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Abstract

In the last years car-sharing has been growing as a valid alternative in urban mobility. Since its beginning the focus has been posed on relocation costs and stations positioning. On the counter part its well known that price discrimination can improve significantly companies' revenues.
Our idea is to apply price discrimination to a particular case of car sharing by adapting tools from classical Revenue Management (e.g. airline models). We firstly propose a pricing strategy for car-sharing, and we propose a mixed-integer linear program to evaluate the best strategy. Furthermore we propose alternative MILP formulations corresponding to different relocation strategies. Finally we performs numerical simulation based on real data to support our approach.

Keywords: car sharing; Revenue management; dynamic pricing

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1. Introduction

Rapid growing in urbanization and miles driven in the city will triple urban mobility by 2050. This explosion in demand requires to switch to Mobility-as-a-Service models, such as car sharing (see e.g. [16], [10], [1] and [15] for an overview on of main business models and operating modes of car-sharing services and their economic impact).
A spread business-to-consumer car-sharing system relies on the one-way free-floating operating mode, where the driver can pickup and return the vehicle wherever in a predefined service area. A critical issue for the car sharing one-way free-floating service providers is the imbalance between vehicles position and customers availability as it may lead to additional costs due to the need of relocation. In this paper we deal with the problem of optimizing the economic performance of the car sharing one-way free-floating service operator by exploiting both price discrimination and costs due to relocation strategies. In order to get this goal we propose a three phases approach.
As a first step, by applying the procedure proposed in [6] we consider a grid partition of the served area and we associate to each resulting cell a value depending on car sharing demand expectations. As in [4], car-sharing demand is determined by analyzing the data of online real car-sharing maps. In [4] data are taken for about one month from large station-based car-sharing operators in France, while in [6] for about two months from one-way free-floating providers in Italy.
On the basis of the grid partition, as a second step we propose an incentive price mechanism which tunes the trip fares according to the "value" of the final destinationthat can be specified by the user at the departure time. The prices are adjusted on a discount/charge basis. On one hand, the mechanism proposes discounts with respect to the basic tariff to induce customers to choose destinations which allow the provider to reduce relocation costs. On the other hand, operator applies an additional charge over the standard tariff whenever the trip is not useful for relocation.
As a third step, by taking inspiration from the network airline revenue management models developed in [7, 8], we propose a mixed-integer linear model (MILP) to maximize the economic performance of the operator by selecting the optimal price discrimination under different relocation strategies. See [20] and [19] for an overview on revenue management problems and solutions. Research on dynamic pricing models for revenue management can be found in [3] too.
So far optimization approach has been applied in different ways to deal with the relocation problem. However, no work simultaneously deal with the complexities arising from the flexibility to users offered by the free-floating operating mode and from providing the users with economic incentives to mitigate the occurrences of the relocations during the production of the car-sharing service. For instance, in [5] an efficient planning of one-way electric vehicle-sharing systems is proposed. However, the considered system is station-based and the relocation problem is taken into account ex ante to optimally design the system. A stations-based system is considered also in [11], where a relocation algorithm is proposed in order to guarantee location balances of cars into distinct parts of the service area. Again, authors in [9] and [2] assume station-based systems. However, similarly to our approach, they use clients' behavior to balance the vehicle

stocks and make car-sharing systems more profitable. In [9] the authors develop a mixed integer non-linear programming model which sets the optimal trip prices, while in [2] a dynamic pricing strategy is applied.
To reduce the negative effects due to vehicle imbalance in one-way car-sharing systems, authors in [12], [18] and [17] propose optimization models to determine efficient relocation plans, but there is no economic incentive to users to reduce the number of necessary relocation. In [13] the relocation problem is extended to the staff members as the vehicle relocation can lead to an imbalance of staff members among stations.

The paper is organized as follows. In Section 2 we discuss the mathematical model, then in Section 3 we present the results of our simulation using data concerning the city of Rome and presented in [6]. The paper ends with the conclusions and hints for future work in Section 4.

2. The Optimization Model

Revenue Management problems are usually related to the definition of booking limits and therefore their objective is to maximize the expected revenue. In this contest linear model are well suited as deterministic version of stochastic models where the focus is on expected values.
Our aim here is to determine weather or not a certain pricing can be applied successfully. The model must be seen as a tool to evaluate different scenarios (varying on prices and relocation strategies). Demands and willingness represents their actual values and so they are not realizations of random variables or their expected values. With this preamble it seems unreasonable to use continuous variables to represent actual sales, which will lead to wrong and too optimistic forecasts.

