Robotics 1

Kinematic control

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Robot motion control

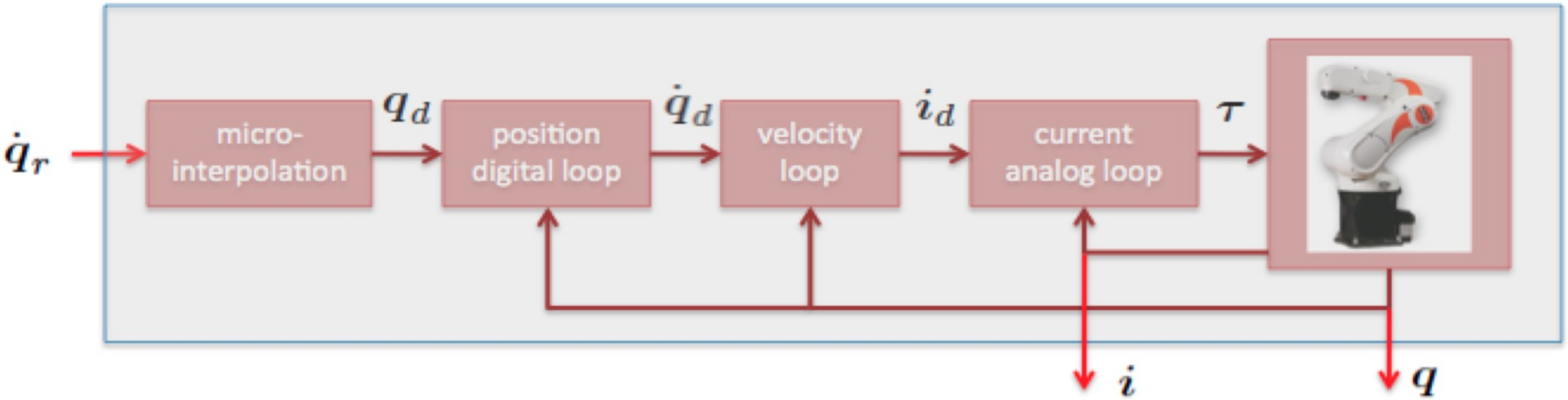
- need to "actually" realize a desired robot motion task ...
  - regulation of pose/configuration (constant reference)
  - trajectory following/tracking (time-varying reference)
- ... despite the presence of
  - external disturbances and/or unmodeled dynamic effects
  - initial errors (or arising later due to disturbances) w.r.t. desired task
  - discrete-time implementation, uncertain robot parameters, ...
- we use a general control scheme based on
  - feedback (from robot state measures, to impose asymptotic stability)
  - feedforward (nominal commands generated in the planning phase)
- the error driving the feedback part of the control law can be defined either in Cartesian or in joint space
  - control action always occurs at the joint level (where actuators drive the robot), but performance has to be evaluated at the task level

Kinematic control of robots

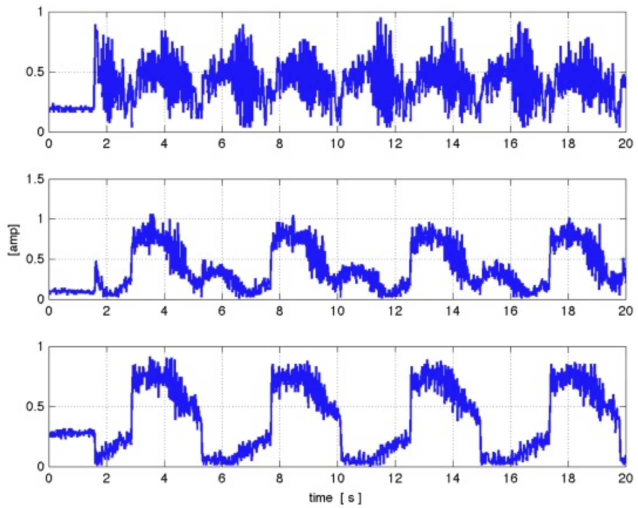
- a robot is an electro-mechanical system driven by actuating torques produced by the motors (with voltage/current inputs)
- it is possible, however, to consider a kinematic command (most often, a velocity) as control input to the system...
- ...thanks to the presence of low-level feedback control at the joints, which imposes to the robot the commanded velocities (at least, in the “ideal case”)
- these feedback loops are present in industrial robots within a “closed” control architecture, where users can only specify reference commands of the kinematic type
- in this way, performance can still be very satisfactory, provided the desired motion is not too fast and/or does not require too large accelerations (or transparent haptic feedback is not needed)

Closed control architecture

KUKA KR5 Sixx R650 robot



- low-level motor control laws are not known nor accessible by the user
- user programs based also on other exteroceptive sensors (vision, Kinect, F/T sensor) can be implemented on an external PC via the RSI (RobotSensorInterface), communicating with the KUKA controller every 12ms
- available robot measures: joint positions (by encoders) and (absolute value of) applied motor currents
- the control reference is given as a velocity or a position in joint space (Cartesian commands are also accepted)

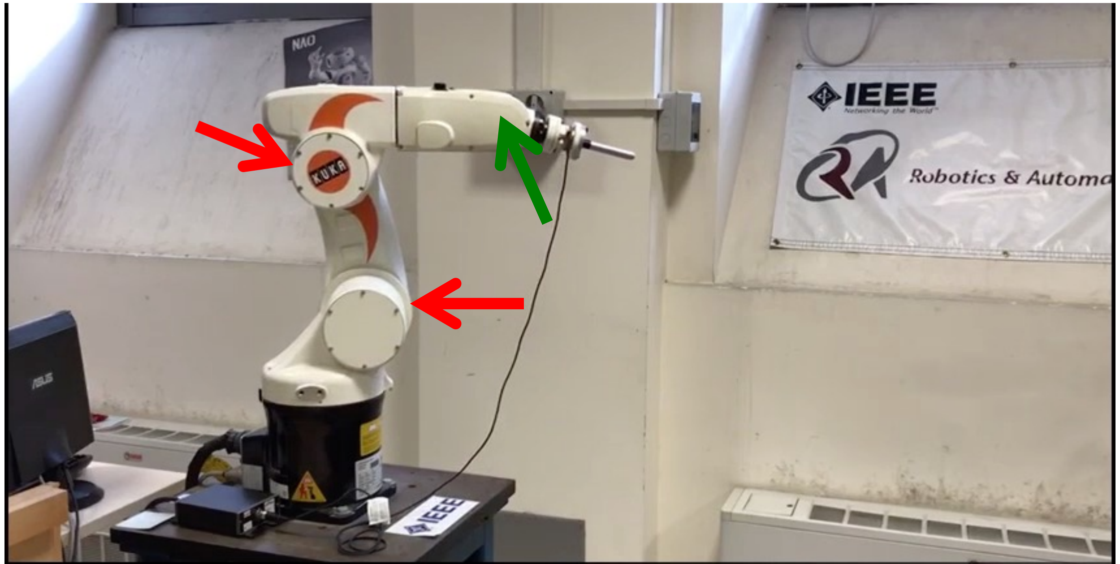


typical motor currents on first three joints

... *from* *pHRI*

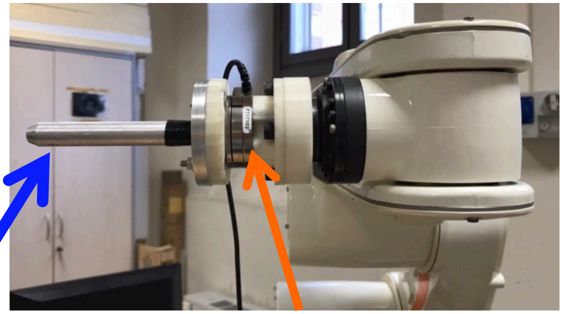
physical Human-Robot Interaction

combining F/T sensor data and motor currents



Robot in cyclic motion between four Cartesian positions

collaborative forces at E-E

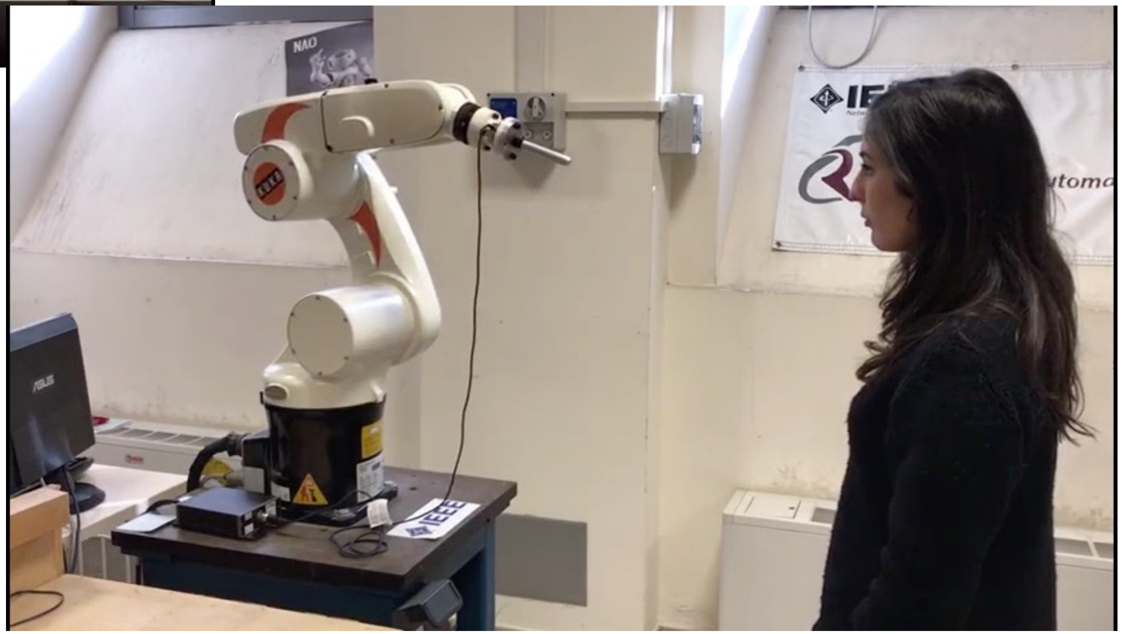


ATI Mini45 F/T sensor

https://youtu.be/SfZcG1Y713w

6R KUKA KR5 Sixx with **closed control** architecture and RSI interface at $T_c = 12\text{ms}$

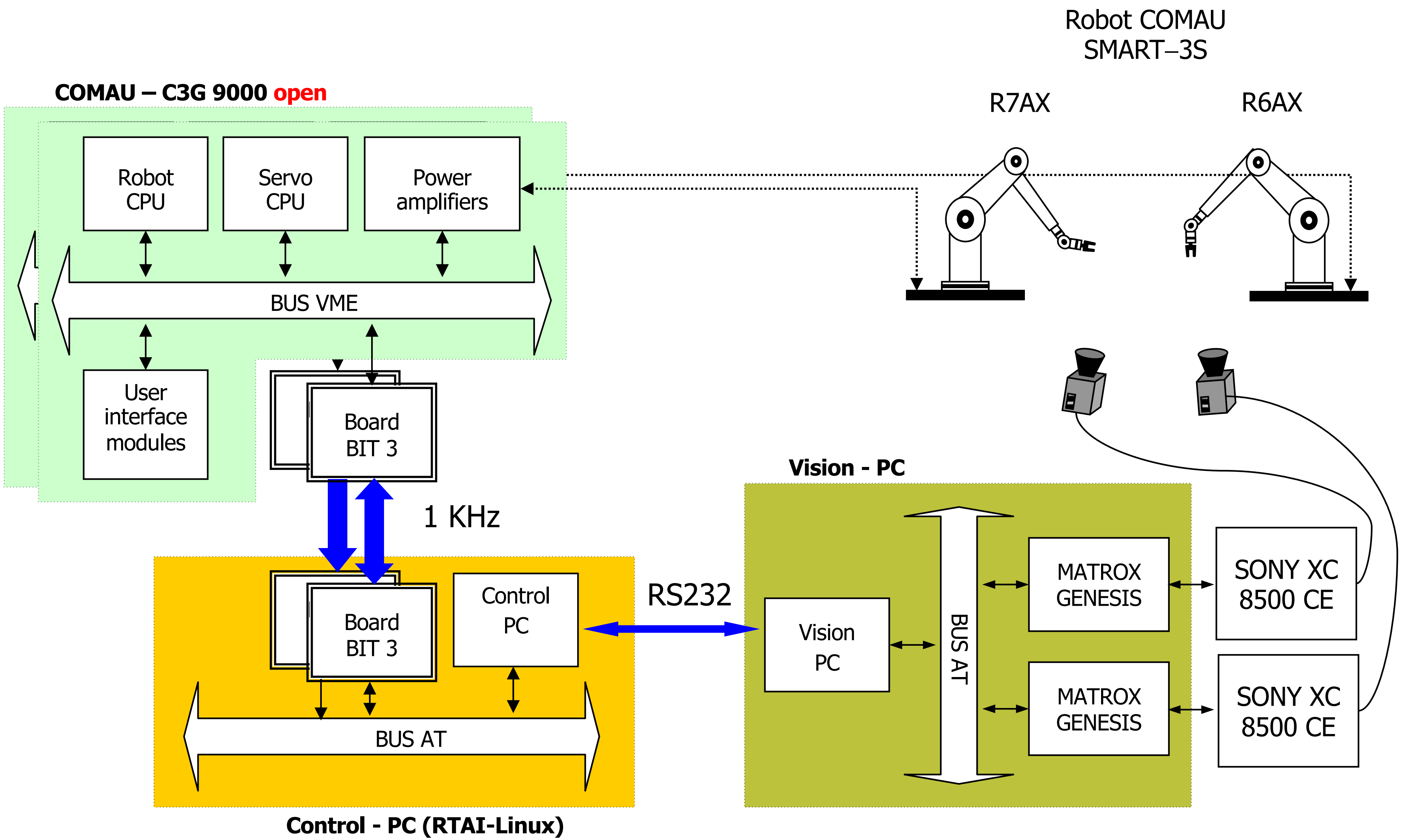
intentional contacts (soft) and/or collisions (hard) may occur anywhere along the robot structure …



