Express (and reasoning over) UML diagrams in OWL(-DL)

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Overview

- OWL as a description logic: OWL-DL
- Functional-Style Syntax (FSS) for OWL-DL
- How to translate UML diagrams to OWL-DL ontologies in FSS
- [Protégé]* as a visual editor to create OWL ontolgies
- [Pellet]** as a reasoner for OWL-DL ontologies

* http://protege.stanford.edu/download/protege/4.0/installanywhere/
** http://pellet.owldl.com/downloads/pellet-2.0.0-rc5.zip

OWL as a Desription Logic

OWL DL is very close to the $\mathcal{SHOIQ}(\mathcal{D})$ Description Logic

- $\mathcal{S} = \mathcal{ALC}_{\mathcal{R}+}$ — $\mathcal{ALC}$* with transitive roles
  TBox axioms of the form $transitive(\boldsymbol{R})$
- $\mathcal{H}$ — role Hierarchies
  TBox axioms of the form $\boldsymbol{R} \sqsubseteq \boldsymbol{S}$, where $\boldsymbol{R}, \boldsymbol{S}$ are roles
- $\mathcal{O}$ — nOminals
  concept constructor $\{\boldsymbol{a}\}$, where $\boldsymbol{a}$ is an individual name
- $\mathcal{I}$ — Inverse roles
  role constructor $\boldsymbol{R}^{-}$, where $\boldsymbol{R}$ is a role name
- $\mathcal{Q}$ — Qualified number restrictions
  concept constructors $\leq n\ \boldsymbol{R.C}$ and $\geq n\ \boldsymbol{R.C}$
- $(\mathcal{D})$ — Datatypes (datatype properties)

* $\mathcal{ALC}$ = $\sqcap$, $\sqcup$, $\neg$, $\exists R.C$, $\forall R.C$

OWL-DL FSS: descriptions(C)

OWL-DL FSS	German DL Syntax	Semantics
A (URI reference)	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
owl:Thing	$\top$	$\text{owl:Thing}^{\mathcal{I}} = \Delta^{\mathcal{I}}$
owl:Nothing	$\bot$	$\text{owl:Nohing}^{\mathcal{I}} = \emptyset$
ObjectIntersectionOf(C_1 C_2)	$C_1 \sqcap C_2$	$C_1{}^{\mathcal{I}} \cap C_2{}^{\mathcal{I}}$
ObjectUnionOf(C_1 C_2)	$\mathrm{C}_1 \sqcup \mathrm{C}_2$	$C_1{}^{\mathcal{I}} \cup C_2{}^{\mathcal{I}}$
ObjectComplementOf(C)	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
ObjectOneOf(o_1 ,...)	$\{o_1, \ldots\}$	$\{o_1{}^{\mathcal{I}}, \ldots\}$
ObjectSomeValuesFrom(R C)	$\exists R.C$	$\{x \mid \exists y\ (x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
ObjectAllValuesFrom(R C)	$\forall R.C$	$\{x \mid \forall y\ (x,y) \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$
ObjectHasValue(R o)	$R : o$	$\{x \mid (x,o^{\mathcal{I}}) \in R^{\mathcal{I}}\}$
ObjectMinCardinality(n R [C])	$\geq nR\ [.C]$	$\{a \in \Delta^{\mathcal{I}} \mid \lvert\{b \mid (a,b) \in R^{\mathcal{I}}\ [\wedge\ y \in C^{\mathcal{I}}]\}\rvert \leq n\}$
ObjectMaxCardinality(n R [C])	$\leq nR\ [.C]$	$\{a \in \Delta^{\mathcal{I}} \mid \lvert\{b \mid (a,b) \in R^{\mathcal{I}}\ [\wedge\ y \in C^{\mathcal{I}}]\}\rvert \geq n\}$

OWL-DL FSS: descriptions(C) (cont.)

OWL-DL FSS	**German DL Syntax**	**Semantics**
DataSomeValuesFrom(U D)	$\exists U.D$	$\{x \mid \exists y\ (x,y) \in U^{\mathcal{I}} \wedge y \in D^{\mathcal{D}}\}$
DataAllValuesFrom(U D)	$\forall U.D$	$\{x \mid \forall y\ (x,y) \in U^{\mathcal{I}} \rightarrow y \in D^{\mathcal{D}}\}$
DataHasValue(U v)	$U : v$	$\{x \mid (x,v^{\mathcal{I}}) \in U^{\mathcal{I}}\}$
DataMinCardinality(n U [D])	$\geq nU\ [.D]$	$\{a \in \Delta^{\mathcal{I}} \mid \lvert\{b \mid (a,b) \in U^{\mathcal{I}}\ [\wedge y \in D^{\mathcal{D}}]\}\rvert \leq n\}$
DataMaxCardinality(n U [D])	$\leq nU\ [.D]$	$\{a \in \Delta^{\mathcal{I}} \mid \lvert\{b \mid (a,b) \in R^{\mathcal{I}}\ [\wedge y \in D^{\mathcal{D}}]\}\rvert \geq n\}$

OWL-DL FSS: data ranges (D) – object properties (R) – data properties (U)

data ranges (D)

D (URI reference)	D	$D^{\mathcal{D}} \subseteq \Delta^{\mathcal{I}}_{\mathcal{D}}$
DataOneOf(v_1, …)	$\{v_1, \ldots\}$	$\{v_1^{\mathcal{I}}, \ldots\}$

object properties (R)

