actuation device with flexible transmission

- Explain the role to each component numbered (from 1 to 5 ) in the figure.
- Using an energy (Lagrangian) approach, derive a dynamic model of the system that is appropriate for trajectory planning, motion control, and simulation. Assume that the flexible transmission can be modeled as a visco-elastic element, i.e., it behaves as a damped spring of constant stiffness. In addition, viscous friction is present on the motor and load sides of the transmission. The DC motor is commanded in current. Write down the complete dynamic equations and draw an associated block diagram for simulation purposes (in Simulink style).
- Neglect all dissipative effects in the following. Plan a smooth rest-to-rest trajectory that rotates the load from an initial angle $q_{0}$ to a final $q_{f}$ in given time $T$, without residual vibrations.
- Determine next the input current profile that should command the motor so that the planned trajectory is realized exactly when the system is initially undeformed and at rest.


## Exercise 2

The Newton-Euler method is used for the dynamic modeling of serial robots. In inverse dynamics problems, its most efficient implementation is a numerical algorithm (NE) containing a forward recursive (FR) part, which computes from the base to the tip all relevant differential kinematic terms associated to the links, and a backward recursive (BR) part, which computes from the tip to the base the exchanged forces/torques between links. Suppose now that we compute the (linear/angular) acceleration vector $\ddot{\boldsymbol{p}} \in \mathbb{R}^{6}$ of the end-effector by

$$
\begin{equation*}
\ddot{\boldsymbol{p}}=\operatorname{NEFR}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})=\boldsymbol{J}(\boldsymbol{q}) \ddot{\boldsymbol{q}}+\dot{\boldsymbol{J}}(\boldsymbol{q}) \dot{\boldsymbol{q}}, \tag{1}
\end{equation*}
$$

where $N E F R$ in eq. (1) denotes compactly the FR part only of the NE algorithm. How could the $N E F R$ algorithm be used to evaluate numerically and separately the Jacobian matrix $\boldsymbol{J}$ and the product term $\dot{\boldsymbol{J}} \dot{\boldsymbol{q}}$ ? With the same algorithm, can we also evaluate the matrix $\dot{\boldsymbol{J}}$ alone?

