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**Vertical Integration in Two-Sided Markets:
Exclusive Provision and Program Quality**

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Vertical Integration in Two-Sided Markets: Exclusive Provision and Program Quality*

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Abstract

We study distribution and investment in content quality in a two-sided media market. We show that a content provider prefers to provide the premium content exclusively to a platform, no matter what the vertical structure of the industry is. However, a vertically integrated content provider has fewer incentives to invest in quality than an independent one. When downstream platforms are asymmetric, the platform with a competitive advantage on the advertising market gets the exclusive content and the content provider invests even less in quality when it is integrated with it. When we endogenize the vertical structure of the industry, we find that the content provider acquires the platform with a competitive advantage on the advertisers market. Vertical integration reduces both consumer surplus and total welfare. Our results suggest that, in merger control, authorities should carefully assess the effects of the integration on the incentives to invest in content quality. Moreover, a policy intervention at the distribution stage that enforces non-exclusive provision might have adverse effects on consumer surplus and welfare. Also advertising cap could have the effect of reducing quality.

Keywords: exclusive contracts, premium content, investment, quality, media, two-sided markets, vertical integration, merger, advertising cap

JEL Classification: D43, L13, L22, L40, L42, L82

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1 Introduction

Television companies deliver (freely or not) information goods to viewers. They can also air ads, selling on viewers' attention caught with programs to advertisers. When viewers and advertisers interact through the media platform, the market is two-sided: since viewers usually dislike ads while advertisers are interested in reaching a large public, viewers exert a positive externality on advertisers and advertisers exert a negative externality on viewers.

A media platform can increase its profitability by providing premium contents to final users. Premium contents are very attractive contents for viewers and, unlike basic ones, they have few substitutes. Moreover, their production and/or the acquisition of their transmission rights entail high fixed costs. Such contents are important sporting events, blockbuster movies, important television formats, successful television series. The so-called “must-have” content, due to a superior technology and well-known brand names, has a big power in affecting platforms performances. Acquiring exclusive rights over these contents is an important strategy for television companies, since in this way they can differentiate from their competitors and they can be very attractive for viewers and, as a consequence, for advertisers.

Content providers and platforms work more and more in close collaboration. In the last years, many mergers and acquisitions among producers and distributors of contents occurred. Just to quote few examples, consider the following cases: AOL/Time Warner, Comcast/NBCU, Vivendi/Canal+/Seagram, News Corporation/Premiere and the BSkyB's attempt to purchase Manchester United. In these cases, authorities were highly worried by input foreclosure. They concentrated on the anti-competitive effects of exclusive contracts and evaluated whether the merging firm could deny access to important inputs to the rivals in the downstream market.¹

In this paper, we show that the main issue in vertical mergers among content providers and platforms may not arise at the distribution stage but at the production stage. In particular, we show that the incentives of a content provider to offer exclusive or non-exclusive contracts might not change if it is independent or vertically integrated. However, an independent and an integrated content provider might have different incentives to invest in content quality.

We build a model with a monopolist upstream content provider and two downstream platforms. Platforms finance themselves through advertising and subscription fees from viewers.² When the contract for the provision of the premium content allows to extract the maximum willingness-to-pay of the platforms for the content, an independent and a vertically inte-

¹See Crawford (2013) for a discussion on the economic issues in cases of vertical integration in the media market.

²In Section 5.3, we extend the model to consider alternative business models for downstream platforms. We show that our main results hold.

grated content provider always choose to provide the content exclusively to one downstream platform.³ However, when it chooses how much to invest in quality, a vertically integrated firm only takes into account the effects of quality on its downstream profits. Instead, an independent content provider also wants to minimize the revenues left to the platform that receives the exclusive content, so as to maximize the upstream revenues from selling the premium content. We show that a vertically integrated content provider always invests less in the quality of the content than an independent content provider. These results are obtained under quite general conditions: we need that the profit of the highest quality firm increases in the quality asymmetry more than the profit of the lowest quality firm decreases in it. This occurs because the competitive pressure is lower when there is vertical differentiation.

Then, we extend the model to consider asymmetric platforms on the downstream market, since asymmetry seems to be an important factor for explaining control over premium contents. We take into account that an advertiser may get a different benefit from interacting with viewers on different platforms. Indeed, viewer market size is not the only determinant of the attractiveness of a platform for advertisers. We introduce a parameter, linked to platform's characteristics, that can be related to the quality of the service offered to advertisers, to the advertising strategy employed by a platform or to the horizontal quality offered (see Depken II, 2004). We find that this advantage on the advertising market amplifies the effect of quality on profits. This occurs because there exists a complementarity between the viewers' and the advertising markets: if an efficient platform on the advertising market airs a premium content, it gains more revenues from this content than a less efficient platform. Hence, the former platform always gets the premium content exclusively. Still, a vertically integrated content provider invests less than an independent one. Moreover, a content provider invests even less when it integrates with the most efficient platform on the advertising market. Compared to the model with symmetric platforms, quality increases given the market structure. This means that the resources derived from a higher efficiency on the advertising market are used to invest in program quality. This result suggests that an advertising cap might have negative effects on the investment in content quality.

Both consumer surplus and welfare are lower under vertical integration than under vertical separation. More specifically, they are the lowest when the content provider integrates with the most efficient platform on the advertising market. This depends on the investment choices made by the content provider under different industry structures.

In another extension we study the incentives to merge of the content provider with a downstream firm. We find that the content provider gains higher profits by acquiring the

³In Section 5.4, we consider alternative contracts and assumptions on the contractual power of the players, in order to check the robustness of our results.

most efficient platform on the advertising market. Hence, the worst scenario for consumers and society realizes.

Our results suggest that, in merger control, policy makers should not only pay attention to the effects of vertical integration on content exclusivity, but also on the incentives to invest in content quality. These effects should carefully be assessed, and the merger should be blocked if the reduction of the quality provided is large. We also show that the imposition of non-exclusive provision of the quality content always results in lowers quality and may have adverse effects on consumer surplus and welfare, both when it is imposed to an independent content provider or as a remedy for a vertical merger. This depends on the extent of horizontal differentiation..

This paper is organized as follows. Section 2 analyzes the relevant literature. Section 3 presents the formal model where platforms finance themselves through advertising and subscription fees for viewers and solves it. Section 4 studies welfare effects. Section 5 studies some extensions to the basic model, and checks the robustness of the results. Section 6 concludes. Appendix 1 presents the proofs of the basic model and Appendix 2 the ones of the free-to-air model presented in Section 5.

2 Related literature

Our paper relates to the large theoretical literature on two-sided media markets.⁴ The first papers in this branch of literature deal with program mix choices. They find that the maximum differentiation principle found in the one-sided literature could be contradicted in two-sided media markets. This because advertising can push toward minimum platform differentiation (see Gabszewicz, Laussel and Sonnac, 2001, 2002, 2004; Gal-Or and Dukes, 2003). Other papers study market provision of advertising (see, among others, Anderson and Coate, 2005; Peitz and Valletti, 2007), that can be too low or too high compared to the socially optimal choice, depending on the nuisance cost of advertising for viewers. When consumers strongly dislike advertising, platforms tend to air less ads. This result can be influenced by the business model of the platform, by single- and multi-homing assumptions and by the number of active platforms. Another important issue is entry in media markets (see, among others, Choi, 2006, and Crampes, Haritchabalet and Jullien, 2009). These papers find that excessive entry may be an issue.

In the present paper, we focus on a different research question. We study how the distri-

⁴For general papers on two-sided markets, see Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2006). For empirical works on two-sided media markets, see Argentesi and Filistrucchi (2007), Kaiser and Song (2009), Kaiser and Wright (2006).

bution and the production of premium contents are influenced by the vertical structure of the industry in a two-sided media market. For this purpose, we use the framework proposed by the literature, namely, a combination of the Hotelling and the Shaked and Sutton models.⁵

There are few papers that study exclusive strategies in media markets. The first to focus on this issue has been Armstrong (1999). He studies the supply of a premium content provided by an independent content provider to pure pay-TV (that is, TVs without ads financed through subscription fees) under different contractual arrangements. He finds that lump-sum payments for content push for exclusive contracts more than per-subscriber fees. Harbord and Ottaviani (2001) study a similar issue. They find that a content provider finds profitable to sell the premium content exclusively for a lump-sum payment. Moreover, they find that the platform that receives the content chooses to resell it using a per-subscriber fee. Both these models are one-sided, hence they do not consider the effects of competition on the advertising market on the distribution of premium contents.

There are some papers on two-sided markets dealing with various aspects of exclusive distribution of premium contents. Hagiu and Lee (2011) assert that the incentives of a content provider to exclusively provide the quality content depend on whether it keeps the control over the retail price (and revenues) of the content or not. They find that total selling of control rights brings to exclusive provision. Hogendorn and Ka Yat Yuen (2009) analyze how the level of platform interconnection influences exclusivity choices of an independent content provider. None of the previous papers deal with vertical integration of content providers and downstream media platforms. Moreover, they do not consider investments in content production.

The closest papers to ours are the ones by Weeds (2012a, 2012b) and by Stennek (2007). Weeds (2012a) compares the incentives to provide exclusive contents when platforms finance themselves only through advertising or also through subscription fees. She considers an independent content provider that imposes a lump-sum fee for the premium content. While under the pay-TV model exclusive distribution always occurs, this could not be the case under the free-to-air model. In another paper, Weeds (2012b) studies a model where an integrated content provider chooses whether to resell the premium content to a rival content provider using a per-subscriber fee or not. In a static context, the content provider resells the content to its rival. In a dynamic model, the content provider may prefer to keep the exclusive right over the content. The setting of Weeds (2012a) resembles ours with symmetric firms. However, in both papers the focus of the analysis is not a comparison between the vertical separation and the vertical integration case. Moreover, the investment stage is disregarded.

Stennek (2007) studies the relationship between investments in program quality and ex-

⁵For a survey, see Anderson and Gabszewicz (2006).

clusivity, in a bargaining game with alternating offers. He concludes that, since exclusivity can increase quality, it should not be prevented. In this paper, investments in the premium content is taken into account. However, the focus is different from ours. While he wants to investigate whether exclusivity over a premium content can boost investment, we study the effects of the vertical structure of the market over the incentives to invest in quality. Stennek (2007) provides a short comparison on the incentives to resell the premium content in the vertical integration and vertical separation case, but he does not consider how the incentive to invest in quality are affected.

Moreover, the quoted papers always consider symmetric platforms, and disregard the fact that advertisers could perceive the platforms as differentiated and could have different benefits from interacting with viewers on different platforms. The impact of this dimension on the production and distribution of premium contents has been disregarded by the previous literature on two-sided media markets.⁶

More generally, there is a large literature about exclusive dealings, vertical contracting and access to an essential input. This literature, in one-sided markets, has been surveyed by Rey and Tirole (2007) in a paper that analyzes the economics of foreclosure. Moreover, there is a literature dealing with licensing of a cost reduction/quality-enhancing innovation, that is relevant to our work (the closest paper to our model with symmetric platforms is the one by Katz and Shapiro, 1986). However, all this wide literature deals with one-sided markets, while here the two-sidedness of the market plays an interesting role.

3 The model

We consider a two-sided market, where two media platforms compete to attract advertisers and viewers. We consider a pay-TV model, where viewers pay a subscription fee to watch a channel and advertisers purchase advertising space to reach viewers.⁷ Each platform airs a basic channel, and it can improve the quality of its offer by airing a premium content. The premium content is a very valuable content, provided by a monopolist upstream operator. The content provider decides how much quality to provide, and it negotiates with platforms for exclusive or non-exclusive provision of the premium content. We compare the form of

⁶Roson (2008) analyzes the role of a similar parameter on horizontal and vertical differentiation choices of platforms. However, his model is quite different from ours. He does not consider a premium content provider, but each platform internally chooses how much quality to provide. Moreover, the timing he chooses annihilates many two-sided effects.

⁷In Section 5.3, we consider a free-to-air model, where viewers watch channels for free and platforms finance themselves through advertising, and a pure pay-TV model, where viewers pay to watch a channel without ads.

distribution chosen by the content provider and its incentives to invest in quality when it is independent or vertically integrated with one of the platforms.

3.1 Basic assumptions

Platforms. Each platform provides a channel. The two platforms, indexed by $i \in \{1, 2\}$, are horizontally differentiated and are located at the two extremes of a Hotelling line: platform $i = 1$ is located in zero and platform $i = 2$ in one.⁸ Platforms finance themselves through subscription fees from viewers and advertising revenues.⁹ Hence, platform i 's profit function is $\pi_i = p_i q_i + P_i(q_i, a_i) a_i$, where p_i is the subscription fee for viewers, q_i is the mass of viewers joining platform i , a_i the amount of advertising and $P_i(q_i, a_i)$ is the inverse demand for advertising.¹⁰ We normalize the production cost of the basic channel and the marginal cost of distribution to zero. Each platform sets p_i and a_i .¹¹

Viewers. There is a large mass (normalized to one) of viewers, with a preference parameter x for horizontal quality, uniformly distributed over the $[0, 1]$ interval. Each viewer watches one and only one channel.¹² The net utilities of a consumer of type $x \in [0, 1]$ from platform 1 and 2 respectively are:

$$U_1 = V + \gamma_1 - xt - \delta a_1 - p_1 \quad (1)$$

$$U_2 = V + \gamma_2 - (1 - x)t - \delta a_2 - p_2 \quad (2)$$

where V is the gross surplus from the basic content of platform i , assumed large enough that all viewers watch a channel. A viewer of type x stands a dis-utility from watching a channel

⁸As Weeds (2012a) points out, horizontal differentiation may not just arise from the type of the basic programs offered by the platforms, but also from the transmission technology used to broadcast the signal and/or from other services bundled with the basic channel.

⁹We do not endogenize the business model of the platform, so we do not consider the case where the business model collapses into a pure pay-TV case or to a free-to-air case. It is like assuming to work in the region where the deviation toward a pure pay-TV or a free-to-air business models is not profitable for the platforms.

¹⁰We assume that the premium content is a film, a television format or a sport event, hence advertising revenues are collected by downstream platforms. If the premium content were a channel, the upstream firm would receive advertising revenues.

¹¹As it will be clear in the following, in this model advertisers multi-home while consumers single-home. Hence, each platform has monopoly power in delivering the attention of its consumers to advertisers (for the competitive bottleneck model see Armstrong, 2006). This entails that it is equivalent to assume that a platform sets the per-viewer advertising price or the advertising level a_i .