To apply an effective pricing it is fundamental to know where a customer is directed. This leads us to differentiate among origin-destination pairs. Despite in airlines the point-to-point representation of origin-destination pairs is almost natural, in car sharing we have to force a partition on the served areas. We found advantageous to consider taxable areas as a start. Generally, these cells are irregular, so we converted them into squares. *Urban Grid* is the set of squared cells.
Timing is also an important aspect. We focus on a fixed time period, say a single day. This is the common time chosen at operational level, since the night offers a natural break point (demands in the night are negligible). Each day is divided into time-slots, as usual in transportation management systems, giving the chance to divide into peak and off-peak slots. Users can make trips inside a time-slot, but not across two of them. Of course this is a simplification that leads to not optimize trips crossing time slots. The model can be modified to include such a situation, but the complexity derived will overcome our purposes.
Users can take or leave a vehicle only in fixed points, called stations. A station can be seen as a wide area where it is possible to park or take the car, it's not necessarily a physical location owned by the company.

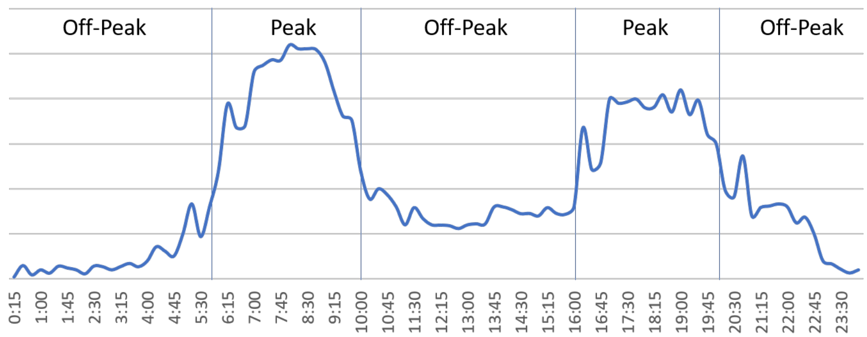


Figure 1: The typical total flow (in + out) of a city zone over time. *Peak* and *Off-peak* labels suggest a possible division into time slots.

Users behavior is driven by their willingness to pay that we assume to know. In other words the willingness to pay can be seen as the percentage of trips that are accepted by the customers at a certain price.
We do not take into account the availability of parking spots. Let's assume infinite capacity for them. By the way to impose a physical limit is straightforward and it is sufficient to add some upper bounds on capacities. Our simplification is justified by the fact that cells are large and the probability to not find a parking spot is too low to be considered. In figure 1 is reported the typical total flow (in + out) of a city area over time.
One of the major points of applying a pricing strategy to car sharing is to save relocation costs by guiding customers trips. We refer to relocation as the physical transfer of a car from a location to another, in order to check future demand needs and therefore improve revenues. Differently from bike sharing, in car sharing relocation costs can be very expensive. In Section 3 we report the relocation cost for the city of Rome, which is 15 $€/car$. We suppose to make available relocation at the end of each time slot, allowing the model to work in a more general framework. This is not the usual strategy adopted by companies, since they move cars only during the night for economic reasons. As it is marked in section 3 the model confirms relocating during the night when it is possible. In the model we want also to determine initial and final distribution of cars. This aspect is strictly related to relocation, since cars are moved in order to match a certain distribution defined by the company. We redirect the reader to [21] for a further description of relocation strategies.

In the following we will show details about the model, as sets variables, objective and constraints.
We will use the following sets:

- $\mathcal{C}$ set of all cells, where a cell is an area where a vehicle can arrive, depart or both
- $\mathcal{H}$ is the set of time slots
- $\mathcal{O} \subseteq \mathcal{C}$ is the set of origins

- $\mathcal{D} \subseteq \mathcal{C}$ is the set of destinations

In order to be synthetic we adopt the following rule for indexes: $i \in \mathcal{O}$, $j \in \mathcal{D}$ and $h \in \mathcal{H}$. For instance if we write d_{ij}^h, it means that the parameter d is indexed from origin i to destination j at time h.
We present some parameters:

- V is the total number of vehicles
- d_{ij}^h is the estimated demand
- $\bar{d}_{ij}^h$ is the not-demand. It's an estimation of all the unsatisfied or refused trips. Of course we assume that $d_{ij}^h - \bar{d}_{ij}^h > 0 \quad \forall\ i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H}$, otherwise the market is closed
- α_{ij}^h is the attractiveness to accept/pay a run. It is a real value between 0 and 1
- γ_{ij}^h is the percentage of customers that will reject the trip and its alternatives. It is a real value between 0 and 1
- p_{ij}^h is the tariff. It is a non-negative real value measured in [€/min].
- t_{ij}^h it is the driving time, it depends on the level of traffic congestion and it is calculated as the ratio between the distance and the driving speed
- $f_{ij}^h = \dfrac{\Delta T^h}{t_{ij}^h}$ is the maximum number of trip that can be carried out in ΔT^h by a single car (trip frequency). where ΔT^h is the time duration in minutes. For instance in the case where the time period is divided into slots of same duration we have $f_{ij}^h = \dfrac{24 \cdot 60}{|\mathcal{H}| \cdot t_{ij}^h}$.

