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A SUFFICIENT CONDITION FOR FULL LINEARIZATION VIA DYNAMIC STATE FEEDBACK

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Abstract

The purpose of this paper is to show that any square invertible nonlinear system whose inverse is "state-free" can be turned into a fully linear controllable and observable system by means of dynamic state-feedback and coordinates transformations. A nonlinear system has an inverse which is "state-free" if the value of the input (at time t) can be expressed as a function of the values (at t) of the output and a finite number of its derivatives.

1. Introduction

Consider a nonlinear system described by equations of the form

$$\dot{x} = f(x) + g(x)u \tag{1.1a}$$

$$y = h(x) \tag{1.1b}$$

where $x \epsilon R^n$, $u \epsilon R^m$ and $y \epsilon R^m$. f and the m columns of g are analytic vector fields, h is an analytic mapping. Note that this system has the same number of input and output components.

The purpose of this paper is the design of a dynamic state-feedback compensator, namely a system described by equations of the form

$$\dot{\xi} = a(\xi,x) + b(\xi,x)v \tag{1.2a}$$

$$u = c(\xi,x) + d(\xi,x)v \tag{1.2b}$$

where $\xi \epsilon R^\nu$ and $v \epsilon R^m$, in such a way as to obtain a closed loop (i.e. the composition of (1.1) and (1.2)) which, after a suitable (local) change of coordinates in the state space $z = \phi(\xi,x)$, is described by equations of the form

$$\dot{z} = Az + Bv \tag{1.3a}$$

$$y = Cz \tag{1.3b}$$

The principle of the linearization method developed herein is the following one. It is known that any square invertible linear system can always be decoupled (from an input-output point of view) via dynamic state-feedback (see [2]) and that any decoupled system, modulo an unobservable part, can always be transformed (via feedback and change of coordinates) into a system of the form (1.3) (see [4]). Thus, loosely speaking, the matter is to identify systems in which such a decoupling procedure does not induce unobservable parts.

It is known that feedback can induce unobservability if and only if Δ^* (the so-called maximal controlled invariant distribution contained in the differential of h) is nonzero. Thus, the issue is to consider systems which have $\Delta^* = 0$ and to be sure that such a condition is not lost along the decoupling procedure.

Based on a recent understanding of the nonlinear equivalents of the notion of "transmission zeros," described in [6], it is shown here that the right class of systems to look at is that of systems whose inverses are "state-free." As we shall see more precisely in the next section, a system has a state-free inverse if its own state (at time t) can be recovered from the values of a finite number of derivatives of the output (at time t). As a result of this, it is shown that invertible systems whose inverses are state-free can be fully linearized via dynamic feedback and coordinates transformations.

2. Computation of a reduced inverse via Singh's algorithm

Following [6], we briefly describe how the generalized inversion procedure developed by Singh [7] which consists of a modification of the so-called structure algorithm, can be used in order to compute also a "reduced" inverse of (1.1).

Consider the mapping:

$$S_0(y,x) = h(x)-y$$

and set

$$\dot{S}_0(y^{(1)},x,u) = [(\partial S_0/\partial x)f(x)-y^{(1)}]+[(\partial S_0/\partial x)g(x)]u$$
$$= F_0(y^{(1)},x) + G_0(x)u$$

Note that $\dot{S}_0$ is linear in u. Let ρ_0 denote the rank of g_0 and set $p_0=m$, $p_1=p_0-\rho_0=m-\rho_0$. Let $K_0(x)$ be a $p_1 x p_0$ matrix of rank p_1 such that

$$k_0(x)\, G_0(x) = 0$$

and set

$$S_1(y^{(1)}, x) = K_0(x)F_0(y^{(1)}, x)$$

This concludes the zero-th step. At the k-th step, consider the mapping $S_k(y^{(1)},\text{--},y^{(k)},x)$ and set:

$$\dot{S}_k(y^{(1)},\dots,y^{(k)},y^{(k+1)},x,u) = \Big[(\partial S_k/\partial x)f(x) + \sum_{\ell=0}^{k}(\partial S_k/\partial y^{(\ell)})y^{(\ell+1)}\Big] + \big[(\partial S_k/\partial x)g(x)\big]u = H_k(y^{(1)},\dots,y^{(k+1)},x) + K_k(y^{(1)},\dots,y^{(k)},x)u$$

Set also:

$$F_k = \begin{bmatrix} F_{k-1} \\ H_k \end{bmatrix}, \qquad G_k = \begin{bmatrix} G_{k-1} \\ K_k \end{bmatrix}$$

Let ρ_k denote the rank of G_k and set $p_{k+1} = p_k-(\rho_k-\rho_{k-1}) = m-\rho_k$.