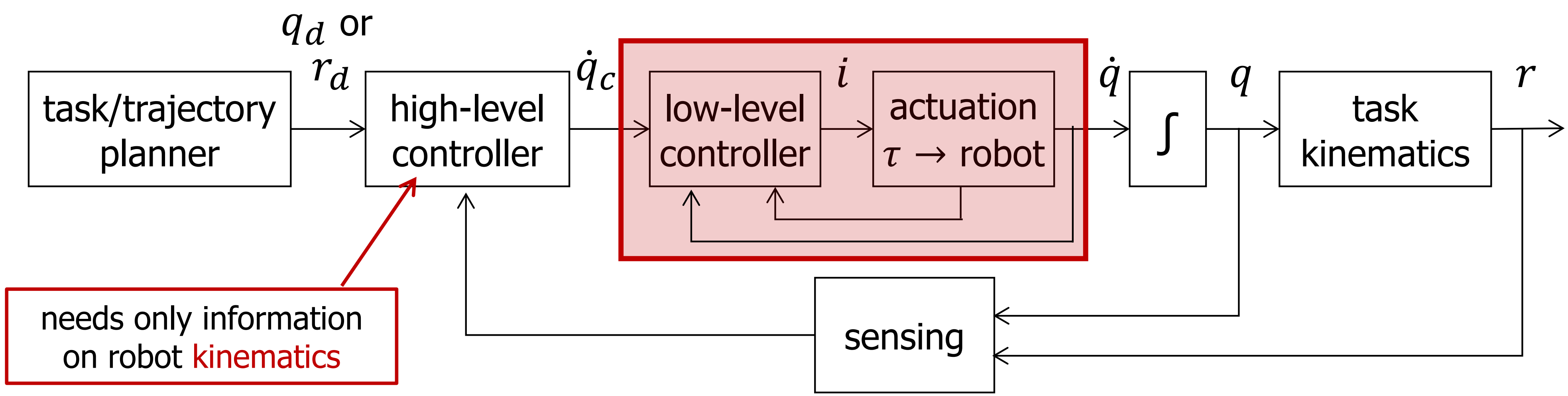
E. Mariotti, E. Magrini, A. De Luca: ICRA2019

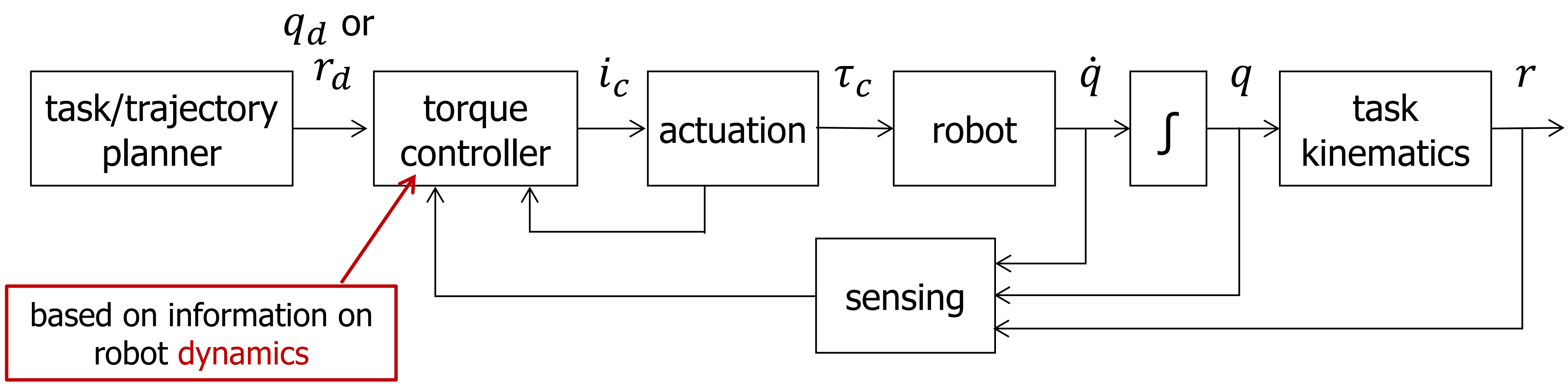
Hardware architecture

example including vision in an open controller



Kinematic vs. dynamic control of robots

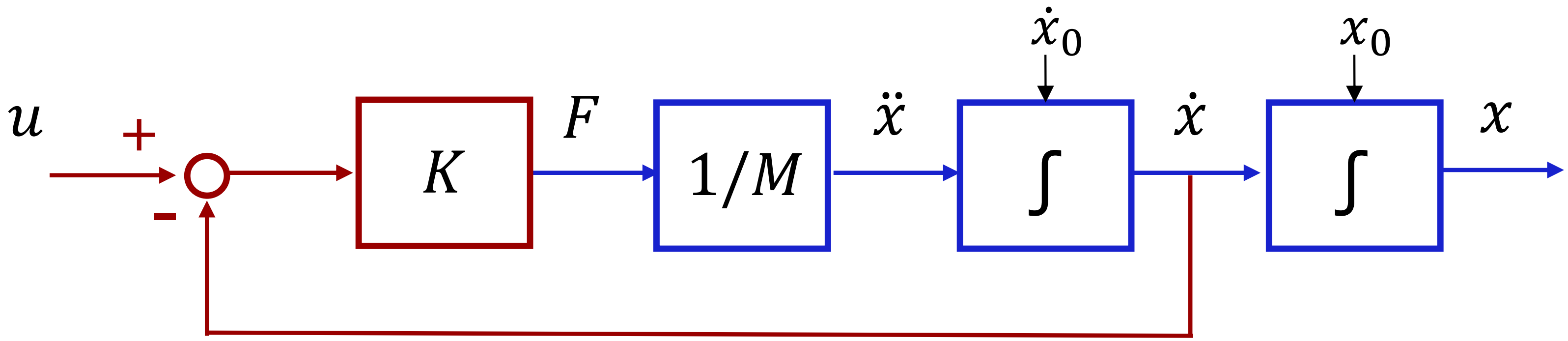




more on this in Robotics 2 ...

An introductory example

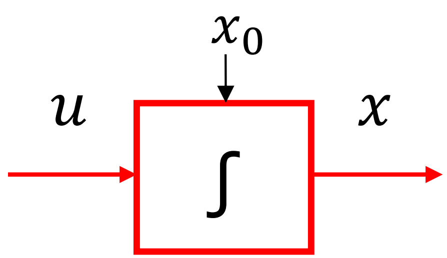
- a mass M in linear motion: $M\ddot{x} = F$ (dynamic model)
- low-level feedback: $F = K(u - \dot{x})$, with u = reference velocity
- equivalent scheme for $K \to \infty$: $\dot{x} \approx u$
- in practice, valid in a limited frequency "bandwidth" $\omega \leq K/M$



inner loop exact solution in continuous time
for a constant input $\bar{u}$

$$\dot{x}(t) = \dot{x}_0 + (\bar{u} - \dot{x}_0)\left[1 - \exp\left(-\frac{K}{M}t\right)\right]$$

$\approx$



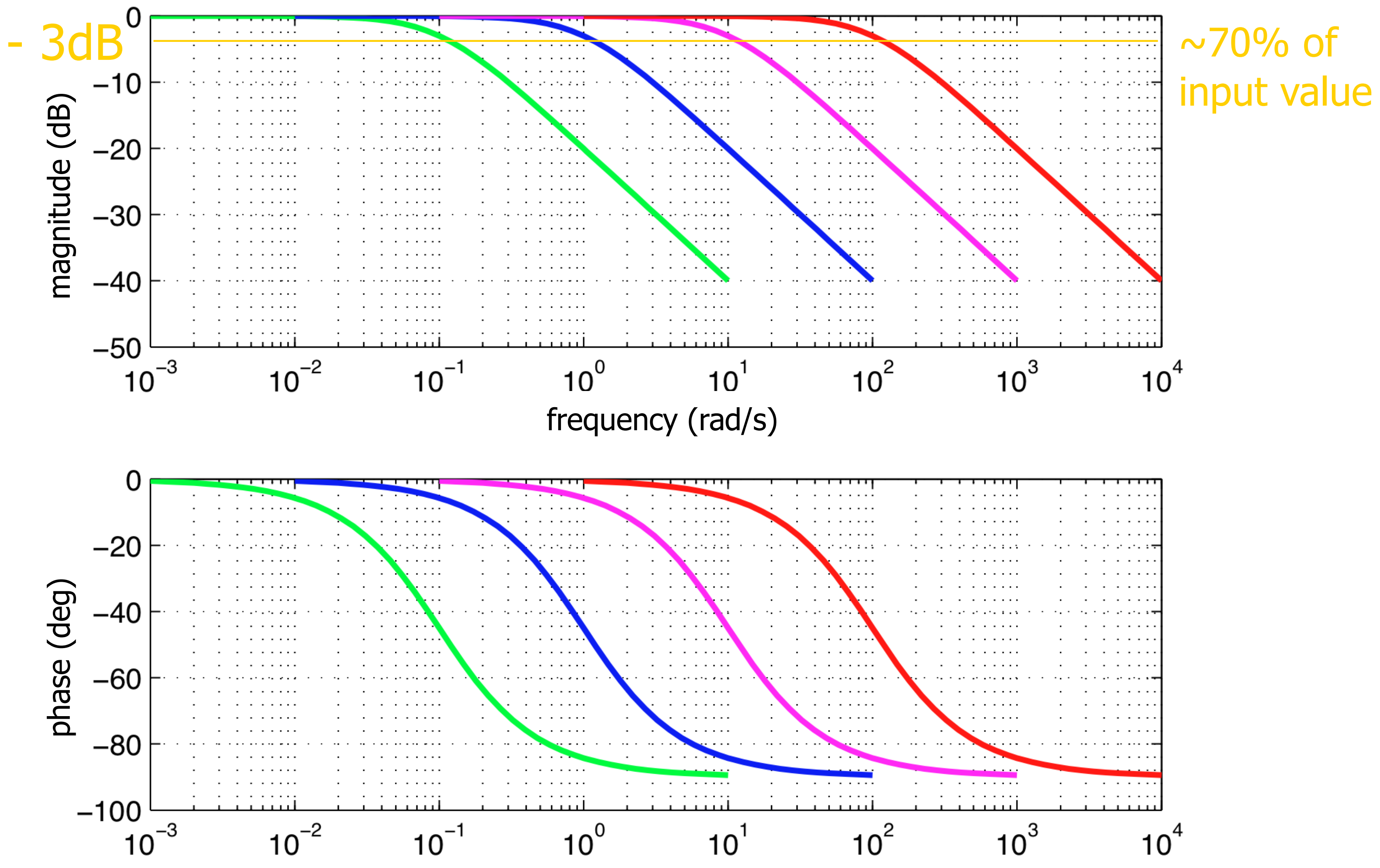
Frequency response
of the closed-loop system

