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AOU-Policlinico Umberto I

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Abstract

Nowadays, Operating Rooms (ORs) scheduling and planning represent one of the most challenging aspects in healthcare systems management. In particular, one of the main concern is doubtless the under-use of the surgery rooms, that one can tackle by modelling different management decision aspects. The number of ORs available in the operating theatre and their daily opening time, the ideal OR Utilization Rate (UR) that the hospital aims to fulfil, the possibility to transfer patients between different surgical units (SUs) are just some of them. Taking into account these management and economic factors, we develop a multi-criteria mixed integer linear optimization model that effectively contributes to the scheduling of surgeries. The model is practically relevant and addresses the particular case of the *Azienda Ospedaliera - Universitaria Policlinico Umberto I* of Rome. The problem is formulated according to an open scheduling strategy, which allows to schedule a weekly list of surgeries by determining the day, time, and operating room needed. Extensive experiments are carried on real data collected during twelve weeks in two SUs and four ORs. The impact of several policies of opening times on the hospital Key Performance Indicators is also tested and we further stress the model introducing randomly generated data in order to take into account surgery priorities. Results demonstrate the significant contribution of using a mathematical model to improve ORs utilization.

Keywords: Operating Room Scheduling; Mixed Integer Linear Optimization; Multi-criteria Optimization; Health Care Management

1 Introduction

Especially in large public hospitals, the management of clinical processes is the weak point of the internal production chain. The concept of "maximum production" is often applied to the individual unit instead of focusing on optimizing the entire flow along the chain. To achieve an overall improvement in the management of the hospital's core activities, it is therefore important to merge Operations Management approaches with Operation Research models (Agnetis et al. (2019)). For this reason, from exact approaches to heuristics and meta-heuristics, a large variety of methods have been applied to planning and scheduling of surgeries depending on the size and complexity of the problem under analysis (Cardoen et al. (2010), Freeman et al. (2015), Rahimi and Gandomi (2020), Samudra et al. (2016), Zhu et al. (2019)).

A key aspect of an efficient hospital facility is the optimal management of operating theatres (Marjamaa et al. (2008)). In particular, Surgical Units (SUs) represent the beating heart of a healthcare facility. Providing approximately 60% of all hospital admissions (Fügener et al. (2017), Gupta (2007)), Operating Room (OR) scheduling and planning have acquired a crucial role in managing the activities of the entire structure in recent decades. As stated in the literature (Cardoen et al. (2009), Guerriero and Guido (2011), Gür and Eren (2018), Samudra et al. (2016), Zhu et al. (2019)), and in the recent and comprehensive review of Rahimi and Gandomi (2020), three levels of action can be defined in the OR scheduling process: *strategic*, *tactical* and *operational*. The strategic level covers all those decisions demanding a long-term planning horizon (from several months up to one year), such as capacity planning, allocation, or the case-mix problem (CMP). By forecasting the hospital demand, the aim is to efficiently distribute the available resources and the budget among the different surgical specialties (see Fügener et al. (2017), Marques and Captivo (2015), Koppka et al. (2018)).

The tactical level, or *Master Surgical Scheduling* problem (MSSP), is a simple cyclic and repetitive schedule used to assign OR times to groups of surgeons, according to their specific needs Bovim et al. (2020). It is a medium-term decision level and it usually controls an interval of three or four months. Undoubtedly, this stage plays a key role in decision making by providing operational guidelines for short-term decisions. A wide range of approaches have been tested in the literature, see for example Aringhieri et al. (2015), Guido and Conforti (2016), Mannino et al. (2012), Penn et al. (2017).

According to the MSSP output, the operational level solves the Surgical Process Scheduling Problem, that can be further divided in the sub-problems of *advance* (deciding the matching *OR-day*) and *allocation* (interesting the

matching *OR-day-time*) scheduling. In this final phase, the resources and the patients on the waiting lists are assigned to specific surgical slots and thus associated with specific Operating Rooms (ORs), days, and starting times (Agnetis et al. (2013), Day et al. (2012)). The OR scheduling problems generally adopt one of the following scheduling strategies: block strategy, open strategy, or modified block strategy. In the first case, OR capacity is divided into blocks and then assigned exclusively to a specific surgical group. This means that surgeries can only be allocated to blocks associated with surgeons of the same specialty. On the other hand, an open strategy guarantees a more flexible solution in which no block assignment exists. It allows different surgical specialties to be assigned in the same operating room session without any priority, following some scheduling principle (for example, a *first come first served* approach). Intuitively, both strategies have their advantages and disadvantages. In the case of block scheduling, if an OR block is assigned to one surgeon group, others cannot perform any surgery in it, even if that slot is free. On the contrary, due to its flexible arrangements, open scheduling leads most of the time to long waiting periods. To overcome these drawbacks, several modified block scheduling strategies have been provided in recent decades, combining the strengths of the two previous strategies.

However, the operating theatre has become highly dynamic with complex decision-making processes which involve many participants, such as OR managers, head doctors, surgeons, and patients. Due to conflicting priorities and preferences, it is hence very hard to satisfy all stakeholders' interests and propose a unique simple method that is capable to improve all performance measures. To find a compromise and take care of all these concerns simultaneously, a considerable number of multi-criteria approaches have been proposed in the last years (see Cardoen et al. (2010), Gul et al. (2011), Meskens et al. (2012), Rachuba (2017)).

In addition, a second issue to be introduced relates to patient classification. A first classification refers to *elective* or *non-elective* patients, that are respectively those patients whose surgeries can be planned in advance or whose arrivals are unexpected, and thus need to be performed urgently (Marques and Captivo (2015), Riet and Demeulemeester (2014)). The policy to be adopted for their scheduling is still debated in the literature (Duma and Aringhieri (2019)), especially for operating rooms that suffer from the non-programmability of non-elective patient flow. A second classification concerns instead patients who must remain in the clinic overnight (*inpatients*) and those who can be dismissed the same day of arrival (*outpatients*).

In this work we address, for a set of ORs located in different SUs, the planning of an open schedule for elective surgeries. The model proposed takes into account multiple performance criteria related to both an OR management improvement and the preservation of the service quality of-

fered to the patients. We develop a multi-criteria mixed integer linear optimization model for the elective patients scheduling at *Azienda Ospedaliera - Universitaria Policlinico Umberto I of Rome* (later simply denoted as *AOU-Policlinico*). It is worth noticing that the aforementioned is the largest European hospital considering the total area and, with a total of 1235 beds (at 31/12/2018), the third in Italy by capacity (AOU-Policlinico (2017, 2018)) with more than 30 operating rooms. It therefore represents a great opportunity and is more than qualified for the effective application of Operations Management tools. Acting at the operational level, the proposed model is able to provide a feasible weekly allocation scheduling for each OR. Thus, following an open scheduling strategy, for each surgical case placed on a waiting list, it determines a scheduled date, a time, and the OR resources needed. We aim to integrate this approach into a wider scheme of functional reorganization of the whole surgical area of the hospital.