Let:

$$[T_k(y^{(1)},\dots,y^{(k)},x)\ V_k(y^{(1)},\dots,y^{(k)},x)]$$

be a matrix in which T_k is $p_{k+1} \times (p_0+\dots+p_{k-1})$ and V_k is $p_{k+1} \times p_k$ and has rank p_{k+1}, such that:

$$T_k(y^{(1)},\dots,y^{(k)},x)G_{k-1}(y^{(1)},\dots,y^{(k-1)},x) + V_k(y^{(1)},\dots,y^{(k)},x)K_k(y^{(1)},\dots,y^{(k)},x) = 0$$

and set:

$$S_{k+1}(y^{(1)},\dots,y^{(k+1)},x) = T_k(y^{(1)},\dots,y^{(k)},x)F_{k-1}(y^{(1)},\dots,y^{(k)},x) + V_k(y^{(1)},\dots,y^{(k)},x)H_k(y^{(1)},\dots,y^{(k+1)},x)$$

concluding the k-th iteration.

If at some k^* the matrix G_{k^*} has rank $\rho_{k^*} = m$, then the algorithm stops and it is easy to conclude that the equation:

$$F_{k^*}(y^{(1)},\dots,y^{(k^*+1)},x) + G_{k^*}(y^{(1)},\dots,y^{(k^*)},x)u = 0 \quad (2.1)$$

is solvable in u (see [7]). Moreover, it is also possible to show (see [6]) that the Jacobian matrix:

$$(\partial/\partial x)\begin{bmatrix} S_0(y,x) \\ S_1(y^{(1)},x) \\ \vdots \\ S_{k^*}(y^{(1)},\dots,y^{(k^*)},x) \end{bmatrix}$$

has rank $\mu = p_0 + p_1+\dots+p_{k^*}$ (namely, equal to the number of its rows). Thus, using the Implicit Function Theorem, from the equation:

$$\begin{bmatrix} S_0(y,x) \\ \vdots \\ S_{k^*}(y^{(1)},\dots,y^{(k^*)},x) \end{bmatrix} = 0$$

one can recover μ components of x, expressed as a function of $y,y^{(1)},\dots,y^{(k^*)}$ and of the remaining $n-\mu$ state components denoted by z. Substituting these into (2.1) and then into (1.1a), one obtains a "reduced" inverse system in the form:

$$\dot{z} = F(y,\dots,y^{(k^*+1)},z) \quad (2.2a)$$

$$u = G(y,\dots,y^{(k^*+1)},z) \quad (2.2b)$$

This inverse system is defined for almost all output functions. The dimension of its dynamics i.e. dim z, loosely speaking, is a reduced number of differential equations needed to recover the input function u of (1) starting from the knowledge of its output function y and of its initial state x_0.

If $\mu=n$ then x(t) can be completely expressed as a function of $y(t),\dots,y^{(k^*+1)}(t)$. Accordingly, in the reduced inverse (2.2) the dynamics (2.2a) disappears and u(t) can be completely expressed as a function of $y(t),\dots,y^{(k^*+1)}(t)$. Whenever this happens, the system is said to have a "state-free" dynamics.

3. Main Results

It is well known that any square invertible system can always be turned, by means of a suitable dynamic extension, into a system which can be decoupled via static state-feedback. A dynamic extension consists of addition of integrators on some input channel and state-dependent coordinates transformations in the input space [2]. The overall procedure of dynamic extension and static state-feedback (on the extended system) is sometimes referred to as dynamic state-feedback.