- Bode diagrams of $P(s) = \dfrac{\dot{x}(s)}{u(s)} = \dfrac{s x(s)}{u(s)}$ for $K/M = 0.1, 1, 10, 100$

actual velocity

$$\frac{\dot{x}}{u}$$

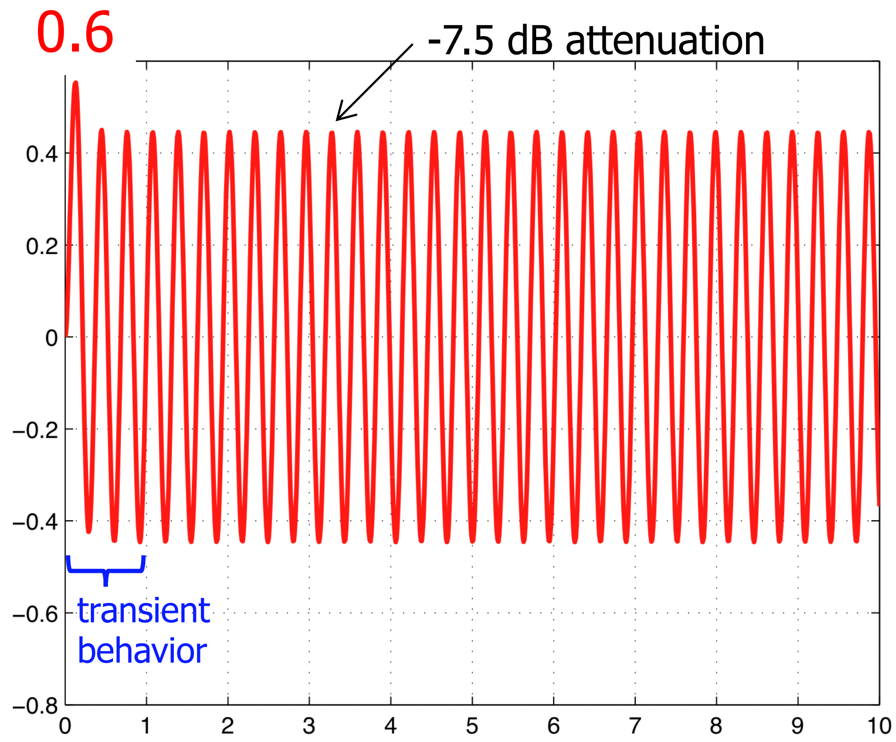
commanded reference velocity



Time response

- setting $K/M = 10$ (bandwidth), we show two possible time responses to unit sinusoidal velocity reference commands at different ω

$\omega = 2$ rad/s

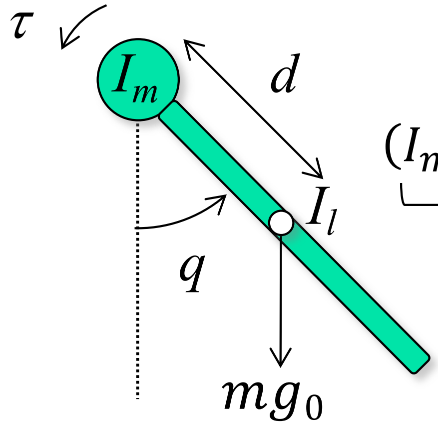


$\omega = 20$ rad/s

realized output velocities

A more detailed example
including nonlinear dynamics

- single link (a thin rod) of mass m, center of mass at d from joint axis, inertia M (motor + link) at the joint, rotating in a vertical plane (the gravity torque at the joint is configuration dependent)



dynamic model

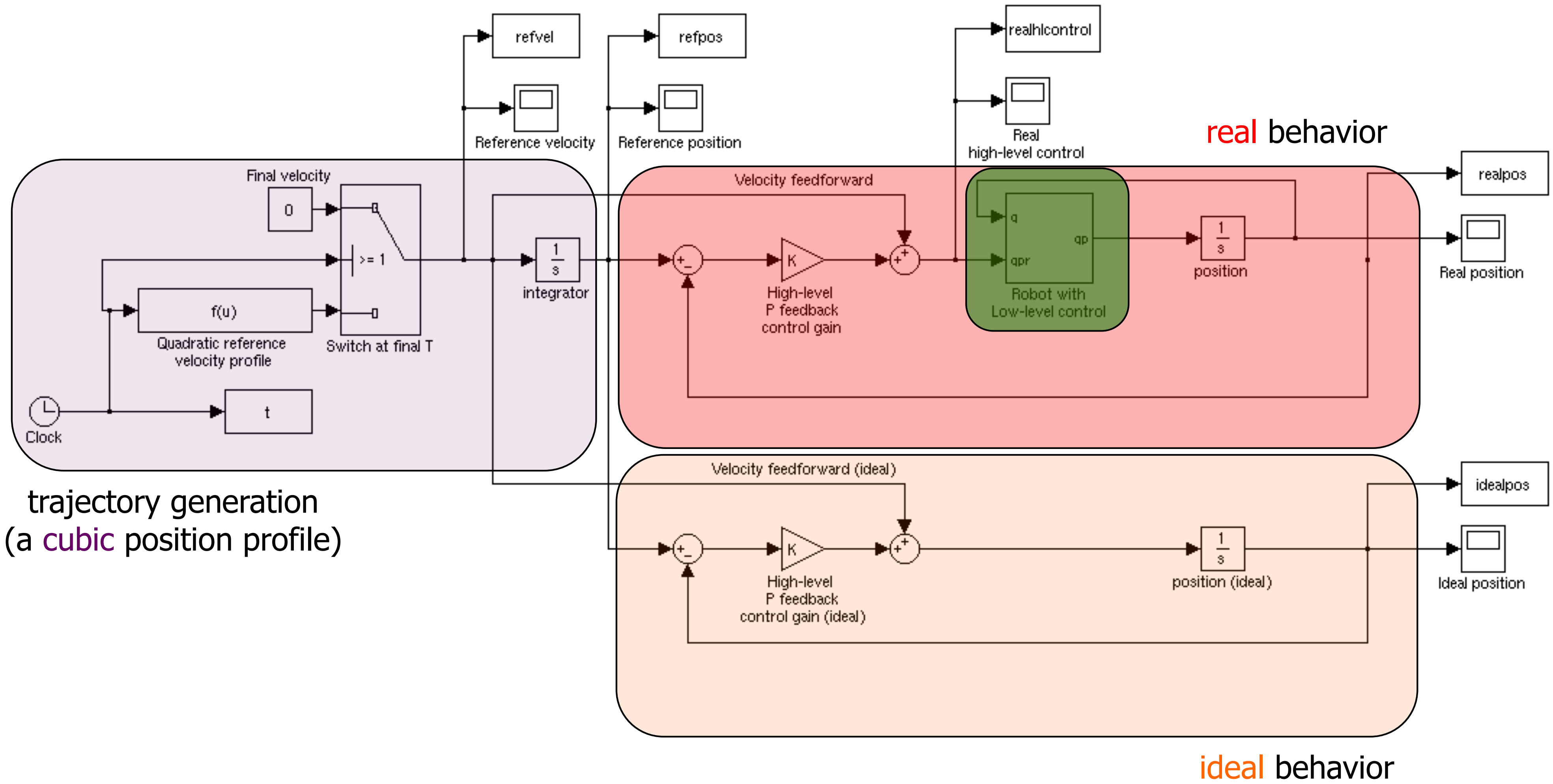
$$\underbrace{(I_m + I_l + md^2)}_{M}\ddot{q} + mg_0 d \sin q = \tau$$

$g_0 = 9.81\ [m/s^2]$
$m = 10\ [kg]$
$d = l/2 = 0.2\ [m]$
$I_l = ml^2/12 = 0.1333[kgm^2]$
$I_m = 0.5333[kgm^2]$
$(= I_l + md^2)$
$\Longrightarrow M = 1.0667[kgm^2]$

- fast low-level feedback control loop based on a PI action on the velocity error + an approximate acceleration feedforward
- kinematic control loop based on a P feedback action on the position error + feedforward of the velocity reference at the joint level
- evaluation of tracking performance for rest-to-rest motion tasks with “increasing dynamics” = higher accelerations

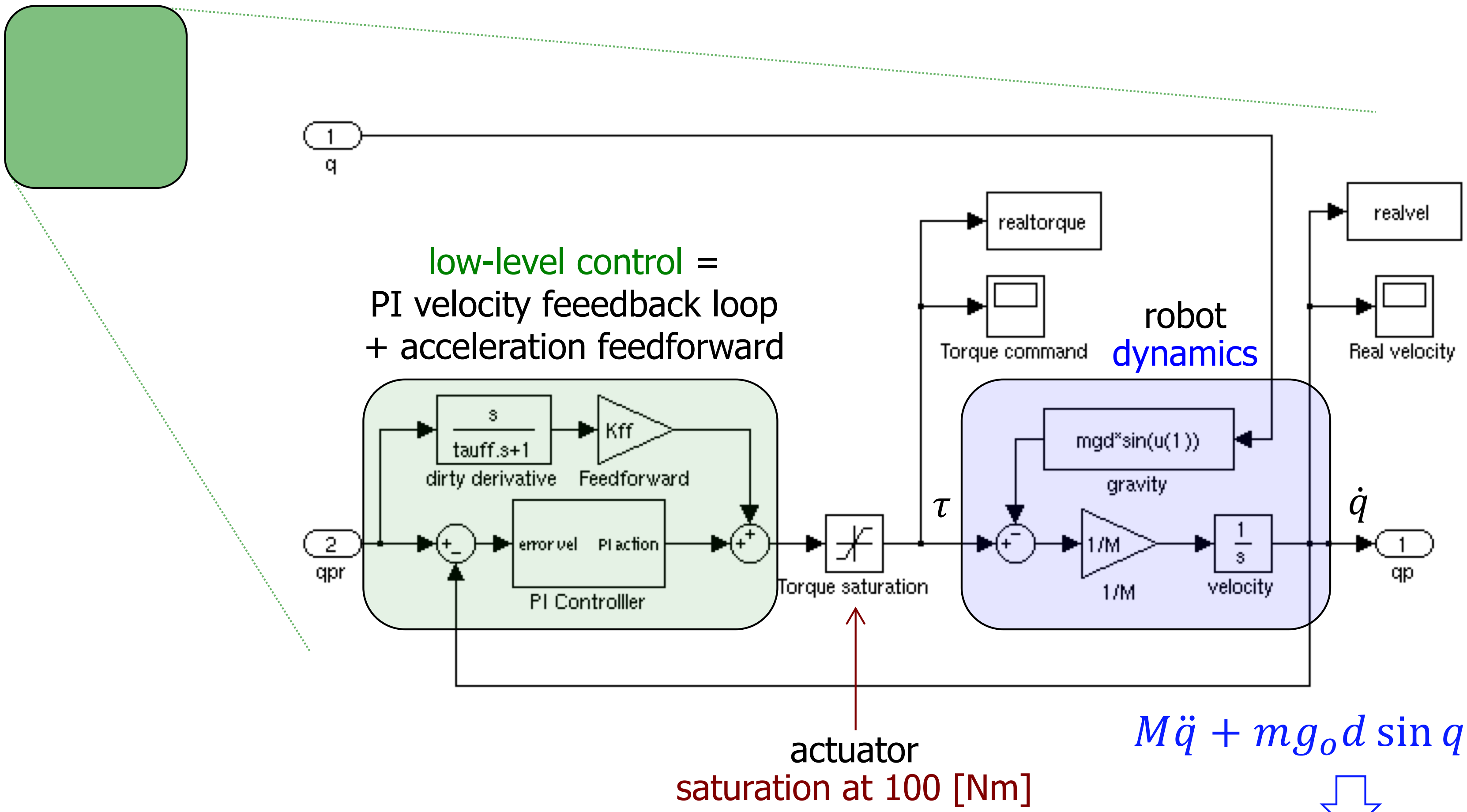
A more detailed example
differences between the ideal and real case