We now present the decision variables. We remark that we want to determine the best possible configuration of sold trips and relocation with respect to total revenues and relocation costs.

- $x_{ij}^h \in \mathbb{N}$ represents the number of sold and accepted trips.
- $z_{ij}^h \in N$ is associated with the total quantity of lost rides.
- $w_i^h \in N$ is the capacity of node i at the beginning of time slot h.
- $v_{ij}^h \in \mathbb{N}$ represents the number of cars needed.
- $\delta_{ij}^h \in \mathbb{N}$ is involved in modeling relocation costs. In particular when $\delta_{ij}^h - 1$ is positive it represents exactly the number of cars relocated.

The objective is to maximize profits:

$$f(\mathbf{x}, \mathbf{v}, \mathbf{w}, \boldsymbol{\delta}, \mathbf{z}) = \sum_{i \in \mathcal{O}} \sum_{j \in \mathcal{D}} \sum_{h \in \mathcal{H}} \left(t_{ij}^h \; p_{ij}^h \; x_{ij}^h - c_{ij}^h \max\{0, \delta_{ij}^h - 1\} \right) \tag{1}$$

The objective says that the total profit is given by the sum of trips sold times their fare per minute times the average trip time minus the cost of relocation.

For each origin i and time slot h, the sold trips from i to every destination j must be lower than or equal to the trip capacity in the origin during the slot time h. It's an availability constraint, meaning that the system can not sell more trips than the number of cars in the origin area.

$$\sum_{j \in D} v_{ij}^h \leq w_i^h \quad \forall \; i \in \mathcal{O}, h \in \mathcal{H}$$

Other constraints relies on the total demand satisfaction, which implies that the number of seats sold plus the number of trips missed must be equal to the total of trips required. This has to be replied for each origin, destination and time slot, in formulas:

$$x_{ij}^h + z_{ij}^h = d_{ij}^h \quad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \tag{2}$$

For the assumption of the model $z_{ij}^h \neq 0$, this because otherwise the associated triple (i, j, h) is closed, in the sense that there is no economic advantage to allocate resources. For this technical reason we need to fix a lower bound on z_m^o. In order to be fair with the simulation we put this bound to $\bar{d}_{ij}^h$. In other words we can consider $\bar{d}_{ij}^h$ as a pessimistic value, leading the solution to not outperform the observed data. Without loss of generality we can substitute $\bar{d}_{ij}^h$ with any positive value and the model will still stand.

$$z_{ij}^h \geq \bar{d}_{ij}^h \quad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \tag{3}$$

We introduce an acceptation constraint, reflecting the fact that the user can choose to accept or reject a proposal of the system

$$\gamma_{ij}^h x_{ij}^h - \alpha_{ij}^h z_{ij}^h \leq 0 \quad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H}$$

Since we are considering a network with integer flows we must add flow constraints

$$w_i^{h+1} = w_i^h - \sum_{j \in D} v_{ij}^h + \sum_{j \in O} v_{ij}^h, \quad \forall \; i \in \mathcal{O}, h = 1, \ldots, |\mathcal{H}| - 1 \tag{4}$$

and the additional constraints assuring the consistence among the number of cars, trips and relocations

$$v_{ij}^h \geq \frac{x_{ij}^h}{f_{ij}^h}, \quad v_{ij}^h - \frac{x_{ij}^h}{f_{ij}^h} \leq \delta_{ij}^h, \qquad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H}$$

We introduce also the condition for the final state

$$w_i^1 = w_i^{|\mathcal{H}|} - \sum_{j \in D} v_{ij}^{|\mathcal{H}|} + \sum_{j \in O} v_{ji}^{|\mathcal{H}|} \geq 0, \quad \forall\, i \in \mathcal{O} \tag{5}$$

