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An optimal non–uniform piecewise constant approximation for the patient arrival rate for a more efficient representation of the Emergency Departments arrival process

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Abstract

Overcrowding represents an increasing and well studied phenomenon which afflicts Emergency Departments all over the world. Shortage of staff, flu season and lack of hospital beds are among the possible causes. As consequence, waiting times are enlarged and life of critical patients can be endangered. This urges ED managers to improve performance of healthcare services. Modeling approaches used to tackle this problem, are often based on Discrete Event Simulation, hence needing to accurately represent the patient arrival process to the ED. Since the arrival rate is time-dependent, suitable non stationary process models must be considered, such as the nonhomogeneous Poisson process.

In this paper we focus on this arrival process, in order to determine the best piecewise-constant approximation of the arrival rate function. A proper

number of non equally spaced intervals are used aiming at accurately representing the time-varying arrival rate. This is obtained by solving an integer non linear black box optimization problem with black box constraints. Data from a large Italian hospital ED are used to show the effectiveness of the proposed approach.
Keywords: Emergency Department, arrival process, non homogeneous Poisson process, black box optimization

1. Introduction

The importance of emergency medical services is very well known . Their effectiveness and timeliness allow, in most cases, to save human lives (see [1]). In this framework, The Emergency Department (ED) can be considered the first service provided to patients which require urgent medical treatments. Unfortunately, the performance of an ED is often compromised by overcrowding. Today it is a worldwide phenomenon which can cause dangerous delays in patient treatment, even endangering their lives. Several healthcare management studies have been devoted to analyze the patient flow through an ED aiming at finding possible countermeasures to overcrowding. This is witnessed by the large number of papers published in the last years dealing with this phenomenon. Some examples are [2, 3, 4, 5, 6, 7, 8, 9, 10]. In particular, [2] reports a survey on causes, effects and possible solutions for the overcrowding problem. According to this review, the main causes are insufficient staffing, inpatient boarding, flu season, request for nonurgent visits, "frequent–flyer" patients, unavailability of hospital beds. The effects are identified in delays and reduced quality of treatments, ambulance diversion, higher patient mortality and number of patients which leave the ED without been visited. Some indices measuring the degree of overcrowding have been also proposed (NEDOCS, READI, EDWIN) [3].

In [11] a review of modelling approaches adopted for studying ED patient flow and overcrowding is reported. Among these, *Discrete Event Simulation* (DES) is the most commonly used (see e.g. [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]). In particular, the recent paper [12] reports a systematic literature review on simulation applied to EDs, along with emerging trends.

Simulation–based optimization approach is also adopted in dealing with ED patient flow (se e.g. [27, 28, 29]). DES outputs are integrated to define a suitable objective function used by an optimizer to find the best resource configuration, taking into account appropriate constraints. It is clear that the objective function does not possess an analytical form since its values are obtained through simulation runs. As a consequence the optimization problem to be solved for determining the optimal ED settings often has challenging characteristics, such as the presence of both continuous and discrete variables, nonlinear objective and constraint functions of black–box type and multiple objectives to be optimized. Other examples of application of this approach in healthcare contexts can be found in [30, 31, 32].

Both these approaches are based on the use of an on purpose constructed DES model, accurately reproducing the ED operation. The key point consists in modeling the complex structure of an ED at a proper level of detail, taking into account the different nature of the processes, describing the various activities of a patient through the ED, since the arrival. DES models provide all of a set of possible responses according to a large number of realizations of patient arrival stochastic process and of the ED activities processes. Therefore, modelling the ED patient arrival represents the very first building block of each DES system adopted for reproducing an ED operation and it strongly affects any analysis based on a simulation study.

In this paper we focus on modeling the ED patient arrival process and, in order to prove the effectiveness of the proposed methodology, a real case study regarding the ED of *Policlinico Umberto I*, a hospital located in the center of Rome, is considered.