- Simulink scheme



A more detailed example
robot with low-level control

- Simulink scheme



$$M\ddot{q} + mg_o d \sin q = \tau$$

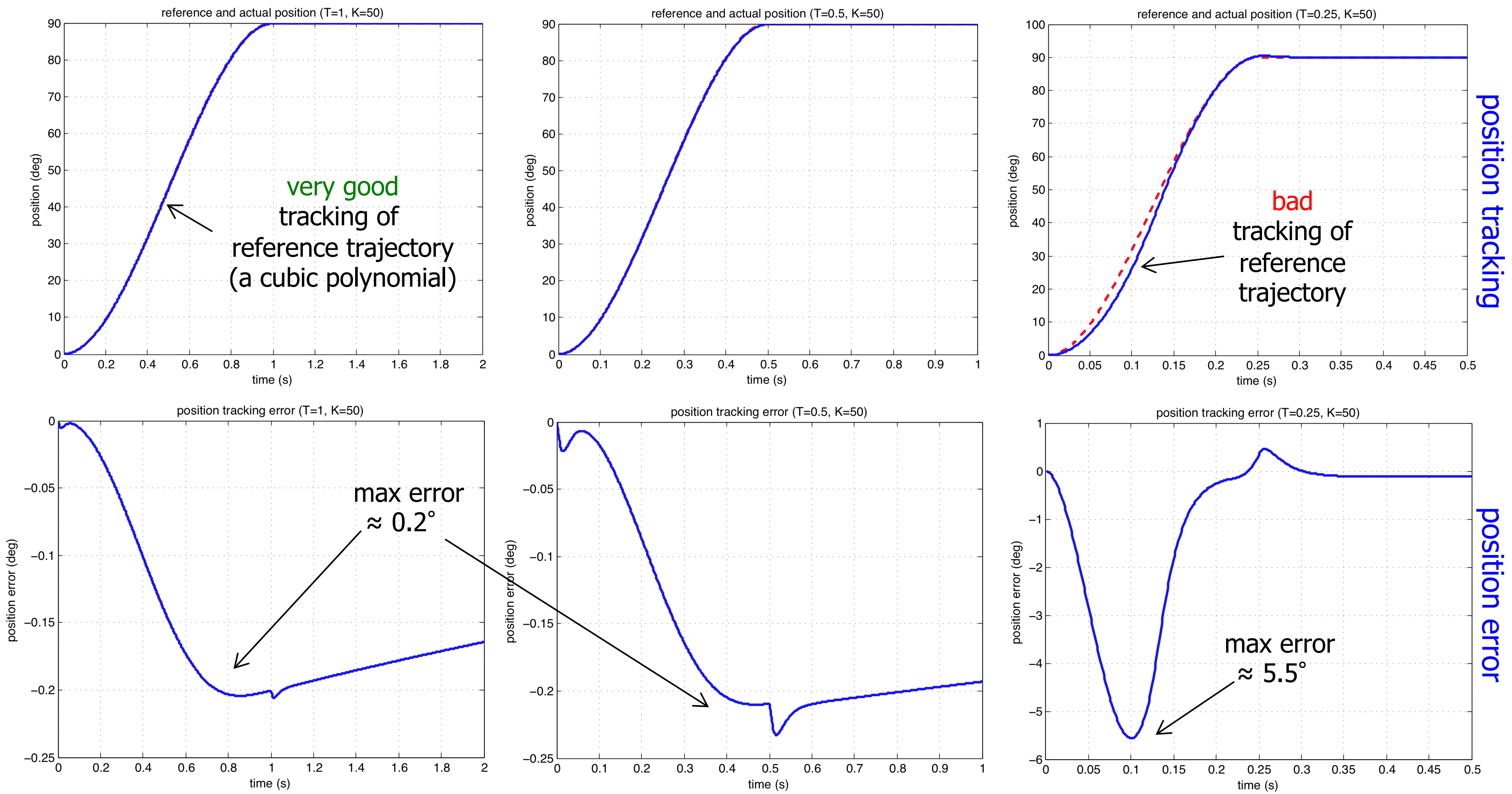
$$\Downarrow$$

$$\ddot{q} = \frac{1}{M}(\tau - mg_o d \sin q)$$

Simulation results

rest-to-rest from downward to horizontal position

- in $T = 1$ s
- in $T = 0.5$ s
- in $T = 0.25$ s



Simulation results

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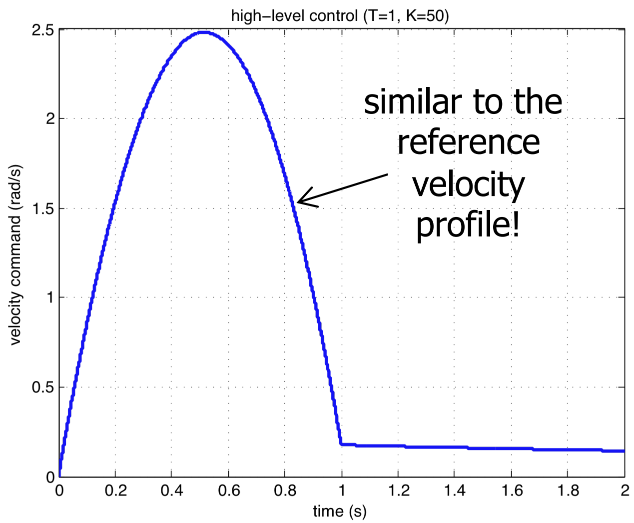
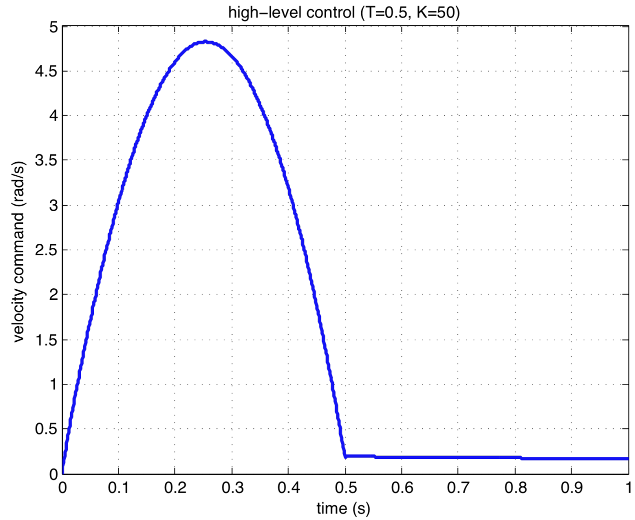
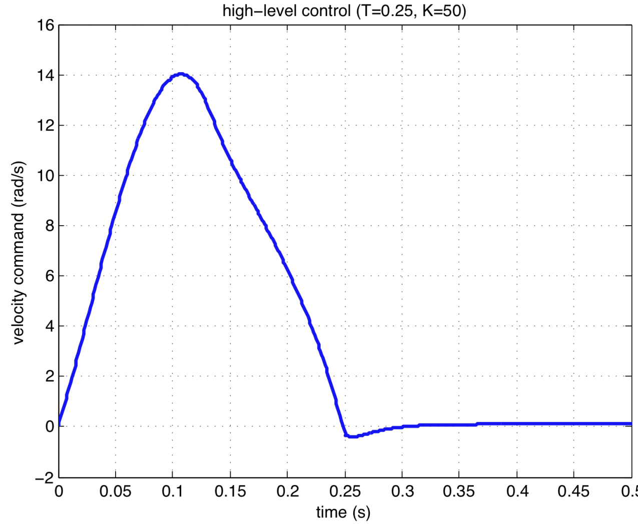
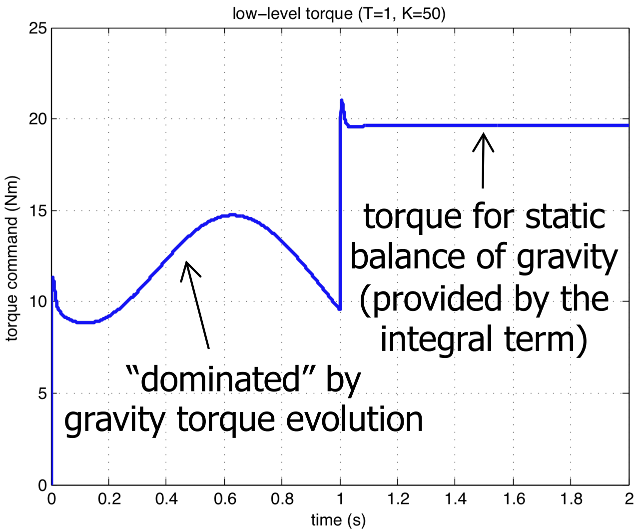
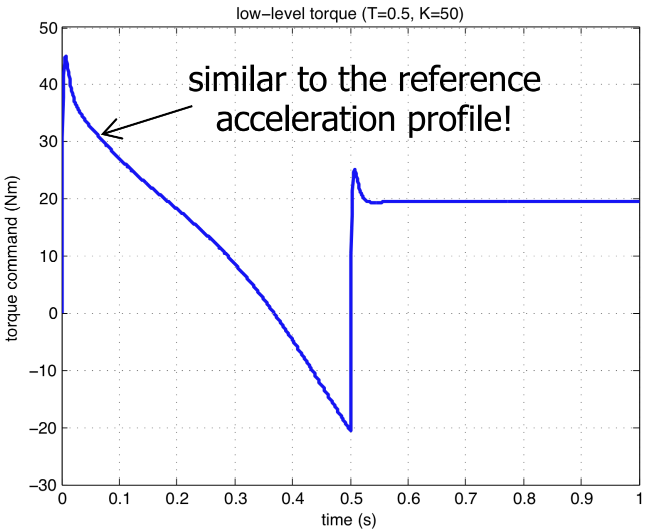


