Robotics 1
February 7, 2025

Exercise 1

For the 6-dof robot YASKAWA Motoman UP500 shown in Fig. 1, assign the link frames and fill in the associated table of parameters according to the Denavit–Hartenberg (DH) convention (use the *extra sheet*). The 0-th DH frame is placed on the floor, with the axis $\boldsymbol{y}_0$ going *inside* the sheet. With the robot in the configuration shown, all the axes of the DH frames that are not in the sheet plane should go instead *outside* the sheet. The origin O_6 of the last DH frame is placed at the center of the final flange. Provide the numerical values of all constant DH parameters, as well as of the joint variables in the configuration shown in the sheet. Compute then the numerical value of the position of Point P specified in the sheet, when the robot is in the configuration $\boldsymbol{q} = \mathbf{0}$.

Figure 1: The YASKAWA Motoman UP500 robot.

Exercise 2

Consider the RRP planar robot in Fig. 2, with the joint coordinates defined therein. The last link is twisted by an angle $\pi/2$ rad with respect to the second link. Derive the direct kinematics of the end-effector pose $\boldsymbol{r} = (\boldsymbol{p}, \alpha) = (p_x, p_y, \alpha) = \boldsymbol{f}(\boldsymbol{q})$. Solve then the inverse kinematics (IK) problem in closed form, i.e., find all IK solutions $\boldsymbol{q} = \boldsymbol{f}^{-1}(\boldsymbol{r})$ in the regular case. Next, set $l_1 = 1$ and $l_2 = 0.5$ [m]. For the pose $\boldsymbol{r} = (1, 1, \pi/4)$ [m,m,rad], provide the numerical values of the IK solutions. Verify that this is not a singular case by analyzing the Jacobian of the direct kinematics.

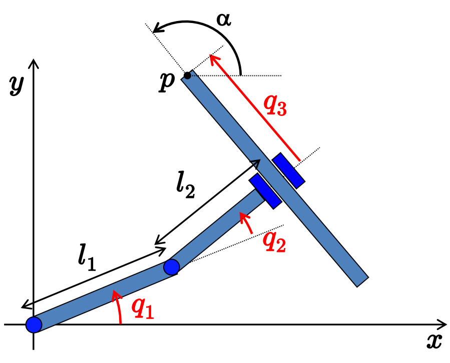

Figure 2: An RRP planar robot.

Exercise 3

With reference to the cooperative task in Fig. 3, the RP robot a and the PP robot b (two 2-dof planar robots) should jointly move a payload along a parametrized Cartesian path. The path is an arc of a circle to be traced counterclockwise, centered at the origin of the world frame, starting at $P_i = (0.4, 0.3)$ [m], and of length $L = 0.8$ m. The base frame of robot b is placed at $O_b = (0.7, 0.6)$ [m]. The RP robot has symmetric joint velocity and acceleration bounds given by

$$V_{a,1} = 1 \text{ rad/s} \qquad V_{a,2} = 0.7 \text{ m/s} \qquad A_{a,1} = 3 \text{ rad/s}^2 \qquad A_{a,2} = 5 \text{ m/s}^2,$$

while the joints of the PP robot have symmetric (and equal) velocity bounds

$$V_{b,1} = V_{b,2} = 0.6 \text{ m/s},$$

while their acceleration is in practice unlimited. Determine the rest-to-rest *coordinated* trajectory that traces the given path in *minimum time* (coordination means that both robots start and end their motions at the same time). Plot the resulting time profiles of the position and velocity of the joints for the two robots.

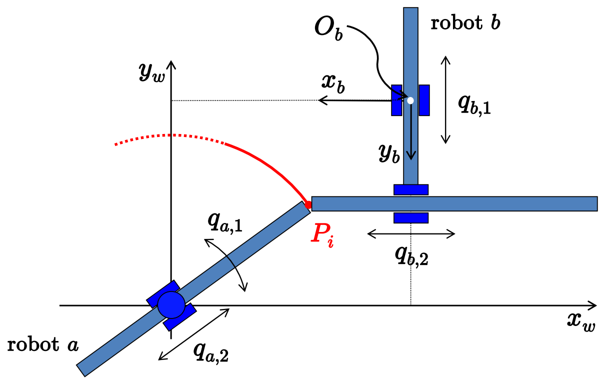

Figure 3: Cooperative motion task of two robots along a Cartesian path.

Exercise 4

A robot joint has maximum (absolute) speed $v_{max} > 0$ and acceleration $a_{max} > 0$. It should move in minimum time from an initial value q_i to a final value $q_f > q_i$, starting with an initial speed $v_i \geq 0$ and ending with a final speed $v_f \geq 0$ (which may be larger, equal or smaller than v_i). Both v_i and v_f are assumed to be feasible. Find the symbolic expression of the minimum time T and draw the associated position, velocity and acceleration profiles. The result is a generalization of the known rest-to-rest case ($v_i = v_f = 0$), in which a bang-coast-bang (b-c-b) profile is found as a solution. Determine also the condition under which a b-c-b solution is found in the present case. Discuss briefly what happens in the limit case and when such condition is violated.

Provide then the numerical value of T and of the other characterizing parameters of this trajectory for the following data

$$q_i = -2 \qquad q_f = 1 \;\; [\text{rad}] \qquad\quad v_i = 0.5 \qquad v_f = 1 \;\; [\text{rad/s}],$$

with the bounds

$$v_{max} = 2 \;\; \text{rad/s} \qquad a_{max} = 3 \;\; \text{rad/s}^2.$$

[240 minutes (4 hours); open books]