Any ED is recognized to have strongly time–varying arrivals, so that a largely accepted model for the arrival process is a non-homogeneous Poisson process (NHPP) with arrival rate $\lambda(t)$, [23, 20, 21, 28, 10, 27]. Consequently, the average number of arrivals in an interval $[t, t + T]$ is given by $\int_0^T \lambda(t + s)\, ds$. Such an assumption usually stems from some experimental evidences: i) patients arrive independently one to another; ii) they hardly come simultaneously; iii) in any whatever small interval Tt the chance to have just one arrival is proportional to Δt, while the chances to have more then one arrival are negligible. On the other end, as discussed in [33], there are situations that depart from the Poisson assumption when the arrivals are, so to say, not spontaneous: arrivals due to accidents or overflow from other structures, scheduled appointments with the specialists, etc. Therefore a proper statistical test must be applied to the considered case study to support the NHPP assumption. Usually a piecewise constant approximation of the arrival rate on equally spaced intervals is considered, and the well known Conditional Uniform property of the Poisson process is exploited: conditional on the number of arrivals in any interval $[t, t + \Delta t]$, the arrival times $\{t_i\}$ rescaled to $\tau_i = (t_i - t)/\Delta t$, are independent and uniformly distributed over $[0, 1]$. This allows to use data from any interval of any day of observation of the ED patient arrival process. Then, as suggested in [33] the Kolmogorov–Smirnov (K-S) test can be applied to check if the NHPP hypothesis can be accepted, providing the rounding of the arrival times is avoided and a proper width of the intervals is adopted.

In this work the piecewise constant approximation of the arrival rate is accomplished with non equally spaced intervals. This choice is suggested by the typical situation that occurs in EDs where the arrival rate is low and varying during the night hours, and it is higher and more stable in the day time, this is indeed what happens in the chosen case study. Therefore, it is more efficient to have a finer approximation during the night hours as opposite to day time. The proposed method, finds the best partition of the 24 hours in intervals, not necessarily equally spaced. To this aim a maximum likelihood method is proposed to estimate the best partition of the day, using experimental data regarding two months of observations of the arrival times at *Policlinico Umberto I*. The objective function is the likelihood of the number of arrivals observed in any interval according to the NHPP hypothesis. This way, the optimal values of the sampling intervals find a trade off between opposite requirements: a good piecewise constant approximation of the arrival rate would require shorter sampling steps, but the number of arrivals in any interval might therefore be too small to give a good sampling approximation of the uniform cumulative distribution function for the rescaled arrival times; on the contrary, larger sampling steps would give a rough piecewise constant approximation of the arrival rate function. The balance between these two competing requirements should avoid an improper rejection of the NHPP hypothesis through the K-S test.

The method used to solve the resulting optimization problem belongs to the class of Derivative–Free Optimization methods. In particular we use the new algorithmic framework proposed in [34] which handles black box problems with integer variables and black box constraints. In our formulation, the black box constraints arise in satisfying the K-S test in each interval. As far as we are aware, this is a novel approach within arrival process modeling in the ED framework.

The results show that this approach enables to determine intervals which accurately approximate the arrival rate function, ensuring the consistency between the NHPP hypothesis and the arrival data.

The paper is organized as follows: in Section 2 some details on the ED considered in this work are briefly reported. The statistical model we adopt is described in Section 3. Section 4 reports the black box optimization problems we consider along with experimental results. Finally, some concluding remarks are in Section 5.

2. Case study

In order to prove the effectiveness of our methodology, we applied it to a real case study regarding the ED of *Policlinico Umberto I* in Rome, that we briefly describe in this section. It is a large dimension ED, the largest in the city, in terms of number of patient arrivals, which are on average 140,000 per year. It suffers from the overcrowding problem and, as a consequence, the average patient waiting times and Length Of Stay (LOS) exceed the maximum values recommended by the guidelines of the Italian Ministry of Health. In particular, new guidelines have been recently issued requiring compliance with more tight threshold values for LOS, on the basis of patient urgency level. Moreover, in order to ensure more appropriate clinical paths and following the main current international trend a new triage which considers five numerical urgency codes has been introduced. However, since we collect data for the year 2018, they still refer to the triage based on standard four colors tags. From ED database, we anonymously collected all the timestamps available regarding patient flow through the ED. In particular, for this study we focus on patient arrival times, for two months (January and February, 2018). After a data cleaning, we use them in our study.