¹²One can think that viewers have idiosyncratic preferences for channels, and that they subscribe only to the channel they prefer. See Ambrus, Calvano and Reisinger (2013) and Anderson, Foros and Kind (2013) for models with multi-homing consumers and advertisers.

that is not of its preferred horizontal specification (see Hotelling, 1929). This dis-utility depends on the “distance” of consumer x from the channel and on the transportation cost t . Parameter γ_i represents the quality of the premium content offered by platform i : $\gamma_i = 0$ when platform i does not offer the premium content, while $\gamma_i = \gamma$ when platform i airs it. Consumers dislike advertising, so they suffer a utility loss that depends on the advertising level a_i and on the nuisance cost δ .¹³ We assume that each viewer has the same marginal utility from the premium content and the same marginal dis-utility from ads.¹⁴

Advertisers. Advertisers use ads in order to inform viewers about their products, since viewers are also consumers of their products. Advertisers can join none, one or both platforms. There is a mass one of advertisers. Following Anderson and Coate (2005), each advertiser produces a product of quality $k \in [0, \bar{k}]$. k is distributed according to a p.d.f. F on this interval. We assume that $F(0) = 0$ and that F is increasing and continuously differentiable with a strictly log concave density. On platform i there is a fraction α_i of viewers who have willingness-to-pay $k > 0$ for a good of quality k , while a fraction $(1 - \alpha_i)$ who has willingness-to-pay equal to zero. We assume that $\alpha_1 = \alpha_2$.¹⁵ Since each producer has monopoly power, it imposes a price for the good that extracts all consumer surplus. In formal terms, the profit function of advertiser k on platform i is $k\alpha_i q_i - P_i(q_i, a_i)$.

The inverse demand function for advertising can be rewritten as $P_i(q_i, a_i) = q_i r_i(a_i)$, where $r_i(a_i)$ is the price for one ad that reaches one viewer. This means that each producer’s willingness-to-pay to reach a viewer is independent of the number of viewers reached. In the following, $R_i(a_i) = a_i r_i(a_i)$ denotes platform i ’s revenues per-viewer from advertising. The assumptions on F imply that $R'_i(a_i)$ is decreasing when positive. Advertisers’ production costs are normalized to zero.

The upstream operator. The monopolist upstream operator U produces a premium content of quality $\gamma > 0$ and sells it to downstream platforms.¹⁶ The upstream firm may

¹³The assumption that viewers dislike advertising is empirically documented by Wilbur (2008) in the US TV market and by Jeziorski (2011) in the US radio market.

¹⁴Premium contents are an important driver of viewers’ subscription. In order to underline their importance, we assume that all consumers equally like the premium content. For simplicity, we assume that horizontal taste is independent of the vertical one. Liu, Putler and Weinberg (2004) make a similar assumption. This might not be the case in reality, and some consumers might not be interested in the premium content at all. Our qualitative results would be robust to a setting where only a portion of consumers on the line is interested in the premium program, if the comparative statics of profits with respect to quality continue to hold, as it will be clear in the following.

¹⁵In Section 5.1, we consider an extension of the main model where advertisers have different benefits from interacting with viewers on different platforms, i.e. $\alpha_1 < \alpha_2$.

¹⁶We assume that a platform that receives the exclusive content does not resell it to the rival. Armstrong (1999) shows the validity of this assumption with lump sum fees.

offer this premium content exclusively to platform $i = \{1, 2\}$ or non-exclusively to both platforms.¹⁷ A contract among the upstream operator and platform i specifies the quality γ_i of the offer and a fixed price T_i .¹⁸ In Section 3.3, we describe in details the contract we use. We assume that the production of the premium content entails a quadratic fixed cost $\frac{\mu\gamma^2}{2}$ proportional to the square of the quality provided γ .¹⁹ Parameter μ is the “cost of quality”, in the sense that the impact of quality on cost increases with μ . We normalize marginal costs of production and distribution to zero.

Welfare. Total welfare is given by the sum of gross surplus from content and from advertising at the net of the fixed cost of production of the premium content, i.e. $W = W^c + W^a - \frac{\mu\gamma^2}{2}$. Gross surplus from content is defined as:

$$W^c = \int_0^{q_1} (V + \gamma_1 - xt) dx + \int_{q_1}^1 (V + \gamma_2 - (1-x)t) dx \quad (3)$$

Gross surplus from advertising as:

$$W^a = q_1 \int_{\underline{k}_1}^{\bar{k}} (k\alpha_1 - \delta) dF(k) + q_2 \int_{\underline{k}_2}^{\bar{k}} (k\alpha_2 - \delta) dF(k) \quad (4)$$

Consumer surplus is defined as the integral over all purchasing consumers of their utility:

$$CS = \int_0^{q_1} U_1 dx + \int_{q_1}^1 U_2 dx \quad (5)$$

Timing We consider a game in three stages. First, the upstream operator invests in the production of the premium content, determining its quality. Second, the upstream firm contracts with platforms for the provision of the premium content. Third, platforms simultaneously compete for advertisers and viewers; advertisers and viewers simultaneously make their consumption choices.

¹⁷A remark is in order at this point. A platform could strategically buy a premium content to horizontally differentiate itself from the rival platform. Hence, the level of horizontal differentiation could be influenced by the platform buying or not the premium content. In this model, we assume that the horizontal dimension is independent of the vertical one. Hence, airing or not the premium content does not modify the position of the platform on the Hotelling line.

¹⁸Armstrong (1999) provides examples for the use of lump-sum fees in contracts among premium content providers and media platforms. Harbord and Ottaviani (2001) describe under which conditions fixed fees are the best option for the upstream content provider. In our analysis, we do not consider contracts that specify per-subscriber fees. We discuss them in Section 5.4. By using fixed fees, we abstract from the effects of these per-subscriber fees on downstream competition.

¹⁹In the present model, we consider only “controllable elements for program quality”, as Liu, Putler, Weinberg (2004) call them. Those are the aspects of quality that depend on the monetary investment in the quality of the program. There also exist “uncontrollable elements” of quality, that we do not consider here.

We solve the game by backward induction.

3.2 Third stage: equilibrium for given quality level

At stage 3, platforms sell contents to viewers and advertising space to advertisers. In this Section, we compute equilibrium demands, prices and profits as a function of γ_i with $i \in \{1, 2\}$.

First, we determine viewers' demands. We find the viewer \bar{x} who is indifferent between the two channels equalizing (1) and (2). Solving for x , we obtain:

$$\bar{x} = \frac{1}{2} + \frac{\gamma_1 - \gamma_2 + p_2 - p_1 + \delta(a_2 - a_1)}{2t} \quad (6)$$

All viewers to the left of \bar{x} join platform 1, while all viewers to the right join platform 2. Hence, viewers' implicit demands are $q_1 = \bar{x}$ and $q_2 = (1 - \bar{x})$.

Then, we determine the inverse per-viewer demand for advertising. The marginal advertiser on platform i is the one which makes zero profits, that is $\underline{k}_i = \frac{P_i(q_i, a_i)}{\alpha_i q_i}$. This entails that the demand for ads on platform i is $a_i = 1 - F\left(\frac{P_i}{\alpha_i q_i}\right) = 1 - F\left(\frac{r_i}{\alpha_i}\right)$. Thus, the inverse per-viewer demand for advertising of platform i is:

$$r_i(a_i) = \alpha_i F^{-1}(1 - a_i) \quad (7)$$

Using the definition in Section 3.1, the profit function of platform i can be rewritten as $\pi_i = p_i q_i + q_i R_i(a_i)$. We maximize platform i 's profits with respect to the subscription fee for viewers p_i and the advertising level a_i . The system of the four first order conditions is:

$$\frac{\partial \pi_i}{\partial a_i} = p_i \frac{\partial q_i}{\partial a_i} + q_i \frac{\partial R_i}{\partial a_i} + R_i \frac{\partial q_i}{\partial a_i} = 0 \quad i = \{1, 2\} \quad (8)$$

$$\frac{\partial \pi_i}{\partial p_i} = q_i + p_i \frac{\partial q_i}{\partial p_i} + R_i \frac{\partial q_i}{\partial p_i} = 0 \quad i = \{1, 2\} \quad (9)$$

Solving the system of the two first order conditions (8), we find that

$$R'_i(a_i^*) = \delta \quad (10)$$

The level of advertising chosen by platform i only depends on the nuisance cost δ and on the shape of the revenue function from advertising $R_i(a_i)$, and is independent of the decision of the rival platform j . Indeed, in the competitive bottleneck model, each platform has monopoly power over its audience and decides the level of advertising so as to maximize the joint surplus of the platform and its consumers (see Armstrong, 2006). Using the Implicit

Function Theorem, it is easy to show that a_i^* decreases in δ and increases in α_i . This discussion implies that a_i^* is independent of the quality of the channel for viewers.

Solving the system of the two first order conditions (9), we find that the subscription fee for viewers on platform i as a function of a_i^* is

$$p_i^* = t + \frac{\gamma_i - \gamma_j}{3} + \frac{\delta(a_j^* - a_i^*)}{3} - \frac{2R_i(a_i^*) + R_j(a_j^*)}{3} \quad (11)$$

The first term in (11) represents the classical Hotelling term. The second one is due to the introduction of a quality differentiation parameter in the Hotelling model: if platform i has a quality advantage over the rival platform j , i.e. $\gamma_i - \gamma_j > 0$, it can ask for a higher price to viewers. The third term is due to the dis-utility from advertising: since advertising is a nuisance for consumers, a platform lowers its subscription fee as it increases the advertising time and the rival decreases its.²⁰ Then, the fourth term is linked to the fact that viewers receive a discount that depends on their “value” on the advertising market. High per-viewer advertising revenues make price competition tougher, since each viewer is very valuable on the advertising market. The subscription fee decreases in α_i , i.e. as a platform becomes more efficient on the advertising market. It also decreases in α_j , i.e. as the rival platform becomes more efficient, since competition in price for viewers becomes tougher. The direct effect is stronger than the indirect one, that is, p_i decreases more rapidly in α_i than in α_j .²¹

From equation (9) we know that $a_1^* = a_2^*$ when $\alpha_1 = \alpha_2$. Hence, (11) can be rewritten as $p_i^* = t + \frac{\gamma_i - \gamma_j}{3} - R_i(a_i^*)$. By substitution, we derive viewers’ demand for platform i :

$$q_i^* = \frac{1}{2} + \frac{\gamma_i - \gamma_j}{6t} \quad (12)$$

The demand for platform i depends on the quality gap between the two platforms ($\gamma_i - \gamma_j$) and this quality gap plays a more important role when t is low. We concentrate the analysis in the region where platforms have positive demands from viewers (i.e. $1 > q_i > 0$ for $i \in \{1, 2\}$, under the covered market assumption $q_1 + q_2 = 1$, which is the region where $t > \frac{\gamma_i - \gamma_j}{3}$ with $i, j \in \{1, 2\}$ $i \neq j$) and from advertisers (i.e. $a_i > 0$ for $i \in \{1, 2\}$, that is $\bar{k}\alpha_i > \delta$ for $i \in \{1, 2\}$). In this region, second order conditions hold.

Then, platform i ’s third stage equilibrium profit is:

$$\pi_i^* = \frac{1}{2t} \left(t + \frac{\gamma_i - \gamma_j}{3} \right)^2 \quad (13)$$

²⁰For a platform, airing less ads than the rival has an impact on the subscription fee similar to a quality advantage.

²¹Observe that, in the feasible interval, prices for viewers can be negative. Indeed, the platform can find profitable to subsidize the viewers’ side of the market with revenues from the advertising market.

Platform i 's profits are convex in $(\gamma_i - \gamma_j)$. Hence, the profit of the highest quality firm increases with the asymmetry more than the profits of the lowest quality firm decreases in it. This occurs since the competitive pressure is lower when there is vertical differentiation.²² In the Hotelling model, only the quality gap matters. This entails that, when $\gamma_i = \gamma_j$, quality does not play a role in profits.

3.3 Second stage: distribution of the premium content

At this stage, the content provider contracts with downstream platforms for exclusive or non-exclusive provision of the premium content of quality γ produced at the first stage.²³

First, let us introduce some notations. In the following, Π denotes first and second stage equilibrium profits. Superscript *ei* denotes equilibrium variables when there is exclusive provision of the premium content to platform i , *ne* when there is non-exclusive provision of the content, while 0 when no platform receives the premium content. Moreover, we use (VIi) to denote the scenario of vertical integration with platform i and (VS) to denote the one of vertical separation.

We model a three stage bargaining process, where the upstream firm sequentially offers an exclusive contract to platform $i \in \{1, 2\}$ and a non-exclusive contract to both platforms.²⁴ It makes these three offers in its preferred order, and each of these contracts can be offered just once. The negotiation stops when a contract is accepted, or at the end of these three stages if no platform accepts an offer. As Armstrong (1999) points out, this is a credible procedure that allows the content provider to obtain the maximum payoff. At each stage of the negotiation, the contract offered to platform i specifies the quality of the offer γ_i and a fixed price T_i for it. The content provider sets the tariff T_i so as to fill the individual rationality constraint of the platform to which it offers the contract.²⁵ The maximum tariff that the upstream firm can ask to a platform depends on both the profits that the platform might have from the provision of the quality content under the form of the contract under negotiation and on its outside option. Note that the outside option of the platform depends

²²This last feature is common to models of product differentiation with linear demands under Bertrand and Cournot competition, as Bester and Petrakis (1993) point out.

²³For simplicity, we assume that under non-exclusive provision the upstream firm provides the same quality to both firms. This would be the result in a game where the upstream firm can offer different qualities to the two platforms when they sign a non-exclusive contract.

²⁴This contract gives the same outcome as a first price auction, where the minimum bid is fixed by the upstream operator and it is equal to the maximum willingness-to-pay of the platforms (Katz and Shapiro, 1986).

²⁵We assume that the contract is enforceable. That is, once the contract is signed, an authority verifies its enforcement, imposing high sanctions if it is not honored. It is like assuming that there is a reputation cost from not honoring the contract. This hypothesis is intended to give some dynamic to the static model.

on the order in which the offers are done. We assume that, if a platform is indifferent between accepting or refusing the contract, it accepts it. Formally, platform i 's maximum willingness to pay for the premium content is

$$\pi_i(\gamma_i = \gamma; \gamma_j) - \pi_i^{ej} \quad (14)$$

where $\pi_i(\gamma_i = \gamma; \gamma_j)$ is the profit of platform i when it airs the premium content, with $\gamma_j \in \{0, \gamma\}$, and π_i^{ej} the profit of platform i when the rival platform j airs the premium content exclusively. Hence, the maximum tariff a content provider can set is $T_i^e = \pi_i^{ei} - \pi_i^{ej}$ for the exclusive content to platform i (in this case $\gamma_j = 0$) and $T_i^{ne} = \pi_i^{ne} - \pi_i^{ej}$ for the non-exclusive content (in this case $\gamma_j = \gamma$), with $i, j \in \{1, 2\}$ and $i \neq j$.²⁶ In all these cases, the platform is left with its minimum outside option, that is π_i^{ej} . Using equation (13), it is easy to verify that $\pi_i^{ei} > \pi_i^{ne} = \pi_i^0 > \pi_i^{ej}$.

The upstream firm decides its preferred form of representation in order to maximize its total profits.²⁷ In our setting, this contract is the one that Segal (1999) calls the “efficient contract”, in the sense that the allocation of the content that arises in equilibrium maximizes industry profits (subject to stage 3 price competition). This contract allows us to abstract from all inefficiencies that could arise at the contracting stage. In the following, first we consider the case where the content provider is independent and then the one where it is integrated with a downstream platform.

