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Abstract

Airlines Revenue Management (RM) Departments pay remarkable attention to many different applications based on Sales Based Integer Program (SBIP). In fact, optimal solutions of SBIP are mainly used by airlines to evaluate the performance of their RM systems, as well as it plays the role of optimization core for some RM Decision Support System. We consider an a Sales-Based Integer Linear Program (SBILP) formulation following [4]. This SBILP is hard to solve to optimality on real problems. We propose a new formulation based on Market-Service decomposition that allows to solve smaller problems. We analyze properties of the decomposed problems.

Keywords: Revenue Management Models, Sales Based models, integer programming, decomposition methods

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1 Introduction

Nowadays Revenue Management (RM) models are very popular and represent one of the most effective methodologies in which maths penetrates in the operative economic decisions. Broadly speaking the Revenue Management systems (RMS) constitute a complex network which involves different fields in applied maths, from statistic to computer science, operations research, data mining and much more, understanding that innovation and research are continuously driven to improve results.
An overview about Revenue Management can be found at [7] and [8].
Probably the most popular sector where RMS are applied is the Airline Industry. Historically, after the deregulation in 1978, the great quantity of fixed costs related to air transportation has convinced a lot of companies of the advantages to invest in such systems.
In this contest one of the latest models proposed has been the Sales-Based Linear Program (SBLP) proposed from [3], which this article starts with. The SBLP model is widely smaller than the Choice-Based Linear Program, see [2] and [5], that was the most recent model proposed.
The principal contribution of this paper is the definition of a new integer formulation for the Sales-Based model.
In real applications, SBILP is a large-scale integer programming problem, that cannot be solved effectively in practice in the most of cases. Even its linear relaxation, well-known as SBLP, requires a remarkable computational effort by state-of-art commercial solvers. However, Airlines RM Departments are interested in SBILP solutions, rather than of SBLP, since the SBLP solutions have low quality due to the considerable integrality gap that is often observed.
Our work aims to improve and investigate important properties of SBILP, in order to meet the practical needs of airlines. We study possible methods to improve the linear relaxations SBLP and the properties due to the particular structure of the problem. In particular, we introduced simple cuts that can highly improve upper bound provided by SBLP. Moreover, we investigate a simple market-based decomposition model that allows first to allocate capacities over different markets (master problem) and then to allocate the capacities among different flight alternatives (Slave problem), thus leading to a cost-effective method for the solution of SBILP.
Up to our knowledge, few other researchers have developed decomposition approaches to solve efficiently large scale of real problems. One for all consider [1] that use the great knowledge in literature for the Single-Leg Revenue Management for defining a good decomposition scheme. However it does not consider the spill and recapture effects, as done e.g. in the Customer Choice Model of [3], so that the resulting the model is unrealistic. Leg-Based Decomposition does not take into account the true nature of the problem; indeed as a matter of fact a leg (direct OD connection) could concern more Services and Markets.
Another relevant observation is that this decomposition is not unique but it depends on the single-leg model adopted.
This paper is divided in four sections. In section 2 we state standard model settings

and terminology for Revenue Management Problem; in section 3 we present the Sales Based Integer Linear Program. In section 4 we focus on some appealing cuts that allows to improve the bound obtained by the linear relaxation SBLP. In section 5 we propose the scalable Master-Slave decomposition approach. This last section is the central one where different approaches, properties and developments are explained.

2 Model settings

Let's enter details of the setting and introduce the notation.

- *Flight Leg.* A leg ℓ defines a direct connection between a certain origin and destination (OD).

  Let $\mathcal{L}$ be the complete set of flight legs.

- *Flight Leg Cabin.* A flight cabin b represents a subset of seats in the flight. Each flight leg ℓ is characterized by a set of cabins B_ℓ. Each cabin $b \in B_\ell$ is restricted by its capacity c_b. Cabin are implicitly ordered: if $b_i < b_j$, then passengers sold to cabin b_i can be upgraded to b_j.

- *Service.* A service s identifies a, possible multi-leg, itinerary to go from one origin O to one destination D which represent respectively the starting and end point of a travel. Let S be the complete set of services. Let S_ℓ be the set of services using the flight leg ℓ.

- *Travel Alternative.* A travel alternative a represents a possible option to travel from O to D. An alternative a is characterized by its first choice demand d_a and fare p_a. An alternative can be owned by the host-airline or it can be related to not fly and other airlines options ("no-purchase or null alternative)

  Let $\mathcal{A}$ be the complete set of host travel alternatives, while $\overline{\mathcal{A}}$ are the "no-purchase or null alternatives.

- *Market.*

  A market defines a set travel alternatives including the no-purchase/null alternative at a given time period. Hence a market m is the set of the demand dependent alternatives

  $$\mathcal{A}_m = \{\bar{a}_m, a_1 \ldots, a_{f_m}\}$$

  where $f_m = |\mathcal{A}_m|$ (cardinality of $\mathcal{A}_m$) and $\bar{a}_m \in \overline{\mathcal{A}}$ denotes the null alternative for market m.

  We denote by $\mathcal{A}_m \subseteq \mathcal{A}$ the set of host market alternatives

  Total market demand is computed as

  $$d_m = \sum_{a \in \mathcal{A}_m} d_a + \bar{d_m} = \rho_m + \bar{d_m}$$

where $\bar{d_m}$ is the demand associated with the null alternative $\bar{a}_m$ and we denote by ρ_m the aggregate host market demand.

Let $\mathcal{M}$ be the complete set of markets.
Let $\mathcal{M}_s$ be the set of markets referring to service s.
Let $\mathcal{M}_{\ell b}$ be the set of markets using flight leg ℓ and cabin b.

We point out that alternatives are interchangeable due to recapture and spill effects. Indeed if all products are available for sale, the first-choice, or natural, demand is satisfied for each of the products. However, when a product is unavailable, its first-choice demand is redirected to other available alternatives (including the no-purchase alternative).

- Spill refers to redirected lost demand to competition or to the no-purchase alternative.
- Spilled demands are recaptured from the open alternatives according to the attractiveness, more simply respect to the first choice demand. Recapture refers to redirected demand in the sale of a different available product.
- NoFly-NoOtherAilrines option, that is also called no-purchase or null alternative, is assumed always available.

3 Sales-Based Model

The problem is to allocate seats to market alternatives such that:

- Revenue is maximized
- Sales of cabin seats are consistent respect to the capacity restrictions.
- Sales are compliant respect to the available demand and respect to spill/recapture effect between close and open alternatives.