3. Statistical model

In most works of the relevant literature, the arrival process at EDs is modeled as a Nonhomogeneous Poisson Process (NHPP) (see for instance [23, 20, 21, 28, 10, 27]). The adoption of a nonstationary process is necessary since the patients arrival rate is time-dependent. In Figure 3.1 the plot of the hourly average arrival rate over the two considered months is shown, while a plot of the interarrival times for an arbitrary day of the year 2018 is reported in Figure 3.2. In both cases, a strong within-day variation is observed: rapid changes in the number of arrivals are featured in the night hours, whereas in the day time a smoother profile is evidenced.

No analytical function is available for the arrival rate, which can be approximated only by average evaluations computed through experimental data

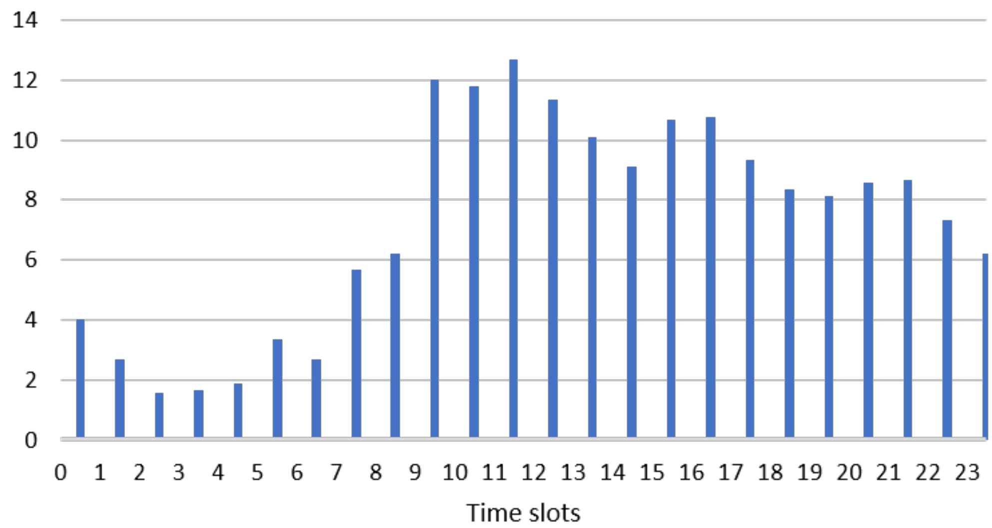

Figure 3.1: Plot of the hourly average arrival rate for a specific day of the week.