Vertical separation. Assume that the content provider is an independent firm. Its profits are simply given by revenues collected from the sale of the premium content to downstream platforms. The content provider decides the allocation of the content of quality γ so as to maximize its profits. Using the incentive constraints we can show that:

$$\Pi_U^{e1}(VS) = \pi_1^{e1} - \pi_1^{e2} = \pi_2^{e2} - \pi_2^{e1} = \Pi_U^{e2}(VS) \quad (15)$$

$$\Pi_U^{ne}(VS) = (\pi_1^{ne} - \pi_1^{e2}) + (\pi_2^{ne} - \pi_2^{e1}) < \pi_i^{ei} - \pi_i^{ej} = \Pi_U^{ei}(VS) \quad (16)$$

Hence, we can state the following Proposition:

²⁶Non-exclusive provision of the quality content allows the content provider to create a prisoner's dilemma on the downstream market: both platforms would prefer not to accept the contract, but they cannot coordinate on that choice. If platform i rejects the contract, platform j is always better off by accepting it.

²⁷In the bargaining process that we model, the upstream firm holds all the bargaining power. In this way we take into account the fact that the producer of a premium content possesses significant power in bargaining with platforms. In Section 5.3 we discuss the assumptions on the bargaining power and the contractual arrangement.

Proposition 1. *An independent content provider always provides the premium content exclusively to one downstream platform.*

We give now some intuitions for this result. Since platforms are symmetric on the downstream market, (15) is verified since $\pi_1^{e1} = \pi_2^{e2}$ and $\pi_1^{e2} = \pi_2^{e1}$. Inequality (16) follows from convexity of downstream profits of platform i in $(\gamma_i - \gamma_j)$, that implies that $\pi_j^{ne} - \pi_j^{ei} < \pi_i^{ei} - \pi_i^{ne}$.

By conveniently rearranging the terms in the incentive constraint of the upstream firm in (15) and (16) and taking into account symmetry of the platforms, it can be shown that at the second stage the content provider chooses the scenario where total industry profits are maximized (given competition at stage 3). Indeed, we find that, by (16), $\Pi_U^{ne} < \Pi_U^{ei} \iff \pi_1^{ne} + \pi_2^{ne} < \pi_i^{ei} + \pi_j^{ei}$ and, by (15), $\Pi_U^{e1} = \Pi_U^{e2} \iff \pi_1^{e1} + \pi_2^{e1} = \pi_1^{e2} + \pi_2^{e2}$.

In order to induce platform i to accept the exclusive offer paying T_i^e , the upstream firm has to threaten it to provide the content exclusively to its rival platform j in case it rejects the offer, so that the outside option of platform i is π_i^{ej} . The credibility of this threat crucially depends on the order in which the offers are done by the content provider. In order to make the threat credible, the content provider first offers the premium content under a non-exclusive contract for an infinite price to both platforms. No platform accepts.²⁸ Second, it offers the quality content to platform i for a price $T_i^e = \pi_i^{ei} - \pi_i^{ej}$. Platform i knows that, if it rejects the offer, the upstream firm will offer the quality content to platform j for a tariff that makes it indifferent between airing the quality content or not, that is for a tariff $\pi_j^{ej} - \pi_j^0$, and that platform j would always accept this offer. Hence, platform i accepts the offer of the upstream firm. The payoffs at stage 2 of the independent content provider, of platform 1 and of platform 2 respectively are: $\Pi_U^{ei}(VS) = \frac{2\gamma}{3}$, $\Pi_i^{ei}(VS) = \pi_i^{ej}$ and $\Pi_j^{ei}(VS) = \pi_j^{ei}$.

Vertical integration. Now, assume that the upstream operator is vertically integrated with platform $i \in \{1, 2\}$. In this case, additionally to the upstream revenues, the integrated firm gains the downstream profit of the affiliated platform i . The integrated firm decides whether to air the premium content exclusively, to sell it exclusively to the rival platform j or to provide it non-exclusively. We assume that the transfer price for the content to the subsidiary platform i is zero. We verify that:

$$\Pi_i^{ei}(VIi) = \pi_i^{ei} = \pi_i^{ej} + \pi_j^{ej} - \pi_j^{ei} = \Pi_i^{ej}(VIi) \quad (17)$$

$$\Pi_i^{ne}(VIi) = \pi_i^{ne} + \pi_j^{ne} - \pi_j^{ei} < \pi_i^{ei} = \Pi_i^{ei}(VIi) \quad (18)$$

²⁸The offer of the non-exclusive content for an infinite price is one possible strategy that the upstream firm can use to induce platform i to refuse the non-exclusive offer and to accept the exclusive contract having as an outside option the exclusivity to the rival platform.

Hence, we can conclude that:

Proposition 2. *An integrated content provider always provides the premium content exclusively to one downstream platform.*

Equality (17) tells us that the integrated platform is indifferent between airing the premium content or giving it exclusively to the rival. Indeed, platforms are symmetric and the contract allows a vertically integrated content provider to extract the maximum revenue from the sale of the exclusive content to the rival platform, so as to compensate the losses on the downstream market from not airing the premium content. Inequality (18) follows from convexity of downstream profits of platform i in $(\gamma_i - \gamma_j)$.

As for the vertical separation case, we can rearrange the terms of these inequalities, showing that the integrated firm chooses the contract that maximizes industry profits, subject to competition at stage 3. Indeed, (17) can be rewritten as $\pi_1^{e1} + \pi_2^{e1} = \pi_1^{e2} + \pi_2^{e2}$, while (18) as $\pi_1^{ne} + \pi_2^{ne} < \pi_i^{ei} + \pi_j^{ei}$.

We can sum up the results of this Section in the following Corollary:

Corollary 1. *The distribution of the quality content is not affected by the vertical structure of the industry: a content provider always provides the premium content exclusively to one downstream platform.*

It is important to note that our results on the distribution of the quality content are quite general. Even if we consider an extended Hotelling model, the main results follow from the comparative statics of profits with respect to quality and thus apply more generally. Results derive from the fact that the profit of the platform with an advantage in quality increases in quality more than the profit of the lowest quality platform decreases in it. This occurs since the competitive pressure is lower in more asymmetric situations. Hence, industry profits are maximized when one downstream platform gets the exclusive content. This implies that, when the contract is efficient (in the sense of Segal, 1999), it is always optimal for the content provider to exclusively provide the quality content to only one platform.

3.4 First stage: investment in quality

At stage 1 the content provider invests in content quality, anticipating platforms competition at stage 3 and the distribution decision at stage 2. The content provider's choice of investment is determined by the point where the marginal benefit and the marginal cost with respect to γ are equal, that is $\frac{\partial \Pi}{\partial \gamma} = \mu\gamma$.²⁹ The content provider faces the same marginal cost under

²⁹Second order conditions hold for $9t\mu > 1$.

all industry structures. As concerns the benefits, we have shown that the content provider always provides the premium content exclusively, no matter what the vertical structure of the industry is (see Corollary 1). However, the vertical structure of the industry affects the profits of the content provider, thus its incentives to invest at stage 1. By comparing the marginal benefits from quality for the content provider under different vertical structures, we verify that the following relation holds:

$$\frac{\partial \Pi_U^{ei}(VS)}{\partial \gamma} > \frac{\partial \Pi_i^{ei}(VIi)}{\partial \gamma} = \frac{\partial \Pi_i^{ej}(VIi)}{\partial \gamma} \quad (19)$$

Hence, we can state the following Proposition:

Proposition 3. *An independent content provider provides a content of higher quality than a vertically integrated one.*

At stage 2, the payoff of an independent content provider is $\Pi_U^{ei}(VS) = \pi_i^{ei} - \pi_i^{ej}$, the one of a vertically integrated platform $i \in \{1, 2\}$ is $\Pi_i^{ei}(VIi) = \Pi_i^{ej}(VIi) = \pi_i^{ei}$. A vertically integrated content provider chooses the quality level that maximizes its downstream profits π_i^{ei} . An independent one not only wants to maximize the downstream profits π_i^{ei} of the platform that airs exclusively the content, but also wants to minimize the profits π_i^{ej} left to it, so as to be able to increase the profits extracted from i through the fee. Since π_i^{ei} increases in γ and π_i^{ej} decreases in it, then (19) holds. Our results just depend on the sign of the comparative statics with respect to the quality γ , hence they can be generalized. The quality levels provided at equilibrium under the different vertical structures of the industry are

$$\gamma(VS) = \frac{2}{3\mu} \quad (20)$$

$$\gamma(VIi) = \frac{3t}{9t\mu - 1} \quad (21)$$

Both decrease in μ , and (21) also decreases in t .

4 Welfare analysis

In this Section, we compare consumer and total surplus under different vertical structures of the industry. First, we calculate consumer surplus as it is defined in (5), with $\gamma_i = \gamma$ and $\gamma_j = 0$ (see Proposition 1 and 2). Then, we calculate the derivative of it with respect to

γ . Since it is positive, consumers are better off when they receive a higher quality content. Using this result and Proposition 3, we can state the following

Proposition 4. *Consumer surplus is higher when the content provider is independent than when it is vertically integrated.*

All consumers are better off under vertical separation. Indeed, consumers who join the platform airing the premium content both under vertical integration and vertical separation are better off when they enjoy a content of higher quality. Also consumers who switch from the platform without the quality content under vertical integration to the platform with the quality content under vertical separation are better off in the latter scenario. Indeed, the higher price that all those consumers pay for this content does not overcome the advantage they derive from quality, since they do not have all their surplus extracted by the platform. Consumers who stick to the platform without the quality content under both scenarios never watch the premium content, but pay a lower price under vertical separation, since the subscription fee decreases with the quality offered by the rival platform.

As concerns advertisers surplus, it does not vary with the market structure. Indeed, neither the demand nor the price for advertising are affected by the quality of the program.

Let us look at the change in social welfare. Both gross consumer surplus and the fixed cost for the content increase with the quality level γ , hence results are not a priori clear. We find that the higher fixed costs implied by a higher quality does not offset the gross surplus created for consumers. Hence, we can state the following

Proposition 5. *Total welfare is higher when the content provider is independent than when it is vertically integrated.*

It is useful to study whether there is under- or over-provision of quality under different market structures, given platforms' choices at stage 2 and 3. In order to do so, we compute total welfare given prices and demands decided at stage 3 and the distribution decision at stage 2, i.e. $\gamma_i = \gamma$ and $\gamma_j = 0$. Then, we study the sign of the first derivative of welfare with respect to γ .³⁰ We find that $\frac{\partial W}{\partial \gamma} = \frac{1}{2} + \frac{5\gamma}{18t} - \mu\gamma$. By simple algebra, it can be shown that under vertical separation there can be over- or under-provision, since $\frac{\partial W}{\partial \gamma}$ can be negative or positive when calculated for $\gamma = \gamma(VS)$. Instead, when $\gamma = \gamma(VI)$, there is always under provision of quality, since $\frac{\partial W}{\partial \gamma}$ is positive.³¹ Hence, an independent content provider and a

³⁰We perform this analysis in the region where the welfare function is concave, i.e. for $t > \frac{5}{18\mu}$. Indeed, only in this region the quality that maximizes total welfare is positive.

³¹Indeed, $\frac{\partial W}{\partial \gamma}(\gamma = \gamma(VS)) = -\frac{1}{6} + \frac{5}{27t\mu}$ can be negative or positive, while $\frac{\partial W}{\partial \gamma}(\gamma = \gamma(VI)) = \frac{9t\mu+2}{6(9t\mu-1)}$ is always positive.

vertically integrated one could deviate from the optimal provision of quality in a different way. In any case, the independent content provider deviates from the optimum less than a vertically integrated one.

5 Extensions

In this Section we consider some extensions of the basic model. In Section 5.1 we consider the case where the two downstream platforms are asymmetric in the advertising market. In Section 5.2 we endogenize the vertical structure of the industry, studying a merger between the content provider and a downstream platform. In Section 5.3 we model two different business model for the platforms: a pure pay-TV and a free-to air model. Finally, in Section 5.4, we discuss the assumptions on the bargaining power and the form of contract used in the basic model, and how results could be affected by a change in these assumptions.

5.1 Asymmetric platforms

We now extend the Anderson and Coate (2005)'s framework by taking into account that advertisers can perceive platforms as differentiated. Formally, we solve the game described in Section 3, taking now into account that each advertiser gets a higher benefit from interacting with viewers on platform 2 than on platform 1, i.e. $\alpha_1 < \alpha_2$ (see Depken II, 2004, and Wilbur, 2008). This inequality may be linked to a more effective advertising strategy employed by platform 2, to a better service offered to advertisers, to horizontal differentiation between platforms, to a reputation effect. We perform this analysis since asymmetry between downstream platforms is an important dimension for explaining exclusive contracts.

Now, we solve the game by backward induction. At stage 3, the solution follows the one in Section 3.2. By the system of first order conditions, we find (10) and (11). Differently from the main model, the two platforms air different levels of advertising at equilibrium. Since $R'_i(a_i)$ is decreasing when positive, from (10) we find that a_i^* increases with α_i . Hence, at equilibrium, $a_1^* < a_2^*$. By substitution, viewers' demand for platform i is:

$$q_i^* = \frac{1}{2} + \frac{\gamma_i - \gamma_j}{6t} + \frac{\delta(a_j^* - a_i^*)}{6t} + \frac{R_i(a_i^*) - R_j(a_j^*)}{6t} \quad (22)$$

As before, platform i 's market share increases with the quality advantage. Moreover, now viewers' demands depend on advertising. Since advertising is a nuisance for viewers, the market share of platform i decreases in δa_i^* and increases in δa_j^* . Moreover, since platforms subsidize viewers using revenues on the advertising market, the market share of platform i

increases in $R_i(a_i^*)$ and decreases in $R_j(a_j^*)$. The advantage of platform 2 on the advertising market is represented by the term $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*)$, which is positive given the assumptions on R_i .³² Platform i 's third stage equilibrium profits are:

$$\pi_i^* = \frac{1}{2t} \left(t + \frac{\gamma_i - \gamma_j}{3} + \frac{\delta(a_j^* - a_i^*)}{3} + \frac{R_i(a_i^*) - R_j(a_j^*)}{3} \right)^2 \quad (23)$$

Differently from the main model, the parameters related to the advertising market play a role in the analysis. We find that π_i^* increases in α_i and decreases in α_j . Moreover, π_1^* increases in δ , while π_2^* decreases in it.