We consider a Sales-Based model that is derived from the SBLP proposed by [4].

The decision variables are the seats sold for each host market alternatives x_a, $a \in \mathcal{A}$ and the seats allocated to null market alternatives z_m, $m \in \mathcal{M}$.
Seats x_a must be integer variables:

$$x_a \in \mathbb{N}, \quad \forall\, a \in \mathcal{A}$$

Formally given a set $\bar{\mathcal{A}}_m \subseteq \mathcal{A}_m$, whichever $m \in M$, the following vectorial notation is introduced :

$$\mathbf{x}_{\bar{\mathcal{A}}_m} = (x_{a_1}, \dots, x_{a_n})^{\intercal}$$

where $n := \mid \bar{\mathcal{A}}_m \mid$, $\bar{\mathcal{A}}_m = \{a_1, \dots, a_n\}$. Another notation used is :

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_{\mathcal{A}_{m_1}} \\ \vdots \\ \mathbf{x}_{\mathcal{A}_{|\mathcal{M}|}} \end{pmatrix}$$

where $\mathcal{M} = \{m_1, \dots, m_{|\mathcal{M}|}\}$.
Alternatives sales x_a are bounded

$$l_a \le x_a \le u_a, \quad \forall\ a \in \mathcal{A}$$

Generally l_a is simply zero and u_a is defined by the total market demand. In some cases, lower and upper bounds may be more restrictive because of some business requirements.

Sales (indirect) decisions z_m for the null market alternatives can be considered as continuous variables $z_m \in \mathbb{R}$ since they do not correspond to physical resources. Market alternatives sales are bounded too

$$z_m \ge \bar{d}_m \quad \forall\ m \in M$$

A vectorial notation is introduced:

$$\mathbf{z} = (z_{m_1}, \dots, z_{m_{|\mathcal{M}|}})^\intercal$$

where $\mathcal{M} = \{m_1, \dots, m_{|\mathcal{M}|}\}$.

A first set of constraints takes into account that the demand must be distributed within all market alternatives, that is:

$$\sum_{a \in \mathcal{A}_m} x_a + z_m = d_m \quad \forall m \in \mathcal{M}.$$

Constraints must be added that model the fact that demand of a not available alternative can be recaptured by another market alternative. Recapturing is proportional to the ratio between the demand for a and for the null alternative.

$$\bar{d}_m \cdot x_a - d_a \cdot z_m \le 0 \qquad \forall m \in \mathcal{M} \quad \forall a \in \mathcal{A}_m$$

It is relevant to observe that in this model the attractiveness proposed in [4] is approximated with demand. This choice is made principally because actual software have not incorporated routines those can compute these values, on the other hand these simplification helps exposition.
Independently from that it' s easy to return to the model proposed by Gallego et al. only substituting the demands in this constraints with attractivenesses.

For each leg ℓ and for each cabin $b \in B_\ell$ we must impose that the total number of seats allocated do not exceed the capacity of b and of all the lower cabin.

Of course total sales on a flight leg must satisfy the seat capacity among all cabins.

$$\sum_{t \in B_\ell : t \leq b} \sum_{m \in \mathcal{M}_{\ell t}} \sum_{a \in \mathcal{A}_m} x_a \leq \sum_{t \in B_\ell : t \leq b} c_t, \qquad \forall \ell \in \mathcal{L}, \forall b \in B_\ell$$

The objective function is the total revenue to be maximized:

$$\max \sum_{m \in \mathcal{M}} \sum_{a \in \mathcal{A}_m} p_a \cdot x_a$$

So the standard Sales-Based Integer Linear Program as the following MILP :

$$\begin{aligned}
\max_{\mathbf{x} \in \mathbb{N}^{|\mathcal{A}|}, \mathbf{z} \in \mathbb{R}^{|\mathcal{M}|}} \quad & \sum_{m \in \mathcal{M}} \sum_{a \in \mathcal{A}_m} p_a \cdot x_a \\
& \sum_{t \in B_\ell : t \leq b} \sum_{m \in \mathcal{M}_{\ell t}} \sum_{a \in \mathcal{A}_m} x_a \leq \sum_{t \in B_\ell : t \leq b} c_t, \qquad \forall \ell \in \mathcal{L}, \forall b \in B_\ell \\
& \bar{d}_m \cdot x_a - d_a \cdot z_m \leq 0, \qquad \forall m \in \mathcal{M}, \quad \forall a \in \mathcal{A}_m \\
& \sum_{a \in \mathcal{A}_m} x_a + z_m = d_m, \qquad \forall m \in \mathcal{M} \\
& l_a \leq x_a \leq u_a, \qquad \forall\ a \in \mathcal{A} \\
& z_m \geq \bar{d}_m, \qquad \forall\ m \in \mathcal{M}
\end{aligned} \tag{1}$$

where x_a, for all $a \in \mathcal{A}_m$, $m \in \mathcal{M}$ and z_m for all $m \in \mathcal{M}$ are the decision variables.

4 Improving formulation

In order to get a bound, integrality can be removed from SBILP considering the linear relaxation SBLP.

$$\begin{aligned}
\max_{\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}, \mathbf{z} \in \mathbb{R}^{|\mathcal{M}|}} \quad & \sum_{m \in \mathcal{M}} \sum_{a \in \mathcal{A}_m} p_a \cdot x_a \\
& \sum_{t \in B_\ell : t \leq b} \sum_{m \in \mathcal{M}_{\ell t}} \sum_{a \in \mathcal{A}_m} x_a \leq \sum_{t \in B_\ell : t \leq b} c_t, \qquad \forall \ell \in \mathcal{L}, \forall b \in B_\ell \\
& \bar{d}_m \cdot x_a - d_a \cdot z_m \leq 0, \qquad \forall m \in \mathcal{M}, \quad \forall a \in \mathcal{A}_m \\
& \sum_{a \in \mathcal{A}_m} x_a + z_m = d_m, \qquad \forall m \in \mathcal{M} \\
& l_a \leq x_a \leq u_a, \qquad \forall\ a \in \mathcal{A} \\
& z_m \geq \bar{d}_m, \qquad \forall\ m \in \mathcal{M}
\end{aligned} \tag{2}$$

Some simple inequalities that can improve the bounds obtainable solving the relaxation SBLP can be derived from the resolution of some easier problems. These bounds can be

obtained easily and their use improves significantly the quality of the relaxed solution as we will show on a small example.
A first bound is obtained by solving the problem of maximizing the seats x_a allocated to an alternative $a \in \mathcal{A}_m$, when all the other alternatives are closed, given its own demand d_a and the recaptured one.