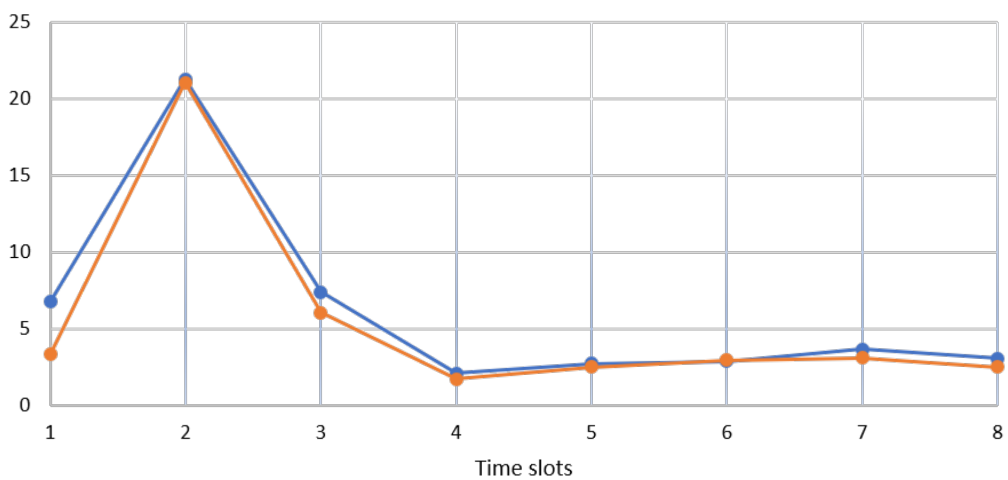

Figure 3.2: Plot of the average (in blue) and standard deviation (in orange) of the inter-arrival times for an arbitrary day of year 2018. On the abscissa axis 3-hours time slots are considered

on suitable intervals over the 24 hours of the day. Consequently, a proper partition $\{T_i\}$ of the observation period $T = [0, 24]$ must be found to define a good local average approximation of the arrival rate $\lambda(t)$ in each T_i. In other words, our goal is to find the best piecewise constant approximation of $\lambda(t)$, $t \in T$. Therefore, we need a criterion to select the best partition $\{T_i\}$ to be used for computing the local average of $\lambda(t)$ in each T_i. Such criterion must ensure the validity of the NHPP hypothesis and, accordingly, maximize the likelihood of the experimental data.

Let $\{t_i\}$ be the set of arrival times within one day. Given any partition $\{T_i\}$ of the observation period T in N intervals, let $\{A_i\}$ the collection of random variables representing the number of arrivals in each interval T_i. For the sake of simplicity, we will denote by T_i either the partition interval or its length $T_i = x_{i+1} - x_i$, $i = 1, \ldots N$.

The likelihood of the number of arrivals A_i in any T_i can be computed by

$$\begin{aligned} P(A_1 = k_1, \ldots, A_N = k_N) &= \prod_{i=1}^{N} P(A_i = k_i) \\ &= \prod_{i=1}^{N} e^{-\lambda_i} \frac{(\lambda_i)^{k_i}}{k_i!}, \end{aligned} \tag{3.1}$$

where k_i is the actual number of arrivals in T_i. The value λ_i is the average arrival rate μ_i/T_i, where μ_i is obtained by averaging the number of arrivals in the given interval for the same day of the week over the two considered months. This is possible because the overdispersion test described in [33] is accepted, confirming that data are consistent with NHPP with fixed rate, enabling to combine arrivals from different days.

The likelihood function (3.1), to be maximized, implicitly depends on the partition $\{T_i\}$ defined by the boundary points $x_1, x_2, \ldots, x_{N+1}$, where $x_1 = 0$

and $x_{N+1} = 24$.

Note that function (3.1) has not analytical structure with respect to the independent variables x_i and can only be computed by a procedure once the x_i values are given. In particular, such function evaluates the likelihood of the experimental data $k_1, k_2, \ldots, k_N$ according to the NHPP hypothesis: in each interval the number of arrivals A_i follows a Poisson distribution law with average arrival rate λ_i that is different for different intervals due to the non-stationarity of the process.

Vector $x = (x_1, \ldots, x_{N+1})' \in \mathbb{R}^N$ collects the unknowns to be determined in order to obtain the best piecewise-constant approximation of the arrival rate which makes the observed data $k_1, \ldots, k_N$ most likely to occur according to the NHPP hypothesis. Nevertheless, some constraints on x must be considered to ensure the validity of the NHPP hypothesis. Therefore, within any T_i, the arrival times must satisfy a statistical test to guarantee that the shape of the distribution is Poisson. To this aim, for any interval $T_i = [x_i, x_{i+1}]$ of the given partition, we consider the rescaled arrival times $\tau_{ij} = \dfrac{t_{ij} - x_i}{T_i}$, where $t_{ij}, j = 1, \ldots, k_i$, are the arrival times within T_i. The rescaled arrival times, conditionally to the value k_i of the number of arrivals, are a collection of i.i.d. random variables uniformly distributed over $[0, 1]$ (see [35]). Hence, in any interval we compare the theoretical cumulative distribution function (cdf) $F(t) = t$ with the empirical cdf