At the second stage, the content provider decides whether to provide the premium content exclusively or non-exclusively to downstream platforms. First, let the content provider be an independent firm. Studying its incentive constraints, we find that

$$\Pi_U^{e1}(VS) = \pi_1^{e1} - \pi_1^{e2} < \pi_2^{e2} - \pi_2^{e1} = \Pi_U^{e2}(VS) \quad (24)$$

$$\Pi_U^{ne}(VS) = \pi_1^{ne} - \pi_1^{e2} + \pi_2^{ne} - \pi_2^{e1} < \pi_2^{e2} - \pi_2^{e1} = \Pi_U^{e2}(VS) \quad (25)$$

Second, let the content provider be vertically integrated with platform $i = 1$. We verify that the following inequalities hold:

$$\Pi_1^{e1}(VI1) = \pi_1^{e1} < \pi_1^{e2} + \pi_2^{e2} - \pi_2^{e1} = \Pi_1^{e2}(VI1) \quad (26)$$

$$\Pi_1^{ne}(VI1) = \pi_1^{ne} + \pi_2^{ne} - \pi_2^{e1} < \pi_1^{e2} + \pi_2^{e2} - \pi_2^{e1} = \Pi_1^{e2}(VI1) \quad (27)$$

Third, let the content provider be integrated with platform $i = 2$. We find that:

$$\Pi_2^{e1}(VI2) = \pi_2^{e1} + \pi_1^{e1} - \pi_1^{e2} < \pi_2^{e2} = \Pi_2^{e2}(VI2) \quad (28)$$

$$\Pi_2^{ne}(VI2) = \pi_2^{ne} + \pi_1^{ne} - \pi_1^{e2} < \pi_2^{e2} = \Pi_2^{e2}(VI2) \quad (29)$$

Inequalities (25), (27) and (29) hold because π_2^{e2} increases in quality more rapidly than π_1^{e2} decreases in it: the gains of platform 2 more than compensate the losses of platform 1. Differently from the model with symmetric platforms, now inequalities (24), (26) and (28) hold strictly because π_2^{e2} increases in quality faster than π_1^{e1} and π_2^{e1} decreases in quality faster

³²We assume positive demands for viewers and advertisers and covered market on the viewer side. We find that $1 > q_i > 0$ if $t > \frac{\gamma_i - \gamma_j}{3} + \frac{\delta(a_j^* - a_i^*)}{3} + \frac{R_i(a_i^*) - R_j(a_j^*)}{3}$ with $i, j \in \{1, 2\}$ $i \neq j$.

than π_1^{e2} . This occurs because a higher efficiency on the advertising market of platform 2 amplifies the effect of quality on profits. Platform 1 could not perform better than platform 2 with the quality content. Formally, the term $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*) > 0$ drives the results. Hence, the premium content is provided exclusively to the most efficient platform.

In order to induce platform 2 to accept its offer and to pay the maximum price $T_2^e = \pi_2^{e2} - \pi_2^{e1}$ for it, the upstream operator can use the same strategy as in the basic model.³³ Observe that now the upstream firm asks to platform 2 a higher price for the content compared to the one it could ask to platform 1, both for the exclusive and the non-exclusive content. Hence, the content provider is able to price discriminate among the two platforms.

We can sum up the results at the second stage in the following:

Proposition 6. *When platforms are asymmetric, the content provider provides the premium content exclusively to the most efficient platform on the advertising market. Results are not affected by the vertical structure of the industry.*

The content provider always chooses the scenario that maximizes stage 2 industry profits, given competition at stage 3. Indeed, by rearranging the terms of the incentive constraints of the content provider under different industry structures, we always find that $\pi_1^{e1} + \pi_2^{e1} < \pi_1^{e2} + \pi_2^{e2}$ and $\pi_1^{ne} + \pi_2^{ne} < \pi_1^{e2} + \pi_2^{e2}$. Industry profits are the highest when the firm with a preexisting advantage on the advertising market exclusively airs the quality content. Indeed, in this case the asymmetry between the two downstream platforms increases and the competitive pressure on the downstream market decreases. There exists some complementarity between the two “quality” advantages, the one on the advertising market linked to $\alpha_1 < \alpha_2$ and the one on the viewers’ market linked to the control over the premium content, that lies in the two-sidedness of the market. Controlling the premium content, platform 2 can expand its market share on the viewers market, and then sell on the attention of many viewers, conquered through the premium content, to advertisers.

At stage 1, the content provider invests in content quality. We calculate the marginal benefit of the content provider with respect to γ under vertical separation and vertical integration, anticipating that at stage 2 the premium content is provided exclusively to platform 2. We find that:

$$\frac{\partial \Pi_U^{e2}(VS)}{\partial \gamma} > \frac{\partial \Pi_1^{e2}(VI1)}{\partial \gamma} > \frac{\partial \Pi_2^{e2}(VI2)}{\partial \gamma} \quad (30)$$

³³The vertically integrated platform 1 may induce the rival platform 2 to accept an exclusive contract also by offering the contract at the third stage of the bargaining, while making non affordable offers before. Indeed, differently from an independent content provider, the integrated platform 1 can ask to the rival platform to pay the tariff $T_2^e = \pi_2^{e2} - \pi_2^{e1}$ also at the last stage of the bargaining. This is because the threat of using the content if the rival rejects the offer is always credible.

Hence, we conclude that

Proposition 7. *An independent content provider has higher incentives to invest in quality than a vertically integrated one. Under vertical integration, the content provider has the lowest incentives to invest in quality when it is integrated with the most efficient platform on the advertising market.*

The stage 2 payoff of an independent content provider is $\Pi_U^{e2}(VS) = \pi_2^{e2} - \pi_2^{e1}$, the one of the vertically integrated platform 1 is $\Pi_1^{e2}(VI1) = \pi_1^{e2} + \pi_2^{e2} - \pi_2^{e1}$ and the one of the vertically integrated platform 2 is $\Pi_2^{e2}(VI2) = \pi_2^{e2}$. Hence, $\frac{\partial \Pi_U^{e2}(VS)}{\partial \gamma} > \frac{\partial \Pi_1^{e2}(VI1)}{\partial \gamma}$, since π_1^{e2} decreases in γ , and $\frac{\partial \Pi_1^{e2}(VI1)}{\partial \gamma} > \frac{\partial \Pi_2^{e2}(VI2)}{\partial \gamma}$ since π_2^{e1} decreases in quality faster than π_1^{e2} . When the integrated platform 2 chooses the quality level, it only considers its downstream profits. Instead, an independent content provider and the integrated platform 1 take into account the revenues from selling the premium content on the downstream market to platform 2. The price $T_2^e = \pi_2^{e2} - \pi_2^{e1}$ for the premium content is set under the threat of giving the content to platform 1 if platform 2 does not accept. This threat increases the incentives to invest in quality of the independent content provider and the integrated platform 1, since they want to extract platform 2's profits, leaving it with the lowest possible profits. Platform 1 provides less quality than an independent content provider since it takes into account also the negative effect on its downstream profit of providing the premium content exclusively to the rival platform 2. However, it still provides more quality than the integrated platform 2, since the efficiency of the latter on the advertising market amplifies the effect of quality on profits ($\frac{\partial(\pi_1^{e2} - \pi_2^{e1})}{\partial \gamma} > 0$).

At equilibrium, the quality level provided are $\gamma(VS) = \frac{2}{3t\mu} \left(t + \frac{\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*)}{3} \right)$, $\gamma(VI1) = \frac{3(t + \delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*))}{9t\mu - 1}$ and $\gamma(VI2) = \frac{3t + \delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*)}{9t\mu - 1}$. Qualities are always increasing in α_2 and decreasing in α_1 , t and μ . From the comparison of these results with the ones in (20) and (21) obtained for $\alpha_1 = \alpha_2$, we find that, given the vertical structure of the industry, the quality of the premium content increases with the asymmetry on the advertising market, hence, with the efficiency on the advertising market of the platform that airs the content. This depends on the fact that the revenues that platform 2 gets on the advertising market due to its efficiency increase the incentives to invest in quality, proportionally to the term $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*)$.

To conclude, we evaluate consumer surplus and welfare under different vertical structures of the industry. By comparing them, we can state the following Proposition:

Proposition 8. *Consumer surplus and welfare are higher when the content provider is independent than when it is vertically integrated. Consumer surplus and welfare are the lowest when the content provider is integrated with the most efficient platform on the advertising market.*

This result occurs because under vertical separation a larger portion of viewers watch the premium content, and that this content is of higher quality. Moreover, advertisers can reach a larger public through the most efficient platform on the advertising market. The higher surplus of both groups more than compensate the higher fixed cost under vertical separation.

5.2 Control over Premium Content

In this Section, we add a stage zero to the timing described in Section 3.1. In this stage the upstream content provider makes an offer, when interested, to one downstream platform in order to acquire control over it.³⁴ We are interested in analyzing the structure of the industry that is chosen by firms. We perform this study for the model with asymmetric platforms presented in Section 5.1. In this way we are able to study the role of the efficiency on the advertising market in merger decisions.

At stage zero, all players know by backward induction the decisions concerning prices, exclusivity and investment taken under different scenarios. First, we verify whether firms prefer to be separated or integrated. The upstream content provider makes an offer to platform i if the profits it makes when it is vertically integrated with the latter, at the net of the price paid for acquiring platform i , are higher than the ones it makes when it is vertically separated. Platform i accepts only if the payment it gets from the upstream content provider is at least as high as its outside option, that is, its downstream profit under vertical separation. Hence, the minimum price that satisfies the incentive constraint of platform i is $\Pi_i^{e2}(VS)$. We find that

$$\left(\Pi_i^{e2}(VIi) - \frac{\mu\gamma(VIi)^2}{2} \right) - \Pi_i^{e2}(VS) > \left(\Pi_U^{e2}(VS) - \frac{\mu\gamma(VS)^2}{2} \right) \quad i \in \{1, 2\} \quad (31)$$

Hence, the upstream content provider is always willing to pay the minimum price at which a downstream firm i is willing to give up the control over the platform. Rearranging the terms, one can say that the sum of the upstream and downstream profits of an independent content provider and an independent platform i (i.e. $\Pi_U^{e2}(VS) + \Pi_i^{e2}(VS) - \frac{\mu\gamma(VS)^2}{2}$) are lower than the

³⁴We rule out the case where the upstream operator wants to buy both downstream platforms. Considering the case where the merger produces a monopoly on the downstream market would entail antitrust concerns that are not the issue of this work.

profit of the integrated platform i (i.e. $\Pi_i^{e2}(VIi) - \frac{\mu\gamma(VIi)^2}{2}$). This occurs because the vertical integrated content provider internalizes the effect of the provision of the quality content on the downstream profit of the platform it controls.

Second, we study whether the content provider prefers to acquire platform 1 or 2. In order to do so, we compare the profits of the content provider when it is integrated with each platform at the net of the price paid for acquiring that platform. We find that:

$$\left(\Pi_1^{e2}(VI1) - \frac{\mu\gamma(VI1)^2}{2}\right) - \Pi_1^{e2}(VS) < \left(\Pi_2^{e2}(VI2) - \frac{\mu\gamma(VI2)^2}{2}\right) - \Pi_2^{e2}(VS) \quad (32)$$

This means that the content provider is willing to acquire platform 2. The profits from integrating with platform 2 are higher than the ones from integrating with 1, that is $\Pi_1^{e2}(VI1) - \frac{\mu\gamma(VI1)^2}{2} < \Pi_2^{e2}(VI2) - \frac{\mu\gamma(VI2)^2}{2}$, since the same firm directly controls the production and the distribution of the premium content. However, the content provider has to pay a higher price to integrate with platform 2 rather than with 1, that is, $\Pi_1^{e2}(VS) < \Pi_2^{e2}(VS)$. However, this higher price does not offset the higher profits generated from integrating with it. Hence, we can state:

Proposition 9. *The upstream content provider merges with the most efficient platform on the advertising market.*

The content provider and platform 2 are better off when they merge than when they stay separated. Also platform 1 is better off when platform 2 merges with the content provider. This is because π_1^{e2} decreases in γ (by Propositions 1 and 2, in any case platform 1 does not get the premium content) and, by Proposition 3, the quality provided to platform 2 is lower when it is vertically integrated with the content provider.³⁵

5.3 Other platform business models: pure pay-TV and free-to-air

In this Section, we consider two different business models for the downstream platforms: a pure pay-TV case, where viewers pay a subscription fee to join platforms that do not air ads, and a free-to-air model, where platforms distribute free contents to viewers and sell viewers' attention to advertisers.

First, we study the pure pay-TV case. The profit function of platform i is $\pi_i = p_i q_i$. This is a sub-case of the pay-TV model studied in the main Section. Indeed, it can be easily shown that platforms' equilibrium profits at stage 3 are the same as in (13). Hence, all

³⁵The results would not change if platforms made offers to acquire the content provider.

conclusions of the symmetric model hold, since they are based on comparative statics on the profit function.

Second, we study the free-to-air case. We perform the analysis considering symmetric platforms, i.e. $\alpha_1 = \alpha_2$. Platform i 's profit function is $\pi_i = a_i P_i(q_i, a_i) = q_i R_i(a_i)$. We consider the basic assumptions in Section 3.1, with the only difference that at stage 3 platforms decide only over a_i , since subscription fees for viewers are zero. We proceed solving the game by backward induction.³⁶

At stage 3, we specify equilibrium demands, prices and profits as a function of quality levels γ_i . In order to calculate platforms' demands, we determine the consumer \bar{x} who is indifferent between subscribing to platform 1 and 2. She can be obtained by equation (6), taking into account that $p_1 = p_2 = 0$. Hence, viewers' demand for platform i is $q_i = \frac{1}{2} + \frac{\gamma_i - \gamma_j + \delta(a_j - a_i)}{2t}$. The inverse per-viewer demand of advertisers is the same as equation (7). Then, we maximize platforms' profits with respect to the advertising demand a_i . From the system of the two first order conditions $\frac{\partial \pi_i}{\partial a_i} = q_i R'_i(a_i) + R_i(a_i) \frac{\partial q_i}{\partial a_i} = 0$ $i = \{1, 2\}$, we find that

$$R'_i(a_i^*) = \frac{R_i(a_i^*) \delta}{2tq_i} \quad (33)$$

Differently from the pay-TV model, advertisers' demands do depend on the quality level of the premium content. By the Implicit Function Theorem, it can be shown that a_i^* increases in γ_i and decreases in γ_j . Also q_i^* increases in γ_i and decreases in γ_j . By the Envelope Theorem, we show that π_i^* increases in γ_i and decreases in γ_j . Since only the quality gap matters, when $\gamma_i = \gamma_j$ platforms' profits are the same as without any quality content. By rewriting the revenue per-viewer as $R_i(a_i) = \alpha_i a_i F^{-1}(1 - a_i)$ and using (33), we find that a_i^* is independent of α_i . Even if α_i does not have any effect on the equilibrium level of advertising, it plays a role in equilibrium profits, that are given by $\pi_i^* = q_i^* R_i(a_i^*) = q_i^* \alpha_i a_i^* F^{-1}(1 - a_i^*)$. Hence, π_i^* increases in α_i .³⁷

At stage 2 the content provider decides whether to provide the quality content of a given quality γ exclusively to one platform or non-exclusively to both. We use the same contract as the one described in Section 3.3. This contract allows the content provider to impose a price for the premium content equal to the maximum willingness-to-pay of the platform for it, as specified in equation (14). In so doing, as we have already shown for the pay-TV model, the content provider always chooses the scenario where industry profits are maximized. We find that

³⁶ All proofs for the results on the free-to-air scenario are in Appendix B.