Definition 1 (Max Single Sales Upper Bound (MSSP)). *Given a market $m \in \mathcal{M}$ and an alternative $a \in \mathcal{A}_m$ we define Max Single Sales Upper Bound (MSSP) the following value:*

$$\bar{u}(a,m) = \max_{x_a \in \mathbb{N}, z_m \in \mathbb{R}} \{ \quad x_a : \begin{array}{l} x_a + z_m = d_m \\ \bar{d}_m \cdot x_a - d_a \cdot z_m \leq 0 \\ z_m \geq \bar{d}_m \} \end{array} \tag{3}$$

The solution of this problem is given by the formula:

$$\bar{u}(a,m) = \left\lfloor \frac{d_a}{d_a + \bar{d}_m} \cdot d_m \right\rfloor$$

We have that $x_a \leq \bar{u}(a,m)$, that can be used as a cut constraint in (2). Further the total revenue

$$\sum_{a \in \mathcal{A}_m} p_a \cdot x_a \leq \sum_{a \in \mathcal{A}_m} p_a \cdot \bar{u}(a,m)$$

However the bound obtained is too weak.
We improve it by introducing a Max Group-sales bound. The second type of bound takes into account the fact that alternatives are always correlated to a market. The idea is to maximize the allocations to a market considering only demand constraints.
In order to solve efficiently each sub-problem, we have to distinguish three different bounds.

Definition 2 (Max Group Sales Upper Bound (MGSP)). *Given a market $m \in \mathcal{M}$ and a subset $I \subseteq \mathcal{A}_m$ we define the Max Group Sales Upper Bound (MGSP) as the following value:*

$$\bar{u}_N(m,I) = \max_{\mathbf{x}_I \in \mathbb{N}^n, z_m \in \mathbb{R}} \{ \quad \sum_{a \in I} x_a : \begin{array}{ll} \sum_{a \in I} x_a + z_m = d_m & \\ \bar{d}_m \cdot x_a - d_a \cdot z_m \leq 0 & a \in I \\ z_m \geq \bar{d}_m \} & \end{array}$$

Usually $|\mathcal{A}_m|$ is not so big so that the number of variables for this type of problems is tractable; additionally the structure of the problem is very simple and so the solution can be easily computed by a standard solver for linear mixed integer program.
It can be used as additional constraints to be add to (1) as

$$\sum_{a \in I} x_a \leq \bar{u}_N(m,I).$$

Despite these good aspects, it could happen that the number of markets is large. So the pre-process needed to compute MGSP bounds could be potentially too expensive. A smart relief is to use the linear relaxation of the MGSP.

Definition 3 (Max Group Sales Upper Bound Linear Program (MGSP-LP))**.** *Given a market $m \in \mathcal{M}$ and a set $I \subseteq \mathcal{A}_m$ we define the Max Group Sales Upper Bound Linear Program (MGSP-LP) as the following value:*

$$\bar{u}_N(m, I) = \max_{\mathbf{x}_I \in \mathbb{R}^n, z_m \in \mathbb{R}} \{ \sum_{a \in I} x_a :$$
$$\textstyle\sum_{a \in I} x_a + z_m = d_m$$
$$\bar{d_m} \cdot x_a - d_a \cdot z_m \leq 0 \quad a \in I$$
$$x_a \geq 0, \qquad a \in I$$
$$z_m \geq \bar{d_m} \}$$

The good structure of the formulation helps us in the fact that is possible to build an analytical solution for this problem.

Proposition 4.1. *Given a market $m \in \mathcal{M}$ the optimal solution of MGSP-LP is given by*

$$x_a^* = \frac{d_m \cdot d_a}{\rho_I + \bar{d_m}} \qquad \textit{for all } a \in I$$
$$z_m^* = \frac{d_m \cdot \bar{d_m}}{\rho_I + \bar{d_m}}$$

where $\rho_I = \sum_{a \in I} d_a$

Proof. Given the primal in definition (3), the dual formulation follows:

$$\text{Minimize}: \; d_m \cdot \lambda_c - \bar{d_m} \cdot \lambda_z$$
$$\lambda_c + \bar{d_m} \cdot \lambda_a \geq 1 \qquad , \quad a \in I \tag{4}$$
$$\lambda_c - \sum_{a \in I} d_a \cdot \lambda_a - \lambda_z \geq 0 \tag{5}$$
$$\lambda_c \geq 0$$
$$\lambda_z \geq 0$$

We can relax the Feasible Region observing some simple structures of the constraints. We can transform the constraints (4) as it follows:

$$\lambda_c + \bar{d_m} \cdot \lambda_a \geq 1 \qquad a \in I$$
$$\Updownarrow$$
$$\lambda_a \geq \frac{1 - \lambda c}{\bar{d_m}} \qquad a \in I$$

Multiplying $d_a \quad \forall a \in I$ and summing by $a \in I$,

$$\sum_{a \in I} d_a \cdot \lambda_a \geq \sum_{a \in I} d_a \cdot \frac{1 - \lambda c}{\bar{d_m}}$$

Then we can remove $\lambda_m := (\lambda_{a_1}, \lambda_{a_2}, ..., \lambda_{a_{|I|}})^T$ variables relaxing the constraint (5), obtaining :

$$\begin{aligned} \text{Minimize :} \ & d_m \cdot \lambda_c - \bar{d_m} \cdot \lambda_z \\ & \lambda_c - \sum_{a \in I} \frac{1 - \lambda c}{\bar{d_m}} \cdot d_a - \lambda_z \geq 0 \\ & \lambda_c \geq 0 \\ & \lambda_z \geq 0 \end{aligned}$$

Given that there is only one vertex, for the *Fundamental Theorem of Linear Programming* if a solution exists, then it is situated in a vertex. It is possible now to compute an easy solution for the dual. Note that there are also different solutions in the sense of λ_I but the optimal value of the objective function does not change.

$$\begin{aligned} \lambda_c^* &= \frac{\rho_I}{\bar{d_m} + \rho_I} \\ \lambda_a^* &= \frac{1}{\bar{d_m} + \rho_I} \qquad , \quad a \in I \\ \lambda_z^* &= 0 \end{aligned}$$

This solution is optimal for the Dual because is optimal for an its relaxation and still feasible for the non-relaxed formulation.