Starting from the data provided by the hospital, we performed a preliminary analysis to identify the most crucial items to intervene on. We also examined clinical processes in depth in order to redefine the relationships between the different SUs, optimize the use of clinical resources, and consequently increase the efficiency of the entire surgical area. This study study led us to consider two SUs that are geographically close to each other and consisting of two ORs each. The goal is to use the ORs at their ideal utilization rate (*UR*), defined by the policy of the hospital, and to balance the different ORs daily opening time. In addition, minimizing patient transfers between wards is part of the optimization criteria. Far from being an innovative theory-oriented work, our main goal is to develop a scalable linear model that can solve real-world problems that need to be addressed when dealing with multiple facilities. We show that the use of optimization provides a great improvement in the management of operating rooms and can lead to several changes in the strategic policy of the AOU-Policlinico.

The paper is organized as follows. In Section 2 we describe the real case problem of AOU-Policlinico and we introduce and analyze data and basics stats on the current use of the hospital SUs and ORs. Then, in Section 3 we give its mathematical formulation as a multi-criteria Mixed Integer Linear Problem (MILP). Section 4 shows the results obtained using the optimization model on different scenarios. Section 5 finally summarises the research outcomes and gives some ideas for future works.

2 The Case Study of AOU-Policlinico of Rome

The AOU-Policlinico, built at the end of the 20th century, is the hospital that covers the largest geographic area in Europe. It consists of "pavilions" spread over $300\,000$ square meters and distributed in 54 buildings; 46 of
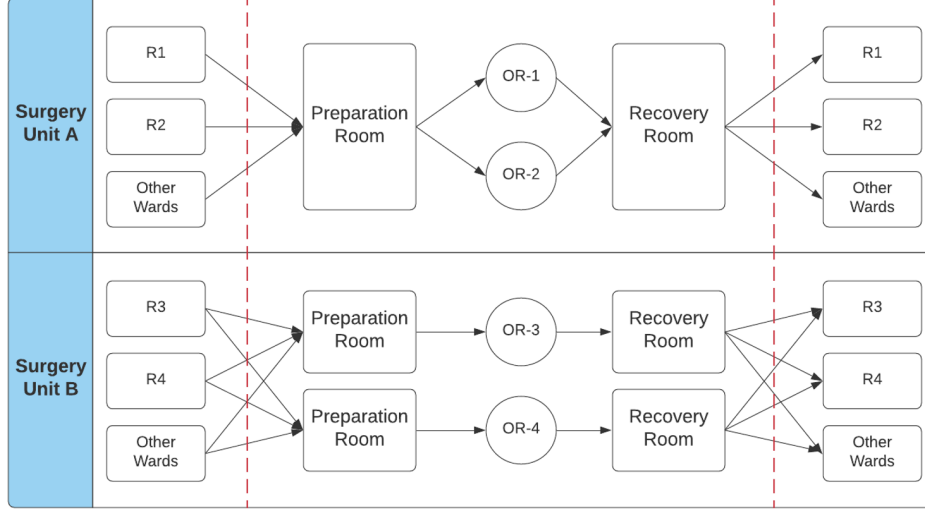
Figure 1: Map of the main enclosure of AOU-Policlinico (from AOU-Policlinico (2018)).

them are located in a large enclosure (see Picture 1) and 8 outside of it (AOU-Policlinico (2018)). It has numerous (more than 30) ORs located in SUs that can accommodate high specialized or generic clinical surgeries related to various diseases. Next we present in detail the hospital facilities, the data acquired and their modelling.

2.1 Surgical Units and Operating Rooms

SUs are located in buildings that cover different clinical specialties for both inpatients and outpatients and have their own wards. Each OR is characterized by the weekly opening days and by a opening time per day. The ORs dedicated to general surgeries are assumed to be interchangeable and there is no preference between them. In this paper, we focus our attention on SUs performing general surgeries since they are the only ones that allow for the improvement of the patient waiting list management. Indeed, specialized SUs usually are dedicated to a small group of surgeons and highly skilled teams and are associated with wards having a limited number of beds.

We consider two Surgical Units managing two ORs each. The choice is motivated by the fact that these SUs account for more than 30% of the total number of clinical surgeries. In addition, the high heterogeneity in the complexity and duration of surgeries opens the way for different schedules. In fact, the managing board of AOU-Policlinico was particularly interested in checking their KPI and exploring different management policies for them. The chosen SUs are located close to each other and allow for easy transfer of patients between them. We denote the SUs with the letters A and B (SUA-A

Figure 2: Schematic representation of the two SUs under analysis and the patient flow throughout the wards.

SU	OR	Opening days	Opening hours
SU-A	OR-1	Monday - Friday	8:00 - 20:00
	OR-2	Monday - Friday	8:00 - 16:00
SU-B	OR-3	Monday - Friday	8:00 - 16:00
	OR-4	Monday - Friday	8:00 - 16:00

Table 1: SUs actual opening time.

and SU-B), their ORs are instead referred as OR-1 and OR-2 (for SU-A), and OR-3 and OR-4 (for SU-B), while the respective wards are indicated by the label R1, R2, and R3, R4. Figure 2 reports a schematic structure of the two SUs and the patient flow throughout the wards. The SUs have also different facilities, different numbers of pre and post-surgery rooms. However, these data do not affect the formulation of our mathematical model.

The ORs are opened 5 days and their operating times are reported in Table 1. Note that the ORs actual opening times, established at the strategic level for each room, do not vary depending on the day of the week.

2.2 Patients workflow management

Patients come both from clinical wards (elective), with different availability of beds for inpatients, and from the Emergency Department. A recently
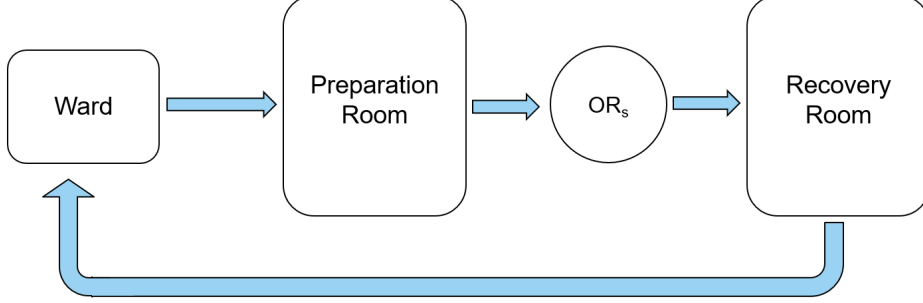
6

Figure 3: Patient's workflow.

implemented planning policy at AOU-Policlino provides a fixed prior allocation of SU beds dedicated to elective patients (AOU-Policlinico (2018)). We can therefore assume that a scheduling is only needed for this class of patients. Normally, the patient is transferred from the clinic department to the SU. Here, he first enters in the preoperative *preparation room*, where the preliminary operations for the surgery are made. In the meantime, the OR is set up for the specific surgery. Then, the patient enters the OR where a team of surgeons performs the scheduled surgery. Once the surgery is concluded, the patient is transferred to the *recovery room* for observation until his full awakening, while the OR needs to be properly cleaned and sanitized before a new surgery can start. A schematic patient's workflow is reported in Figure 3.

We model the possibility of transferring patients from one SU to the other in order to perform the surgery in a specific OR. However, whichever operating room the surgery is performed in, patients' pre- and post-operative hospitalization is retained by the department (thus the SU) where they were admitted. For example, the surgery of a patient coming from ward R1 (SU-A) may be performed in OR-3 (SU-B), however the patient is then transferred back to their hospitalization ward in SU-A.