It is also well known that systems which can be decoupled via static state-feedback, if $\Delta^* = 0$, are feedback-equivalent to linear controllable and observable systems. In view of this, it is clear the interest in seeking whether or not there are cases in which the dynamic extension required in order to fulfill the decoupling conditions is such as to produce a system in which $\Delta^* = 0$. For, if this is the case, then the extended system will be feedback equivalent to a linear controllable and observable one.

A natural candidate is the class of systems which already have $\Delta^* = 0$ (i.e. before dynamic extension). However, this condition alone does not seem to be the good one because, as shown e.g. in [5], the property that $\Delta^* = 0$ may not be preserved under addition of integrators on the input. A first attempt to find additional conditions which make the property $\Delta^* = 0$ invariant under dynamic extension was given in [1][5], where a set of sufficient conditions (based on a property of the so-called maximal controlled invariant distribution algorithm) was found. In this paper we present a full solution to this problem, in the sense that we give necessary and sufficient conditions.

The result in question is a consequence of the following two lemmas.

Lemma 3.1 [6]

Suppose the system (1.1) can be decoupled via static state-feedback. Then the dimension of Δ^* and the dimension of the dynamics of the reduced inverse (2.2) are equal. In particular,

$\Delta^* = 0$ if and only if $\dim(z) = 0$.

Lemma 3.2

Suppose the system (1.1) is invertible. Then the dimension of the dynamics of the reduced inverse (2.2) is invariant under dynamic extension.

The proof of this Lemma, which is simple but a little tedious, can be found in the Appendix.

Remark 3.1

Suppose the following system

$$\dot{\zeta} = \phi(\zeta, y, \ldots, y^{(\gamma)}) \tag{3.1a}$$

$$u = \gamma(\zeta, y, \ldots, y^{(\gamma)}) \tag{3.1b}$$

is an inverse of (1.1). Then it is very easy to find an inverse for the dynamic extension of (1.1) obtained by addition of an integrator on some input, say u_i. For, set

$$\dot{\xi} = v_i \ , \ u_i = \xi$$

Then, differentiate with respect to time the i-th line of (3.1b):

$$\dot{u}_i = v_i = \frac{\partial \gamma_i}{\partial \zeta}\dot{\zeta} + \sum_{j=0}^{(\gamma)} \frac{\partial \gamma_i}{\partial y^{(j)}} y^{(j+1)} = $$
$$= \tilde{\gamma}_i(\zeta, y, \ldots, y^{(\gamma+1)})$$

Thus an inverse for the extended system is provided by (3.1a)-(3.1b), with the i-th line of (3.1b) replaced by the latter expression.

The reason why the proof of Lemma 3.2 was quite longer is that in view of the opportunity of using Lemma 3.1, we had to refer explicitly to Singh's algorithm. □

Knowing that from an invertible square system, after dynamic extension, one can obtain a system which can be decoupled via static state feedback, we can now easily prove the main result.

Theorem 3.1

Suppose the system (1.1) is such that:

i) $\rho_{k^*} = m$

ii) $\dim(z) = 0$

Then it can be fully linearized via dynamic state-feedback.

Proof. Simply use the decoupling procedure described in [2]. At each stage, a dynamic extension is found which, in view of Lemma 3.2, leaves the condition $\dim(z) = 0$ unchanged. Because of the invertibility assumption ($\rho_{k^*} = m$), after a finite number of stages the procedure ends up with an extended system which can be decoupled via static state feedback. This system, in view of Lemma 3.1, has now $\Delta^* = 0$ and therefore is feedback equivalent to a linear controllable and observable one. □

4. Examples

Example 4.1.

Consider the following system, taken from [3]:

$$\begin{aligned} \dot{x}_1 &= u_2 & \qquad y_1 &= x_2 \\ \dot{x}_2 &= x_1 u_1 & \qquad y_2 &= x_3 \\ \dot{x}_3 &= x_2 u_1 \end{aligned}$$

This system has $\Delta^* = 0$ but does not satisfy the sufficient conditions given in [1], as a simple computation shows. However, this system satisfies the condition of Theorem 3.1 and can therefore be fully linearized via dynamic state-feedback (note also that span $\{g_1, g_2\}$ is not involutive and therefore the state equation is not (static)-feedback equivalent to a linear controllable one).