³⁷ We concentrate the analysis in the region where the market for viewers is covered and where each platform has positive demands from viewers and advertisers. Second order conditions hold if $R''_i < R'_i \frac{\delta}{tq_i}$.

Proposition 10. *Under the free-to-air model, the content provider provides the premium content exclusively to platform i if and only if:*

$$\frac{R_i(a_i^*)}{2t} \left(1 + \delta \frac{\partial a_j^*}{\partial \gamma}\right) > \frac{R_j(a_j^*)}{2t} \left(1 - \delta \frac{\partial a_i^*}{\partial \gamma}\right) \quad (34)$$

Otherwise, it provides the premium content non-exclusively to both platforms. Results are independent of the vertical structure of the industry.

Inequality (34) verifies whether the profit of platform i increases with γ more than the profit of platform j decreases with it, when platform i receives the premium content exclusively. This inequality is always verified if the second derivative of the function $R_i(a_i)$ for $i = \{1, 2\}$ is not so negative. If, instead, the second derivative of $R_i(a_i)$ is very small, this inequality may be violated and non-exclusive distribution could arise at equilibrium. Indeed, when the revenues on the advertising market from selling the exclusive content are small compared to the losses of the platform without the quality content, the content provider has higher profits from providing the quality content to both platforms.

Then, at stage 1, the content provider chooses the quality of the premium content γ . From Proposition 10 we know that the content provider chooses the same form of distribution of the premium content when it is independent and vertically integrated. Both when the content is exclusively provided to platform i and non-exclusively to both platforms, we verify that $\frac{\partial \Pi_U(VS)}{\partial \gamma} > \frac{\Pi_i(VIi)}{\partial \gamma}$. Hence, we can state the following

Proposition 11. *Under the free-to-air model, an independent content provider has higher incentives to invest in quality than a vertically integrated one.*

This entails that Proposition 3 is confirmed in the free-to-air model. The intuitions for this result are the same as in the main model.³⁸

Finally, we find that consumers always prefer to have a higher quality content, given the choice of distribution of the premium content. Hence, from Proposition 10 and 11, we conclude that consumer surplus is higher under vertical separation than under integration.

³⁸ Assume that platforms are asymmetric on the advertising market, i.e. $\alpha_1 < \alpha_2$. At stage 2, we find that the premium content is provided exclusively to platform 2 or non-exclusively to both. The premium content is never provided exclusively to platform 1. Indeed, even if the parameter α_i does not affect the choice over a_i^* , it amplifies the effects of quality on profits and industry profits are higher when platform 2 gets the exclusive content than when platform 1 gets it. From equilibrium profits, it can be easily shown that $\frac{\partial \pi_2^e}{\partial \gamma} \leq \frac{\partial \pi_1^e}{\partial \gamma}$ and $\frac{\partial \pi_2^e}{\partial \gamma} \geq \frac{\partial \pi_1^e}{\partial \gamma}$. This entails that, when the content is provided exclusively and platforms are asymmetric, it is provided to platform 2. At stage 1, we confirm that the investment in quality is the highest under vertical separation and the lowest when the content provider is integrated with platform 2. Hence, Proposition 7 is confirmed in the free-to-air model. The proof is the same as the one of Proposition 7, since the same comparative statics of downstream profits with respect to quality hold.

To conclude, even if the mechanisms in the free-to-air model are not the same as in the pay-TV one, the policy implications in Section 6 are still the same.

5.4 Contract form and bargaining power

The negotiation we designed in the basic model allows us to abstract from any (ad hoc) inefficiency at the contracting stage so as to concentrate on the production stage. However, other contractual forms are possible. Moreover, the upstream firm could have a limited bargaining power.

First, it can be shown that the main conclusions of the model hold if downstream platforms have some bargaining power. Assume that there is Nash bargaining among platforms and the content provider. In this case, the fee that the content provider sets to platform i for an exclusive offer is equal to $T_i^e = \lambda (\pi_i^{ei} - \pi_i^{ej})$ and for a non-exclusive contract is $T_i^{ne} = \lambda (\pi_i^{ne} - \pi_i^{ej})$, where $\lambda \in [0, 1]$ represents the bargaining power of the content provider.³⁹ When the upstream content provider is an independent firm, its decision at stage 2 concerning the distribution of the premium content is not affected by parameter λ , that cancels out when we compare the profits under exclusive and non-exclusive distribution of the content. However, the investment decision at stage 1 is affected by parameter λ , and the quality produced is reduced and is now equal to $\lambda\gamma(VS)$. When the content provider is integrated with platform i , it is not anymore indifferent between retaining exclusive rights over the premium content and providing it exclusively to the rival platform j . Hence, the vertically integrated platform i keeps the premium content exclusively. Since its benefit are the same as in the basic model, the quality provided at stage 1 is not affected, and it is equal to (21). If the market power of the upstream platform is low (i.e. λ is low), the price it could ask to the platform under vertical separation for the premium content is very low. In this case, the integrated content provider could provide a higher quality than the independent content provider. Interestingly, we find that the merger decision at stage zero is the same as in the main model, since the profits of the couple producer-distributor of the premium content are higher when the content provider and platform i are integrated. We conclude that, when the premium content is important enough, which implies also a high λ in this model, the policy implications of the basic model are still valid.

In the basic model we designed a three stage contract that gives the same bargaining power to an integrated and an independent content provider. Assume now that the content provider is able to make the second offer to platforms with probability zero. In this case, the bargaining process is one-shot and the upstream firm can propose its preferred contract

³⁹When $\lambda = 1$ the content provider has all the bargaining power. This is the case of the basic model.

(either an exclusive or non-exclusive contract) to downstream platforms just once. The parties know that this is the only chance to reach an agreement. Now, an independent content provider, differently from a vertically integrated one, is not able to threat a platform to give the premium content to the rival in the event the former rejects the exclusive contract. Hence, the highest price it can impose for the exclusive content to platform i is $\pi_i^{ei} - \pi_i^0$, that is lower than the price T_i^e set under a three-stage bargaining (and set by a vertically integrated platform). Now, an independent content provider provides the content to both platforms when horizontal differentiation is high enough, while it provides the content exclusively to platform i otherwise. Under vertical integration there are more exclusive contracts than under vertical separation. Moreover, welfare can be higher under exclusive provision and vertical integration than under non-exclusive provision and vertical separation.

One could also think of a contract where the content provider imposes a per-viewer fee for the premium content. In this case, the content provider finds profitable to increase the number of viewers reached by the premium content. Hence, we could expect to find more non-exclusive provision of the quality content.⁴⁰ However, such a contract could neither be feasible (when the number of viewers cannot be monitored) nor desirable (see Harbord and Ottaviani, 2001, for a discussion). Moreover, a contract that specifies per-viewer fees would distort downstream competition.

Finally, when the content provider earns revenues from advertising, and this is the case when the content provided is a channel, we also find that there is more non-exclusive provision of the premium content compared to our basic setting. Indeed, the content provider can increase its revenues from advertising by providing the content to all the market. In particular, Weeds (2012a) examines the case where the content provider earns the revenues from advertising and sets a per-viewer fee for the premium content. Her results confirm our intuitions.

6 Policy implications

The model presented here highlights that even if the form of distribution of the premium content might not be affected by the vertical structure of the industry, the incentives to invest in the premium content might change with it. In the model, the premium content is always exclusively provided to one platform (which is the most efficient on the advertising market when platforms are asymmetric). However, the incentives of the content provider to invest in quality are higher under vertical separation than under vertical integration. Moreover,

⁴⁰See Weeds (2012a) and Harbord and Ottaviani (2001).

a content provider integrated with a highly efficient firm on the advertising market has the lowest incentives to invest in quality.

In an extension, we find that the content provider always finds profitable to merge with a downstream platform, that is the most efficient platform on the advertising market when platforms are asymmetric. Both consumer surplus and total welfare are higher when the content provider is independent, since it invests in higher quality. Hence, the model predicts that the equilibrium scenario is the one where consumer surplus and total welfare are the lowest. This occurs under quite general conditions: we need downstream competition to relax in more asymmetric situations.

The previous discussion wants to highlight the fact that, in merger control, authorities should not just be worried by the effect of vertical integration on the distribution of premium contents, hence, by input foreclosure. The main issue could arise at the production stage, when the content provider chooses how much to invest in the development and the production of the content. Obviously, a direct intervention of an authority at this stage would be neither feasible nor desirable. However, in this setting, imposing vertical separation would be a feasible and beneficial intervention of the authority. Indeed, both consumer surplus and total welfare are higher when the upstream firm is independent.

It is interesting to investigate the effects of an intervention of the authority at the distribution stage, like the imposition of non-exclusive provision. First, we find that this intervention entails a drop in quality, under all the market structures we consider. A content provider, either separated or integrated, produces always a higher quality when it provides the exclusive content to one platform than when it provides the content to both platforms. Imposing non-exclusive provision may have adverse effects on consumer surplus and welfare. Indeed, under exclusive provision, the content is of higher quality but it is provided only to a part of the market. Moreover, the consumers who enjoy the premium content pay a higher price for it, while the others receive a discount. Under non-exclusive provision, the quality provided is lower, but all the market enjoys it. We find that both consumer surplus and total welfare are higher under non-exclusive distribution than under exclusive distribution when t is high enough. Indeed, when the transportation cost is low, platforms are closer substitutes for viewers and it can be socially beneficial to have a higher premium content aired by only one platform. The model anticipates that an intervention of the authority at the distribution stage could not produce positive effects for consumers and society.

Also a more intrusive intervention, where both vertical separation and non-exclusive provision are imposed, might not be beneficial for consumers and society. In particular, consumer surplus and total welfare are higher compared to the basic model only when t is high enough. The intuition is the same as before.

Moreover, we have an interesting result in Section 5.1. We find that the efficiency of platform 2 in the advertising market increases the quality provided at equilibrium by the content provider under all market structures. This implies that the imposition of a (binding) advertising cap on platforms has a negative effect on the quality of the premium content.

7 Conclusions

In the media market we can observe many mergers among platforms and content producers. In the present work we investigate how vertical integration, as opposed to vertical separation, can affect exclusive distribution and quality investments in premium contents. We find that the premium content is always granted on an exclusive basis to one platform, both under vertical separation and under vertical integration. When platforms are asymmetric, the platform which gets the exclusive contract is the most efficient on the advertising market, no matter what the vertical structure of the industry is: the market power on the downstream market determines the outcome of the bargaining for the exclusivity over the premium content.

However, we find that the vertical structure of the industry plays a role when we study the incentives of the content provider to invest in quality. Indeed, an independent content provider has higher incentives to invest in quality compared to a vertically integrated one. We also find that the investment in quality decreases with the efficiency on the advertising market of the subsidiary platform. When we endogenize the merger decision, we find that vertical integration is always the final outcome. Moreover, the content provider chooses to merge with the most efficient platform on the advertising market.

Vertical integration lowers consumer surplus and total welfare. We find the worst results in terms of consumer surplus and total welfare when the content provider is integrated with the most efficient platform on the advertising market. Since they would receive higher program quality, consumers and society prefer the upstream firm to remain independent. Even if the model is static, some dynamic consideration can be drawn. It can be observed a trend toward concentration, in the sense that firms always prefer the scenario of vertical integration. Moreover, the premium content is always exclusively provided to the most efficient platform on the advertising market. This exacerbates the differences of platforms on the downstream market. The effect of vertical integration on exclusivity over valuable program is one of the questions in the agenda of public authorities. We highlight that other aspects should be kept in mind in merger control, like the effects of vertical integration on the incentives to invest in quality.

Appendix 1

Proof of Proposition 1. It has been showed in the text that the content provider chooses the form of distribution where total industry profits are maximized (given competition at stage 3). Thus, we have to show that stage 2 industry profits are maximized when platform $i = \{1, 2\}$ gets the exclusive content. First, a preliminary result: in a Hotelling model we find that $\pi_1^{ne} + \pi_2^{ne} = \pi_1^0 + \pi_2^0$, and that $\frac{\partial \pi_1^{ne}}{\partial \gamma} = \frac{\partial \pi_2^{ne}}{\partial \gamma} = 0$, since only quality gaps play a role. Assume that $\gamma_i = \gamma$ and $\gamma_j = 0$, with $i, j = \{1, 2\}$ and $i \neq j$. Observe that π_i^{ei} increases in γ , since $\frac{\partial \pi_i^{ei}}{\partial \gamma} = \frac{1}{3t} \left(t + \frac{\gamma}{3} \right) > 0$, while π_j^{ei} decreases in it, since $\frac{\partial \pi_j^{ei}}{\partial \gamma} = -\frac{1}{3t} \left(t - \frac{\gamma}{3} \right) < 0$. In order to show that $\pi_i^{ei} + \pi_j^{ei} > \pi_1^{ne} + \pi_2^{ne}$, we need to show π_i^{ei} increases in quality more rapidly than π_j^{ei} decreases in it. It can be easily shown that $\left| \frac{\partial \pi_i^{ei}}{\partial \gamma} \right| = \frac{1}{3t} \left(t + \frac{\gamma}{3} \right) > \left| \frac{\partial \pi_j^{ei}}{\partial \gamma} \right| = \frac{1}{3t} \left(t - \frac{\gamma}{3} \right)$, since viewers' demands are positive. Hence, the upstream content provider offers the exclusive content to platform $i = \{1, 2\}$. ■

Proof of Proposition 2. It is the same as the Proof of Proposition 1. ■

Proof of Proposition 3. At stage 1, the content provider produces the quality that maximizes its profits under exclusive provision to platform i (see Proposition 1 and 2). The content provider's choice of investment is determined by the point where the marginal benefit $\frac{\partial \Pi}{\partial \gamma}$ and the marginal cost $\mu\gamma$ are equal. The marginal benefit depends on the vertical structure of the industry. First, assume that the content provider is an independent firm. Its revenues from the sale of the exclusive content to platform i are $\Pi_U^{ei}(VS) = \pi_i^{ei} - \pi_i^{ej}$. Assume now that platform i is vertically integrated with the content provider. It has the same revenues from keeping the premium content exclusively ($\Pi_i^{ei}(VI) = \pi_i^{ei}$) and selling it exclusively to the rival platform j ($\Pi_i^{ej}(VI) = \pi_i^{ej} + \pi_j^{ej} - \pi_j^{ei}$) since, by symmetry, $\pi_i^{ej} = \pi_j^{ei}$ and $\pi_i^{ei} = \pi_j^{ej}$. We conclude that the independent platform invests more than an integrated platform, i.e. $\frac{\partial \Pi_U^{ei}(VS)}{\partial \gamma} > \frac{\partial \Pi_i^{ei}(VI)}{\partial \gamma} = \frac{\partial \Pi_i^{ej}(VI)}{\partial \gamma}$, since, as shown in the Proof of Proposition 1, $\frac{\partial \pi_i^{ei}}{\partial \gamma} > 0$ and $\frac{\partial \pi_i^{ej}}{\partial \gamma} < 0$. ■