2.3 Data analysis

Hospital data have two main sources: the discharge databases (HDD) and the hardcopy registers of the ORs, both accounting for privacy protection. Data coming from HDD, and sent to the regional public authority (*Regione Lazio*), report:

- the type of the surgery to be performed;

- the type of the patient (elective or coming from the Emergency Department);

- the "hospitalization regime" (ordinary \ day surgery);

- the SU in which the patient is admitted to;

- ward admission and resignation dates;

- other information about the patient medical history.

In addition, thanks to the paper records from the ORs, we are able to obtain the following piece of information for each patient:

- *diagnosis-related group* (DRGs);

- date of surgery;

- patient time of arrival at the preparation room;

- starting time of surgery;

- ending time of the surgery;

- OR exit time.

Thanks to careful analysis and cross-referencing of all the information, we get the detailed patients' surgery times needed. In particular, for each surgery the following data are available:

- *Setup time*: the time needed to set up the OR for the surgery;

- *Surgical time*: the duration time of the surgery; this time component, considered as the real operative time (Davila (2013)), suffers from a great variability deriving from several causes, such as the surgery complexity and specialty, the patient conditions and response and the occurrence of complications arising during the surgery;

- *Cleaning time*: the time for cleaning and sanitizing the OR;

- *Dismiss time*: the time spent by the patient in the *recovery room*.

Note that, for each surgery, the overall time of use of an OR can be obtained by summing up the Setup, Surgical, and Cleaning times. We will refer to this sum as *Occupation time*.

We consider surgeries carried out in the first 12 weeks of the year 2019, from January 1st to March 30th, in the two clinical SUs of AOU-Policlinico described previously. In agreement with the medical staff, we made some assumption on the *setup* and *cleaning times*. Indeed, the doctors and the OR

Time Name	Description	Assumptions
Setup time	the room is prepared for a specific surgery	constant, 15 min
Surgical time	a team of surgeons performs the surgery on the patient	obtained from historical data and/or doctors' experience
Cleaning time	the OR is properly cleaned and sanitized post-surgery	constant, 20 min
Dismiss time	the time patient remains in the recovery room	obtained from historical data
Occupation time	Overall OR occupation time	sum of the *Setup*, *Surgical* and *Cleaning time*

Table 2: Surgery times: definitions and value assumptions.

chief nurse confirm these values to be independent from the type of surgery performed and poorly influenced by uncertainty. The presence of outliers in the available data appears to be due to other organizational and logistic problems that could be removed by careful surgery planning. We therefore consider them as fixed and respectively equal to 15 and 20 minutes. Instead, for the *surgical time* we round the effective duration time of the surgery. Table 2 summarizes all time components presented above and specifies their values set in accordance with both historical data and physician prediction. In Table 3 we report, for each of the 12 weeks under study (numbered from 1 to 12), the following basic stats:

- the number of surgeries performed in the SU-A and the SU-B (#*I* SU-A and #*I* SU-B respectively);

- the *average surgical time*, namely the average duration of the surgeries scheduled for the week (Avg T) in minutes;

- the standard deviation of this times (StDev T) in minutes;

- the number of *underused* ORs (#ORs UU), namely their workload percentage follows below 50% of its operating time;

- the number of ORs in which the percentage of the daily workload exceeds 80% of the operating time, i.e. the number of ORs that are *overused* (#ORs OU).

For the numerical experiments, we assume that the number of surgeries performed in the week corresponds to the number of patients inserted in the SUs waiting lists. We highlight that the ORs are often under-utilized (under 50%) while in some other cases the total occupation time of one OR exceeds its opening time. Figure 4 reports the GANTT chart of the surgeries performed in the OR-3 during weeks 5 and 6. In this case, both the ORs under and over usage can be easily observed. More details on the historical surgery schedule are reported in Section 4, where the planned schedule by

9

Week	#I SU-A	#I SU-B	Avg T	StDev T	#ORs UU	#ORs OU
1	31	12	123	67	7	1
2	29	17	137	74	7	4
3	31	18	110	49	9	4
4	31	10	122	78	8	1
5	34	15	121	65	9	6
6	29	11	144	90	8	4
7	25	13	144	86	8	5
8	28	13	129	84	7	1
9	27	12	122	69	10	2
10	29	17	126	81	9	4
11	35	14	124	63	6	5
12	32	11	120	53	8	3

Table 3: Dataset stats.

the hospital is used as a baseline for the comparison with the model outputs schedules.

2.4 Hospital goals

When dealing with OR scheduling and planning, most of the performance metrics concern under and over-utilization of the resources. Moreover, among the multi-criteria models, almost two-thirds of the mathematical programs show at least a utilization-related metric (as shown in the review work Cardoen et al. (2010)). AOU-Policlinico shares this optimization goal.

The policy of the AOU-Policlinico is to set a target utilization rate (UR) for each OR. A portion of the OR time capacity is hence set aside to manage either possible emergency surgeries on non-elective cases, or to face uncertainties that may occur during the working day and that may cause scheduled surgeries to be extended beyond their scheduled duration. This approach is justified since emergency operations seem to be performed more efficiently on elective ORs instead of predefined rooms (Wullink et al. (2008), Hans and Vanberkel (2012)).

After a lengthy discussion with the governance of AOU-Policlinico, we identified as one of the aims the definition of ORs optimized workloads that are as close as possible to the ideal UR, while avoiding occasional ORs daily closures. Indeed, the difference in the cost for opening an OR for just a few days or the full week is negligible, so that when an OR works for a day the policy of AOU-Policlinico is to make it work all the days of the week. On the contrary, the hospital is interested in checking if the closure of an OR is possible for all the week.
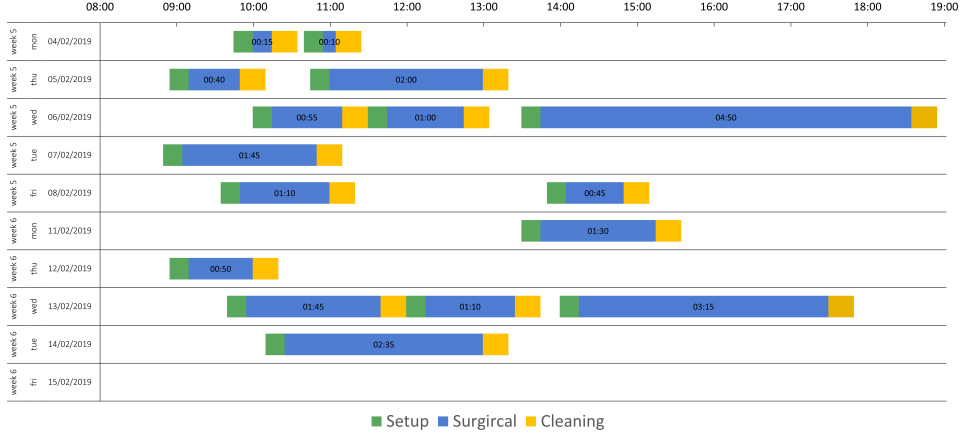
Figure 4: GANTT chart of historical schedule planned for OR-3 (opening times 8.00 - 16:00) during weeks 5 and 6.