Noting that λ_c^* *and* $\lambda_a^*, \quad a \in I$ are greater than 0, we can use the *Complementarity Formulas* to compute the primal solution.
We have to simply solve a linear system of equations, with square coefficient matrix .

$$\begin{aligned} & \bar{d_m} \cdot x_a - d_a \cdot z_m = 0, \qquad a \in I \\ & \sum_{a \in I} x_a + z_m = d_m \end{aligned}$$

$$\Updownarrow$$

$$\begin{aligned} x_a^* &= \frac{d_m \cdot d_a}{\rho_I + \bar{d_m}}, \qquad a \in I \\ z_m^* &= \frac{d_m \cdot \bar{d_m}}{\rho_I + \bar{d_m}} \end{aligned}$$

□

We report a simple example to show the values of the different bounds allow to add different cuts to the original formulation.

Example 4.2. *Given a set of alternatives $\{A, B, C\}$ calling $\{0\}$ the null alternative and the related demands, we can compute the bounds.*

Alternatives	*Demand*	*MSSP*
A	*0.6*	*1*
B	*1*	*1*
C	*0.6*	*1*
0	*1.5*	

Alternatives	*MGSP*	*MGSP-LP*
A+B	*1*	*2*
B+C	*1*	*2*
A+C	*1*	*2*
A+B+C	*2*	*3*

With MSSP no alternative can be removed from the problem.
Using MGSP-LP, no restrictions are added and there's no benefit to use them instead of MSSP's ones.
Finally with MGSP we have that at most two alternatives can be used, reducing the feasible region.

5 Decomposition Approach

Problem (1) can have a huge number of variables so that it cannot be solved at optimality even using improved formulation. In literature and in practical applications there are a lot of Decomposition methods proposed for Network Based Revenue Management problem. The usual practice is to decompose the network by legs and then to optimize each single leg separately using standard methods, introducing some factors that take into account network relations.
In the following we assume that at each market m is associated a single service s.
We introduce a decomposition by market and not by single leg. The main idea is very simple, the solution of the problem is obtained by solving by a sequence of Master - Slave pair problems. The principal problem is divided as follows :

- *Master Problem:* is the problem to allocate capacities over markets, maximizing network profit;
- *Slave Problem:* is the problem to allocate the capacity allocated to a market by the Master problem over the different alternatives, maximizing the profit.

The decision variables of the Master problem are the capacities v_m dedicated to the m-th market where $m \in \mathcal{M}$. They are constrained to be integers and they cannot exceed the aggregate host demand for the market $\rho_m = \sum_{a \in \mathcal{A}_m} d_a$.
We first introduce the Slave problem where we assume that the capacities v_m dedicated to the m-th market, $m \in \mathcal{M}$, are assigned. The decision variables of the Slave problems are analogous to those of the basic model (1) but restricted to a single market m. Since each market has its own capacity allocated in the Master decision step, it is always satisfied the relation:

$$\sum_{a \in \mathcal{A}_m} x_a = v_m \tag{6}$$

The Slave formulation for market m is:

$$\max_{\mathbf{x}_{\mathcal{A}_m} \in \mathbb{N}^{|\mathcal{A}_m|}, z_m \in \mathbb{R}} \quad \sum_{a \in \mathcal{A}_m} p_a \cdot x_a$$

$$\begin{aligned} & \sum_{a \in \mathcal{A}_m} x_a = v_m \\ & \sum_{a \in \mathcal{A}_m} x_a + z_m = d_m \\ & \bar{d_m} \cdot x_a - d_a \cdot z_m \leq 0, \qquad \forall\ a \in \mathcal{A}_m \\ & z_m \geq \bar{d_m} \end{aligned} \tag{7}$$

For given fixed allocation $v_m, m \in \mathcal{M}$ the optimal objective value of the slave problem $p_m(v_m)$ is defined as follows.

Definition 4 (Revenue Market Function)**.** *For a given $m \in \mathcal{M}$ associated with a value v_m such that $v_m \in \mathbb{N}$ and $v_m \leq \rho_m$, we define the **Revenue Market Function** $p_m(v_m)$ as follows:*

$$p_m(v_m) := \max_{\mathbf{x}_{\mathcal{A}_m} \in \mathbb{N}^{|\mathcal{A}_m|}, z_m \in \mathbb{R}} \left\{ \begin{aligned} & \sum_{a \in \mathcal{A}_m} p_a \cdot x_a : \\ & \sum_{a \in \mathcal{A}_m} x_a = v_m \quad , \\ & \sum_{a \in \mathcal{A}_m} x_a + z_m = d_m \quad , \\ & \bar{d_m} x_a - d_a z_m \leq 0 \qquad a \in \mathcal{A}_m, \\ & z_m \geq \bar{d_m} \end{aligned} \right\} \tag{8}$$

We are now ready to define the Master formulation:

$$\text{Maximize}: \quad \sum_{m \in \mathcal{M}} p_m(v_m)$$

$$\begin{aligned}
&\sum_{t \in B_l : t \leq b} \sum_{m \in M_{lt}} v_m \leq \sum_{t \in B_l : t \leq b} c_t, && \forall\, l \in L, \quad \forall\, b \in B_l && (9)\\
&0 \leq v_m \leq \rho_m, && \forall\, m \in \mathcal{M}\\
&v_m \in \mathbb{N}, && \forall\, m \in \mathcal{M}
\end{aligned}$$

5.1 Slave problem

It's very interesting to show that it's possible to find a trivial solution for the Slave problem.
We are ready to show the main result about the slave problem.

Theorem 5.1. *Given a market $m \in \mathcal{M}$ and a value v_m. Only one of the two following sentences is true:*

1. *the Slave problem is unfeasible ;*
2. *the Slave problem admits a solution. This solution is obtainable using a Greedy-type algorithm.*

Proof. Given a market $m \in \mathcal{M}$ and a value v_m, consider the Slave formulation , definition (5.4). Now we show that a solution exists if and only if a certain condition is verified.
Given a market $m \in \mathcal{M}$ we will say that v_m satisfies the feasibility condition if satisfies:

$$0 \leq v_m \leq \sum_{a \in \mathcal{A}_m} \left\lfloor \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor \tag{10}$$

So definetely a solution for the Slave problem exists if and only if the feasibility condition is satisfied.

