Introduction to Model Checking

These slides are based on those of
Tevfik Bultan for CS 267
University of California, Santa Barbara
http://www.cs.ucsb.edu/~bultan/
http://www.cs.ucsb.edu/~bultan/courses/267/

Transformational vs. Reactive Systems

Transformational systems

```
get input;
{pre-condition}
compute something;
{post-condition}
return result;
```

- Earlier work in verification uses the transformational view:
  - halting problem
  - Hoare logic
  - pre and post-conditions
  - partial vs. total correctness

Reactive systems

```
while (true) {
   receive some input,
   send some output
}
```

- For reactive systems:
  - termination is not the main issue
  - pre and post-conditions are not enough

Temporal Logics for Reactive Systems

[Pnueli FOCS 77, TCS 81]

Transformational systems

```
get input;
compute something;
return result;
```

- Transformational view follows from the initial use of computers as advanced calculators: A component receives some input, does some calculation and then returns a result.

Reactive systems

```
while (true) {
   receive some input,
   send some output
}
```

- Nowadays, the reactive system view seems more natural: components which continuously interact with each other and their environment without terminating

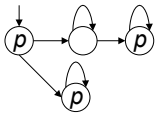
Temporal Logics

Temporal Logics

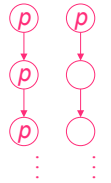
- Invariant p (G p, AG p, $\Box p$)
- Eventually p (F p, AF p, $\Diamond p$)
- Next p : (X p, AX p, $\bigcirc p$)
- p Until q : (p U q, A(p U q))

Branching vs. Linear Time

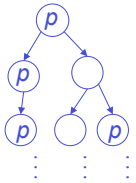
Transition system:



LTL view
G(p) F(p)



CTL view
AF(p), EG(p)



Automated Verification of Finite State Systems

[Clarke and Emerson 81], [Queille and Sifakis 82]

Transition Systems

- S : Set of states (finite)
- $I \subseteq S$: Set of initial states
- $R \subseteq S \times S$: Transition relation

Model checking problem: Given a temporal logic property, does the transition system satisfy the property?

- Complexity: linear in the size of the transition system

Verification vs. Falsification

Verification:

show: initial states $\subseteq$ truth set of p

Falsification:

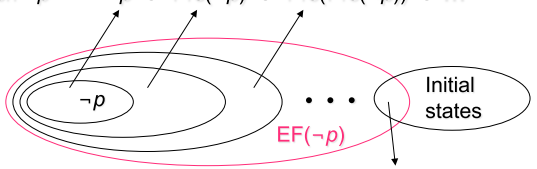
find: a state $\in$ initial states $\cap$ truth set of $\neg p$

generate a counter-example starting from that state

Temporal Properties ≡ Fixpoints

[Emerson and Clarke 80]

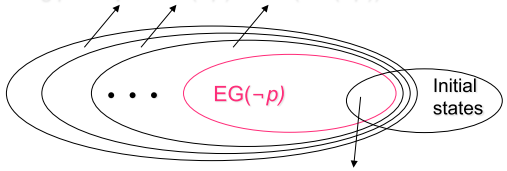
$EF(\neg p) \equiv$ states that can reach $\neg p \equiv \neg p \cup Pre(\neg p) \cup Pre(Pre(\neg p)) \cup \ldots$



initial states that satisfy $EF(\neg p)$

≡ initial states that violate $AG(p)$

$EG(\neg p) \equiv$ states that can avoid reaching $p \equiv \neg p \cap Pre(\neg p) \cap Pre(Pre(\neg p)) \cap \ldots$



initial states that satisfy $EG(\neg p)$

≡ initial states that violate $AF(p)$

Symbolic Model Checking

[McMillan et al. LICS 90]

- Represent sets of states and the transition relation as Boolean logic formulas
- Fixpoint computation becomes formula manipulation
  - pre and post-condition computations: Existential variable elimination
  - conjunction (intersection), disjunction (union) and negation (set difference), and equivalence check
- Use an efficient data structure
  - Binary Decision Diagrams (BDDs)

SMV [McMillan 93]