In order to gain more flexibility in the ORs use, AOU-Policlinico agreed to consider the transfers of patients in charge of one SU to ORs located in a different SU. However, these transfers between SUs located in separate buildings require an effort both in terms of personnel needed and patient comfort. Thus, we identify as a further criterion the minimization of the number of transfers between SUs.

3 Model Definition

We now give a formal definition of the multi-criteria optimization model for the weekly ORs scheduling problem at AOU-Policlinico. The model plans the daily surgeries of the ORs and it is designed as a mathematical support tool for the planning decisions of the senior health administrator and the medical staff of the SUs. Note that since crew tournaments are not influencing the hospital goals in determining an optimal schedule, we do not include this aspect and any surgeon preferences in the model. In fact, being in a public hospital, the surgeons do not have their own set of patients and all surgeries can be therefore performed by the specialist on duty.

Notation. We use lower cases to denotes vector x, and x_i to denote the $i-$th component of vector x. Calligraphic characters, such as $\mathcal{I}$, denote instead the sets.

3.1 Parameters

We are given a list $\mathcal{I}$ of all elective surgery procedures planned for the week (*Week Surgery Program*). Each element i of the list $\mathcal{I}$ comes with a fixed time parameter t_i, that is the *surgery occupation time*. The latter is defined as the sum of preparation, surgical, and cleaning time and is inferred from the historical registers (for more detail see Table 2).

Among all the surgeries $i \in \mathcal{I}$, some of them can have different priorities. These priorities can depend on patients care cases (urgency or routine), medical staff needs, or hospital wards space requirements. We distinguish between:

- *day priority*: surgeries that have to be scheduled as the first surgery of the day because of patient's requirements (e.g. child, elder) or other doctors commitments. The position priority surgeries are inserted in a list $\mathcal{I}_{\mathcal{D}} \subset \mathcal{I}$;

- *mandatory day*: surgeries that, due to some patient's or doctor's needs, are bound to a specific day of the week $\widehat{d}_i$. The list of these surgeries is $\mathcal{I}_{\mathcal{M}} \subset \mathcal{I}$.

Now, let $\mathcal{C}$ be the set of the available SUs and $\mathcal{S}$ the set of all accessible ORs. $\mathcal{D} = \{1, 2, 3, 4, 5\}$ is the set of the days of the week (from Monday to Friday) while $\mathcal{P} = \{1, ..., p^{\max}\}$ represents the possible schedule positions of the surgery within a day. One can observe that the effective number of positions available in a day depends on the opening time O_s of the OR s and on the duration of the surgeries scheduled in day d, so that we should have $p_{sd}^{\max}$. For sake of simplicity, we assume that the number $p^{\max}$ is constant on all the days of the week and for all the ORs. We overestimated it considering the highest possible number of daily positions as follows:

$$\mathbb{Z} \ni p^{\max} = \left\lfloor \frac{\max\limits_{s \in S} O_s}{\min\limits_{i \in I} t_i} \right\rfloor \tag{1}$$

To guarantee the feasibility of the problem we assume that $|\mathcal{I}_{\mathcal{D}}| \leq |\mathcal{D}| \times |\mathcal{S}|$ and that $\sum_{i \in \mathcal{I}_{\mathcal{M}} | \hat{d}_i = d} t_i \leq \sum_{s \in \mathcal{S}} O_s$ for each day $d \in \mathcal{D}$.

As previously stated, we do not consider in the model the stochastic nature of the surgery times. Following the policy of the AOU-Policlinico, unforeseen events that may cause delays in the schedules are taken into account by considering a limitation on the maximum daily percentage of OR workload. In particular, the total daily occupation time for an OR, given by the sum of all the occupation times of the surgeries performed in that room on that day, must not exceed the percentage workload U of the daily opening time O_s.

Since each surgery must be associated with the SU where the patient has

Set	Description
$\mathcal{I}$	Set of all elective surgeries planned for the week
$\mathcal{I}_\mathcal{D}$	Subset of surgeries with a position priority
$\mathcal{I}_\mathcal{M}$	Subset of surgeries with a mandatory day
$\mathcal{C}$	Set of the SUs
$\mathcal{S}$	Set of the ORs
$\mathcal{D}$	Set of the days of the week
$\mathcal{P}$	Set of the time slots of a surgery day

Parameters	Description
t_i	Surgery total occupation time
U	Maximum daily percentage of ORs workload
O_s	Total daily opening time of OR s
a_{ic}	Boolean value representing if the surgery i origins in the unit c
f_{sc}	Boolean value representing if the SU s belong to the unit c
$\widehat{d_i}$	Mandatory day to perform the surgery $i \in I_M$
$p^{\max}$	Maximum number of surgeries per day obtained as in (1)

Table 4: List of sets and parameters used in the model.

been hospitalized, we define a parameter $a_{ic} \in \{0,1\}$, with $i \in \mathcal{I}$ and $c \in \mathcal{C}$, such that:

$$a_{ic} = \begin{cases} 1 & \begin{array}{l}\text{if patient undergoing surgery } i \\ \qquad \text{is hospitalized in SU } c\end{array} \\ 0 & \text{otherwise.} \end{cases} \qquad (2)$$

Similarly, we introduce the parameter $f_{sc} \in \{0,1\}$, with $s \in \mathcal{S}$ and $c \in \mathcal{C}$ to indicate the SU c where the OR s is located, namely:

$$f_{sc} = \begin{cases} 1 & \text{if OR } s \text{ is located in SU } c \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

For sake of clarity, the complete list of sets and parameters used in the model is reported in Table 4.

3.2 Variables

We use positional assignment binary activation variables for the problem formulation. For each surgery i we need to decide where ($s \in \mathcal{S}$) and when ($d \in \mathcal{D}$) it is scheduled, together with its position ($p \in \mathcal{P}$) in the daily schedule. Therefore, we introduce a binary variable called *surgery assignment variable* x_i^{sdp} indicating if the surgery i occurs in the OR $s \in \mathcal{S}$ on day d at position p.

$$x_i^{sdp} = \begin{cases} 1 & \begin{array}{l}\text{if } i \text{ is performed in OR } s \text{ on day } d \\ \qquad \text{as the } p^{th} \text{ surgery}\end{array} \\ 0 & \text{otherwise.} \end{cases}$$

We will refer to the triple (sdp) with the term *slot*, since it identifies exactly when and where the surgery i is performed. We can say that $x_i^{sdp} = 1$ denotes that surgery i is performed in slot (sdp). We further denote by x the vector of dimension $|\mathcal{I}| \times |\mathcal{S}| \times |\mathcal{D}| \times |\mathcal{P}|$ made up of elements x_i^{sdp}.

We also need to consider a further binary *OR weekly opening variable* z_s stating if the OR s is active during the week, i.e. at least one surgery is performed in s on any day $d \in \mathcal{D}$.

$$z_s = \begin{cases} 1 & \text{if OR } s \text{ is active during the week} \\ 0 & \text{otherwise.} \end{cases}$$

We denote with z the vector of dimension $|\mathcal{S}|$ made up of elements z_s.

3.3 Constraints

In this section we present the constraints considered in the model.