We remove z_m from the formulation. In fact we can substitute what appear in the *fixed allocation constraint* in the *total demand* one, obtaining :

$$z_m = d_m - v_m$$

If the feasibility condition is valid $v_m \leq \rho_m$, thus this value for z_m is always feasible and constant. We remove it.
We now reduce the Slave formulation and we rewrite the *relative demand* constraints.

$$\max_{\mathbf{x}_{\mathcal{A}_m} \in \mathbb{N}^{|\mathcal{A}_m|}, z_m \in \mathbb{R}} \sum_{a \in \mathcal{A}_m} p_a \cdot x_a$$

$$\sum_{a \in \mathcal{A}_m} x_a = v_m$$

$$x_a \leq \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m), \qquad a \in \mathcal{A}_m$$

Combining the integrality with the *relative demand* constraints, we can easily improvethe upper bounds for all $a \in A_{\mathcal{M}}$ as follow:

$$\left\{x_a \in \mathbb{N} : x_a \leq \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m)\right\} \Leftrightarrow \left\{x_a \in \mathbb{N} : x_a \leq \left\lfloor \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor\right\}$$

It's simple to understand that there's not loss of information.
The solution of this problem is trivial, in fact all the variables are superior bounded and each bound is constant. So it's sufficient to use a Greedy-type algorithm to reach the solution.
We have to use all the fixed capacity v_m allocating resource until the bound is reached or the remain capacity is terminated. Obviously this procedure begins from the alternative $a \in \mathcal{A}_m$ with maximum price, when it's full then it consider the second maximum price alternative, then the third and so on iteratively.

Now we have to show that a solution for the Slave problem exists only if v_m satisfies the feasibility condition. We decide to demonstrate the negation of the what we've just said (observe that this is a note and valid method to conduce demonstrations). So definitively we have to demonstrate that if v_m doesn't satisfy the feasibility condition the Slave problem is unfeasible .
If the feasibility condition is not valid we can distinguish 3 cases:

1. $v_m < 0$
2. $v_m \geq 0 \quad \wedge \quad v_m > \rho_m$
3. $v_m \geq 0 \quad \wedge \quad v_m \leq \rho_m$

Case 1 : If $v_m < 0$ the problem is infeasible because $\sum_{a \in \mathcal{A}_m} x_a \geq 0$ and so $\sum_{a \in \mathcal{A}_m} x_a > v_m$.
Case 2 : If $v_m \geq 0 \quad \wedge \quad v_m > \rho_m$ we have again that the problem is infeasible because $z_m = \rho_m + \bar{d_m} - v_m$ and so $z_m < \bar{d_m}$.
Case 3 : Finally suppose that $v_m \geq 0 \quad \wedge \quad v_m \leq \rho_m$. Known that $v_m \leq \rho_m$ we can

repeat the same steps made before to reduce the Slave formulation, re obtaining :

$$\max \sum_{a \in \mathcal{A}_m} p_a \cdot x_a$$

$$\sum_{a \in \mathcal{A}_m} x_a = v_m$$

$$x_a \leq \left\lfloor \frac{da}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor, \qquad a \in \mathcal{A}_m$$

Summing by $a \in \mathcal{A}_m$ all the *relative demand constraints* we observe:

$$\sum_{a \in \mathcal{A}_m} x_a \leq \sum_{a \in \mathcal{A}_m} \left\lfloor \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor$$

But remembering the *fixed allocation constraint* we have:

$$v_m \leq \sum_{a \in \mathcal{A}_m} \left\lfloor \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor$$

This is exact the feasibility condition and we are supposing that it is not satisfied, but for the 3rd case this means that $v_m > \sum_{a \in \mathcal{A}_m} \left\lfloor \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor$ and thus finally the Slave problem is Infeasible.

Actually we've just demonstrated that a solution exists if and only if the feasibility condition is satisfied, so the point (2) of the theorem is accomplished.

To conclude the demonstration we have to show that if the solution does not exist, then the problem is unfeasible . But as we've just seen a solution does not exist if and only if the feasibility condition is violated. Thus we have definitely two cases:

- Case 1 :

  $$v_m < 0$$

  Trivially the feasible region is empty because both the following condition have to be verified:

  $$\sum_{a \in \mathcal{A}_m} x_a = v_m < 0 \quad \wedge \quad x_a \in \mathbb{N}, \quad a \in \mathcal{A}_m$$

  In fact a sum of positive numbers cannot be negative.

- Case 2 :

  $$v_m > \sum_{a \in \mathcal{A}_m} \left\lfloor \frac{d_a}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor$$

  But looking at the problem it is clear that

  $$x_a \leq \left\lfloor \frac{da}{\bar{d_m}} \cdot (d_m - v_m) \right\rfloor, \qquad a \in \mathcal{A}_m \quad \wedge \quad \sum_{a \in \mathcal{A}_m} x_a = v_m$$

It's impossible that $\sum_{a\in\mathcal{A}_m} x_a$ is greater than the sum of the bounds of the terms those compose it. So again the feasible region is empty.

This conclude the proof. ∎

More formally we can introduce a simple Greedy-type algorithm scheme for the computation of the solution for the Slave problem.

Slave solution Greedy Algorithm

Initialization. Set $P_0 = \mathcal{A}_m$, $s_0 = v_m \in \mathbb{N}$, $x_a^0 = 0$ for all $a \in \mathcal{A}_m$ and $p_m^0 = 0$.

For $h = 0, 1, \dots$

1. Set $p^* = \max_{a\in P_h}\{p_a\}$, $\quad a^* = \arg\max_{a\in P_h}\{p_a\}$.
2. If $s_h \geq 1 \quad \wedge \quad x_{a^*}^h < \left\lfloor \frac{da^*}{d_m} \cdot (d_m - v_m) \right\rfloor$
   then $\quad x_{a^*}^{h+1} = x_{a^*}^h + 1, \quad x_a^{h+1} = x_a^h \quad \forall a \in \mathcal{A}_m \setminus \{a^*\}, \quad s_{h+1} = s_h - 1$
   and $\quad p_m^{h+1} = p_m^h + p^*$
3. If $s_h \geq 1 \quad \wedge \quad x_{a^*}^h \geq \left\lfloor \frac{da^*}{d_m} \cdot (d_m - v_m) \right\rfloor$
   then $P_{h+1} = P_h \setminus \{a_*\}$
4. If $s_h = 0$
   put $x_a = x_a^{h+1}, \quad \forall a \in \mathcal{A}_m \quad$ and $\quad p_m(v_m) = p_m^{h+1}$
   then **STOP**

End For

A central result is a particular property that the Revenue Market Function.