Proof of Proposition 4. Consumer surplus is defined in (5). Assume without loss of generality that platform $i = 2$ airs exclusively the premium content, i.e. $\gamma_2 = \gamma$ and $\gamma_1 = 0$. The derivative of consumer surplus with respect to the quality γ is $\frac{\partial CS}{\partial \gamma} = U_1(x = q_1^*) \frac{\partial q_1^*}{\partial \gamma} + \int_0^{q_1^*} \frac{\partial U_1(x)}{\partial \gamma} dx - U_2(x = q_1^*) \frac{\partial q_1^*}{\partial \gamma} + \int_{q_1^*}^1 \frac{\partial U_2(x)}{\partial \gamma} dx$. Since, by definition, q_1^* is equal to the indifferent consumer, then $U_1(x = q_1^*) = U_2(x = q_1^*)$, and we can rewrite $\frac{\partial CS}{\partial \gamma} = \int_0^{q_1^*} \frac{\partial U_1(x)}{\partial \gamma} dx + \int_{q_1^*}^1 \frac{\partial U_2(x)}{\partial \gamma} dx$. Since, $\frac{\partial U_1(x)}{\partial \gamma} = -\frac{\partial p_1^*}{\partial \gamma}$ and $\frac{\partial U_2(x)}{\partial \gamma} = 1 - \frac{\partial p_2^*}{\partial \gamma}$, then $\frac{\partial CS}{\partial \gamma} = \left[-\frac{\partial p_1^*}{\partial \gamma} \right]_0^{q_1^*} + \left[1 - \frac{\partial p_2^*}{\partial \gamma} \right]_{q_1^*}^1 = -q_1^* \frac{\partial p_1^*}{\partial \gamma} +$

$q_2^* - q_2^* \frac{\partial p_2^*}{\partial \gamma} = \frac{1+q_2^*}{3} > 0$ (by the covered market assumption, $q_2^* = 1 - q_1^*$). Consumer surplus increases with the quality of the premium content and by Proposition 3 $\gamma(VS) > \gamma(VIi)$, hence consumer surplus is higher when the content provider is independent than when it is vertically integrated with platform i . ■

Proof of Proposition 5. Total welfare is defined as $W = W^c + W^a - \frac{\mu\gamma^2}{2}$. Assume without loss of generality that platform $i = 2$ airs exclusively the premium content, i.e. $\gamma_2 = \gamma$ and $\gamma_1 = 0$. Gross surplus from content is defined in (3) and it can be rewritten as $W^c = V + \gamma q_2^* - \frac{t}{2} + t q_2^* - t (q_2^*)^2 = V - \frac{t}{4} + \frac{\gamma}{2} + \frac{5\gamma^2}{36t}$, considering that $q_2^* = \frac{1}{2} + \frac{\gamma}{6t}$. Gross surplus from advertising is defined in (4). Since $\int_{\underline{k}_i}^{\bar{k}} \delta dF(k) = \delta (F(\bar{k}) - F(\underline{k}_i)) = \delta a_i^*$ and $\int_{\underline{k}_i}^{\bar{k}} k \alpha_i dF(k) = \int_{\underline{k}_i}^{\bar{k}} k \alpha_i f(k) dk = \alpha_i \left[\frac{r_i a_i^*}{\alpha_i} + \int_{\underline{k}_i}^{\bar{k}} (1 - F(k)) dk \right] = R_i(a_i^*) + \alpha_i \int_{\underline{k}_i}^{\bar{k}} (1 - F(k)) dk$ (since $\frac{d(1-F(k))}{dk} = -f(k)$), it can be rewritten as $W^a = R_i(a_i^*) - \delta a_i^* + \alpha_i \int_{\underline{k}_i}^{\bar{k}} (1 - F(k)) dk$. Gross surplus with respect to advertising is the same under all vertical structure of the industry and it cancels out when we compare welfares. We now show that $W^c(\gamma = \gamma(VS)) - \frac{\mu(\gamma(VS))^2}{2} > W^c(\gamma = \gamma(VIi)) - \frac{\mu(\gamma(VIi))^2}{2}$. This can be rewritten as $\frac{\gamma(VS)}{2} \left(1 + \frac{5\gamma(VS)}{18t} - \mu\gamma(VS) \right) > \frac{\gamma(VIi)}{2} \left(1 + \frac{5\gamma(VIi)}{18t} - \mu\gamma(VIi) \right)$. By substituting the equilibrium values of the quality content, $\gamma(VS) = \frac{2}{3\mu}$ and $\gamma(VIi) = \frac{3t}{9t\mu-1}$, and rearranging the terms, we can rewrite it as $\frac{1053t^2\mu^2-324t\mu+20}{324t\mu^2(9t\mu-1)^2} > 0$. By simple algebra, we find that it is positive for each $t > \frac{2}{9\mu}$, that always holds in the region under analysis. ■

Proof of Proposition 6. Following the same reasoning as in the Proof of Proposition 1, we show that industry profits are maximized (given competition at stage 3) when platform 2 gets the exclusive content. Since only quality gaps create profits, we find that $\pi_1^{ne} + \pi_2^{ne} = \pi_1^0 + \pi_2^0$, and that $\frac{\partial \pi_1^{ne}}{\partial \gamma} = \frac{\partial \pi_2^{ne}}{\partial \gamma} = 0$. Assume that $\gamma_i = \gamma$ and $\gamma_j = 0$. Using the Envelope Theorem, it can be easily shown that $\frac{\partial \pi_i^{ei}}{\partial \gamma} + \frac{\partial \pi_i^{ei}}{\partial a_i} \frac{\partial a_i}{\partial \gamma} + \frac{\partial \pi_i^{ei}}{\partial a_j} \frac{\partial a_j}{\partial \gamma} = \frac{\partial \pi_i^{ei}}{\partial \gamma}$ and $\frac{\partial \pi_j^{ei}}{\partial \gamma} + \frac{\partial \pi_j^{ei}}{\partial a_i} \frac{\partial a_i}{\partial \gamma} + \frac{\partial \pi_j^{ei}}{\partial a_j} \frac{\partial a_j}{\partial \gamma} = \frac{\partial \pi_j^{ei}}{\partial \gamma}$, where, for $i, j \in \{1, 2\}$ and $i \neq j$, $\frac{\partial a_i}{\partial \gamma} = \frac{\partial a_j}{\partial \gamma} = 0$ since demands for advertising do not depend on quality. We find that $\frac{\partial \pi_i^{ei}}{\partial \gamma} = \frac{1}{3t} \left(t + \frac{\gamma}{3} + \frac{\delta(a_j^* - a_i^*)}{3} + \frac{R_i(a_i^*) - R_j(a_j^*)}{3} \right) > 0$ and $\frac{\partial \pi_j^{ei}}{\partial \gamma} = -\frac{1}{3t} \left(t - \frac{\gamma}{3} - \frac{\delta(a_j^* - a_i^*)}{3} + \frac{R_i(a_i^*) - R_j(a_j^*)}{3} \right) < 0$ since viewers' demands are positive.

First, we prove that $\pi_1^{e2} + \pi_2^{e2} > \pi_1^{ne} + \pi_2^{ne}$. In order to do so, we need to show that π_2^{e2} increases in quality more rapidly than π_1^{e2} decreases in it. By the Envelope Theorem, we find that $\left| \frac{\partial \pi_1^{e2}}{\partial \gamma} \right| = \frac{1}{3t} \left(t - \frac{\gamma}{3} + \frac{\delta(a_2^* - a_1^*)}{3} + \frac{R_1(a_1^*) - R_2(a_2^*)}{3} \right) < \left| \frac{\partial \pi_2^{e2}}{\partial \gamma} \right| = \frac{1}{3t} \left(t + \frac{\gamma}{3} + \frac{\delta(a_1^* - a_2^*)}{3} + \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right)$. The inequality holds since $\gamma > 0$ and $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*) > 0$. Second, we prove that $\pi_1^{e2} + \pi_2^{e2} > \pi_1^{e1} + \pi_2^{e1}$. In order to do so, we proceed by steps. On one hand, we show that π_2^{e2} increases with γ more rapidly than π_1^{e1} . By the Envelope Theorem, we find that

$\frac{\partial \pi_2^{e2}}{\partial \gamma} = \frac{1}{3t} \left(t + \frac{\gamma}{3} + \frac{\delta(a_1^* - a_2^*)}{3} + \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right) > \frac{\partial \pi_1^{e1}}{\partial \gamma} = \frac{1}{3t} \left(t + \frac{\gamma}{3} + \frac{\delta(a_2^* - a_1^*)}{3} + \frac{R_1(a_1^*) - R_2(a_2^*)}{3} \right)$, that holds since $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*) > 0$. On the other hand, we show that the π_2^{e1} decreases with γ more rapidly than π_1^{e2} . By the Envelope Theorem we find that $\frac{\partial \pi_2^{e1}}{\partial \gamma} = -\frac{1}{3t} \left(t - \frac{\gamma}{3} + \frac{\delta(a_1^* - a_2^*)}{3} + \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right) < \frac{\partial \pi_1^{e2}}{\partial \gamma} = -\frac{1}{3t} \left(t - \frac{\gamma}{3} + \frac{\delta(a_2^* - a_1^*)}{3} + \frac{R_1(a_1^*) - R_2(a_2^*)}{3} \right)$. This inequality holds since $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*) > 0$. Summing up, industry profits are maximized (given competition at stage 3) when platform 2 airs the premium content exclusively. Hence, the upstream content provider always offers the content exclusively to platform 2, independently of the vertical structure of the industry. ■

Proof of Proposition 7. At stage 1, the content provider produces the quality that maximizes its profits under exclusive provision to platform 2 (see Proposition 6). The content provider's choice of investment is determined by the point where $\frac{\partial \Pi}{\partial \gamma} = \mu\gamma$. When the content provider is an independent firm, its revenues from the sale of the premium content to platform 2 are $\Pi_U^{e2}(VS) = \pi_2^{e2} - \pi_2^{e1}$. When it is vertically integrated with platform 1, its revenues are $\Pi_1^{e2}(VI1) = \pi_1^{e2} + \pi_2^{e2} - \pi_2^{e1}$. Finally, when it is vertically integrated with platform 2, its revenues are $\Pi_2^{e2}(VI2) = \pi_2^{e2}$. Since we showed in the Proof of Proposition 6 that $\frac{\partial \pi_i^{ei}}{\partial \gamma} > 0$ and $\frac{\partial \pi_i^{ej}}{\partial \gamma} < 0$ for each $i, j \in \{1, 2\}$ with $i \neq j$, hence $\frac{\partial \Pi_U^{e2}(VS)}{\partial \gamma} > \frac{\partial \Pi_1^{e2}(VI1)}{\partial \gamma}$. Moreover, we showed that $0 > \frac{\partial \pi_1^{e2}}{\partial \gamma} > \frac{\partial \pi_2^{e1}}{\partial \gamma}$, hence $\frac{\partial \Pi_1^{e2}(VI1)}{\partial \gamma} > \frac{\partial \Pi_2^{e2}(VI2)}{\partial \gamma}$. Summing up, we can conclude that $\frac{\partial \Pi_U^{e2}(VS)}{\partial \gamma} > \frac{\partial \Pi_1^{e2}(VI1)}{\partial \gamma} > \frac{\partial \Pi_2^{e2}(VI2)}{\partial \gamma}$. Hence, an independent content provider invests more than a vertically integrated one. Moreover, the content provider integrated with platform 2 provides the lowest level of quality. ■

Proof of Proposition 8. First, consider consumer surplus. From the Proof of Proposition 4, we know that, when $\gamma_1 = 0$ and $\gamma_2 = \gamma$, consumer surplus increases in the quality γ , i.e. $\frac{\partial CS}{\partial \gamma} > 0$. Since, from Proposition 7, $\gamma(VS) > \gamma(VI1) > \gamma(VI2)$, consumer surplus is the highest under vertical separation. Moreover, consumer surplus is the lowest when the content provider is integrated with platform 2.

Second, consider total welfare $W = W^c + W^a - \frac{\mu\gamma^2}{2}$. From Proposition 6 we know that platform 2 airs the premium content, i.e. $\gamma_1 = 0$ and $\gamma_2 = \gamma$. Gross surplus from content is defined in (3) and it can be rewritten as $W^c = V + \gamma q_2^* - \frac{t}{2} + t q_2^* - t (q_2^*)^2$. Gross surplus from advertising is defined in (4) and, following the reasoning in the Proof of Proposition 5, we can rewrite it as $W^a = q_1^* \left(R_1(a_1^*) - \delta a_1^* + \alpha_1 \int_{\underline{k}_1}^{\bar{k}} (1 - F(k)) dk \right) + q_2^* \left(R_2(a_2^*) - \delta a_2^* + \alpha_2 \int_{\underline{k}_2}^{\bar{k}} (1 - F(k)) dk \right)$. Now, we split total welfare in two components $\bar{W} = V + \gamma q_2^* - \frac{t}{2} + t q_2^* - t (q_2^*)^2 + q_1^* (R_1(a_1^*) - \delta a_1^*) + q_2^* (R_2(a_2^*) - \delta a_2^*) - \frac{\mu\gamma^2}{2}$ and $\bar{\bar{W}} = q_1^* \alpha_1 \int_{\underline{k}_1}^{\bar{k}} (1 - F(k)) dk + q_2^* \alpha_2 \int_{\underline{k}_2}^{\bar{k}} (1 - F(k)) dk$. First, we consider \bar{W} . Using (22), we find

that $\overline{W} = \frac{5\gamma(\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*))}{18t} + \frac{5(\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*))^2}{36t} + \frac{\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*)}{2} + (R_1(a_1^*) - \delta a_1^*) + V - \frac{t}{4} + \frac{\gamma}{2} + \frac{5\gamma^2}{36t} - \frac{\mu\gamma^2}{2}$. Now, we show that $\overline{W}(\gamma = \gamma(VS)) > \overline{W}(\gamma = \gamma(VI1))$. By substituting the equilibrium values of the quality content, $\gamma(VS) = \frac{3t+\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*)}{9t\mu-1}$ and $\gamma(VI1) = \frac{3(t+\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*))}{9t\mu-1}$, and rearranging the terms, we can rewrite it as $\frac{(3t(2-9\mu t)+(\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*)))(2-9\mu t)(3t(10-117\mu t)+(\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*)))(10-171\mu t)}{2916t^3\mu^2(9t\mu-1)^2} > 0$. This inequality always holds in the feasible region. Moreover, we show that $\overline{W}(\gamma = \gamma(VI1)) > \overline{W}(\gamma = \gamma(VI2))$. Considering that $\gamma(VI2) = \frac{3t+\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*)}{9t\mu-1}$ and rearranging the terms, we have that $\frac{(\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*))((3t(2+9\mu t)+(\delta(a_1^*-a_2^*)+R_2(a_2^*)-R_1(a_1^*)))(5+9\mu t))}{9t(9t\mu-1)^2} > 0$. Hence, we showed that $\overline{W}(\gamma = \gamma(VS)) > \overline{W}(\gamma = \gamma(VI1)) > \overline{W}(\gamma = \gamma(VI2))$. Second, we consider $\overline{\overline{W}} = q_1^*\alpha_1 \int_{\underline{k}_1}^{\bar{k}} (1 - F(k)) dk + q_2^*\alpha_2 \int_{\underline{k}_2}^{\bar{k}} (1 - F(k)) dk$. By definition $a_i = 1 - F(\underline{k}_i)$. Since $a_1^* < a_2^*$, hence $1 - F(\underline{k}_1) < 1 - F(\underline{k}_2)$, implying that $\underline{k}_1 > \underline{k}_2$. Hence, since $(\underline{k}_1, \bar{k}) \subset (\underline{k}_2, \bar{k})$ and $1 - F(k) > 0$, we can state that $\int_{\underline{k}_1}^{\bar{k}} (1 - F(k)) dk < \int_{\underline{k}_2}^{\bar{k}} (1 - F(k)) dk$. Using this fact and that q_2^* increases with γ , we can conclude that $\overline{\overline{W}}(\gamma = \gamma(VS)) > \overline{\overline{W}}(\gamma = \gamma(VI1)) > \overline{\overline{W}}(\gamma = \gamma(VI2))$. Hence, it is easy to conclude that $W(\gamma = \gamma(VS)) > W(\gamma = \gamma(VI1)) > W(\gamma = \gamma(VI2))$. ■