We can fix the final state as above and let the model decide the best configuration, hypothesizing that the next operating day will have the same structure of the actual one. This lead to a more compact and readable representation. On the other hand w.l.o.g. we can introduce a desired configuration for initial and final state

$$w_i^0 = \tilde{w}_i \quad \forall\, i \in \mathcal{O}$$

$$w_i^{|\mathcal{H}|} = \hat{w}_i \quad \forall\, i \in \mathcal{O}$$

We also have the number of available vehicles is fixed

$$\sum_{i \in O} w_i^h = V \quad \forall\, h \in \mathcal{H}$$

By putting all the constraints together we obtain the following formulation

$$
\begin{aligned}
\max \quad & f(\mathbf{x}, \mathbf{z}) = \sum_{i \in \mathcal{O}} \sum_{j \in \mathcal{D}} \sum_{h \in \mathcal{H}} \left(t_{ij}^h \; p_{ij}^h \; x_{ij}^h - c_{ij}^h \max\{0, \delta_{ij}^h - 1\} \right) \\
\text{s.t.} \quad & \textstyle\sum_{j \in D} v_{ij}^h \leq w_i^h \quad \forall\, i \in \mathcal{O}, h \in \mathcal{H} \\
& v_{ij}^h + z_{ij}^h = d_{ij}^h \quad \forall\, i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \\
& \gamma_{ij}^h x_{ij}^h - \alpha_{ij}^h z_{ij}^h \leq 0 \quad \forall\, i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \\
& w_i^{h+1} = w_i^h - \sum_{j \in D} v_{ij}^h + \sum_{j \in O} v_{ji}^h \quad \forall\, i \in \mathcal{O}, h \in \{h \in \mathcal{H} : h \leq |\mathcal{H}| - 1\} \\
& w_i^1 = w_i^{|\mathcal{H}|} - \sum_{j \in D} v_{ij}^{|\mathcal{H}|} + \sum_{j \in \mathcal{O}} v_{ji}^{|\mathcal{H}|} \quad \forall\, i \in \mathcal{O} \\
& v_{ij}^h \geq \frac{x_{ij}^h}{f_{ij}^h} \quad \forall\, i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \\
& v_{ij}^h - \frac{x_{ij}^h}{f_{ij}^h} \leq \delta_{ij}^h \quad \forall\, i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \\
& \sum_{i \in O} w_i^h = V \quad \forall\, h \in \mathcal{H} \\
& z_{ij}^h \geq \bar{d}_{ij}^h \quad \forall\, i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H} \\
& \mathbf{x}, \mathbf{z}, \mathbf{w}, \mathbf{v}, \delta \geq 0 \;\; \text{integers}
\end{aligned} \tag{6}
$$

2.1. Relocation strategies

Relocation is a strategic decision for car-sharing companies. Our purpose here is to introduce modeling ideas for some of the most common solutions.
As mentioned before, the assumption is that relocation is made instantaneously or during an ideal time where no requests arise, i.e. during the night.
We synthesize as it follows:

- Uniform, a fixed amount of cars at the beginning and at the end of the time window
- Threshold, distribution must respect two thresholds, i.e. minimum of 10 cars and maximum of 90
- Free, the model as it is

Note that the threshold strategy can be seen also as a way to introduce parking availability in the model.

Uniform

The uniform strategy is the easiest, both theoretically and practically. We need only to put the same amount of resources among macroareas at the first time slot. It is sufficient to add the constraint

$$\sum_{i \in \mathcal{O}} w_i^1 = \bar{c}$$

where $\bar{c} = \frac{V}{|\mathcal{O}|}$. We observe that since the number of cars in each station is integer, we accept slight modifications of $\bar{c}$ among zones.

Threshold

Here the relocation must respect two bounds on the number of cars, respectively lb and ub. With this control the relocation starts when the number of cars is lower or upper in a certain zone. In general this process is continuous but since we are considering time-slots we must assume that this relocation arises at the end of each time-slot and it's instantaneous.
As said before the model will find the best scheme distribution/relocation under the above hypothesis. To menage this we simply add the bounds on capacity constraints i.e.

$$lb_i^h \leq w_i^h \leq ub_i^h, \qquad \forall\ i \in \mathcal{O}, h \in \mathcal{H}$$

Threshold relocation is useful in all the cases where there is high demand variability. Upper and lower bounds must be seen as safe intervals mostly like to satisfy trips requests.

2.2. Generalization based on alternative choices

Sometimes it could happen that customers are willing to change their former destination to another suggested by the car sharing operator at a lower price. This is the case of adjacent areas with different relevance. To menage this fact we introduce the concept of *alternative*, i.e. a trip to a destination adjacent to the original one, but cheaper.

To address this situation we add to the problem some new variables, representing the choice alternatives. We call $\mathcal{A}_j^h \subseteq \mathcal{D}$ for $i \in \mathcal{D}$ and $h \in \mathcal{H}$, the set of available alternative trips in the neighborhood of j. We introduce variables y_{ij}^{hi}, representing the alternative trips sold for going from i to i at time h, when the original arriving position was in j. We also include variables $u_{ij}^h \in \mathbb{N}$ modeling the number of cars needed and derived by alternative trips. Variables $\mu_{ij}^{ha} \in \mathbb{N}$ are involved in modeling relocation costs associated with alternative trips. Finally call β_{ij}^{hi} the attractiveness associated with trips y_{ij}^{hi}.