Proposition 5.2. *For any $m \in \mathcal{M}$, let v_m satisfying* (10) *then*

$$\frac{p_m(v_m+1)}{v_m+1} \leq \frac{p_m(v_m)}{v_m}$$

Proof. Suppose by contradiction that $\exists m \in \mathcal{M}$ such that $\exists v_m$ satisfying (10) and such that

$$\frac{p_m(v_m+1)}{v_m+1} > \frac{p_m(v_m)}{v_m} \tag{11}$$

We know that

$$p_m(v_m+1) = p_m(v_m) + \Delta p_m(v_m+1)$$

where

$$\Delta p_m(v_m+1) = p_m(v_m+1) - p_m(v_m)$$

Substituting in (11) we have that

$$\frac{p_m(v_m)}{v_m+1} + \frac{\Delta p_m(v_m+1)}{v_m+1} > \frac{p_m(v_m)}{v_m}$$

$$\Downarrow$$

$$\Delta p_m(v_m+1) \cdot v_m > p_m(v_m) \tag{12}$$

On the other hand we are incrementing only one unit of capacity so the increment is less or equal than the price of the alternative immediately disposal when one unit of capacity is added.

$$\Delta p_m(v_m+1) = p_m(v_m+1) - p_m(v_m) \leq \bar{p}$$

But we know that

$$p_m(v_m) = p_{a_1}x_{a_1} + p_{a_2}x_{a_2} + \ldots + p_{a_r}x_{a_r}$$

Where $r \leq f_m$ and $p_{a_1} \geq p_{a_2} \geq \ldots \geq p_{a_r}$. Easily we can write

$$p_m(v_m) \geq p_{a_r} \cdot v_m$$

Remembering that $v_m < v_m + 1$ we have that necessarily $p_{a_r} \geq \bar{p}$. Finally we have that

$$p_m(v_m) \geq p_{a_r} \cdot v_m \geq \bar{p} \cdot v_m \geq \Delta p_m(v_m+1) \cdot v_m$$

this is in contradiction with the absurd assumption (12), and so the thesis is verified. □

Roughly speaking we say that the unitary Revenue is non-increasing. Using empirical data it turns out that the structure of the Market Revenue Function is basically concave, with a noise that modifies locally its behaviour.
We can make a lower and an upper approximation.

- *Upper Approximation*

  The main idea is to use some results from the Global Optimization Theory. A famous branch of Global Optimization refers to the concept of *Convex Hull* (speaking about minimization problems). For a better comprehension see [6].

  Briefly from now we will write more coherently about the *Concave Hull* of the Market Revenue Function.

  A note definition of this curve follows.

Proposition 5.3. *Given $f(x) : \Omega \to \mathbb{R}$, a continous function always defined and finite on the set $\Omega \subseteq \mathbb{R}^n$ that is closed. We can define the ConcaveHull $co(f(x))$ of the function $f(x)$ as:*

$$co(f(x)) = inf\{a^T x + b : a^T y + b \geq f(y) \quad \forall y \in \Omega, \quad a \in \mathbb{R}^n, \quad b \in \mathbb{R}\}$$

Our approach is very simple. We apply this definition to a discretized set, and the problem degenerate to a simple bi-dimensional linear programming problem.

In other words we compute $p_m(v)$ $\forall v \in \{0, 1, \dots, ub_m\}$, the reason of this are twice: first we are only interested on integer values, second the Market Revenue Function is attributable to a continuous piecewise linear function on $[0, ub_m]$.

After this calculus we can easily find the *ConcaveHull* $co(p_m(v))$ solving the following problem:

$$co(p_m(v)) = min\{a \cdot v + b : a \cdot u + b \geq p_m(u) \quad \forall u \in \{0, 1, \dots, ub_m\}, \quad a \in \mathbb{R}, \quad b \in \mathbb{R}\}$$

Observe that we write min instead of inf because this is a linear programming minimization problem lower bounded and non-empty (a feasible solution is given by $a = max\{p_m(v)/v : v \in \{1, \dots, ub_m\}\}$ and $b = 0+$).

Our application is fitted on the Airline Revenue Management, so generally speaking the capacities are in the order of 500-1000 available units. Definitely this gives us the opportunity to effectively build and solve the problem we've just presented.

- *Lower Approximation*

  The main idea here is to use the *ConcaveHull* shown before to realize a lower function. This computation is very easy and can be made just rescaling opportunely what we've already collect computing the Upper Bound.

  Thus once calculated $p_m(v)$ $\forall v \in \{0, 1, \dots, ub_m\}$ and the respective *ConvexHull* $co(p_m(v))$ $\forall v \in \{0, 1, \dots, ub_m\}$ (eventually reducing $ub_m \to u^*_m$), we can solve the following problem:

$$\alpha = min\{a : a \cdot co(p_m(v)) \leq p_m(v) \quad \forall v \in \{0, 1, \dots, ub_m\}, a \in [0, 1]\}$$

  The problem is well posed because is linear and non-empty ($\alpha = 0$ is a feasible solution).

  This problem have only one decision variable and have to be solved one time for Market.

  Once alpha is computed we can easily write the *ConcaveLowerBoundFunction* $co_L(p_m(v))$ as:

$$co_L(p_m(v)) = \alpha \cdot co(p_m(v)) \qquad \forall v \in \{0, \dots, ub_m\}$$

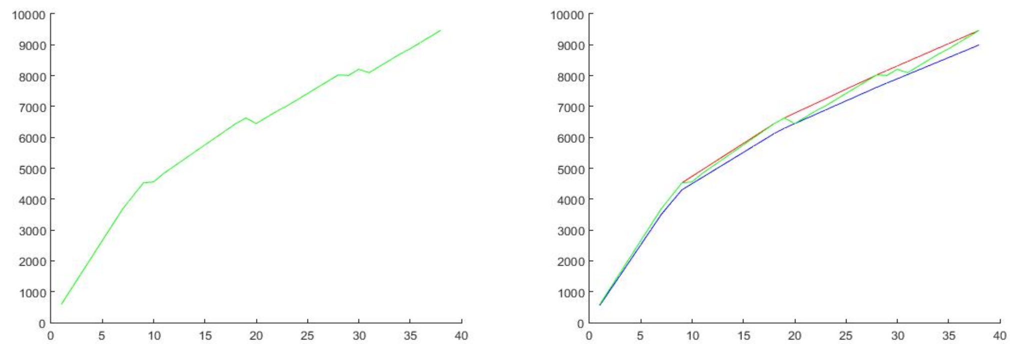

Figure 1: Left: the Market Revenue Function. Right: the Market Revenue Function, its upper approximation and the lower one.