Proof of Proposition 9. First, we verify whether firms prefer to be separated or integrated. The content provider makes an offer to platform i if the profits it makes when it is vertically integrated with the latter at the net of the price it pays for acquiring platform i are higher than the ones it makes when it is vertically separated. The minimum price at which platform i accepts the offer is $\Pi_i^{e2}(VS)$, that is, the downstream profits when it is independent. Formally, we verify if $(\Pi_i^{e2}(VIi) - \mu \frac{\gamma(VIi)^2}{2}) - \Pi_i^{e2}(VS) > (\Pi_U^{e2}(VS) - \mu \frac{\gamma(VS)^2}{2})$ for $i \in \{1, 2\}$ holds. For $i = 1$, this inequality can be rewritten as $\pi_1^{e2}(\gamma(VI1)) + \pi_2^{e2}(\gamma(VI1)) - \pi_2^{e1}(\gamma(VI1)) - \mu \frac{\gamma(VI1)^2}{2} - \pi_1^{e2}(\gamma(VS)) > \pi_2^{e2}(\gamma(VS)) - \pi_2^{e1}(\gamma(VS)) - \mu \frac{\gamma(VS)^2}{2}$, where $\gamma(VI1) = \operatorname{argmax}(\pi_1^{e2} + \pi_2^{e2} - \pi_2^{e1} - \mu \frac{\gamma^2}{2})$ and $\gamma(VS) = \operatorname{argmax}(\pi_2^{e2} - \pi_2^{e1} - \mu \frac{\gamma^2}{2})$. By rearranging the terms, we rewrite $\pi_1^{e2}(\gamma(VI1)) + \pi_2^{e2}(\gamma(VI1)) - \pi_2^{e1}(\gamma(VI1)) - \mu \frac{\gamma(VI1)^2}{2} > \pi_1^{e2}(\gamma(VS)) + \pi_2^{e2}(\gamma(VS)) - \pi_2^{e1}(\gamma(VS)) - \mu \frac{\gamma(VS)^2}{2}$. This inequality is always verified since $\pi_1^{e2} + \pi_2^{e2} - \pi_2^{e1} - \mu \frac{\gamma^2}{2}$ is maximized for $\gamma = \gamma(VI1)$. For $i = 2$, the previous inequality can be rewritten as $\pi_2^{e2}(\gamma(VI2)) - \mu \frac{\gamma(VI2)^2}{2} - \pi_2^{e1}(\gamma(VS)) > \pi_2^{e2}(\gamma(VS)) - \pi_2^{e1}(\gamma(VS)) - \mu \frac{\gamma(VS)^2}{2}$, where $\gamma(VI2) = \operatorname{argmax}(\pi_2^{e2} - \mu \frac{\gamma^2}{2})$. By rearranging the terms, we have $\pi_2^{e2}(\gamma(VI2)) - \mu \frac{\gamma(VI2)^2}{2} > \pi_2^{e2}(\gamma(VS)) - \mu \frac{\gamma(VS)^2}{2}$, that is verified since $\pi_2^{e2} - \mu \frac{\gamma^2}{2}$ is maximized for $\gamma = \gamma(VI2)$.

Second, we verify if the content provider prefers to acquire platform 1 or 2. It prefers to acquire 2 if $\Pi_1^{e2}(VI1) - \mu \frac{\gamma(VI1)^2}{2} - \Pi_1^{e2}(VS) < \Pi_2^{e2}(VI2) - \mu \frac{\gamma(VI2)^2}{2} - \Pi_2^{e2}(VS)$. By substituting

tution, we rewrite $\pi_1^{e2}(\gamma(VI1)) + \pi_2^{e2}(\gamma(VI1)) - \pi_2^{e1}(\gamma(VI1)) - \mu \frac{\gamma(VI1)^2}{2} - \pi_1^{e2}(\gamma(VS)) < \pi_2^{e2}(\gamma(VI2)) - \mu \frac{\gamma(VI2)^2}{2} - \pi_2^{e1}(\gamma(VS))$, that is $\left[\pi_2^{e2}(\gamma(VI2)) - \mu \frac{\gamma(VI2)^2}{2} - \pi_2^{e2}(\gamma(VI1)) + \mu \frac{\gamma(VI1)^2}{2} \right] + [\pi_1^{e2}(\gamma(VS)) - \pi_2^{e1}(\gamma(VS)) + \pi_2^{e1}(\gamma(VI1)) - \pi_1^{e2}(\gamma(VI1))] > 0$. The first term in squared brackets is positive since $\pi_2^{e2} - \mu \frac{\gamma^2}{2}$ is maximized for $\gamma = \gamma(VI2)$. The second term in squared brackets is also positive, since π_2^{e1} decreases in quality faster than π_1^{e2} . Indeed, it can be rewritten as $\frac{1}{2t} \left(t - \frac{\gamma(VS)}{3} + \frac{\delta(a_2^* - a_1^*)}{3} - \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right)^2 - \frac{1}{2t} \left(t - \frac{\gamma(VS)}{3} - \frac{\delta(a_2^* - a_1^*)}{3} + \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right)^2 + \frac{1}{2t} \left(t - \frac{\gamma(VI1)}{3} - \frac{\delta(a_2^* - a_1^*)}{3} + \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right)^2 - \frac{1}{2t} \left(t - \frac{\gamma(VI1)}{3} + \frac{\delta(a_2^* - a_1^*)}{3} - \frac{R_2(a_2^*) - R_1(a_1^*)}{3} \right)^2 = \frac{2}{9t} (\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*)) (\gamma(VS) - \gamma(VI1)) > 0$, that is positive since $\delta(a_1^* - a_2^*) + R_2(a_2^*) - R_1(a_1^*) > 0$ and by Proposition 3 $\gamma(VS) \geq \gamma(VI1)$. Hence, we conclude that the content provider prefers to acquire platform 2 rather than platform 1. ■

Appendix 2

First, we show that a_i^* increases in γ_i . By the Implicit Function Theorem, we can write:

$$\frac{\partial a_i^*}{\partial \gamma_i} = - \frac{\det \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial a_i \partial \gamma_i} & \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} \\ \frac{\partial^2 \pi_j}{\partial a_j \partial \gamma_i} & \frac{\partial^2 \pi_j}{\partial a_j \partial a_j} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial a_i^2} & \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} \\ \frac{\partial^2 \pi_j}{\partial a_j \partial a_i} & \frac{\partial^2 \pi_j}{\partial a_j^2} \end{bmatrix}} = - \frac{\left(R_i' \frac{\partial q_i}{\partial \gamma_i} \right) \left(R_j'' q_j + 2R_j' \frac{\partial q_j}{\partial a_j} \right) - \left(R_i' \frac{\partial q_i}{\partial a_j} \right) \left(R_j' \frac{\partial q_j}{\partial \gamma_i} \right)}{\left(R_i'' q_i + 2R_i' \frac{\partial q_i}{\partial a_i} \right) \left(R_j'' q_j + 2R_j' \frac{\partial q_j}{\partial a_j} \right) - \left(R_i' \frac{\partial q_i}{\partial a_j} \right) \left(R_j' \frac{\partial q_j}{\partial a_i} \right)}. \text{ Using the fact}$$

that $\frac{\partial q_i}{\partial a_i} = \frac{\partial q_j}{\partial a_j} = -\frac{\delta}{2t}$, $\frac{\partial q_i}{\partial a_j} = \frac{\partial q_j}{\partial a_i} = \frac{\delta}{2t}$, and $\frac{\partial q_j}{\partial \gamma_i} = -\frac{1}{2t}$ for $i, j \in \{1, 2\}$ with

$i \neq j$, this can be rewritten as: $-\frac{\frac{R_j'' q_j R_i'}{2t} - \frac{\delta R_j' R_i'}{4t^2}}{R_j'' q_j R_i'' q_i - \frac{R_i'' q_i R_j' \delta}{t} - \frac{R_j'' q_j R_i' \delta}{t} + \frac{3R_j' R_i' \delta^2}{4t^2}}$. Since $R_i' > 0$ and $R_i'' < 0$,

we can easily conclude that the numerator is negative, while the denominator is positive. Hence, $\frac{\partial a_i^*}{\partial \gamma_i} > 0$. Then, it can be shown that a_j^* is decreasing in γ_i . By the Implicit Function

$$\text{Theorem } \frac{\partial a_j^*}{\partial \gamma_i} = - \frac{\det \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial a_i^2} & \frac{\partial^2 \pi_i}{\partial a_i \partial \gamma_i} \\ \frac{\partial^2 \pi_j}{\partial a_j \partial a_i} & \frac{\partial^2 \pi_j}{\partial a_j \partial \gamma_i} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial a_i^2} & \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} \\ \frac{\partial^2 \pi_j}{\partial a_j \partial a_i} & \frac{\partial^2 \pi_j}{\partial a_j^2} \end{bmatrix}} = - \frac{\left(R_j' \frac{\partial q_j}{\partial \gamma_i} \right) \left(R_i'' q_i + 2R_i' \frac{\partial q_i}{\partial a_i} \right) - \left(R_i' \frac{\partial q_i}{\partial \gamma_i} \right) \left(R_j' \frac{\partial q_j}{\partial a_i} \right)}{\left(R_i'' q_i + 2R_i' \frac{\partial q_i}{\partial a_i} \right) \left(R_j'' q_j + 2R_j' \frac{\partial q_j}{\partial a_j} \right) - \left(R_i' \frac{\partial q_i}{\partial a_j} \right) \left(R_j' \frac{\partial q_j}{\partial a_i} \right)}$$

$= - \frac{-\frac{R_i'' q_i R_j'}{2t} + \frac{\delta R_j' R_i'}{4t^2}}{R_j'' q_j R_i'' q_i - \frac{R_i'' q_i R_j' \delta}{t} - \frac{R_j'' q_j R_i' \delta}{t} + \frac{3R_j' R_i' \delta^2}{4t^2}}$. Since $R_i' > 0$ and $R_i'' < 0$, both the numerator and the denominator are positive, hence $\frac{\partial a_j^*}{\partial \gamma_i} < 0$.

Now, we show that the profit of platform i increases in γ_i . By the Envelope Theorem, we can write that $\frac{\partial \pi_i^*}{\partial \gamma_i} = \frac{\partial \pi_i}{\partial \gamma_i} + \frac{\partial \pi_i}{\partial a_i} \frac{\partial a_i^*}{\partial \gamma_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j^*}{\partial \gamma_i} = \frac{\partial \pi_i}{\partial \gamma_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j^*}{\partial \gamma_i}$, since $\frac{\partial \pi_i}{\partial a_i} = 0$ by first order conditions. Hence, we can rewrite $\frac{\partial \pi_i^*}{\partial \gamma_i} = \frac{R_i}{2t} \left(1 + \delta \frac{\partial a_j^*}{\partial \gamma_i} \right)$. We verify that

$\frac{\partial a_j^*}{\partial \gamma_i} = -\frac{\left(-R'_j \frac{1}{2t}\right)\left(R''_i q_i - R'_i \frac{\delta}{t}\right) - \left(R'_i \frac{1}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)}{\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(R'_j q_j - R'_j \frac{\delta}{t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)} > -\frac{\left(-R'_j \frac{1}{2t}\right)\left(R''_i q_i - R'_i \frac{\delta}{t}\right) - \left(R'_i \frac{1}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)}{\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(-R'_j \frac{\delta}{2t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)} = -\frac{1}{\delta}$, where the inequality follows from the fact that, at the denominator $\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(R'_j q_j - R'_j \frac{\delta}{t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right) > \left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(-R'_j \frac{\delta}{2t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)$. Hence, since $\frac{\partial a_j^*}{\partial \gamma_i} > -\frac{1}{\delta}$, we can conclude that $\frac{\partial \pi_i^*}{\partial \gamma_i} > 0$. Then, we show that the profit of platform i decreases in γ_j . As before, by the Envelope Theorem, we have that $\frac{\partial \pi_i}{\partial \gamma_j} = \frac{\partial \pi_i}{\partial \gamma_j} + \frac{\partial \pi_i}{\partial a_i} \frac{\partial a_i^*}{\partial \gamma_j} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j^*}{\partial \gamma_j} = \frac{\partial \pi_i}{\partial \gamma_j} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j^*}{\partial \gamma_j}$. We obtain that $\frac{\partial \pi_i^*}{\partial \gamma_j} = \frac{R_i}{2t} \left(-1 + \delta \frac{\partial a_j^*}{\partial \gamma_j}\right)$. Since $\frac{\partial a_j^*}{\partial \gamma_j} = \frac{\left(R'_j \frac{1}{2t}\right)\left(-R''_i q_i + R'_i \frac{\delta}{t}\right) - \left(R'_i \frac{1}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)}{\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(R'_j q_j - R'_j \frac{\delta}{t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)} < \frac{\left(R'_j \frac{1}{2t}\right)\left(-R''_i q_i + R'_i \frac{\delta}{t}\right) - \left(R'_i \frac{1}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)}{\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(-R'_j \frac{\delta}{2t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)} = \frac{1}{\delta}$, we conclude that $\frac{\partial \pi_i^*}{\partial \gamma_j} < 0$.