Carrying out Singh's algorithm one obtains

$$S_0 = \begin{bmatrix} x_2 - y_1 \\ x_3 - y_2 \end{bmatrix} \qquad K_0 = (x_2 - x_1)$$

$$S_1 = x_1\dot{y}_2 - x_2\dot{y}_1$$

$$G_1(x, \dot{y}) = \begin{bmatrix} x_1 & 0 \\ x_2 & 0 \\ -x_1\dot{y}_1 & \dot{y}_2 \end{bmatrix}$$

The matrix G_1 has generically rank $m=2$ and $\mu = p_0 + p_1 = 3$ (i.e. $\dim(z)=0$). Thus, both conditions of Theorem 3.1 are satisfied.

In order to get full linearization via dynamic feedback we use first the decoupling procedure described in [2]. Since the decoupling matrix of the system has the form

$$A(x) = L_g h(x) = \begin{bmatrix} x_1 & 0 \\ x_2 & 0 \end{bmatrix}$$

we have to add an integrator on the first input channel.

The system thus extended

$$\begin{aligned} \dot{x}_1 &= w_2 & \qquad y_1 &= x_2 \\ \dot{x}_2 &= x_1 x_4 & \qquad y_2 &= x_3 \\ \dot{x}_3 &= x_2 x_4 \\ \dot{x}_4 &= w_1 \end{aligned}$$

has now a (generically) nonsingular decoupling matrix

$$A^e(x^e) = L_{g^e} L_{f^e} h^e = \begin{bmatrix} x_1 & x_4 \\ x_2 & 0 \end{bmatrix}$$

and $\Delta^{*e} = 0$. Thus, the latter is feedback equivalent to a system consisting of two chains of two integrators each. Namely, the feedback

$$w = [A^e(x^e)]^{-1} (-L_{f^e}^2 h^e + v)$$

and the (local) change of coordinates $(\xi_0, \xi_1) = (h^e, L_{f^e} h^e)$ turn the system into

$$\begin{aligned} \dot{\xi}_0 &= \xi_1 \\ \dot{\xi}_1 &= v \\ y &= \xi_0 \end{aligned}$$

Example 4.2

Consider the system, taken from [8]

$$\dot{x}_1 = x_1 + x_3 + u_1$$
$$\dot{x}_2 = x_2 + x_4 + 2x_3u_1$$
$$\dot{x}_3 = x_3 + 3u_1$$
$$\dot{x}_4 = x_1x_2x_3 + (1+2x_3)^2u_1 + 2x_4u_2$$

$$y_1 = x_1 \ , \ y_2 = x_2$$

This system has $\Delta^* = 0$. Carrying out Singh's algorithm one obtains

$$S_0 = \begin{bmatrix} x_1 - y_1 \\ x_2 - y_2 \end{bmatrix} \qquad K_0 = (2x_3 \ -1)$$

$$S_1 = 2x_3(x_1 + x_3 - \dot{y}_1) - (x_2 + x_4 - \dot{y}_2)$$

$$G_1(x,\dot{y}) = \begin{bmatrix} 1 & 0 \\ 2x_3 & 0 \\ * & -2x_4^2 \end{bmatrix}$$

The matrix G_1 has generically rank m=2, but $\mu=p_0+p_1 = 3$ and therefore the second condition of Theorem 3.1 is not satisfied. The dynamics of the reduced inverse (2.2) has dimension 1: an easy computation shows that, taking $z = z_3$, one gets the dynamics

$$\dot{z} = 2z + 3(\dot{y}_1 - y_1)$$

Note that the two conditions of Theorem 3.1 are only sufficient ones (for full linearizability). This example shows exactly why they are not necessary. Suppose we use the decoupling procedure of [2]; then, we have to add an integrator on the first input channel, i.e. we have to set $u_1 = x_5$, $\dot{x}_5 = w_1$ and $u_2 = w_1$. The system thus obtained has a decoupling matrix

$$A^e(x^e) = L_{g^e}L_{f^e}h^e = \begin{bmatrix} 1 & 0 \\ 2x_3 & 2x_4^2 \end{bmatrix}$$

which is now generically nonsingular, but $\dim(\Delta^{*e})=1$.