We reformulate the problem taking into account these new quantities. Objective function

$$f(\mathbf{x}, \mathbf{y}, \mathbf{v}, \mathbf{u}, \mathbf{w}, \boldsymbol{\delta}, \mathbf{z}) = \sum_{i \in \mathcal{O}} \sum_{j \in \mathcal{D}} \sum_{h \in \mathcal{H}} \left(t_{ij}^h \; p_{ij}^h \; x_{ij}^h + \sum_{a \in \mathcal{A}_j^h} t_{ia}^h \; p_{ia}^h \; y_{ij}^{ha} - c_{ij}^h \max\{0, \delta_{ij}^h - 1\} \right)$$

Capacity constraints

$$\sum_{j \in D} \left(v_{ij}^h + \sum_{a \in \mathcal{A}_i^h} u_{ij}^{ha} \right) \leq w_i^h \quad \forall \; i \in \mathcal{O}, h \in \mathcal{H}$$

Total demand balance

$$x_{ij}^h + \sum_{a \in \mathcal{A}_i^h} y_{ij}^{ha} + z_{ij}^h = d_{ij}^h \quad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H}$$

CCM

$$y_{ij}^{ha} - \beta_{ij}^{ha} z_{ia}^h \leq 0 \quad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H}, a \in \mathcal{A}_i^h$$

Flows

$$w_i^{h+1} = w_i^h - \sum_{j \in D} \left(v_{ij}^h + \sum_{a \in \mathcal{A}_j^h} u_{ij}^{ha} \right) + \sum_{j \in O} \left(v_{ij}^h + \sum_{a \in \mathcal{A}_j^h} u_{ji}^{ha} \right), \quad \forall \; i \in \mathcal{O}, h = 1, \ldots, |\mathcal{H}| - 1$$

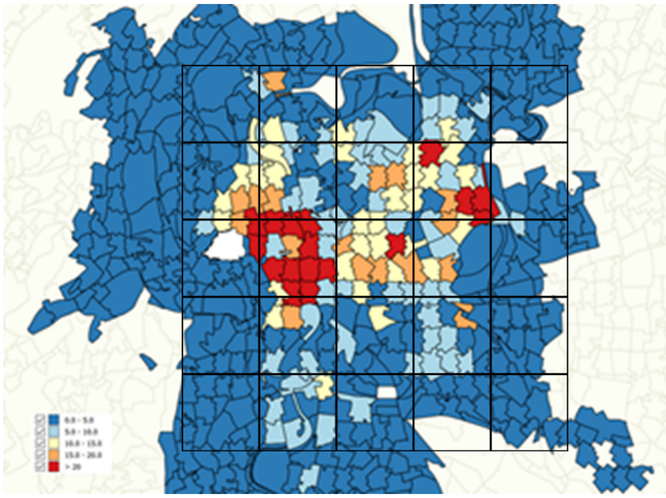
$$w_i^1 = w_i^5 - \sum_{j \in D} \left(v_{ij}^5 + \sum_{a \in \mathcal{A}_j^5} u_{ij}^{5a} \right) + \sum_{j \in O} \left(v_{ij}^5 + \sum_{a \in \mathcal{A}_j^5} u_{ji}^{5a} \right), \quad \forall \; i \in \mathcal{O}$$

Additional constraints

$$v_{ij}^h \geq \frac{x_{ij}^h}{f_{ij}^h}, \qquad u_{ij}^{ha} \geq \frac{y_{ij}^{ha}}{f_{ia}^h} \qquad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, h \in \mathcal{H}$$

$$v_{ij}^h - \frac{x_{ij}^h}{f_{ij}^h} \leq \delta_{ij}^h, \qquad u_{ij}^{ha} - \frac{y_{ij}^{ha}}{f_{ia}^h} \leq \mu_{ij}^{ha} \quad \forall \; i \in \mathcal{O}, j \in \mathcal{D}, a \in \mathcal{A}_j^h, h \in \mathcal{H}$$

Figure 2: Heat map of Rome using UAV indicator with a posed grid on it.



All the other constraints, variables and parameters maintain their meaning.

3. Numerical simulation

Our case study relies on a simulation in the city of Rome, Italy. Data where collected between April 28th and June 18th, 2016. To measure demand and car flows, we tracked vehicles positions using GPS coordinates (latitude and longitude) evolving with time. These are called *stop data* and they are uniquely associated to one cell. From them we derived the *Average Stop Duration* and the *Stop Density*, in different time ranges (see [6] for further details).