$$F_i(t) = \frac{1}{k_i} \sum_{j=1}^{k_i} 1_{\{\tau_{ij} \le t\}}, \qquad 0 \le t \le 1$$

by using

$$D_i = \sup_{0 \le t \le 1} (|F_i(t) - t|), \tag{3.2}$$

where $1_{\{S\}}$ denotes the indicator function of the set S.

On any T_i a Conditional Uniform Kolmogorov-Smirnov (CU K-S) test can be performed to check the validity of the Poisson assumption with average arrival rate λ_i. The critical value for this test is denoted as $T(k_i, \alpha)$ and its values can be found on the K-S test critical values table. Accordingly, the Poisson hypothesis is accepted if $D_i \leq T(k_i, \alpha)$. Therefore, the best piecewise-constant approximation $\{\lambda_i\}$ of the time-varying arrival rate $\lambda(t)$ is obtained by solving the following black-box optimization problem:

$$\begin{aligned} \max \quad & f(x) \\ s.t. \quad & g_i(x) \leq 0, \quad i = 1, \dots, N, \end{aligned} \tag{3.3}$$

where $f(x)$ is given by (3.1), taking into account that k_i is the number of arrivals in the interval $[x_i, x_{i+1}]$, and

$$g_i(x) = D_i - T(k_i, \alpha). \tag{3.4}$$

We highlight that the idea to use as constraint of the optimization problem a test to validate the underlying statistical hypothesis on data is completely novel in the framework of modeling ED patient arrivals process.

4. The optimization problem and experimental results

We start the section by defining more in detail the optimization problem introduced in the previous section. Since the aim is to determine the best number and size of the intervals, we introduce vector of variables $x \in \mathbb{Z}^{25}$ whose any component x_i represents the boundary point of an interval, namely

$$T_i = x_{i+1} - x_i,$$

with $x_1 = 0$ and $x_{25} = 24$.

We adopt as starting point of the optimization algorithm x^0 the following

$$x_i^0 = i - 1, \qquad i = 1, \dots, 25,$$

which corresponds to the case of 24 intervals of unitary length. This choice is a commonly used partition in most of the approaches proposed in literature. We observe that, in our case study, this partition is even unfeasible (i.e. $g_i(x) > 0$ for some i); this corresponds to reject the statistical hypothesis on some T_i. Notwithstanding, the optimization algorithm we use, is able find a feasible solution which maximize the likelihood.

The optimization problem can then be reformulated

$$\begin{aligned}
\max \quad & f(x) \\
s.t. \quad & x_i \le x_{i+1}, \quad i = 1, \dots, 24 \\
& x_0 = 0 \\
& x_{25} = 24 \\
& \tilde{g}_i(x) \le 0 \quad i = 1, \dots, 24,
\end{aligned} \tag{4.1}$$

where

$$\tilde{g}_i(x) = \begin{cases} D_i - T(k_i, \alpha), & \text{if} \quad x_1 < x_{i+1} \\ 0, \quad \text{if} \quad x_i = x_{i+1}. \end{cases}$$

Solution of problem (4.1), also provides us with the best number of intervals, namely

$$N(x) = \left| \{ i \in \{1, \dots, 24\} \ : \ x_i < x_{i+1} \} \right|,$$

where $| \cdot |$ denotes the cardinality of a set.

