Robotics 1
June 10, 2026

Exercise 1

The 6-dof lightweight robot Z1 Pro by Unitree is shown in Fig. 1. The robot has six revolute joints, a spherical wrist, and an elbow offset. Assign a set of frames according to the standard Denavit-Hartenberg (DH) convention, so that all linear parameters a_i and d_i are *nonnegative*. Place the first (base) DH frame on the ground and the origin of the last DH frame at the center of the final flange. Draw the frames in the clearest possible way directly on the extra sheets that have been distributed, and then build the corresponding table of parameters. For the robot configuration shown in the *side view* on the extra sheets, provide also the sign (or, in case, the value) of each joint variable.

Figure 1: The Z1 Pro robot by Unitree

Exercise 2

With reference to the generic situation shown in Fig. 2, consider the following coordinated task for two robots, a 3R arm A and a 2R arm B, both having links of unitary length and moving on a horizontal plane (x, y) —thus, we will use a 2D planar notation for Cartesian vectors and velocities, as well as for rotation matrices. The pose of the base frame of robot B with respect to the base frame of robot A is given by

$$ {}^A\boldsymbol{T}_B = \begin{pmatrix} {}^A\boldsymbol{R}_B & {}^A\boldsymbol{p}_{O_B} \\ 0 \quad 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 2.5 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1} $$

The configuration of robot A is $\boldsymbol{q}_A = (0, \pi/4, \pi/2)$ [rad], with joint velocity $\dot{\boldsymbol{q}}_A = (\pi/4, -\pi/2, 0)$ [rad/s]. Determine the configuration $\boldsymbol{q}_B$ of robot B and its joint velocity $\dot{\boldsymbol{q}}_B$ so that:

- the position of the end-effector point P_B of robot B with respect to point P_A of robot A, when expressed in the end-effector frame EA of robot A, is

$$ {}^{EA}\boldsymbol{p}_{AB} = \begin{pmatrix} -\sqrt{2}/2 \\ -\sqrt{2}/2 \end{pmatrix}; \tag{2} $$

- the end-effector velocity of the two robots is the same, i.e., $\boldsymbol{v}_A = \boldsymbol{v}_B \in \mathbb{R}^2$;
- in this instantaneous situation, there is no collision between the two robots.

List first the logical steps that you have to follow in order to solve the problem, then provide its numerical solution.

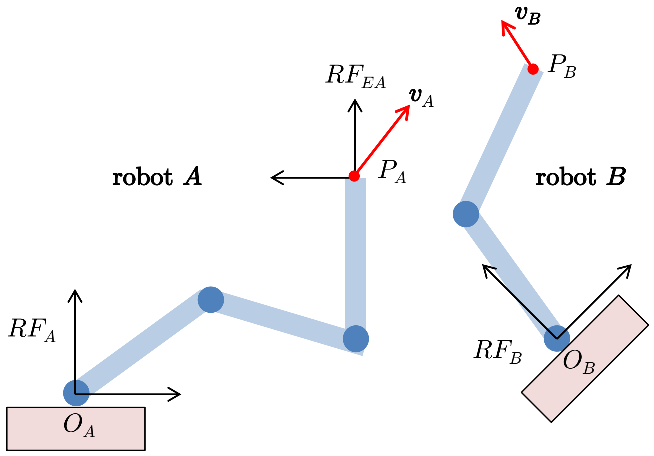

Figure 2: Generic placement and configurations of robots A and B involved in a coordinated task

Exercise 3

A point mass should move on a line in minimum time from rest to rest between $x_i = 0$ and $x_f = 9$ m, under maximum speed and acceleration limits $|\dot{x}| \leq V = 6$ m/s, $|\ddot{x}| \leq A = 4$ m/s^2. The point mass should start immediately at $t_i = 0$ and move away from $x_i = 0$ as fast as possible, since a moving obstacle may possibly enter a small line segment there and produce a collision. Moreover, the mid transfer position $x_m = x_f/2$ is forbidden during the time interval $t \in (t_l, t_u) = (1, 1.6)$ s, since a collision would certainly occur with another moving obstacle traversing orthogonally the line at this point. When the line is permanently free, determine the minimum motion time T and sketch the corresponding optimal speed profile and associated position. Develop then a strategy to complete the assigned task in the least possible total time, without a collision between the point mass and the moving obstacles. Illustrate first qualitatively your solution approach and its motivation. Sketch then the corresponding speed profile and compute the motion time T_{obs}.

[210 minutes (3,5 hours); open books]