- Surgery assignment: each surgery must be assigned exactly to one slot (sdp).

$$\sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{P}} \sum_{d \in \mathcal{D}} x_i^{sdp} = 1 \qquad \forall\, i \in \mathcal{I} \tag{4}$$

- Slot assignment: a slot (sdp) can be assigned to at most one surgery.

$$\sum_{i \in \mathcal{I}} x_i^{sdp} \leq 1 \qquad \forall\, s \in \mathcal{S}, p \in \mathcal{P}, d \in \mathcal{D} \tag{5}$$

- Time utilization: For each OR $s \in \mathcal{S}$ on each day $d \in \mathcal{D}$, the daily occupation time must not exceed the maximum percentage of utilization of the OR s when it is working, namely $z_s = 1$.

$$\sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} t_i \cdot x_i^{sdp} \leq U \cdot O_s \cdot z_s \qquad \forall\, s \in \mathcal{S}, d \in \mathcal{D} \tag{6}$$

- Position priority: the surgeries $i \in \mathcal{I}_{\mathcal{D}}$ must be scheduled as the first operation of the day in the OR s, namely

$$\sum_{s \in \mathcal{S}} \sum_{d \in \mathcal{D}} x_i^{sd1} = 1 \qquad \forall\, i \in \mathcal{I}_{\mathcal{D}} \;. \tag{7}$$

- Mandatory day priority: the surgeries $i \in \mathcal{I}_{\mathcal{M}}$, having a mandatory day priority $\widehat{d_i}$, must be allocated properly.

$$\sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{P}} x_i^{s\widehat{d_i}p} = 1 \qquad \forall\, i \in \mathcal{I}_{\mathcal{M}} \tag{8}$$

We define the feasible region $\mathcal{F} = \{(x, z) : \text{constraints (4)-(8) are satisfied}\}$.

3.4 Optimization criteria

According to the discussion on Section 2, three main optimization criteria are identified as follows.

$F1$: minimize the number of weekly open ORs. Thus, we aim to minimize the function

$$F_1(x, z) = \sum_{s \in \mathcal{S}} z_s \ .$$

$F2$: minimize the number of patients transferred between SUs. We consider the "cost" of transferring patients among SUs independent from the origin/destination. Aiming to minimize the overall number of transfers among SUs, we need to count the number of patients i hospitalized in c and undergoing the surgery in an OR s that is not located in the SU c. Thus, we minimize the following function

$$F_2(x, z) = \sum_{i \in \mathcal{I}} \sum_{c \in \mathcal{C}} \sum_{s \in \mathcal{S}} (1 - f_{sc}) \cdot a_{ic} \left(\sum_{p \in \mathcal{P}} \sum_{d \in \mathcal{D}} x_i^{sdp} \right)$$

where a_{ic}, f_{sc} are defined in (2) and (3).

$F3$: minimize the maximum deviation between the actual workload and the AOU-Policlinico ideal one. The latter is obtained as the product of the maximum daily percentage workload U and the ORs opening time O_s. Therefore, we define the third objective as

$$F_3(x, z) = \max_{s \in \mathcal{S}} \left(|\mathcal{D}| \cdot U \cdot O_s \cdot z_s - \sum_{d \in \mathcal{D}} \sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} t_i \cdot x_i^{sdp} \right) \ .$$

In particular, this criterion can be linearized using standard arguments and introducing an auxiliary variable.

We observe that the three criteria might be at odds with each other. Thus, the problem is a truly multi objective integer optimization problem. We look for a Pareto optimal solution, namely a feasible point $(\widetilde{x}, \widetilde{z})$ such that there exists no other feasible point (x, z) satisfying $F_i(x, z) \leq F_i(\widetilde{x}, \widetilde{z})$ for $i = \{1, 2, 3\}$ and $F(x, z) \neq F(\widetilde{x}, \widetilde{z})$, being F the vector made up of the three components F_1, F_2, F_3. We define the *ideal* objective vector $F^{id} \in \mathbf{R}^3$ component-wise as

$$F_i^{id} = \min_{(x,z) \in \mathcal{F}} F_i(x, z) \qquad i = \{1, 2, 3\}.$$

As usual in multi objective optimization, we consider the vector F^{id} as a reference vector and the feasible solutions $(\widetilde{x}, \widetilde{z})_i^{\star}$, with $i = \{1, 2, 3\}$, are said

Pareto optimal.

In order to tackle a multi-Criteria problem, we adopt a weighted-sum scalarization technique (Yang (2014)). We hence consider a single objective integer optimization problems with the following objective function:

$$\min_{(x,z)\in\mathcal{F}} w_1 F_1(x,z) + w_2 F_2(x,z) + w_3 F_3(x,z), \qquad (9)$$

where $w_i \geq 0$, for $i = \{1,2,3\}$ are finite weights. It is well known (see Proposition 3.9 in Ehrgott (2005)) that if $w_i > 0$ for all i, then each optimal solution of Problem (9) is a Pareto solution for the problem. In principle, one is interested in finding all the Pareto optimal solutions but this is known to be a hard optimization problem even in the case of two objectives (see e.g. De Santis et al. (2020) and references therein). Thus, we are interested in selecting one Pareto solution by fixing the weights w_i to suitable positive values. The choice of the weights w_i has been decided according to the policy of the AOU-Policlinico as reported in Section 4.

3.5   MILP formulation

We report in this section the weight formulation of (9) as a Mixed Integer Linear Problem (MILP). At first step, we introduce an auxiliary variable $u \in \mathbb{Z}_+$ to linearize the criterion $F3$. Note that, without loss of generality, we consider the set of all positive integers as the domain for the new variable. Indeed, it is sufficient to consider integer value for the occupational time parameter t_i. Following standard arguments, we add the group of constraints presented below:

$$|\mathcal{D}|\cdot U \cdot O_s \cdot z_s - \sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} t_i \cdot x_i^{sdp} \leq u \qquad \forall\, s \in \mathcal{S} \qquad (10)$$

and substitute $F3$ as u in (9).

Thus, the integer problem (9) can be written in compact form as the following MILP:

$$
\begin{aligned}
\min_{(x,z,u)} \quad & w_1 F_1(x,z) + w_2 F_2(x,z) + w_3 u, \\
s.t. \quad & |\mathcal{D}|\cdot U \cdot O_s \cdot z_s \ + \\
& -\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}}\sum_{p\in\mathcal{P}} t_i \cdot x_i^{sdp} \leq u \quad \forall\, s \in \mathcal{S} \\
& (x,z) \in \mathcal{F} \qquad u \in \mathbb{Z}_+ \quad .
\end{aligned}
$$

4   Scenario analysis and optimization results

In this section we present the results obtained by running the MILP model described in Section 3. The computational experiments are performed using

an x64 MS Windows 10 machine with an Intel (R) Core (TM) i7-10510U CPU and 16 GB of RAM. The solution of the MILP is carried using the *IBM ILOG CPLEX v12.10* as optimizer. Note that different choices of the weights in (9) will lead to different Pareto solutions. Now, because the three goals have different priorities for AOU-Policlinico we also use different orders of magnitude. After several trials mirroring the hospital preferences, the weights in (9) are set as follows:

w_1	w_2	w_3
10	1	10

We use three different Key Performance Indicators (KPIs) to evaluate the models' outputs:

- *UR*: the OR week average utilization rate in the open days, which is close to the ideal maximum utilization rate $U = 80\%$ when minimizing $F3$;

- *Cls*: the total number of inactive days during the week of the OR, which is minimized when using the objective $F1$;

- *Tr*: the number of patients transferred among SUs, which is minimized when using the objective $F2$.