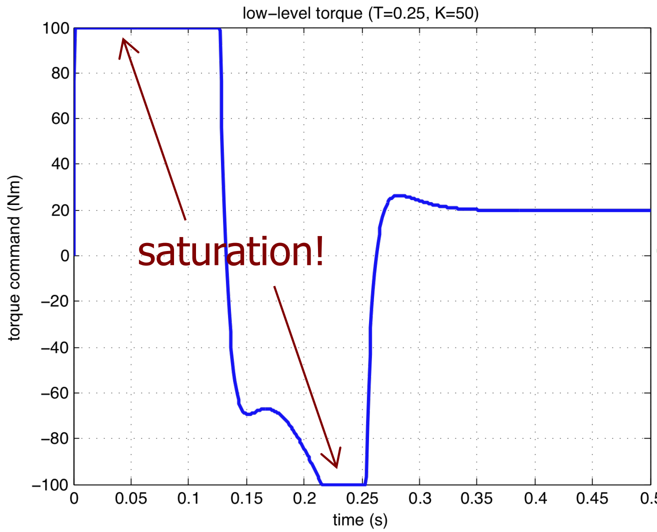




high-level velocity command





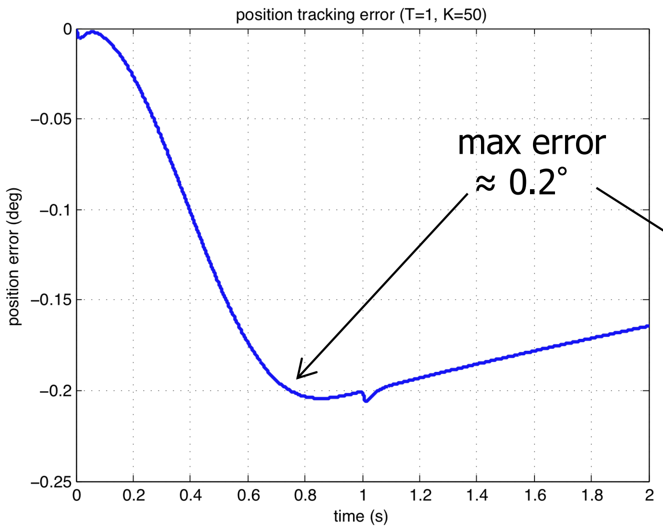


low-level torque command

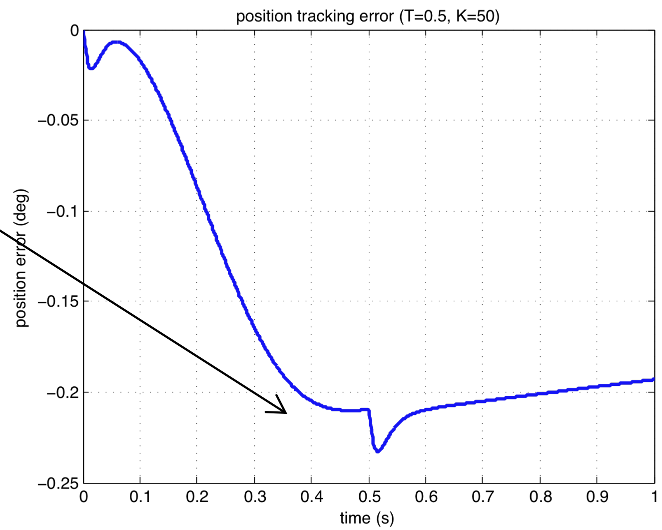
Simulation results

rest-to-rest from downward to horizontal position

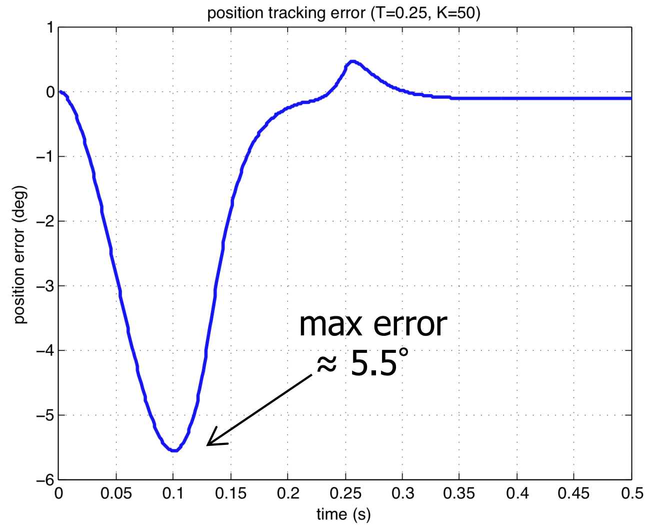
- in $T = 1$ s



- in $T = 0.5$ s

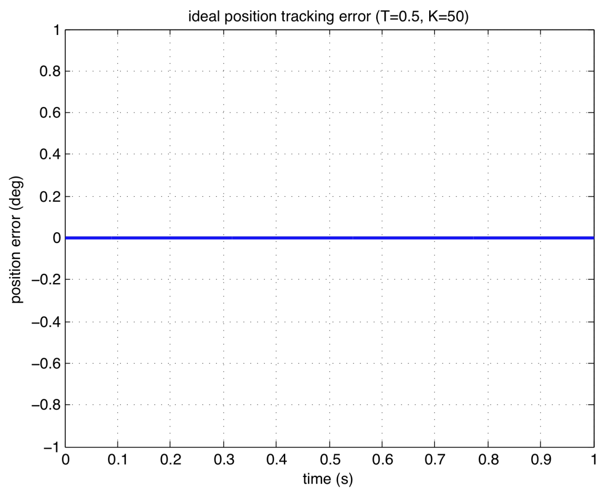


- in $T = 0.25$ s



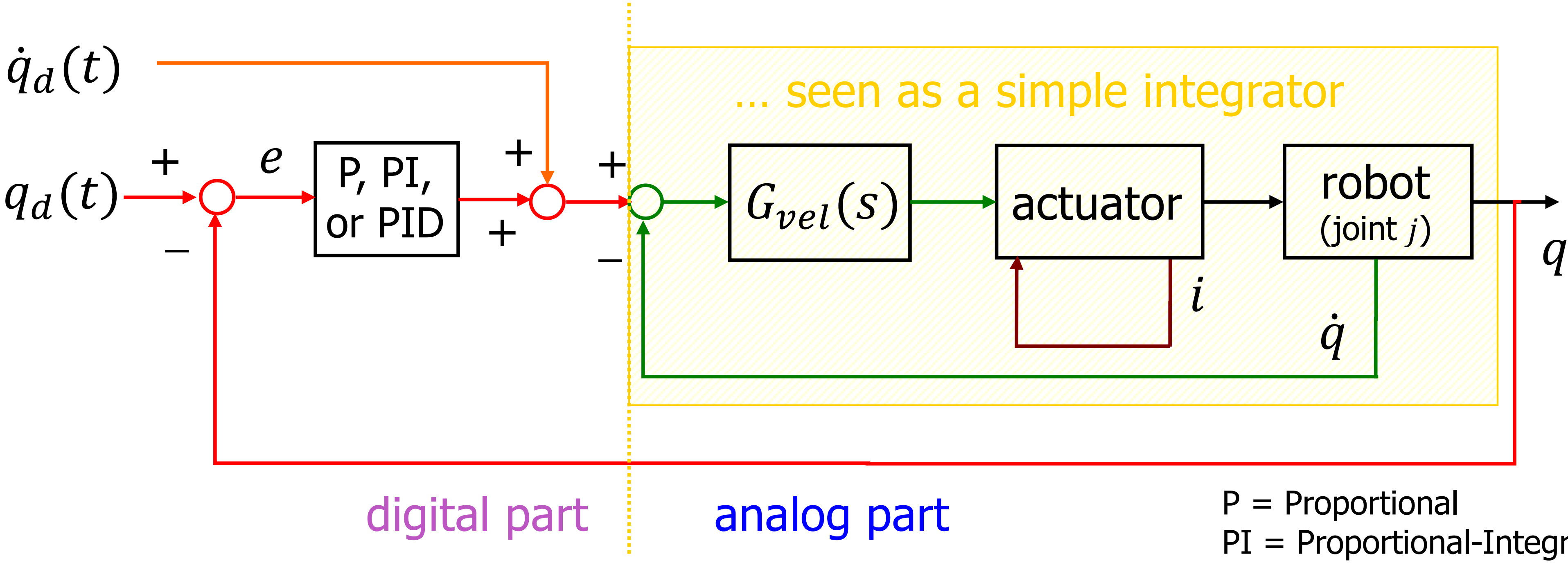
real position errors increase when reducing too much the motion time ($\Rightarrow$ acceleration is too large)

while ideal position errors
(based only on kinematics)
remain always the same!!
here $\equiv 0$, thanks to the initial matching between robot and reference trajectory



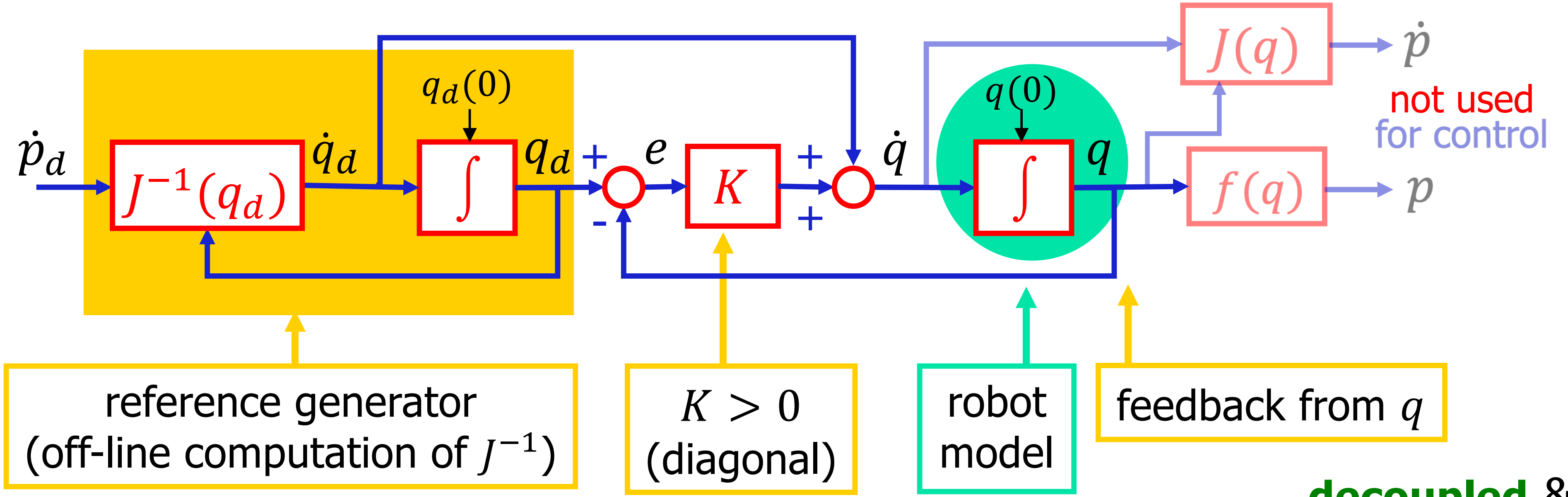
Control loops in industrial robots

- analog loop of large bandwidth on motor current (∝ torque)
- analog loop on velocity ($G_{vel}(s)$, typically a PI)
- digital feedback loop on position, with velocity feedforward
- this scheme is local to each joint (decentralized control)



P = Proportional
PI = Proportional-Integral
PID = PI-Derivative

Kinematic control of joint motion



$$e = q_d - q \;\Rightarrow\; \dot{e} = \dot{q}_d - \dot{q} = \dot{q}_d - (\dot{q}_d + K(q_d - q)) = -Ke$$

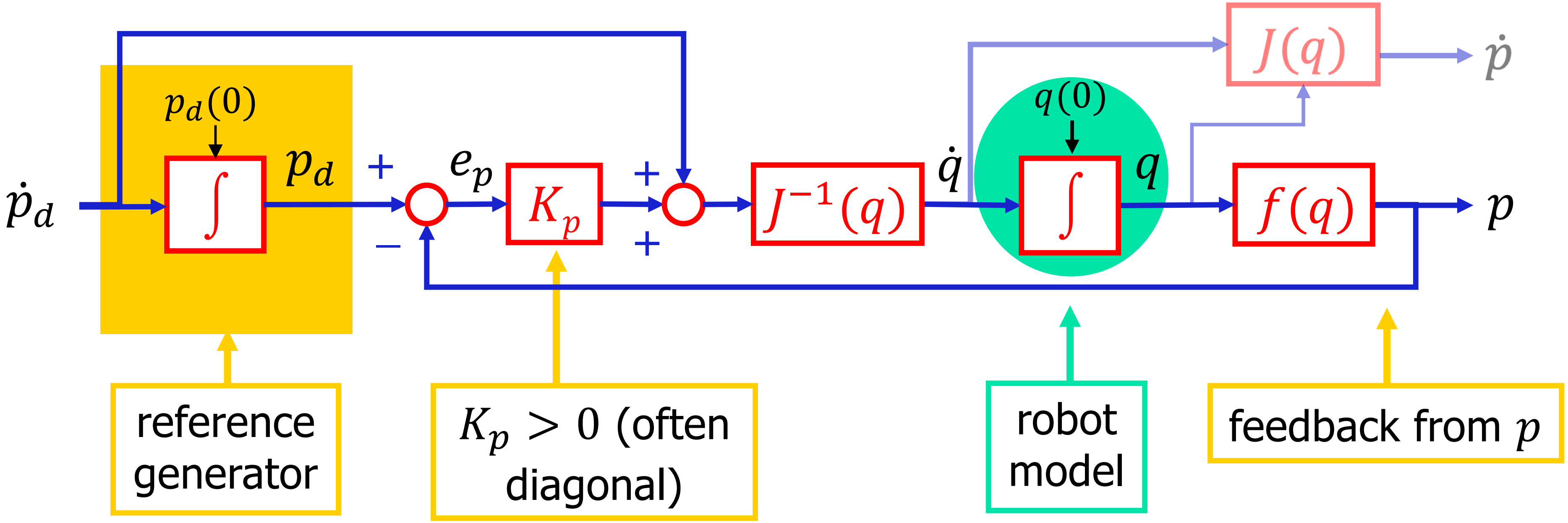
decoupled & **linear**: $e_j \to 0$ $(j = 1, \cdots, n)$ exponentially, $\forall e(0)$

$$e_p = p_d - p \;\Rightarrow\; \dot{e}_p = \dot{p}_d - \dot{p} = J(q_d)\dot{q}_d - J(q)(\dot{q}_d + K(q_d - q))$$

$$q \approx q_d \qquad e_p \to J(q)e \;\Rightarrow\; \dot{e}_p \approx -J(q)K\,J^{-1}(q)e_p$$

coupled, **nonlinear** Cartesian error dynamics

Kinematic control of Cartesian motion



$$e_p = p_d - p \Rightarrow \dot{e}_p = \dot{p}_d - \dot{p} = \dot{p}_d - \cancel{J(q)J^{-1}(q)}\left(\dot{p}_d + K_p(p_d - p)\right) = -K_p e_p$$

- **decoupled** & **linear**: $e_{p,i} \to 0$ $(i = 1, \cdots, m)$ exponentially, $\forall e_p(0)$
- needs on-line computation of the inverse(*) $J^{-1}(q)$
- real-time + singularities issues

(*) or pseudoinverse if $m < n$

Kinematic control at acceleration level

- the second-order control model is now $\ddot{q} = u$
- consider for instance the case of Cartesian kinematic control
- define as control law

$$u = J^{-1}(q)\left(\ddot{p}_d + K_d(\dot{p}_d - \dot{p}) + K_p(p_d - p) + \dot{J}(q)\dot{q}\right)$$

with $K_p > 0, K_d > 0$ both diagonal

$$e_p = p_d - p \Rightarrow \dot{e}_p = \dot{p}_d - \dot{p} \Rightarrow \ddot{e}_p = \ddot{p}_d - \ddot{p}$$

second-order system: acceleration error!

$$\Rightarrow \ddot{e}_p = \ddot{p}_d - \left(J(q)\ddot{q} + \dot{J}(q)\dot{q}\right) = \ddot{p}_d - \left(J(q)u + \dot{J}(q)\dot{q}\right)$$
$$= \cancel{\ddot{p}_d} - \cancel{J(q)J^{-1}(q)}\left(\cancel{\ddot{p}_d} + K_d(\dot{p}_d - \dot{p}) + K_p(p_d - p) + \cancel{\dot{J}(q)\dot{q}}\right) + \cancel{\dot{J}(q)\dot{q}}$$
$$= -K_d\dot{e}_p - K_p e_p$$

decoupled & **linear** 2nd-order differential equations

$\dot{e}_{p,j} \to 0,\ e_{p,j} \to 0\ (j = 1, \cdots, m)$ exponentially, $\forall e_p(0), \dot{e}_p(0)$