Nevertheless, the structure of the objective function may force a low value of $N(x)$, because $f(x)$ is the product of terms smaller than one. Hence, the less the number, the larger the intervals; each one of these collects a higher number of arrivals hence increasing the value of the likelihood. Therefore, the product increases accordingly. To possibly avoid this drawback, we consider also the following modified objective function

$$\tilde{f}(x) = \left[\prod_{i=1}^{N} e^{-\lambda_i} \frac{(\lambda_i)^{k_i}}{k_i!} \right]^{\frac{1}{N(x)}}, \tag{4.2}$$

where each factor in (4.2) is enlarged due to fractional exponent. Thus, the modified optimization problem becomes

$$\begin{aligned} \max \quad & \tilde{f}(x) \\ s.t. \quad & x_i \leq x_{i+1}, \quad i = 1, \ldots, 24 \\ & x_0 = 0 \\ & x_{25} = 24 \\ & \tilde{g}_i(x) \leq 0 \quad i = 1, \ldots, 24. \end{aligned} \tag{4.3}$$

Consider that, both function $f(x)$ and $\tilde{f}(x)$ are not available in analytical form but their values are computed by a data driven computational procedure.

To solve problems (4.1) and (4.3) we use the algorithm proposed in [34]. This is motivated by the fact that the problems in hand are integer nonlinear constrained black–box, and both the objective and the constraints are relative expensive to compute. The results reported in [34] clearly show that this algorithm is particularly efficient in tackling problems with such features. In particular we use all the default parameters, with the stopping criterion based on the maximum number of function evaluations set to 3000.

In the sequel we report the optimal solution for both the problems:

- Problem (4.1):

$$
\begin{aligned}
T_1 &= [0,\ 7] \\
T_2 &= [7,\ 9] \\
T_3 &= [9,\ 17] \\
T_4 &= [17,\ 24]
\end{aligned}
$$

- Problem (4.3):

$$
\begin{aligned}
T_1 &= [0,\ 2] \\
T_2 &= [2,\ 4] \\
T_3 &= [4,\ 6] \\
T_4 &= [6,\ 7] \\
T_5 &= [7,\ 8] \\
T_6 &= [8,\ 9] \\
T_7 &= [9,\ 10] \\
T_8 &= [10,\ 17] \\
T_9 &= [17,\ 20] \\
T_{10} &= [20,\ 24]
\end{aligned}
$$

Figure 4.1 reports the plot of the average hourly arrival rate and its piecewise constant approximation obtained by solving problem (4.1). As expected, a coarse approximation is obtained with 4 intervals. Of course, in any interval Poisson hypothesis is validated. Figure 4.2 reports the plot of the average

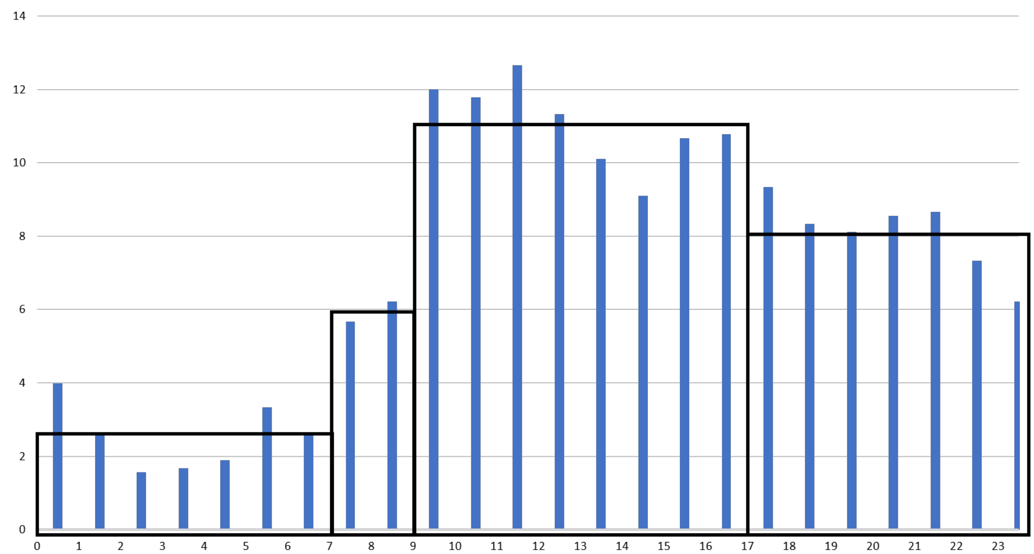

Figure 4.1: Plot of the comparison between the average hourly arrival rate (in blue) and the piecewise constant approximation function (in black), optimal solution of (4.1).