The most relevant advantage of use this approach is we have a gap between Upper and Lower approximation that is always constant in percentage. In formulas:

$$\frac{co(p_m(v)) - co_L(p_m(v))}{co(p_m(v)} = 1 - \alpha \qquad \forall v \in \{1, \dots, ub_m\}$$

or equivalently

$$\frac{co_L(p_m(v))}{co(p_m(v)} = \alpha \qquad \forall v \in \{1, \dots, ub_m\}$$

Example 5.4. *Considering a simplified Market made up of a couple Origin-Destination related to only one service. Sample data are reported in the table.*

Alternative	*Demand*	*Price*
a1	*0*	*985*
a2	*0.004*	*777*
a3	*0.808*	*586*
a4	*4.454*	*517*
a5	*0.026*	*464*
a6	*1.758*	*418*
a7	*0.402*	*339*
a8	*2.399*	*276*
a9	*7.457*	*230*
a10	*23.426*	*201*
a11	*0.487*	*171*
a0	*90.399*	

We can easily generate the approximations.
Here the value of alpha is 95%, so the error committed is no more than 5%.

5.2 Master problem

We discuss in this section properties of the Master problem (9).
It is useful to solve the problem rewriting it in a easier form, that is a linear combinatorial programming reformulation.
Notationally we call $k_m = \lfloor \rho_m \rfloor \quad m \in \mathcal{M}$ and $p_{m,i} = p_m(i) \qquad \forall m \in \mathcal{M}, i \in \{0, 1, \ldots, k_m\}$.

Definition 5. *The Master problem binary reformulation (BPM) is defined as :*

$$\textit{Maximize :} \sum_{m \in \mathcal{M}} \sum_{i=l_m}^{k_m} p_{m,i} \cdot y_{m,i}$$

$$\sum_{t \in B_l : t \leq b} \sum_{m \in \mathcal{M}_{lt}} \sum_{i=0}^{k_m} i \cdot y_{m,i} \leq \sum_{t \in B_l : t \leq b} c_t \qquad l \in L, \quad b \in B_l$$

$$\sum_{i=l_m}^{k_m} y_{m,i} = 1 \qquad m \in \mathcal{M}$$

$$y_{m,i} \in \{0, 1\} \qquad m \in \mathcal{M}$$

here $y_{m,i} \quad m \in \mathcal{M}, i \in \{0, 1, \ldots, k_m\}$ *are the decision variables.*

A binary variable is equal to one if and only if we're allocating i capacity to the respective market. It is also possible to reserve only a variable to a certain market, this is the reason of the additional constraints. Master feasibility is kept in mind by $k_m \quad m \in \mathcal{M}$ parameters.
The only observation here is that the problem, even if it is linear, has computational complications given by the binary structure of the decision variables.

5.3 Market-Service Decomposition

In this section we neglect the assumption one Market- one Service. As a matter of fact real application of these models are based on the key concept of Market-Service.
We want to discuss now how the model could be modified if we pass from the single-service case to the multiple one.
First of all we have to redefine the Slave Problem itself. For a given market $m \in \mathcal{M}$ let be $S_m := \{s_1, \ldots, s_q\}$ the set of services associated to this market. Let now be $P_m := \{P_{s_1}, \ldots P_{s_q}\}$ a partition of alternatives associated to market m, each $P_{s_i} \subseteq \mathcal{A}_m \quad i = 1, \ldots, q$, $P_{s_i} \cap P_{s_h} = \emptyset \quad \forall i, h \in \{1, \ldots, q\} \quad i \neq h$ and $\cup_{i=1}^{q} P_{s_i} = \mathcal{A}_m$.

Definition 6. *For a given $m \in \mathcal{M}$ associated with a value v_m such that $v_m \in \mathbb{N}$ and $v_m \leq \rho_m$, we define the **Multi-Service Revenue Market Function** $p_m^S(v_m, w_{s_1}, w_{s_2}, \ldots, w_{s_q})$*

as follows:

$$p^S_m(v_m, w_{s_1}, w_{s_2}, \dots, w_{s_q}) := \max \sum_{a \in \mathcal{A}_m} p_a \cdot x_a :$$
$$\sum_{a \in s_i} x_a = v_i \qquad i = 1, \dots, q,$$
$$x_a \le \left\lfloor \frac{d_a}{\bar{d}_m} \cdot (d_m - v_m) \right\rfloor \qquad a \in \mathcal{A}_m, \tag{13}$$
$$x_a \in \mathbb{N} \qquad a \in A_{\mathcal{M}} \tag{14}$$

This complication is reflected in the Master problem by using the following constraint:

$$v_m = \sum_{i=1}^{q} w_{s_i} \tag{15}$$

Fortunately we can now further decompose the Slave Problem. In fact the allocation of capacity for each service is outsourced in the Master (maintaining its own structure).

Definition 7. *Given $m \in \mathcal{M}$ associated with a value v_m such that $v_m \in \mathbb{N}$ and $v_m \le \rho_m$. Given also a service $s_j \in S_m$ associated with a value w_{s_j}, we define the **Single-Service Revenue Market Function** $p^{s_j}_m(v_m, w_{s_j})$ as follows:*

$$p^{s_j}_m(v_m, w_{s_j}) := \max \sum_{a \in P_{s_j}} p_a \cdot x_a :$$
$$\sum_{a \in P_{s_j}} x_a = w_{s_j} \qquad ,$$
$$x_a \le \left\lfloor \frac{d_a}{\bar{d}_m} \cdot (d_m - v_m) \right\rfloor \qquad a \in P_{s_j}, \tag{16}$$
$$x_a \in \mathbb{N} \qquad a \in P_{s_j} \tag{17}$$

This decomposition is exact because the problem is Block decomposable, and so we can write:

$$p^S_m(v_m, w_{s_1}, w_{s_2}, \dots, v_{s_q}) = \sum_{i=1}^{q} p^{s_i}_m(v_{m,w}\, s_j)$$

So definitely the only complication in the Master will be to introduce as a constraint the equation (15), maximizing the following objective function:

$$p(v_{m_1}, \dots, v_{m_{|M|}}) = \sum_{m \in \mathcal{M}} p^S_m(v_m, w_{s_1}, w_{s_2}, \dots, w_{s_q}) = \sum_{m \in \mathcal{M}} \sum_{j=1}^{q} p^{s_i}_m(v_m, w_{s_j})$$

The size of the Master problem is now larger but still less than exponential. Indeed in the worst case we have exact one alternative for each service, and at most one with 2 alternatives. This is still significantly less the dimension of the basic model, so it's still good.