An important indicator is the *Urban Area Value* (UAV), which is the ratio between *Stop Density* and *Average Stop Duration*. UAV is the quantity we used to discriminate areas in the Urban Grid. In fig.2 it's shown the original heat map with respect with over 600 cells. We simplified the division by considering the squared partition applied above in the map and resumed in fig.3.

As mentioned in the introduction we considered 25 different zones, grouped by 5 macro-areas: Dark Blue (DB), Light Blue (LB), Yellow (Y), Orange (O) and Red (R). The color (from bue to red) is associated with an area ir related to the UAV indicator: red zones have the highest UAV, blue ones the lowest. There are 5 hourly ranges, indeed $\mathcal{H} = \{1, 2, 3, 4, 5\}$, where 2 and 4 are the peak sessions, and $1, 3, 5$ the off-peak ones.

According to our demand data and experts in the field, we fix the willingness to accept a ride equal to 0.6, and 0.4 to not accept during off-peak sessions. During peak sessions we use the willingness shown in tab.1 and tab.2.

We made a similar reasoning for prices. For the off-peak slots, prices are supposed to be all equal. The value is $0.25€/min$,which is the standard fare in Rome. During peak slots we associate a penalization value τ_i for $i \in \{DB, LB, Y, O, R\}$ to each zone. Let fix $\tau_{DB} = 1$, $\tau_{LB} = 2$, $\tau_Y = 3$, $\tau_O = 4$ and $\tau_R = 5$.

Figure 3: The derived aggregated grip map.

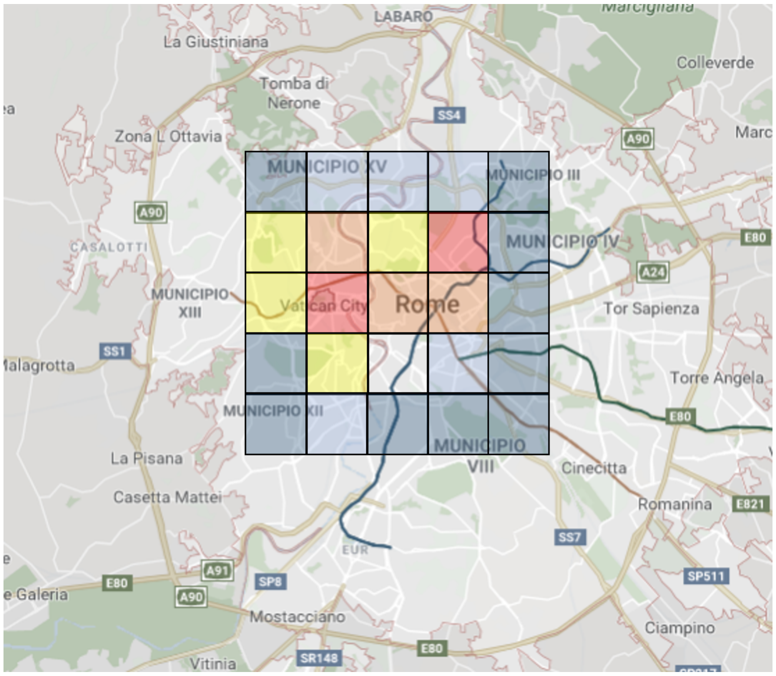


Table 1: Willingness to take a ride during time slot 2.

		Destination				
		DB	LB	Y	O	R
	DB	0.6	0.7	0.75	0.8	0.85
	LB	0.5	0.6	0.7	0.75	0.8
Origin	Y	0.45	0.5	0.6	0.7	0.75
	O	0.4	0.45	0.5	0.6	0.7
	R	0.35	0.4	0.45	0.5	0.6

Table 2: Willingness to take a ride during time slot 4.

		Destination				
		DB	LB	Y	O	R
	DB	0.6	0.5	0.45	0.4	0.35
	LB	0.7	0.6	0.5	0.45	0.4
Origin	Y	0.75	0.7	0.6	0.5	0.45
	O	0.8	0.75	0.7	0.6	0.5
	R	0.85	0.8	0.75	0.7	0.6

Table 3: Price Matrix during peaks.

		Destination				
		DB	LB	Y	O	R
Origin	DB	0.25	0.225	0.31	0.335	0.35
	LB	0.225	0.25	0.275	0.31	0.335
	Y	0.20	0.225	0.25	0.275	0.31
	O	0.175	0.20	0.225	0.25	0.275
	R	0.15	0.175	0.20	0.225	0.25

Given a couple origin-destination (i, j) in a peak time slot we can compute the price p_{ij}^h using the following formula

$$p_{ij}^h = \bar{p} + (\tau_{i_i^h} - \tau_{s_j^h})\Delta p \tag{7}$$

Where s_i^h is the macro-area where the origin zone i is at time slot o. s_j^h is analogous. We fix $\bar{p} = 0.25$, the standard tariff, and $\Delta p = 0.025$ as incentive/disincentive. In tab. 3 we report all the values.