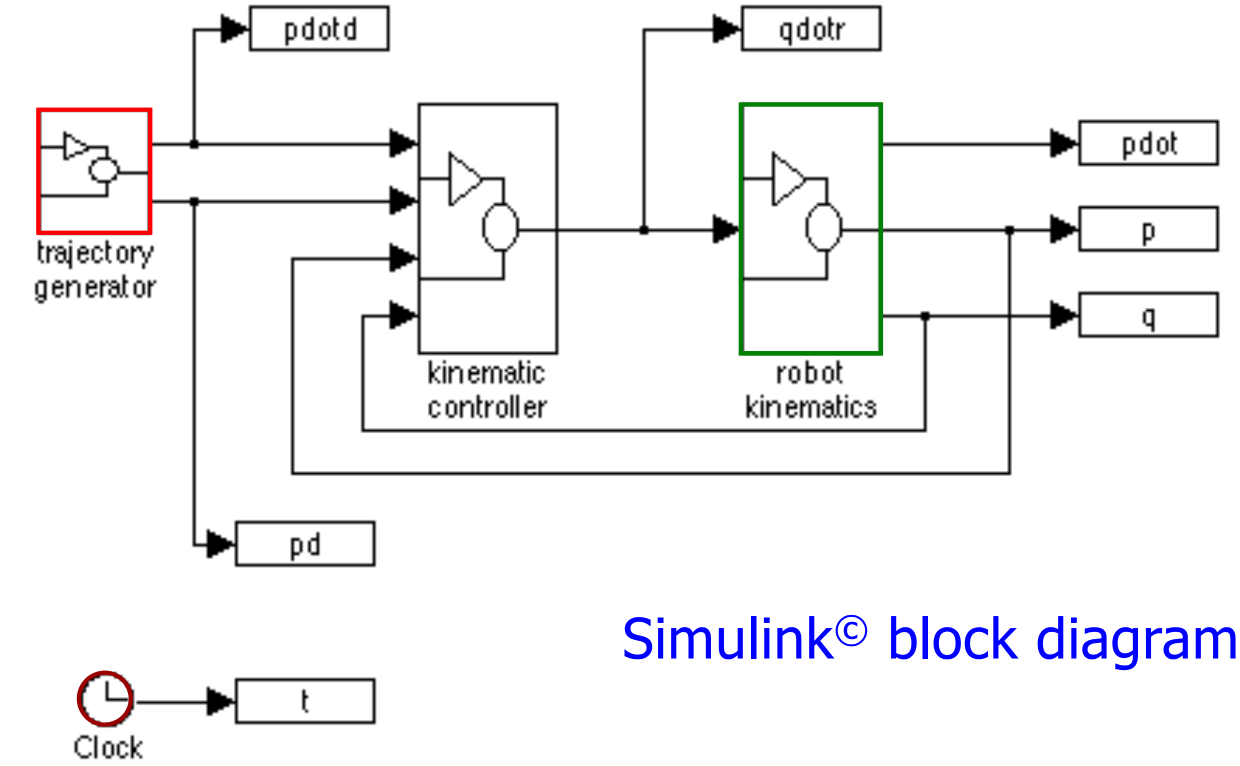
Simulation

features of kinematic control laws

desired reference trajectory:
two types of tasks
1. straight line
2. circular path

both with constant speed

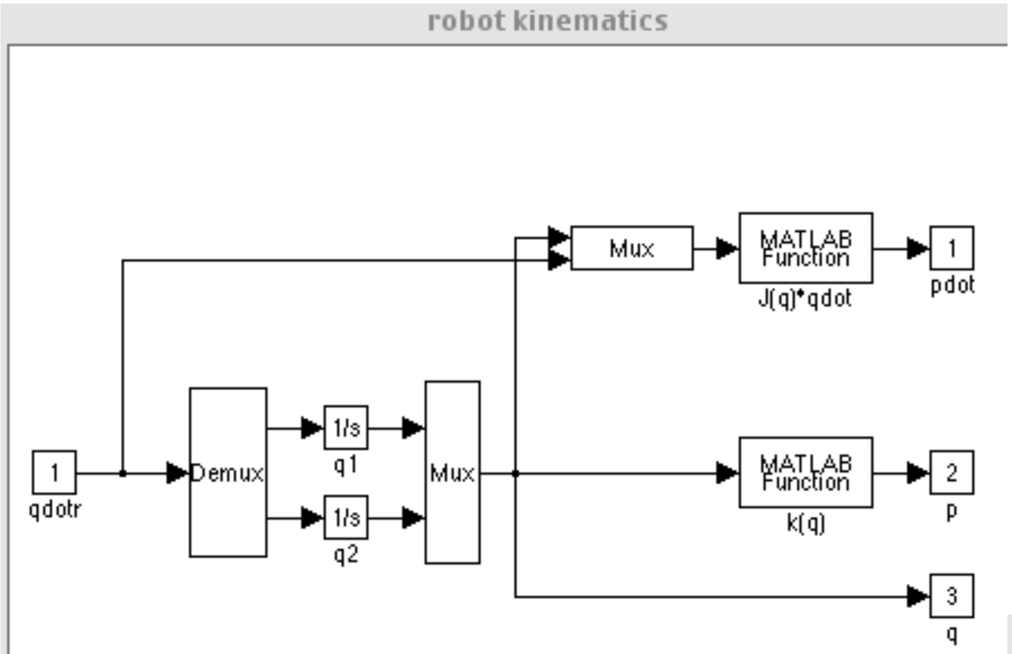
numerical integration method:
fixed step Runge-Kutta at 1 msec



robot:
planar 2R
lengths $l_1 = l_2 = 1$

Simulink© block diagram

Simulink blocks



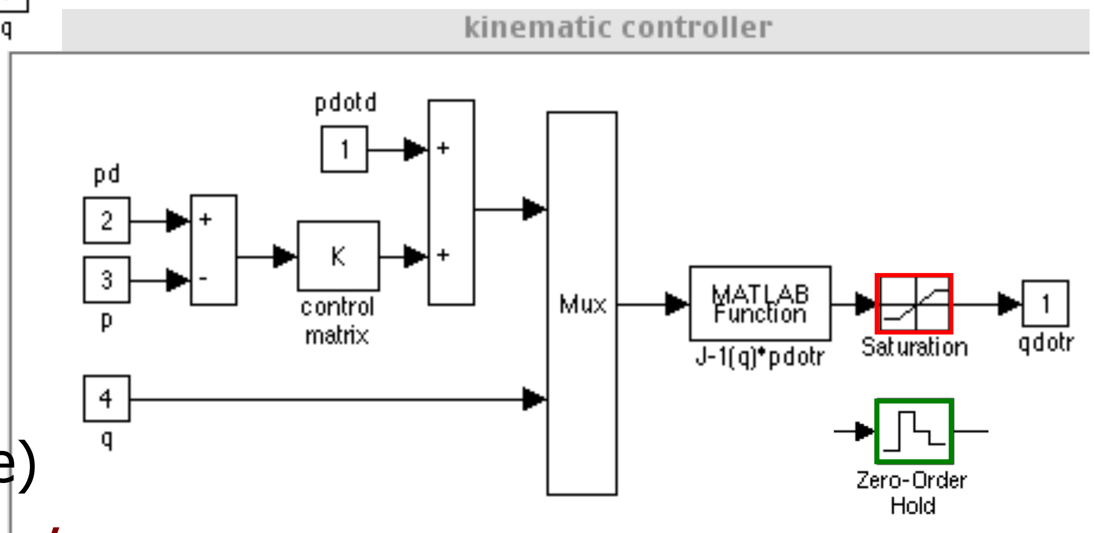
calls to Matlab functions

k(q)=dirkin (user)

J(q)=jac (user)

J^{-1}(q)=inv(jac) (library)

- a saturation (for task 1.) or a sample and hold (for task 2.) added on joint velocity commands
- system initialization of kinematics data, desired trajectory, initial state, and control parameters (in init.m file)



never put "numbers" inside the blocks !

Matlab functions

dirkin.m

```
function [p] = dirkin(q)

global l1 l2

px=l1*cos(q(1))+l2*cos(q(1)+q(2));
py=l1*sin(q(1))+l2*sin(q(1)+q(2));
```

jac.m

```
function [J] = jac(q)

global l1 l2

J(1,1)=-l1*sin(q(1))-l2*sin(q(1)+q(2))
J(1,2)=-l2*sin(q(1)+q(2));
J(2,1)=l1*cos(q(1))+l2*cos(q(1)+q(2));
J(2,2)=l2*cos(q(1)+q(2));
```

init.m

```
% controllo cartesiano di un robot 2R
% initialization

clear all; close all
global l1 l2

% lunghezze bracci robot 2R

l1=1; l2=1;

% velocità cartesiana desiderata (costante)

vxd=0; vyd=0.5;

% tempo totale

T=2;

% configurazione desiderata iniziale

q1d0=-45*pi/180; q2d0=135*pi/180;

pd0=dirkin([q1d0 q2d0]');
pxd0=pd0(1); pyd0=pd0(2);

% configurazione attuale del robot

q10=-45*pi/180; q20=90*pi/180;

p0=dirkin([q10 q20]');

% matrice dei guadagni cartesiani

K=[20 20]; K=diag(K);

%saturazioni di velocità ai giunti (input in deg/sec, convertito in rad/sec)

vmax1=120*pi/180; vmax2=90*pi/180;
```

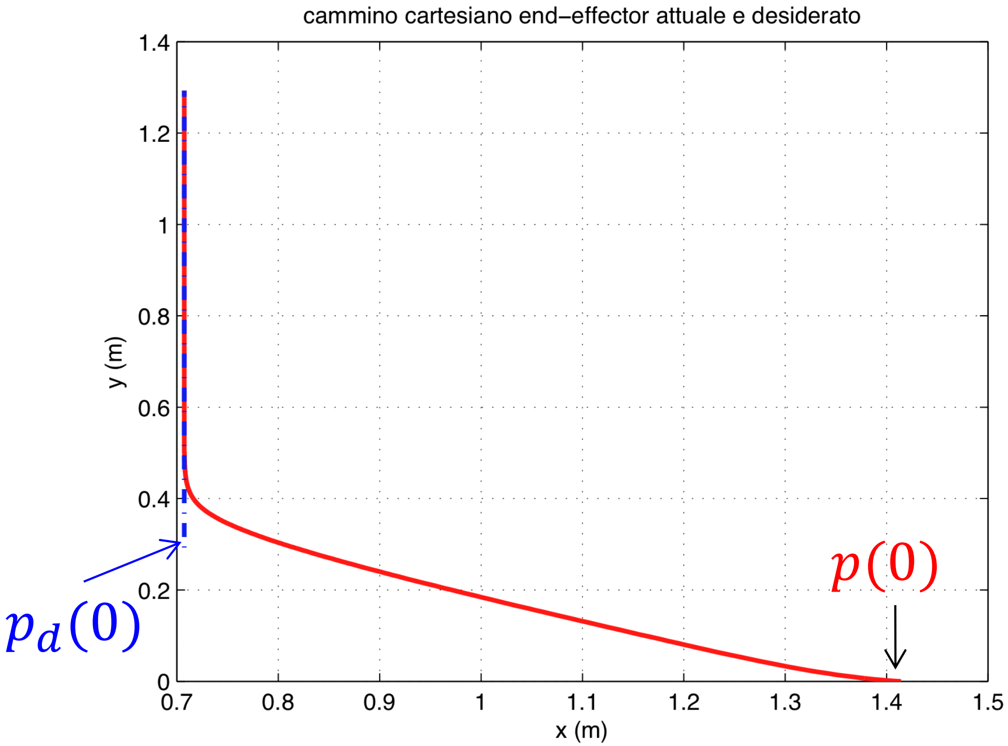
init.m
script
(for task 1.)