hourly arrival rate and its piecewise constant approximation obtained by solving problem (4.3). It can be observed that in the latter case the approximation is finer with 10 intervals with a better fitting of actual patient arrival rate.

5. Conclusions

In this paper we study the ED patient arrival process, which is the first step when modeling approaches, such as a Discrete Event Simulation, are applied for studying efficiency of hospital EDs. Since the arrival process significantly affects all the analyses on the ED model, its accurate represen-

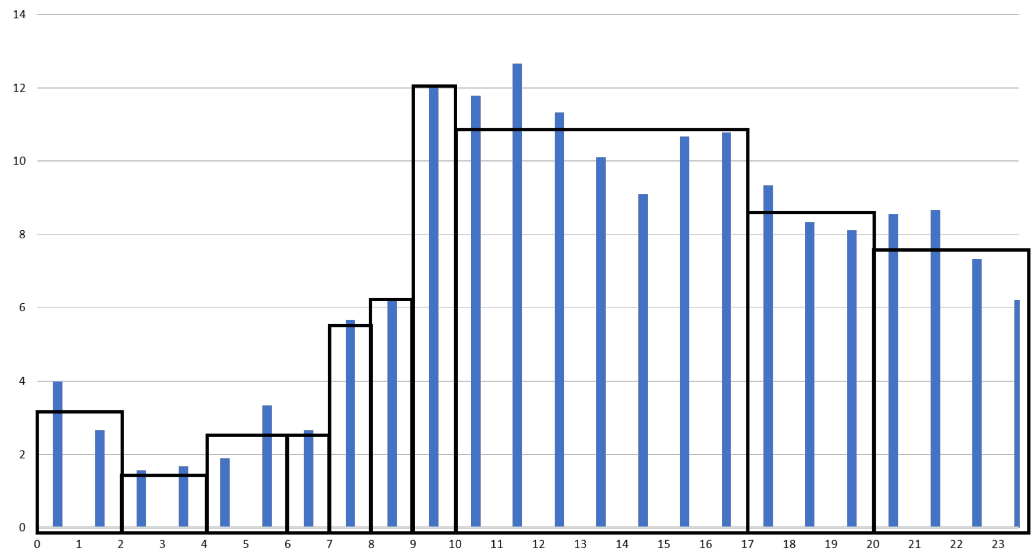

Figure 4.2: Plot of the comparison between the average hourly arrival rate (in blue) and the piecewise constant approximation function (in black), optimal solution of (4.3).

tation is required. According to the relevant literature and to the empirical evidence, the arrival process is usually modeled as non stationary due to the strong within-day variation. In particular, a NHPP is considered. A black box optimization approach is proposed for determining the optimal set of intervals partitioning the 24 hours of the day, allowing to accurately approximate the time-varying arrival rate by a piecewise-constant function. Moreover, to assure that the statistical model is consistent with the data for each interval, proper hypothesis testing is performed. Following [33], the CU K-S is adopted and, in particular, it is used to define the feasible set of the optimization problem. Since both objective and constraint functions are of black box type, a suitable derivative-free optimization algorithm is used. To assess the effectiveness of the proposed approach, we applied it to the patient arrivals of a large Italian ED. The results show that this approach is able to find intervals which enable to accurately approximate the arrival rate function and, at the same time, to assure the consistency between the NHPP hypothesis and the arrival data.

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