However, it happens that the (decoupling) feedback

$$w = [A^e(x^e)]^{-1} \ (-L_{f^e}^2h^e + v)$$

together with the (local) change of coordinates

$$(\xi_0,\xi_1,\xi_2) = (h^e, \ L_{f^e}h^e \ , \ x_5)$$

turn the system into

$$\dot{\xi}_0 = \xi_1$$
$$\dot{\xi}_1 = v$$
$$\dot{\xi}_2 = -2\xi_2 + (1 \ -2)\xi_0 + (0 \ \ 1)v$$

which is fully linear (even though not observable).
As a matter of fact, $\Delta^* = 0$ is not needed in order to have full linearity under feedback equivalence (see [9] for further details) when the dynamics of the reduced inverse is already a linear one.

5. Concluding Remarks

Sufficient conditions for full linearization of nonlinear affine systems via dynamic state feedback have been described. These conditions require the invertibility of the given system ($\rho_{k^*} = m$) and no dynamics for the reduced inverse system (dim z = 0). One could ask whether or not these two conditions could be weakened.

The first one, $\rho_{k^*} = m$, is clearly also necessary if we want eventually to obtain a linear system which is also invertible. The second condition implies that the linear system obtained via dynamic feedback is a system without transmission zeros. Hence, this condition is not necessary for full linearization as one can immediately realize by taking a system which is already linear but with $\dim(z)\neq 0$.

If $\dim(z)\neq 0$, then the dynamic extension procedure developed in [2] yields a system with $\dim(\Delta^*)\neq 0$ (Lemma 3.1). This extended system could still be made linear via feedback and coordinates transformations, provided e.g. that the necessary and sufficient conditions established in [9] are satisfied. The conditions in question essentially say that the dynamics associated with the inverse system must be diffeomorphic to a linear one.

Appendix

Proof of Lemma 3.2

First of all, note that a nonsingular x-dependent transformation in the input space $\beta(x)$ does not affect the computation of the reduced inverse dynamics. For, changing $g(x)$ into $\tilde{g}(x) = g(x)\beta(x)$ implies changing $G_k(x)$ into $\tilde{G}_k(x) = G_k(x)\beta(x)$, while $F_k(x)$, $T_k(k)$ and $V_k(x)$ remain unchanged. In view of this, without loss of generality we can prove the Lemma for the addition of just one integrator on the first input channel. We refer hereafter to the system thus obtained as to the "extended" system.

Let us use the superscript "e" when dealing with such a system and set

$$x^e = \begin{bmatrix} x \\ z \end{bmatrix}, \ f^e = \begin{bmatrix} f + g_1z \\ 0 \end{bmatrix}, \ \ g_1{}^e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ \ g_i^e = \begin{bmatrix} g_i \\ 0 \end{bmatrix} \ \ i\geq 2$$

$$h^e = h$$

Let q denote the least integer such that $\gamma=(\partial S_q/\partial x)g_1\neq 0$. Moreover, let G_k' and K_k' denote the matrices consisting of the last (m-1) columns of G_k and, respectively, K_k.

It is easy to see that one may carry out the inversion algorithm on the extended system in such a way as to obtain

$$S_k^e = S_k \tag{A.1}$$

for all $0 \leq k \leq q$. This is because one may choose at each stage the same transformation matrices as those used when dealing with the original system.

At the q-th iteration one has

$$F_q = \begin{bmatrix} F_{q-1} \\ H_q \end{bmatrix} \qquad\qquad G_q = \begin{bmatrix} 0 & G'_{q-1} \\ \gamma & K'_q \end{bmatrix}$$

and, respectively,

$$F^e_q = \begin{bmatrix} F_{q-1} \\ H_q + \gamma z \end{bmatrix} \qquad G^e_q = \begin{bmatrix} 0 & G'_{q-1} \\ 0 & K'_q \end{bmatrix}$$