For the first set of experiments we consider the actual setting of the ORs opening times O_s, corresponding to $(12, 8, 8, 8)$ opening hours for the respective ORs (for a total of 36 hours per week). Furthermore, since we do not have data on priority day surgeries or surgeries requiring a mandatory day, we first run the model neglecting constraints groups (7) and (8). Hereafter we denote this setting as *Scenario #0*. The historical planned schedules for the surgeries along the 12 weeks is used as a benchmark for the optimized schedules. In particular, we compare the KPIs evaluated on the historical schedule with those obtained by optimizing each of the three single objectives $F1$ $F3$ $F2$, and by using the multi-criteria function with the weight scalarization approach. The results are reported and discussed in Section 4.1. By analyzing them, we found that there was a large possibility of improvements on all the KPIs by changing the actual setting of the ORs opening times O_s. In fact, one of the aims of AOU-Policlinico is to obtain some insight on the possibility of closing one OR or changing the operating times to obtain a better utilization rate. To this purpose, in the second set of experiments, we analyze the MILP results on several different scenarios for the daily ORs opening times O_s. We report them in Table 6, where, for the sake of completeness, we also show Scenario #0 opening hours. The results of these experiments are hence discussed in Section 4.2.

A third set of experiment finally investigate the role of the surgeries position and day priorities. In Section 4.3 we then analyze the results obtained by the model when introducing randomly generated priority restrictions.

4.1 Optimizing with the actual Opening times

In this section, we analyze the results obtained from Scenario #0. First, we solve the optimization problems related to each of the three objectives $F1$, $F2$, $F3$ and then the multi-criteria problem. The baseline used for comparison is the historical schedule on the 12 weeks. We remark that this surgery planning is not always feasible for our optimization model due to violation of the maximum target utilization rate $U = 80\%$ (Over Run) in some cases. In particular, for each of the 12 weeks and each of the four ORs, we report in Table 6 the week average utilization rate (UR) in the open days, the total number of inactive days throughout the week (Cls) as well as the number of transferred patients between the two SUs (Tr). We remark that when Cls is five, it means that the OR is closed for the entire week and its corresponding UR is equal to 0. Instead, for each week and SU, Tr describes the number of transferred patients moved to that SU.

We note that in all the experiments conducted the optimal solution was reached ($MIPGap = 0$ %) in a negligible computational time proving that the model can be solved efficiently. Indeed, solving to optimality one of the single objectives $F1$ or $F2$ requires on average half a second to be solved, while optimizing $F3$ needs about one second to produce the final schedule. On the other hand, the multi-objective model takes about five seconds to optimally solve each instance. Based on the results in Table 7, we observe that in the historical schedules in each week at least an OR has a UR that falls below 50%. It is interesting to note that, although the SU-B (OR-3, OR-4) is opened 4 hours less than SU-A (OR-1, OR-2), the SU-B presents a higher number of daily closed ORs and an overall lower percentage of usage. This highlights an inefficient distribution of surgeries among the two SUs and justifies the considered possibility of transferring patients among them. Note that there are no patient transfers between SUs in the historical schedule.

Getting into details, the minimization of $F1$, namely the daily open ORs, presents a higher UR obtained by the closure of at least an OR on each week (the ninth week presents two closed ORs). Minimizing $F2$ obviously leads to a solution with the lowest number of transferred patients (only 5 over 524 total surgeries) and might be seen as a minimal change with respect to the historical schedules. It further presents the highest number of inactive days (33) distributed among the ORs. In this case, no week closure is planned, thus highlighting the conflict between the two objectives $F1$ and $F2$. Besides, the optimization results for $F3$ are similar to those seen for $F1$, only differing in the choice of the weekly closed OR. Indeed, since $F3$ minimize the OR deviation from the ideal UR, it forces in some weeks the closure of the OR with the longest opening time (OR-1). Furthermore, for both $F1$ and $F3$, the number of patients transferred among SUs accounts for 50% of all the

Week	SU	OR	Historical use			OPT_F1			OPT_F2			OPT_F3			MILP		
			UR	Cls	Tr	UR	Cls	Tr	UR	Cls	Tr	UR	Cls	Tr	UR	Cls	Tr
1	A	1	52.1	0	0	59.2	0	10	63	0	0	0	5	4	0	5	2
		2	68.4	0		57.5	0		65.1	1		73.3	0		73.5	0	
	B	3	47.7	0	0	0	5	16	33.3	2	0	73.3	0	15	73.4	0	16
		4	32.8	1		74.2	0		54	0		73.8	0		73.5	0	
2	A	1	60	0	0	76.1	0	14	71.9	0	0	76.2	0	15	76.2	0	4
		2	76.7	0		75.6	0		58.8	0		74.4	0		74.4	0	
	B	3	51.7	0	0	73.3	0	10	39.8	0	0	74.4	0	8	74.4	0	1
		4	44.8	0		0	5		56.7	0		0	5		0	5	
3	A	1	47.1	0	0	73.9	0	11	59	0	0	0	5	2	0	5	0
		2	70.2	0		0	5		52.3	0		74.8	0		74.8	0	
	B	3	59.6	1	0	59.6	0	20	25.3	1	0	74.8	0	18	74.8	0	10
		4	44.8	1		54	0		63.3	0		74.8	0		74.8	0	
4	A	1	51.4	0	0	64.4	0	6	68.8	1	1	63.9	0	7	64	0	1
		2	64.2	0		46.2	0		75.4	0		55.8	0		69.8	1	
	B	3	39.1	1	0	0	5	11	43.8	4	0	56	0	10	55.8	0	1
		4	44	1		64.8	0		51.3	1		0	5		0	5	
5	A	1	44.3	0	0	75.3	0	11	63.1	0	0	72.4	0	7	72.4	0	3
		2	83.3	0		67.7	0		55.2	0		0	5		68.5	0	
	B	3	45	0	0	65.4	0	12	48.1	0	0	69	0	18	0	5	1
		4	51.3	0		0	5		48.1	0		68.5	0		69	0	
6	A	1	55.4	0	0	72.8	0	3	62.4	0	1	71.1	0	6	71.1	0	2
		2	65	0		0	5		73.5	0		0	5		66.7	0	
	B	3	45.6	1	0	66.5	0	15	55.2	4	0	66.5	0	16	66.9	0	0
		4	55.6	0		64.6	0		62.1	0		67.1	0		0	5	
7	A	1	47.4	1	0	70.5	0	10	58	0	1	66.8	0	12	66.9	0	2
		2	99.7	1		60.4	0		72.1	0		75.3	1		60.2	0	
	B	3	50.5	1	0	0	5	8	31.3	3	0	60.4	0	10	0	5	1
		4	51	0		54.6	0		61.5	1		0	5		60.2	0	
8	A	1	54.7	0	0	70.5	0	10	58	0	1	66.8	0	12	66.9	0	2
		2	60.1	0		60.4	0		72.1	0		75.3	1		60.2	0	
	B	3	46.1	1	0	0	5	8	31.3	3	0	60.4	0	10	0	5	1
		4	52.1	1		54.6	0		61.5	1		0	5		60.2	0	
9	A	1	57.9	0	0	79.9	0	12	53.6	0	0	79.4	0	12	79.4	0	2
		2	65.3	1		78.5	0		58.8	0		79.1	0		0	5	
	B	3	28.1	2	0	0	5	0	41.7	4	0	0	5	0	79.2	0	4
		4	42.3	0		0	5		63.5	1		0	5		0	5	
10	A	1	55.6	0	0	72.8	0	13	69.9	0	1	71.7	0	8	71.7	0	3
		2	55.6	0		64.8	0		51.5	0		0	5		67.3	0	
	B	3	30.6	0	0	0	5	6	44.8	0	0	67.3	0	15	0	5	0
		4	72.5	0		68.1	0		41	0		67.3	0		67.3	0	
11	A	1	63.1	0	0	72.6	0	10	66.4	0	0	74	0	5	74.2	0	1
		2	78.8	0		69.4	0		73.8	0		0	5		0	5	
	B	3	49.7	1	0	75.2	0	9	54.9	2	0	71	0	17	71	0	9
		4	50.5	1		0	5		47.3	0		71.5	0		71.2	0	
12	A	1	56.3	0	0	72.8	0	10	61.8	0	0	0	5	3	0	5	0
		2	69.2	0		61.2	0		60.8	0		71.7	0		71.9	0	
	B	3	29.9	2	0	55.5	1	7	41.7	3	0	71.7	0	18	71.5	0	13
		4	54.2	1		0	5		55.7	1		71.5	0		71.5	0	