Proof of Proposition 10. Following the Proof of Proposition 1, in order to verify whether the upstream content provider offers the content of quality γ exclusively to platform i or non-exclusively to both platforms, we look at industry profits and we verify under which form of distribution they are maximized (given competition at stage 3). In the Hotelling model, we verify that $\pi_1^{ne} + \pi_2^{ne} = \pi_1^0 + \pi_2^0$. In order to show that $\pi_i^{ei} + \pi_j^{ei} > \pi_1^{ne} + \pi_2^{ne}$, we need to show π_i^{ei} increases in quality more rapidly than π_j^{ei} decreases in it. Using the Envelope Theorem, this occurs if $\left|\frac{\partial \pi_i^{ei}}{\partial \gamma} + \frac{\partial \pi_i^{ei}}{\partial a_j} \frac{\partial a_j^*}{\partial \gamma}\right| > \left|\frac{\partial \pi_j^{ei}}{\partial \gamma} + \frac{\partial \pi_j^{ei}}{\partial a_i} \frac{\partial a_i^*}{\partial \gamma}\right|$, that is if and only if $\frac{R_i}{2t} \left(1 + \delta \frac{\partial a_j^*}{\partial \gamma}\right) > \frac{R_j}{2t} \left(1 - \delta \frac{\partial a_i^*}{\partial \gamma}\right)$. By first order conditions $R_i = \frac{2tq_i R'_i}{\delta}$ and by $\frac{\partial a_i^*}{\partial \gamma}$ and $\frac{\partial a_j^*}{\partial \gamma}$ evaluated before, we rewrite the previous inequality as $\frac{1}{\delta} \left(q_i R'_i - q_j R'_j\right) > -\left[\left(\frac{\partial a_j^*}{\partial \gamma_i} q_j R'_j\right) + \left(\frac{\partial a_i^*}{\partial \gamma_i} q_i R'_i\right)\right] = -\left[\frac{\left[\left(-R'_i \frac{1}{2t}\right)\left(R'_j q_j - R'_j \frac{\delta}{t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{1}{2t}\right)\right] q_j R'_j + \left[\left(R'_j \frac{1}{2t}\right)\left(R''_i q_i - R'_i \frac{\delta}{t}\right) + \left(R'_i \frac{1}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)\right] q_i R'_i}{\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(R'_j q_j - R'_j \frac{\delta}{t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)}\right]$. By simple algebra, we find that $\frac{1}{\delta} \left(q_i R'_i - q_j R'_j\right) \left[\left(R''_i q_i - R'_i \frac{\delta}{t}\right)\left(R'_j q_j - R'_j \frac{\delta}{t}\right) - \left(R'_i \frac{\delta}{2t}\right)\left(R'_j \frac{\delta}{2t}\right)\right] > \frac{\delta}{4t^2} \left(R'_i R'_j\right) \left(q_i R'_i - q_j R'_j\right) - \frac{1}{2t} \left(R'_i R'_j\right) \left(R''_i q_i^2 - R''_j q_j^2\right)$. We rearrange the terms and find that $\frac{1}{\delta} q_i R'_i \left(R''_i q_i - R'_i \frac{\delta}{t}\right) \left(R'_j q_j - R'_j \frac{\delta}{2t}\right) - \frac{1}{\delta} q_j R'_j \left(R''_i q_i - R'_i \frac{\delta}{2t}\right) \left(R'_j q_j - R'_j \frac{\delta}{t}\right) > 0$. When this inequality holds, the content provider gives the premium content exclusively to platform i . It is always verified if R''_i and R'_j are not very negative. It is easy to show that it holds for R''_i and R'_j that go to zero. However, it may be violated if R''_i and R'_j are highly negative. It can be shown by assuming a shape for $R_i(a_i)$ and proceeding by simulation. Hence, in this case the content provider provides the premium content non-exclusively to both platforms. ■

Proof of Proposition 11. The content provider's choice of investment is determined by the point where $\frac{\partial \Pi}{\partial \gamma} = \mu \gamma$. The marginal benefit depends on the vertical structure of the industry. In Proposition 10, we find that the content provider provides the premium content

exclusively to one platform if (34) is verified, otherwise it provides the premium content non-exclusively, no matter the vertical structure of the industry. When, at stage 2, the content is provided exclusively to platform i , the payoff of an independent content provider is $\Pi_U^{ei}(VS) = \pi_i^{ei} - \pi_i^{ej}$ and the payoff of the vertically integrated platform i is $\Pi_i^{ei}(VIi) = \pi_i^{ei}$. Since, as we have shown before, π_i^{ei} increases in γ and π_i^{ej} decreases in it for $i, j \in \{1, 2\}$ with $i \neq j$, then $\frac{\partial \Pi_U^{ei}(VS)}{\partial \gamma} > \frac{\partial \Pi_i^{ei}(VIi)}{\partial \gamma}$. Instead, when the content is provided non-exclusively at stage 2, the payoff of an independent content provider is $\Pi_U^{ne}(VS) = \pi_i^{ne} - \pi_i^{ej} + \pi_j^{ne} - \pi_j^{ei}$ and the payoff of the vertically integrated platform i is $\Pi_i^{ne}(VIi) = \pi_i^{ne} + \pi_j^{ne} - \pi_j^{ei}$. By symmetry of the platforms (i.e. $\pi_i^{ne} = \pi_j^{ne}$ and $\pi_i^{ei} = \pi_j^{ej}$), payoffs can be rewritten as $\Pi_U^{ne}(VS) = 2\pi_i^{ne} - 2\pi_i^{ej}$ and $\Pi_i^{ne}(VIi) = 2\pi_i^{ne} - \pi_j^{ei}$. Since π_i^{ej} decreases in γ and π_i^{ne} is not affected by the quality provided, then $\frac{\partial \Pi_U^{ne}(VS)}{\partial \gamma} > \frac{\partial \Pi_i^{ne}(VIi)}{\partial \gamma}$. Hence, the quality provided under vertical separation is always higher than the one provided under vertical integration. ■

Finally, we show that consumer surplus is always higher under vertical integration than under vertical separation. Consumer surplus is $CS = \int_0^{q_1^*} U_1(x) dx + \int_{q_1^*}^1 U_2(x) dx$, where $U_1 = V + \gamma_1 - xt - \delta a_1^*$ and $U_2 = V + \gamma_2 - (1-x)t - \delta a_2^*$. Assume that platform i gets the exclusive content. Consider first the case of exclusive provision. Without loss of generality, assume that $\gamma_1 = 0$ and $\gamma_2 = \gamma$. The derivative of consumer surplus with respect to the quality γ is $\frac{\partial CS}{\partial \gamma} = U_1(x = q_1^*) \frac{\partial q_1^*}{\partial \gamma} + \int_0^{q_1^*} \frac{\partial U_1(x)}{\partial \gamma} dx - U_2(x = q_1^*) \frac{\partial q_1^*}{\partial \gamma} + \int_{q_1^*}^1 \frac{\partial U_2(x)}{\partial \gamma} dx = \int_0^{q_1^*} \frac{\partial U_1(x)}{\partial \gamma} dx + \int_{q_1^*}^1 \frac{\partial U_2(x)}{\partial \gamma} dx$ since $U_1(x = q_1^*) = U_2(x = q_1^*)$. We find that $\frac{\partial U_1(x)}{\partial \gamma} = -\delta \frac{\partial a_1^*}{\partial \gamma}$ and $\frac{\partial U_2(x)}{\partial \gamma} = 1 - \delta \frac{\partial a_2^*}{\partial \gamma}$, then $\frac{\partial CS}{\partial \gamma} = \left[-\delta \frac{\partial a_1^*}{\partial \gamma}\right]_0^{q_1^*} + \left[1 - \delta \frac{\partial a_2^*}{\partial \gamma}\right]_{q_1^*}^1 = -q_1^* \delta \frac{\partial a_1^*}{\partial \gamma} + (1 - \delta \frac{\partial a_2^*}{\partial \gamma})(1 - q_1^*) = -\delta \frac{\partial a_1^*}{\partial \gamma} + q_2^* (1 + \delta \frac{\partial a_1^*}{\partial \gamma} - \delta \frac{\partial a_2^*}{\partial \gamma}) > 0$. This inequality holds since, $\frac{\partial a_1^*}{\partial \gamma} < 0$ and $1 + \delta \frac{\partial a_1^*}{\partial \gamma} - \delta \frac{\partial a_2^*}{\partial \gamma} > 0$. Now, consider that platforms air the content non-exclusively, i.e. $\gamma_1 = \gamma_2 = \gamma$. In this case $a_1^* = a_2^*$ (it is not affected by the quality level) and $U_1 = V + \gamma - xt - \delta a_1^*$ and $U_2 = V + \gamma - (1-x)t - \delta a_2^*$. Hence, $\frac{\partial CS}{\partial \gamma} = 1 > 0$.

References

- [1] Ambrus, A., Calvano, E. and Reisinger, M. (2013), Either or Both Competition: a “Two-Sided” Theory of Advertising With Overlapping Viewerships, Working Paper available at https://www.econstor.eu/dspace/bitstream/10419/79912/1/VfS_2013_pid_326.pdf.
- [2] Anderson, S.P. and Coate, S. (2005), Market Provision of Broadcasting: A Welfare Analysis, *Review of Economic Studies*, 72, 947-972.

- [3] Anderson, S.P., Foros, O. and Kind, H.J. (2013), Competition for Advertisers and Viewers in Media Markets, retrieved from http://www.cresse.info/uploadfiles/2013_S16_PP1.pdf.
- [4] Anderson, S.P. and Gabszewicz, J. (2006), The media and advertising: a tale of two-sided markets, *Handbook of the Economics of Art and Culture*, 1, 567-614.
- [5] Argentesi, E. and Filistrucchi, L. (2007), Estimating market power in a two-sided market: The case of newspapers, *Journal of Applied Econometrics*, 22, 1247-1266.
- [6] Armstrong, M. (1999), Competition in the pay-TV market, *Journal of the Japanese and International Economies*, 13, 257-280.
- [7] Armstrong, M. (2006), Competition in Two-Sided Markets , *The RAND Journal of Economics*, 37, 668-691.
- [8] Bester, H. and Petrakis, E. (1993), The Incentives for Cost Reduction in a Differentiated Industry, *International Journal of Industrial Organization*, 11, 519-534.
- [9] Caillaud, B. and Jullien, B. (2003), Chicken & Egg: Competition among Intermediation Service Providers, *The RAND Journal of Economics*, 34, 309-328.
- [10] Choi, J.P. (2006), Broadcast competition and advertising with free entry: Subscription vs. free-to-air, *Information Economics and Policy*, 18, 181-196.
- [11] Crampes, C., Haritchabalet, C. and Jullien, B. (2009), Advertising, Competition and Entry in Media Industries, *The Journal of Industrial Economics*, 57, 7-31.
- [12] Crawford, G.S. (2013), Cable Regulation in the Internet Era, in Rose, N. (ed.), *Economic Regulation and Its Reform: What Have We Learned?*, Forthcoming, University of Chicago Press.
- [13] Depken II, C.A. (2004), Audience Characteristics and the Price of Advertising in a Circulation Industry: Evidence from U.S. Magazines, *Information Economics and Policy*, 16, 179-196.
- [14] Gabszewicz, J.J., Laussel, D. and Sonnac, N. (2001), Press advertising and the ascent of the 'Pensée Unique' , *European Economic Review*, 45, 641-651.
- [15] Gabszewicz, J.J., Laussel, D. and Sonnac, N. (2002), Press advertising and the political differentiation of newspapers, *Journal of Public Economic Theory*, 4, 317-334.

- [16] Gabszewicz, J. J., Laussel, D. and Sonnac, N. (2004), Programming and Advertising Competition in the Broadcasting Industry, *Journal of Economics and Management Strategy*, 13, 657-669.
- [17] Gal-Or, E. and Dukes, A. (2003), Minimum Differentiation in Commercial Media Markets, *Journal of Economics and Management Strategy*, 12, 291-325.
- [18] Hagiu, A. and Lee, R.S. (2011), Exclusivity and Control, *Journal of Economics and Management Strategy*, 20, 679-708.
- [19] Harbord, D. and Ottaviani, M. (2001), Contracts and Competition in the Pay-TV Market, LBS, Department of Economics, Working Paper No. DP 2001/5, available at SSRN: <http://ssrn.com/abstract=289334> or doi:10.2139/ssrn.289334.
- [20] Hogendorn, C. and Ka Yat Yuen, S. (2009), Platform competition with must-have components, *The Journal of Industrial Economics*, 57, 294-318.
- [21] Hotelling, H. (1929), Stability in Competition, *Economic Journal*, 39, 41-57.
- [22] Jeziorski, P. (2012), Effects of Mergers in Two-Sided Markets: Examination of the U.S. Radio Industry, available at SSRN: <http://ssrn.com/abstract=1700763>.
- [23] Kaiser, U. and Song, M. (2009), Do media consumers really dislike advertising? An empirical assessment of the role of advertising in print media markets, *International Journal of Industrial Organization*, 27, 292-301.
- [24] Kaiser, U. and Wright, J. (2006), Price structure in two-sided markets: Evidence from the magazine industry, *International Journal of Industrial Organization*, 24, 1-28.
- [25] Katz, M. and Shapiro, C. (1986), How to License Intangible Property, *Quarterly Journal of Economics*, 101, 567-589.
- [26] Liu, Y., Putler, D.S. and Weinberg, C.B. (2004), Is Having More Channels Really Better? A Model of Competition Among Commercial Television Broadcasters, *Marketing Science*, 23, 120-133.
- [27] Peitz, M. and Valletti, T.M. (2008), Content and advertising in the media: Pay-tv versus free-to-air, *International Journal of Industrial Organization*, 26, 949-965.
- [28] Rey, P. and Tirole, J. (2007), A Primer on Foreclosure, *Handbook of Industrial Organization*, 3, 2145-2220.

- [29] Rochet, J.C. and Tirole, J. (2006), Two-sided markets: a progress report, *The RAND Journal of Economics*, 37, 645–667.
- [30] Roson R. (2008), Price Discrimination and Audience Composition in Advertising-Based Broadcasting, *Journal of Media Economics*, 21, 234–257.
- [31] Segal, I. (1999), Contracting with Externalities, *The Quarterly Journal of Economics*, 114, 337–388.
- [32] Stennek, J. (2007), Exclusive Quality - Why Exclusive Distribution May Benefit the TV Viewers, CEPR Discussion Paper No. 6072, available at SSRN: <http://ssrn.com/abstract=997354>.
- [33] Weeds, H. (2012a), TV Wars: Exclusive Content and Platform Competition in Pay TV, Working Paper, CEPR Discussion Paper No. 8781, available at SSRN: <http://ssrn.com/abstract=1995964>
- [34] Weeds, H. (2012b), Advertising and the Distribution of Content, retrieved from http://privatewww.essex.ac.uk/~hfweeds/Advertising%20and%20content%20distribution_July%202012.pdf.
- [35] Wilbur, K. C. (2008), A Two-Sided, Empirical Model of Television Advertising and Viewing Markets, *Marketing Science*, 27, 356–378.