Briefly we resume the dimension of our problem:

- Number of vehicles: 1.248
- Number of origins: 25
- Number of destinations: 25
- Number of time-slots: 5
- Number of cells: 25
- Number of tariffs: 5
- Number of variables: 84.500
- Number of constraints: 87.750

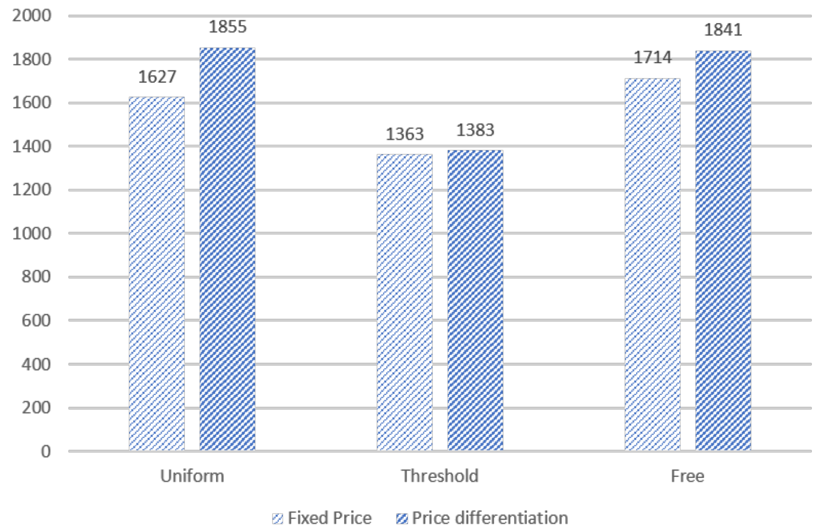
We ran the problem on a Ubuntu 18.04 based Intel® Core™ i7-5500U CPU, dual core (2.40GHz each) with 5.7 GiB of memory capacity.

3.1. Comparison

We tried different simulations in order to prove the effectiveness of price discrimination. It arose the total profit is always better than the fixed price situation. In all our simulations we took into account alternatives.

To understand differences we first measure the total flow of each model. The total flow is given by the total number of cars moved between periods. An higher value of this quantity is strictly related to trips sold.It can be noted that in price discrimination

Figure 4: The total flow among different relocation models.



the flow is generally higher. This is due to the need of control relocation by price, so that a company can save costs by selling trips at lower price. In fig.4 the total flow is reported.

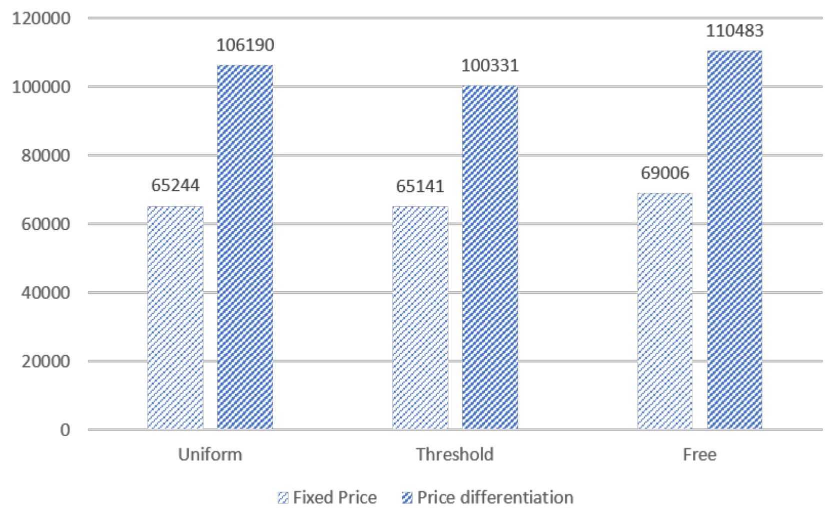
Another unexpected fact is that profits are not significantly ruled by different relocation strategies, as it can be seen in fig.5. The less we influence the structure of the system the more profits will grow, i.e. on average $\mathbb{E}(\mathit{Profit}_{\text{Threshold}}) \leq \mathbb{E}(\text{Profit}_{\mathit{Uniform}}) \leq \mathbb{E}(\text{Profit}_{\mathit{Free}})$. However, the relative differences are quite small. For the price discrimination case we have the Uniform model is 5.5% higher than the Threshold, the Free is 4% higher than Uniform and 10.1% than Threshold. For the fixed price case we have the Uniform model is 0.1% higher than the Threshold, the Free is 5.7% higher than Uniform and 5.9% than Threshold.
The same cannot be said for the differences among price strategies, where the price discrimination is always better. More in detail for the Uniform case, price discrimination is 62.7% higher than fixed price, for the Threshold is 54% and for the Free is 60.1%.