Table 5: KPIs computed for the historical schedule and obtained when optimizing each of the three single objectives $F1$ $F3$ $F2$, and by using the multi-criteria function with the weight scalarization approach.
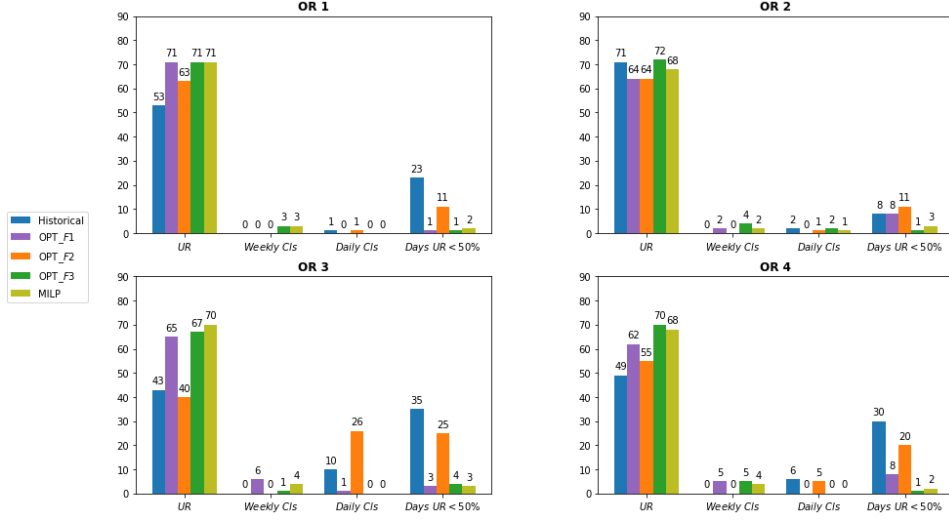
Figure 5: Bar histograms of the ORs aggregated performances over the twelve weeks when minimizing the single goals individually and the weighted scalarized multi-objective function.

surgeries performed. The output schedules of the MILP leave evidence that it is possible to obtain a more uniform distribution in the ORs workload while deciding for the OR closure of the entire week. Without a doubt, the multi-objective model balances the results satisfactorily while fulfilling all the hospital requirements. Indeed, a high average UR of the weekly opened ORs is obtained transferring only a small number of patients from one SU to the other (15% of the patients).

Figure 5 shows the aggregated performance obtained for the four ORs over the twelve weeks in terms of the different objective functions analyzed. Aside from the UR, we report the total number of closed ORs (*Weekly Cls*), the number of inactive days (*Daily Cls*) and the number of days showing an UR lower than 50% (*Days UR < 50%*). Based on the results obtained by the MILP, the AOU-Policlinico could have decided to close one of the ORs with 8 hours of opening time while maintaining the same efficiency level for elective surgeries. However, there is still room for improvement. Note that, the closed OR in each week is not always the same and it can be either one belonging to SU-A or SU-B. In particular, in 3 weeks we observe the closure of the OR-1, i.e. the one with an opening time equal to 12 hours. Given these observations, it is also interesting to check whether a different opening schedule can give better KPIs or if it is convenient to close one OR. To this aim, we performed some additional computational experiments, presented in the following section, in which we analyze different scenarios with a decreasing number of weekly working hours.

# Scenario	Opening time O_s				Total opening time $\sum\limits_{s \in S} O_s$
	OR-1	OR-2	OR-3	OR-4	
0	12	8	8	8	36
1	12	6	10	6	34
2	12	6	8	6	32
3	10	6	8	6	30
4	10	6	6	6	28
5	12	8	8	0	28

Table 6: Scenarios tested for the weekly ORs opening hours O_s. Scenario #0 represents the AOU-Policlinico actual opening time.

	Scenario #0	Scenario #5
Transferred Patients	79	45
Weekly ORs Closures	13	1
Daily ORs Closures	1	0
#Days UR < 50%	31	16

Table 7: Comparison of the metrics obtainded in Scenario #0 and Scenario #5 of ORs opening hours.

4.2 Optimizing with different Opening time

We test the multi-criteria MILP in different scenarios of ORs opening times to optimize the weekly opening hours. Table 6 reports the different combinations of the ORs workload tested. We underline that at least one of the ORs must work for at least 10 hours because of some long surgeries time. For each of the scenarios, it was possible to obtain a feasible schedule for all the weeks. Thus we can reduce the total amount of working hours from 36 to 28. The performances obtained show analogies with the results of the previous experiments and make more efficient use of the clinical resources. In particular, we analyze the results of the last scenario (#5), in which the closure of OR-4 in the SU-B is required. Indeed, closing an OR without reducing the level of service represents a large saving in terms of cost and medical staff. This appears to be the best choice given the three months of data collected.