In real application we have to consider also lower and upper bounds that each alternative, or each couple market-service, could have. Generally this increase only the reading complexity.
We can easily rebuild the Master problem based on Market-Service decomposition.

Definition 8. *The Master problem is defined as solving the following mathematical programming formulation:*

$$\textit{Maximize}: \sum_{m \in \mathcal{M}} \sum_{s \in S_m} p_m(v_m, w_s)$$

$$\begin{aligned}
&\sum_{s \in S_m} w_s = v_m && m \in M \\
&\sum_{t \in B_l : t \leq b} \sum_{m \in M_{lt}} v_m \leq \sum_{t \in B_l : t \leq b} c_t && l \in L, \quad b \in B_l \\
&l_s \leq w_s \leq u_s && m \in \mathcal{M}, \quad s \in S_m \\
&l_m \leq v_m \leq \rho_m && m \in M \\
&w_s \in \mathbb{N} && m \in \mathcal{M}, \quad s \in S_m \\
&v_m \in \mathbb{N} && m \in M
\end{aligned}$$

where $v_m \quad m \in \mathcal{M}$ *and* $w_s \quad m \in \mathcal{M}\ ,\ s \in S_m$ *are the decision variables.*

The Master problem defined above can be formulated in a binary way. In this case we have to introduce two group of constraints that have the function to link capacities between Markets and Services.
We decide to present the model and after that to describe it.

Definition 9. *The Master problem binary Market-Service reformulation (BPMS) is* *defined as :*

$$Maximize : \sum_{m\in\mathcal{M}} \sum_{s\in S_m} \sum_{i=l_m}^{k_m} \sum_{j=l_s}^{u_s} p_{m,s}(i,j)\cdot y_{m,s,i,j}$$

$$\sum_{t\in B_l:t\le b} \sum_{m\in M_{lt}} \sum_{s\in S_m} \sum_{i=l_m}^{k_m} \sum_{j=l_s}^{u_s} j\cdot y_{m,s,i,j} \le \sum_{t\in B_l:t\le b} c_t \qquad l\in L, \quad b\in B_l$$

$$\sum_{i=l_m}^{k_m} \sum_{j=l_s}^{u_s} y_{m,s,i,j} = 1 \qquad m\in\mathcal{M}, \quad s\in S_m \tag{18}$$

$$\sum_{j=l_s}^{u_s} y_{m,\hat{s},i,j} = \sum_{j=l_s}^{u_s} y_{m,\tilde{s},i,j} \qquad m\in\mathcal{M}, \quad \hat{s}\in S_m\setminus\{\tilde{s}\}, \quad i\in\{l_m,\dots,k_m\} \tag{19}$$

$$\sum_{s\in S_m} \sum_{j=l_m}^{k_m} j\cdot y_{m,s,i,j} \le i \qquad m\in\mathcal{M}, \quad i\in\{l_m,\dots,k_m\} \tag{20}$$

$$y_{m,s,i,j}\in\{0,1\} \qquad m\in\mathcal{M}, \quad s\in S_m, \quad i\in\{l_m,\dots,k_m\}, \quad j\in\{l_s,\dots,u_s\}$$

here $y_{m,s,i,j} \quad m\in\mathcal{M}, i\in\{l_m,\dots,k_m\}, s\in S_m, j\in\{l_s,\dots,u_s\}$ *are the decision variables, and* $\tilde{s}\in S_m$ *is a whichever service for each* $m\in M$*.*

The binary formulation becomes rapidly crowded with constraints. Obviously the previous constraints are still necessary and thus here we have to add a summation that covers all the possible new binary variables for each Market-Service. As said before, unfortunately, two big groups of constraints are added to the model:

- *Market's Capacity Coherency constraints :*

  They refer to constraints (19). The idea here is that we have to choose the same level of capacity allocated for a Market, for each couple Market-Service correlated.

  Combined with the constraints of type (18), it's easy to see that for a given level of capacity allocated to a Market, then the summation could be 1 ore 0 (and no other values are admitted). Definitely if a certain value $\hat{i}$ is chosen, then only binary variables with this value are a solution.

  Without these constraints the algorithm tends to solve separately each Market-Service couple , returning solutions non feasible for the original formulation.

- *Market's Capacity bound constraints :*

  They refer to constraints (20). The idea here is that given a fixed value of Market's capacity $\hat{i}$, then the summation of capacities allocated to Market-Service couples have to be equal to the one allocate to the Market. In other terms we try to reproduce the following constraint:

$$v_m = \sum_{s \in S_m} w_s \qquad m \in \mathcal{M}$$

Unfortunately the binary nature of the reformulation does not admit equality in this case, in fact this should imply that a Market's capacity value is fixed. So in the end our constraints force a weaker condition:

$$v_m \geq \sum_{s \in S_m} w_s \qquad m \in \mathcal{M}$$

Realistically speaking this is not a problem, as a matter of fact a solution is optimal if and only if all the Market's capacity is consumed. This is an easy deduction derived from the greedy nature of Market Revenue Function and from the fact that capacity constraints are based on Market decomposition concept.

6 Conclusions and future research

SBILP is one of the more effective Revenue Management models. In real applications, SBILP is a large-scale integer programming problem, that cannot be solved effectively in practice in the most of cases. Even its linear relaxation, well-known as SBLP, requires a remarkable computational effort by state-of-art commercial solvers. The results in this paper investigate important properties of SBILP that allows us to improve the formulation. In particular, we introduced simple cuts that can highly improve upper bound provided by SBLP. Moreover, we investigate a simple decomposition approach leading to a cost-effective method for the solution of SBILP.

Having observed that our research moved on binary formulation. Here we definitely found a strong formulation for the problem, but the Boolean nature of the variables generates a large number of elements that wore out time efficiency.

In our intention we will now move to reduce the size of the Master problem, trying to add more complexity to the Slave. A central idea is to pass from the Market-Service decomposition, to the Service-Cabin one. This setting can reduce dimensions of Master's input even of 75 percent.

Actually we are also continuing to develop an algorithm that can take into account the concave approximation.

We have developed preliminary numerical test that show viability of the approach. Future research will be dedicated to the implementation of the algorithm and test in real data.

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