The rank of G^e_q can be either equal to that of G_q or one unit less. Suppose we are in the first case. Then $p^e_{q+1} = p_{q+1}$ and one may choose

$$(T^e_q \; V^e_q) = (T_q \; V_q)$$

This yields

$$S^e_{q+1} = T_q F_{q-1} + V_q(H_q + \gamma z) = S_{q+1}$$

(because $V_q\gamma = 0$, by definition). At the next stage we encounter a similar situation, because F^e_{q+1}, G_{q+1}, G^e_{q+1} have the form

$$F^e_{q+1} = \begin{bmatrix} F_{q-1} \\ H_q + \gamma z \\ H_{q+1} + \delta z \end{bmatrix}, \; G_{q+1} = \begin{bmatrix} 0 & G'_{q-1} \\ \gamma & K'_q \\ \delta & K'_{q+1} \end{bmatrix}, \; G^e_{q+1} = \begin{bmatrix} 0 & G'_{q-1} \\ 0 & K'_q \\ 0 & K'_{q+1} \end{bmatrix}$$

If the rank of G^e_{q+1} is equal to that of G_{q+1} one may still use the same transformation matrices as the one used when working on the original system and obtain

$$S^e_{q+1} = S_q$$

Thus, the real difference appears whenever, at some $s \geq q$, $\rho^e_s = \rho_s - 1$ (and, accordingly, $p^e_{s+1} = p_{s+1} + 1$). At this stage, F_s, G_s, F^e_s, G^e_s will have the form

$$F_s = \begin{bmatrix} F_{q-1} \\ H \\ H_s \end{bmatrix}, \; F^e_s = \begin{bmatrix} F_{q-1} \\ H+\theta z \\ H_s+\theta_s z \end{bmatrix}, \; G_s = \begin{bmatrix} 0 & G'_{q-1} \\ \theta & K' \\ \theta_s & K'_s \end{bmatrix}, \; G^e_s = \begin{bmatrix} 0 & G'_{q-1} \\ 0 & K' \\ 0 & K'_s \end{bmatrix}$$

(the blocks in the middle of each matrix are present if $s>q$ and consist of $p_{s-1}+\ldots+p_q$ rows).

If $(T_q \; T \; K_s)$ is a matrix which annihilates G_s (K_s is $(p_{s+1}) \times p_s$ and has rank p_{s+1}), then a matrix of the form

$$\begin{bmatrix} T_q & T & K_s \\ \lambda_q & \lambda & \lambda_s \end{bmatrix}$$

will annihilate G^e_s. The extra row $(\lambda_q \; \lambda \; \lambda_s)$ cannot annihilate G_s and, therefore

$$\lambda\theta + \lambda_s\theta_s \neq 0$$

Accordingly this choice, S^e_{s+1} has the form

$$S^e_{s+1} = \begin{bmatrix} S_{s+1}(x) \\ \phi_1(x) + \phi_2(x)z \end{bmatrix} \tag{A.2}$$

(where $\phi_2(x)\neq 0$) and, consequently, G_{s+1} and G^e_{s+1} will have the form

$$G_{s+1} = \begin{bmatrix} 0 & G'_{q-1} \\ \theta & K' \\ \theta_s & K'_s \\ \theta_{s+1} & K'_{s+1} \end{bmatrix} \qquad G^e_{s+1} = \begin{bmatrix} 0 & G'_{q-1} \\ 0 & K' \\ 0 & K'_s \\ 0 & K'_{s+1} \\ \phi_2 & * \end{bmatrix}$$

One may easily conclude that $\rho^e_{s+1} = \rho_{s+1}$ and, therefore, that $p^e_{s+2} = p_{s+2}$.

From this stage on, it is possible to continue keeping

$$S^e_k = S_k$$

for all $k \geq s+2$.

This is because if w is a row vector which annihilates G_{s+1} (namely a row of $(T_{s+1} \; V_{s+1})$), then $(w \; 0)$ will annihilate G^e_{s+1}. Therefore, looking at F^e_{s+1}, whose form is

$$F^e_{s+1} = \begin{bmatrix} F_{s+1} \\ \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \theta z \\ \theta z \\ \theta_{s+1} z \\ * \end{bmatrix}$$

one concludes that $(w \; 0) \; F^e_{s+1} = wF_{s+1}$, i.e. a row of S_{s+2}.

In conclusion, we may say that it is possible to carry out the algorithm on the extended system in such a way that (A.1) holds for all $k\neq s+1$, whereas for $k=s+1$ (A.2) holds. Since ϕ_2 is nonzero, then clearly

$$\mu^e = \mu+1$$

and this concludes the proof. □

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