We report in Figure 6 four histograms, one for each OR, where we represent the average weekly ORs workload of Scenarios #0 and #5. When a bar chart is not reported, it means that in that week the OR is closed. It appears that the weekly workload in Scenario #5 is better distributed among the ORs. From the performance metrics in Table 7, it is clear that there is an overall
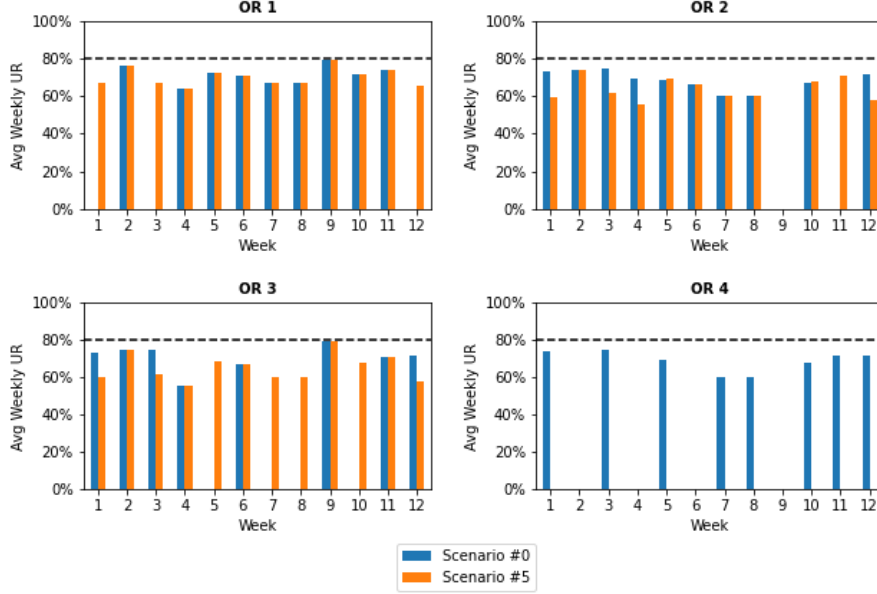
Figure 6: Histogram showing the weekly workload obtained for each week by Scenario #0 and Scenario #5 ORs opening hours. We report on the x axis the number of the week and on the y axis the average weekly *UR*. Note that, in the Scenario #5 the OR-4 is unavailable.

improvement in every single objective. Indeed, the closure of OR-4 allows 34 transfers less than Scenario #0 while ensuring a very low daily deviation from the ideal *UR* and a lower number of days with underused ORs (16). Lastly, in scenario #5 all the ORs perform at least a surgery on each day (*Daily ORs Closures* equal 0).

4.3   Optimizing with priority constraints

Now we present the results of the multi-criteria MILP when introducing sets of position and mandatory day priority constrains (see Section 3.3 for more details). For each week, we generate a position priority with 10% of probability for each surgery, while we add a mandatory day priority at 50%. In particular, the specific day $\hat{d}_i$ of a prioritized surgery i is selected according to a uniform distribution ranging from 1 to 5 (from Monday to Friday). Table 8 reports, for each week, the number of surgeries that must be scheduled as the first surgery of the day ($|\mathcal{I}_{\mathcal{D}}|$), and the number of surgeries that must be performed on a specific day of the week ($|\mathcal{I}_{\mathcal{M}}|$).

In Figure 7 we show the aggregated performance over the 12 weeks of the historical data compared with the one obtained by the multi-criteria MILP formulated with and without priorities. As expected, the results of the prioritized model evidence a reduction of the average *UR* a reduction in the

Week	1	2	3	4	5	6	7	8	9	10	11	12
$|\mathcal{I}_\mathcal{D}|$	10	18	13	7	14	12	17	8	12	13	11	13
$|\mathcal{I}_\mathcal{M}|$	6	3	7	3	3	5	5	3	4	9	7	4

Table 8: Number of position and day priority randomly generated for each test week.
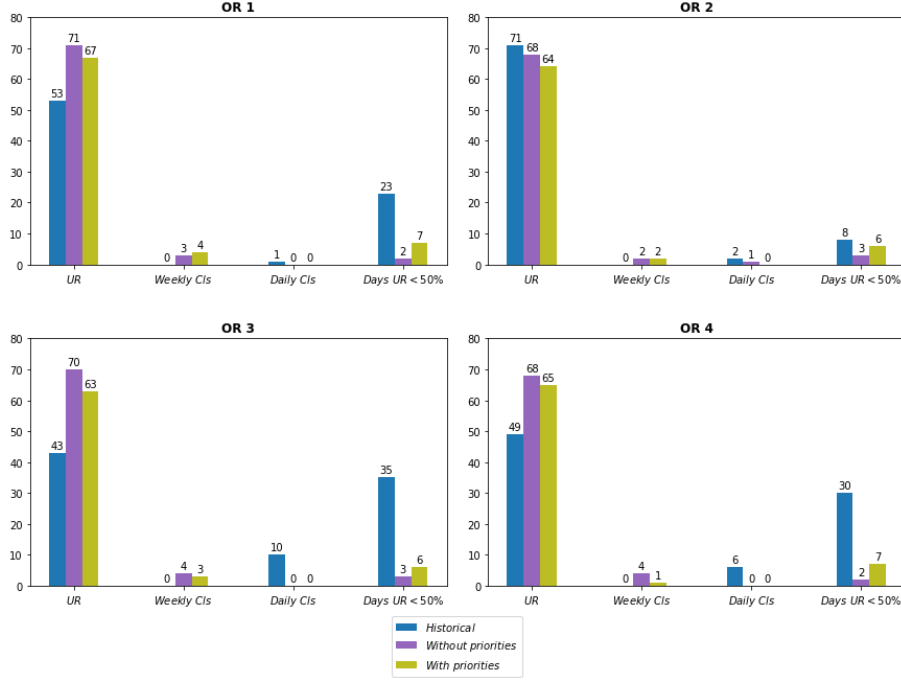
Figure 7: Comparison of aggregated performance over the 12 weeks of the historical data and the multi-criteria MILP with and without position and mandatory day priorities.

average UR compared to its optimized sibling model, and this is due to the presence of more days with under-utilization. We can also record a reduction in the weekly closures determined by the presence of mandatory day priority that may force the activation of an OR on a specific day. However, even though the inclusion of priorities further limits the feasible region of the model, the resulting metrics show a more balanced situation compared to historical schedules. This fact demonstrates the significant contribution of using a mathematical model to improve ORs utilization.

5 Conclusions

The present work aims to make the AOU-Policlinico ORs utilization more efficient exploiting a mathematical formulation for the weekly surgery schedule. We formulate a multi-criteria mixed integer linear model as a support tool for health direction and, more in general, for all the medical staff involved in the surgery scheduling phase of two surgical units at AOU-Policlinico.

We first gave an overview of the data at hand and analyzed their main features then, in collaboration with the hospital team, we identified the most critical optimization criteria. Extensive tests were carried out considering not only the contribution given by each goal (alone and within the multi-objective function), but also varying the weekly opening hours of the ORs under analysis and inserting some priority constraints. The results obtained by running the model on 12 weeks of real data showed that AOU-Policlinico has multiple options to improve the efficiency of its surgical units and to greatly benefit from clinical resources. Indeed, we highlighted the occurrence of some unnecessary opening of the ORs throughout the weeks. As a result, closing an operating room should be considered; the OR can be assigned to another division or it can be dedicated to emergency cases or it can be also used to increase the number of elective patients scheduled. Of course, the last option must take into account the availability of elective beds in the wards and might open to a change of distribution policy among emergency and elective beds.

Regardless of the chosen objective function and constraint bounds, computational tests have shown that the use of mathematical tools in the surgeries scheduling allows for a more efficient use of resources. Our research project not only addresses a real-life scheduling problem but also intends to simplify the overall scheduling process from a practical perspective. The approach presented can be significantly helpful for the administration also to organize more efficiently the surgeon group shifts.

We remark that in the current implementation of the model the effective surgery times have been used as deterministic values but instead surgery times are stochastic. As future work, we plan to use a robust optimization approach to tackle possible variations in the standard times defined by clinicians. Moreover, we want to formulate a more complex model that considers preference-related measures, for both the patients and surgeons. Doctors may want to give different emphasis to weekly and daily priorities; this can be done by inserting priority goals, with different coefficients and orders of magnitude, in the objective function.

Acknowledgements

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