Since we have shown in price discrimination the flow is bigger, we want now to compare a certain measure of the congestion provoked by such policy. Here for congestion is meant the imbalance between areas. To measure this quantity we simply use the Standard Deviation. In fig.6 we can see how more or less all the models have the same behavior. By the way it's interesting to observe the jump of both Uniform models, underlining a mismatch between the initial distribution and the one really needed by the model.
In all the configurations price discrimination leads to slightly higher congestion values, due to the higher volume of sold trips. In tab 4 the global congestion is reported. As expected the lowest values are the ones referring to the Threshold relocation strategy. As said before, this is exactly the objective of this type of relocation, which is used

Table 4: Global congestion.

	Price Discrimination	Fixed Price
Uniform	36.9	40.4
Threshold	28.8	29.0
Free	41.2	43.4

Figure 5: The total profit expressed in euros [€] among different relocation models.



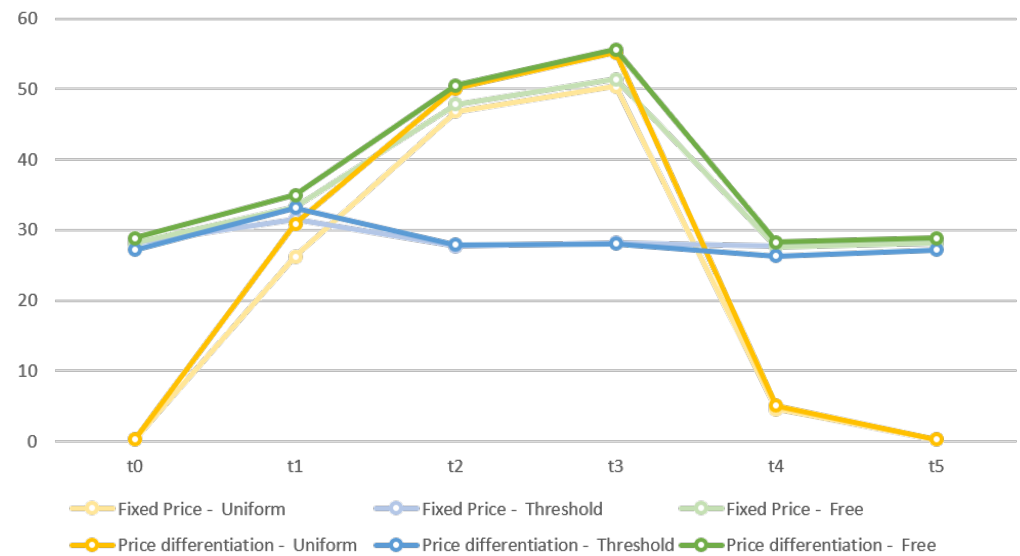
when demands vary frequently, not allowing predictions, and thus a flow control is needed to protect the car sharing system against variability.

4. Conclusions and future research

In this paper we adapted a tool from Revenue Management to car sharing, improving the formulation adding constraints related to flows and differentiating Customer Choice model constraints between pure trips and alternatives. We developed a way to estimate the performance of pricing strategies and we prove empirically the supremacy of these price mechanisms simulating from real data.
It is known far beyond in the past that price discrimination is in general a good way for companies to improve revenues, but under ideal conditions. Our work shows that the volume of trips sold by the company under study is sufficient to justify such a strategy.

Future researches include the the introduction of a Game Theory-based model for a portion of the demand. It is known in economics that a part of the demand is loyal to the company, refusing to change, and another part is more sensible to prices and can decide to use a different mode of transportation (not necessarily a different car-sharing company, but also buses, subways etc.). This remaining part of the demand can be

Figure 6: Congestion among different relocation model over time. With PD we refer to price discrimination, with FP to fixed price.



modeled using Game Theory, where more companies sell their own product trying to catch customers.

Another important development for this kind of problems is the substitution of classical Customer Choice Model with non-parametric and hopefully Machine Learning-based estimation of willingness and relations. The major problem here is the collection of data.

We also remark that the presented results can also be extended to bike-sharing systems, which have different complexities but very similar business model and operating mode (see e.g. [14]).

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