Simulation data for task 1

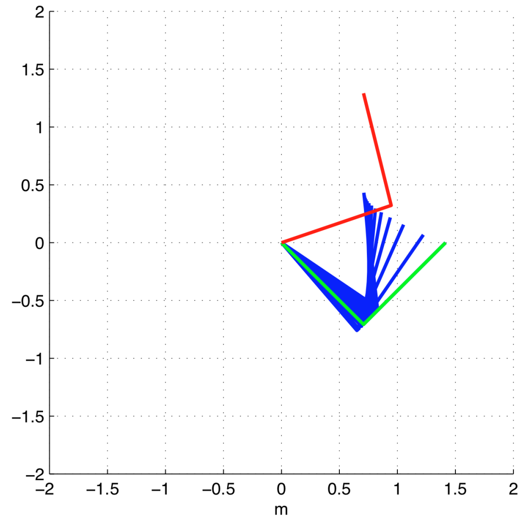
- straight line path with constant velocity
  - $x_d(0) = 0.7$ m, $y_d(0) = 0.3$ m; $v_{d,y} = 0.5$ m/s, for $T = 2$ s
- large initial error on end-effector position
  - $q(0) = (-45°, 90°) \Rightarrow e_p(0) = (-0.7, 0.3)$ m
- Cartesian control gains
  - $K_p = \text{diag}\{20, 20\}$
- (a) without joint velocity command saturation
- (b) with saturation $|\dot{q}_j| \leq v_{\max,j}$, $j = 1{,}2$:
  - $v_{\max,1} = 120°$/s, $v_{\max,2} = 90°$/s

Results for task 1a

straight line: initial error, **no** saturation

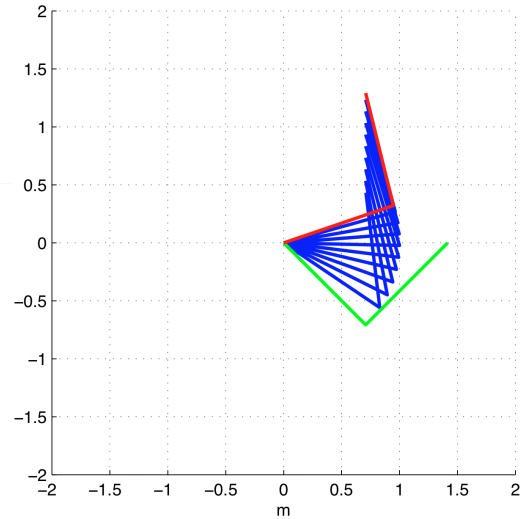


path executed by the robot end-effector (actual and desired)



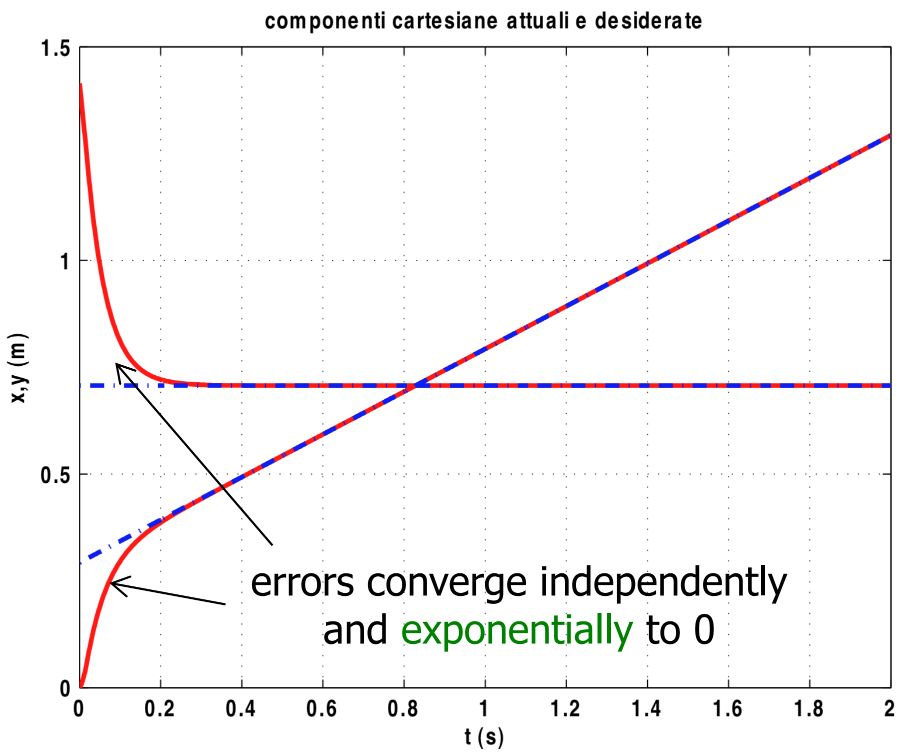
initial transient phase (about 0.2 s)

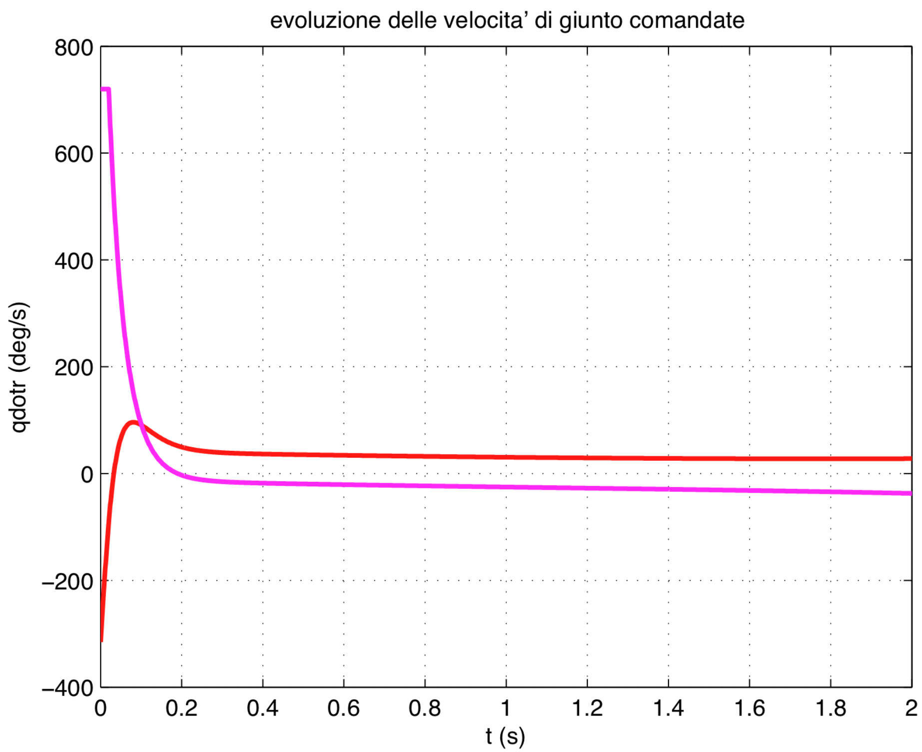
stroboscopic view of motion (start and end configurations)



trajectory following phase (about 1.8 s)

Results for task 1a
straight line: initial error, **no** saturation



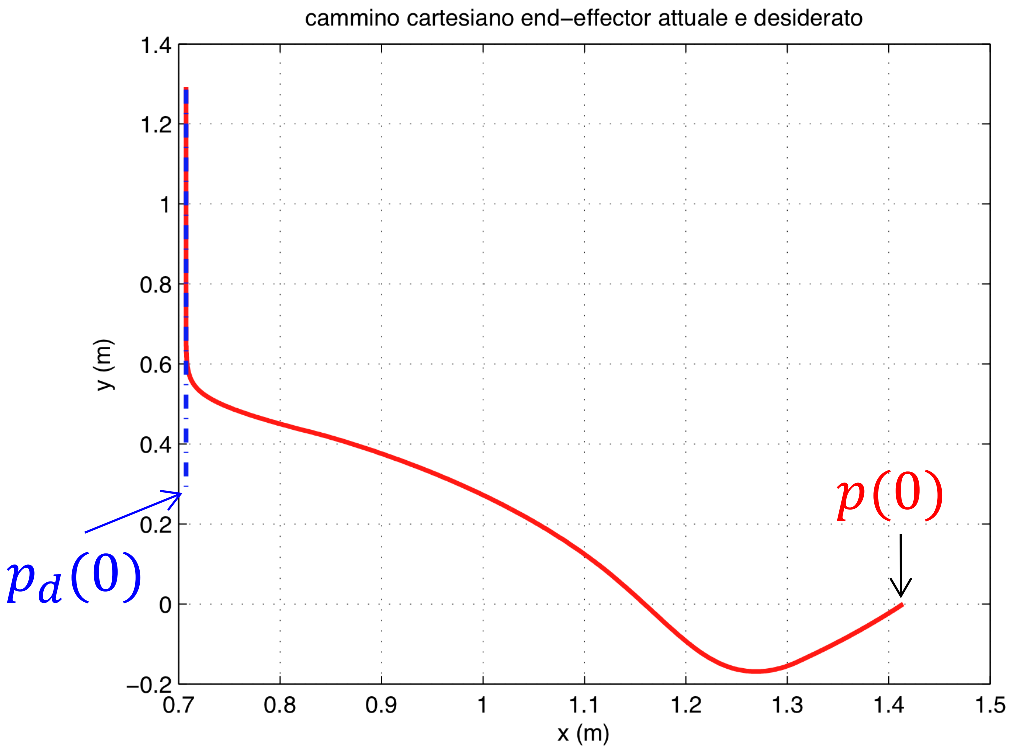


p_x, p_y actual and desired

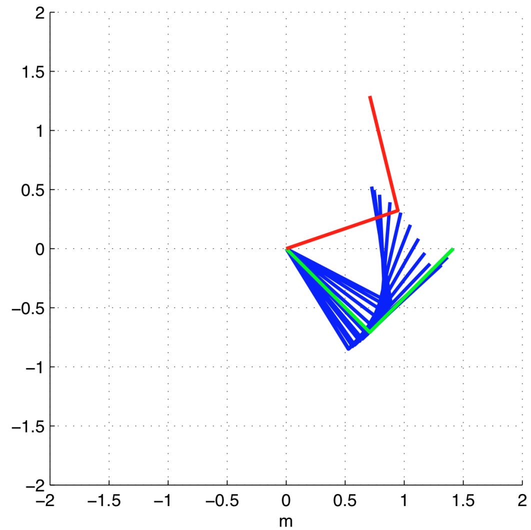
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$

Results for task 1b

straight line: initial error, **with** saturation

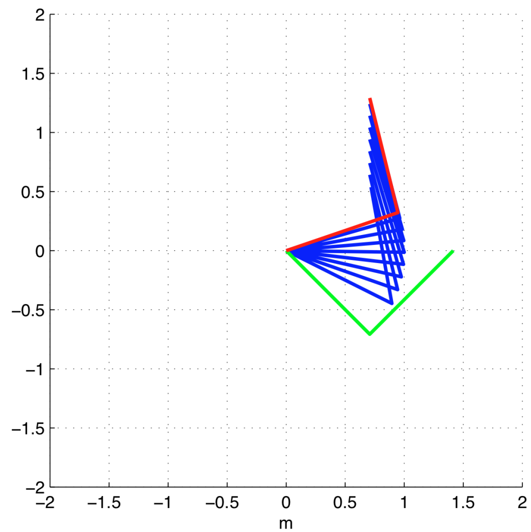


path executed by the robot end-effector (actual and desired)



initial transient phase (about 0.5 s)

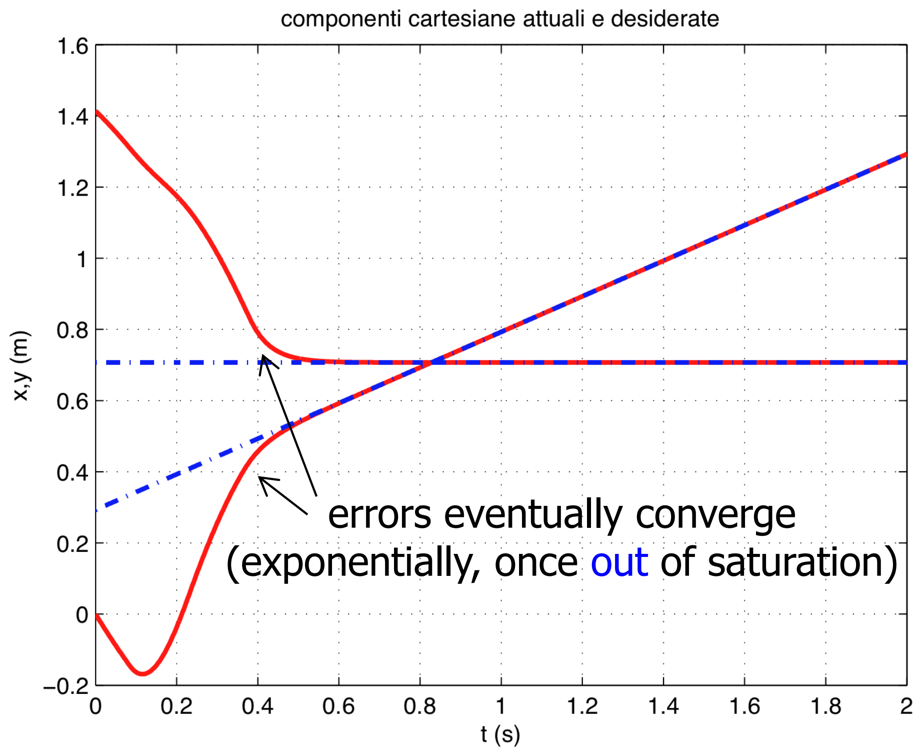
stroboscopic view of motion (start and end configurations)