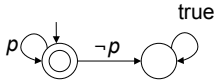
- BDD-based symbolic model checker
- Finite state
- Temporal logic: CTL
- Focus: hardware verification
  - Later applied to software specifications, protocols, etc.
- SMV has its own input specification language
  - concurrency: synchronous, asynchronous
  - shared variables
  - boolean and enumerated variables
  - bounded integer variables (binary encoding)
    - SMV is not efficient for integers, can be fixed

LTL Properties ≡ Büchi automata

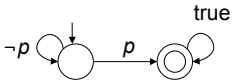
[Vardi and Wolper LICS 86]

- Büchi automata: Finite state automata that accept infinite strings
- A Büchi automaton accepts a string when the corresponding run visits an accepting state infinitely often
- The size of the property automaton can be exponential in the size of the LTL formula

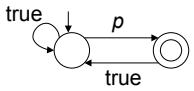
G *p*



F *p*



G (F *p*)



SPIN [Holzmann 91, TSE 97]

- Explicit state, finite state
- Temporal logic: LTL
- Input language: PROMELA
  - Asynchronous processes
  - Shared variables
  - Message passing through (bounded) communication channels
  - Variables: boolean, char, integer (bounded), arrays (fixed size)
- Property automaton from the negated LTL property
- Product of the property automaton and the transition system (on-the-fly)
- Show that there is no accepting cycle in the product automaton
- Nested depth first search to look for accepting cycles
- If there is a cycle, it corresponds to a counterexample behavior that demonstrates the bug

Model Checking Research

- These key ideas and tools inspired a lot of research

[Clarke, Grumberg and Peled, 99]

  - efficient symbolic representations
  - partial order reductions
  - abstraction
  - compositional/modular verification
  - model checking infinite state systems (pushdown automata)
  - model checking real time systems
  - model checking hybrid systems
  - model checking programs
  - ...

Model Checking Impact

- Model checking research had significant impact in other areas. Some examples:
- Software Engineering:
  - Chaki et al. "Modular Verification of Software Components in C" ICSE 03, ACM SIGSOFT distinguished paper
  - Betin Can at al. "Application of Design for Verification with Concurrency Controllers to Air Traffic Control Software" ASE 05 best paper
- Systems:
  - Yang et al. “Using Model Checking to Find Serious File System Errors, OSDI 04 best paper.
  - Killian et al. “Life, Death, and the Critical Transition: Finding Liveness Bugs in Systems Code” NSDI 2007 best paper
- Also conferences in Security and Programming Languages have plenty of model checking papers nowadays!

Other issues

- Abstraction
- Bounded model checking
- Dealing with infinite-state transition system
- Automated synthesis

Abstract Interpretation [Cousot and Cousot POPL 77]

- Abstract interpretation provides a general framework for defining abstractions
- The size of the state space of an abstracted system is smaller than the original system, which makes static analysis of the abstract state space feasible
- Different abstract domains can be combined using the abstract interpretation framework
- Abstract interpretation framework also provides conservative approximation techniques such as widening for computing approximations of fixpoints

Predicate Abstraction [Graf and Saidi CAV 97]

- An automated abstraction technique that reduces the state space of a program by removing some variables from the program and just keeping information about a set of predicates about them
- Given a program and a set of predicates, predicate abstraction abstracts the program so that only the information about the given predicates are preserved
- The abstracted program adds nondeterminism since in some cases it may not be possible to figure out what the next value of a predicate will be based on the predicates in the given set
- One needs an automated theorem prover to compute the abstraction

Counter-example Guided Abstraction Refinement

[Clarke et al. CAV 00][Ball and Rajamani SPIN 00]

The basic idea in counter-example guided abstraction refinement is the following:

- First look for an error in the abstract program (if there are no errors, we can terminate since we know that the original program is correct)
- If there is an error in the abstract program, generate a counter-example path on the abstract program
- Check if the generated counter-example path is feasible using a theorem prover.
- If the generated path is infeasible add the predicate from the branch condition where an infeasible choice is made to the predicate set and generate a new abstract program.

Bounded Model Checking [Biere et al. TACAS 99]

- Represent sets of states and the transition relation as Boolean logic formulas
- Instead of computing the fixpoints, unroll the transition relation up to certain fixed bound and search for violations of the property within that bound
- Transform this search to a Boolean satisfiability problem and solve it using a SAT solver