R (URI reference)	R	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \text{ x } \Delta^{\mathcal{I}}$
InverseObjectProperty(R)	R^{-}	$(R^{\mathcal{I}})^{-}$

data properties (U)

U (URI reference)	U	$U^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \text{ x } \Delta^{\mathcal{I}}_{\mathcal{D}}$

individuals(o)

o (URI reference)	o	$o^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$

data values (v)

v	v	$v^{\mathcal{D}}$

OWL-DL FSS: class axioms

OWL-DL FSS	German DL Syntax	Semantics
SubClassOf(C_1 C_2)	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$
DisjointClasses($C_i \ldots . C_j$)	$C_i \sqcap C_j \equiv \bot$, $i \neq j$	$C_i^{\mathcal{I}} \cap C_j^{\mathcal{I}} = \emptyset$, $i \neq j$
EquivalentClasses($C_i \ldots . C_n$)	$C_1 \equiv \ldots \equiv C_n$	$C_i^{\mathcal{I}} = \ldots = C_n^{\mathcal{I}}$
EquivalentClasses(A ObjectOneOf(o_1 ,…))	$A \equiv \{o_{1,} \ldots., o_n\}$	$A^{\mathcal{I}} = \{o_1^{\mathcal{I}}, \ldots., o_n^{\mathcal{I}}\}$

OWL-DL FSS: (object and data) property axioms

OWL-DL FSS	German DL Syntax	Semantics
SubObjectPropertyOf(R_1 R_2)	$R_1 \sqsubseteq R_2$	$R_1{}^{\mathcal{I}} \subseteq R_2{}^{\mathcal{I}}$
ObjectPropertyDomain(R C)	$\geq 1\ R \sqsubseteq C$	$R^{\mathcal{I}} \subseteq C^{\mathcal{I}}$ x $\Delta^{\mathcal{I}}$
ObjectPropertyRange(R C)	$\top \sqsubseteq \forall R.C$	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ x $C^{\mathcal{I}}$
EquivalentObjectProperties(R_i ….R_n)	$R_1 \equiv \ldots \equiv R_n$	$R_1{}^{\mathcal{I}} = \ldots = R_n{}^{\mathcal{I}}$
FunctionalObjetProperty(R)	$\top \sqsubseteq \leq 1\ R$	$R^{\mathcal{I}}$ is functional
SymmetricObjectProperty(R)	$R \equiv R^{-}$	$R^{\mathcal{I}} = (R^{\mathcal{I}})^{-}$
InverseFunctionalObjectProperty(R)	$\top \sqsubseteq \leq 1\ R^{-}$	$(R^{\mathcal{I}})^{-}$ is functional
TransitiveObjectProperty(R)	$Tr(R)$	$R^{\mathcal{I}} = (R^{\mathcal{I}})^{+}$
InverseObjectProperties(R R_0)	$R \equiv (R_0)^{-}$	$R^{\mathcal{I}} = (R_0{}^{\mathcal{I}})^{-}$

Note 1: Blue means that the axiom holds only for object property expression
Note 2: When you are referring to a data property axiom, substitute “Object” with the word “Data”

OWL-DL FSS: individual axioms

OWL-DL FSS	**German DL Syntax**	**Semantics**
ClassAssertion(C o)	$o \in C$	$o^{\mathcal{I}} \in C^{\mathcal{I}}$
ObjectPropertyAssertion(R o_1 o_2)	$\{o_1, o_2\} \in R$	$\{o_1^{\mathcal{I}}, o_2^{\mathcal{I}}\} \in R^{\mathcal{I}}$
DataPropertyAssertion(U o lt)	$\{o, lt\} \in U$	$\{o^{\mathcal{I}}, lt^{\mathcal{I}}\} \in U^{\mathcal{I}}$
SameIndividual($o_{\mathrm{i}} \ldots . o_{\mathrm{n}}$)	$o_1 = \ldots = o_{\mathrm{n}}$	$o_1^{\mathcal{I}} = \ldots = o_{\mathrm{n}}^{\mathcal{I}}$
DifferentIndividuals($o_{\mathrm{i}} \ldots . o_{\mathrm{n}}$)	$o_1 \neq \ldots \neq o_{\mathrm{n}}$	$o_1^{\mathcal{I}} \neq \ldots \neq o_{\mathrm{n}}^{\mathcal{I}}$

Note: $\top_D = \Delta^{\mathcal{I}}_{\mathcal{D}}$ corresponds to rdfs:Literal in OWL

OWL-DL: *Unique Name Assumption (UNA)*

- The Unique Name Assumption says that any two individuals with different names are different individuals
- OWL and DL Semantics do not make the UNA
- Example: Let $\mathcal{T} = \{ \top \sqsubseteq \leq 1\, R \}$ and
  $\mathcal{A} = \{ (a,b) : R, (a,c) : R \}$

  Is $\mathcal{A}$ consistent w.r.t. $\mathcal{T}$?

  Yes! Consider $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}} = \{x, y\}$,
  $R^{\mathcal{I}} = \{(x, y)\}$, $a^{\mathcal{I}} = x$, $b^{\mathcal{I}} = y$ and $c^{\mathcal{I}} = y$