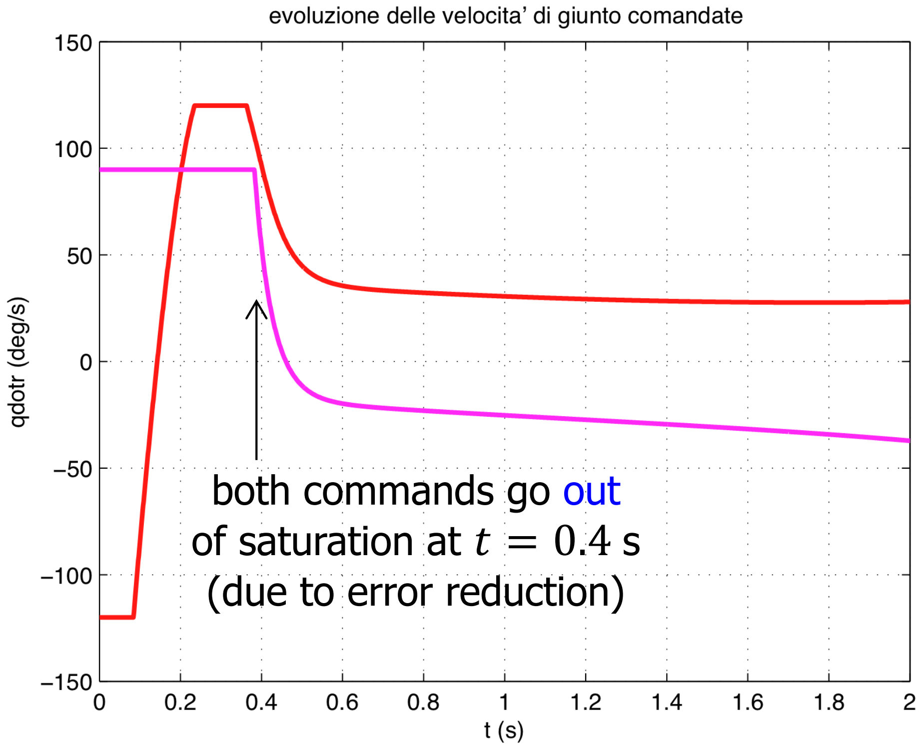
trajectory following phase (about 1.5 s)

Results for task 1b

straight line: initial error, **with** saturation



p_x, p_y actual and desired



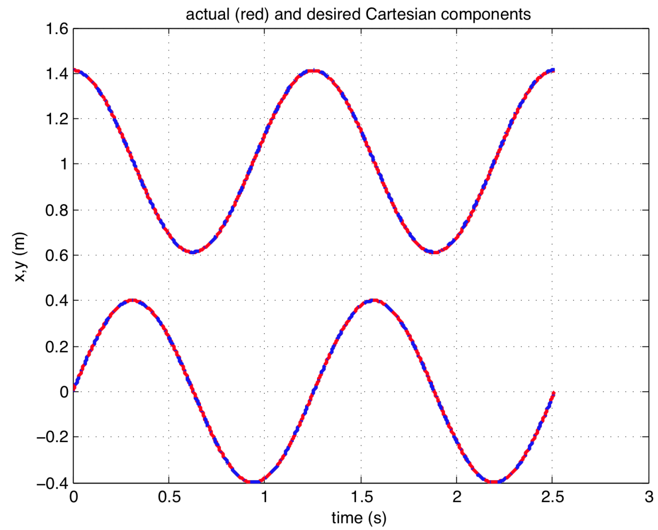
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$
(saturated at $\pm\, v_{\max,1}$, $\pm\, v_{\max,2}$)

Simulation data for task 2

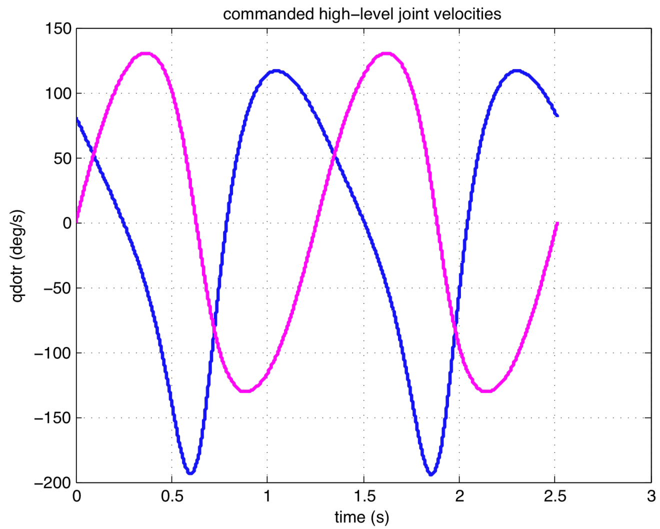
- circular path with constant velocity
  - centered at $(1.014, 0)$ with radius $R = 0.4$ m;
  - $v = 2$ m/s, performing two rounds $\Rightarrow T \approx 2.5$ s
- zero initial error on Cartesian position ("match")
  - $q(0) = (-45°, 90°) \Rightarrow e_p(0) = 0$
- (a) ideal continuous case (1 kHz), even without feedback
- (b) with sample and hold (ZOH) of $T_{\text{hold}} = 0.02$ s (joint velocity command updated at 50 Hz), but without feedback
- (c) as before, but with Cartesian feedback using the gains
  - $K_p = \text{diag}\{25, 25\}$

Results for task 2a

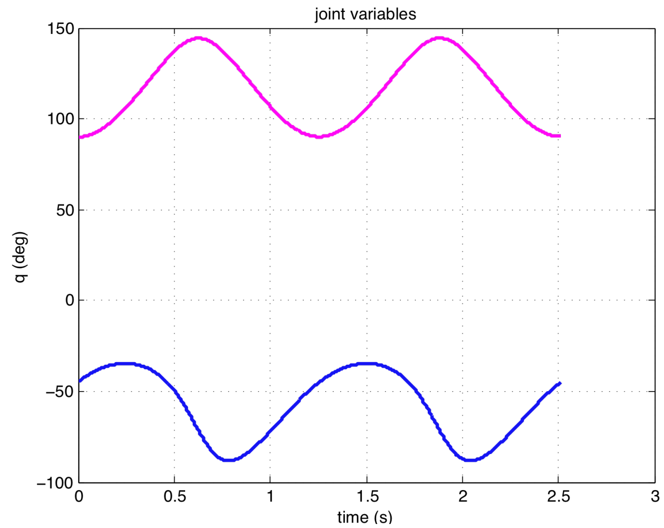
circular path: no initial error, **continuous** control (ideal case)



p_x, p_y actual and desired

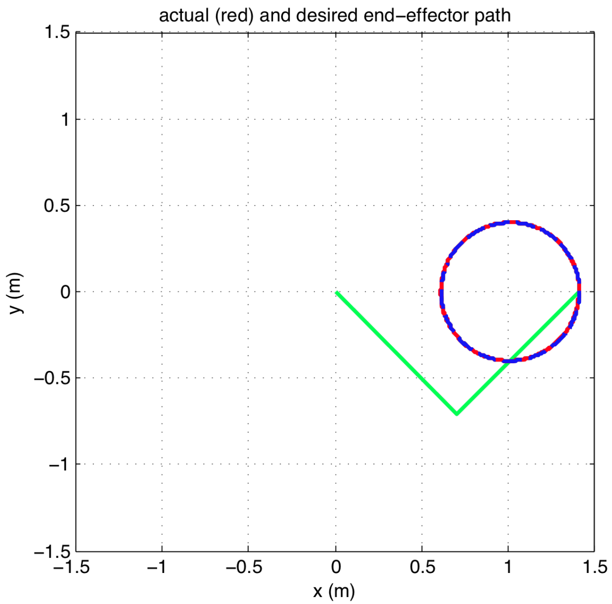


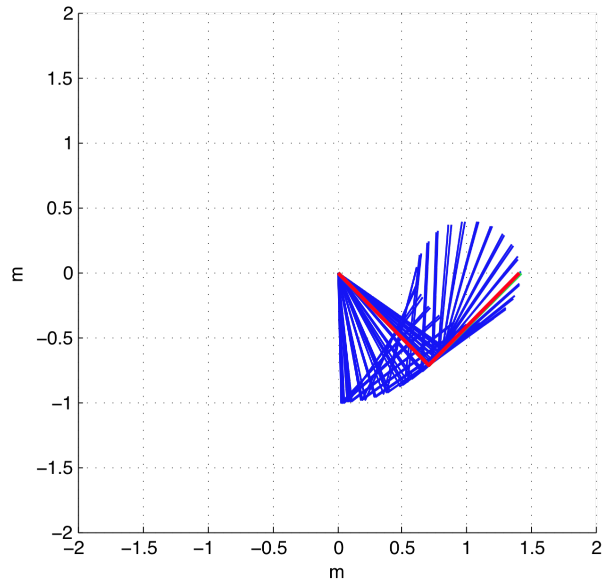
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$



joint variables q_1, q_2

zero tracking error is kept at all times

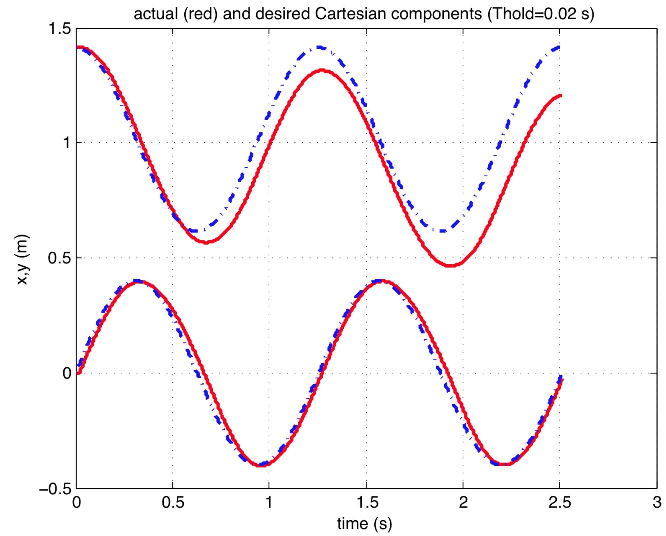




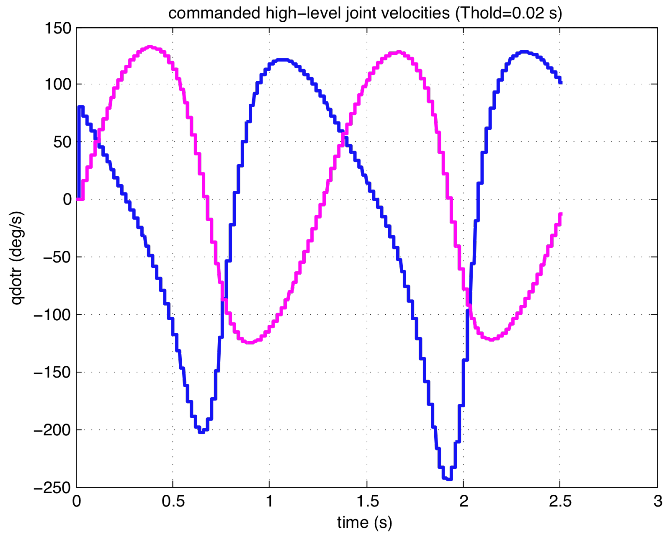
final configuration (after two rounds) coincides with initial configuration

Results for task 2b

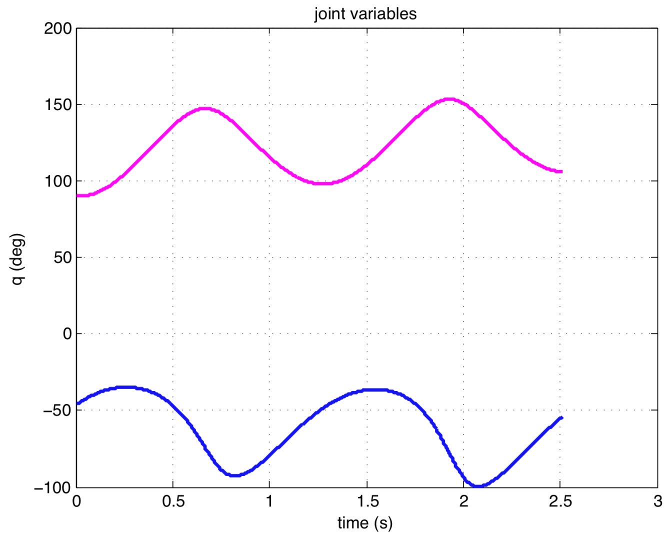
circular path: no initial error, **ZOH** at 50 Hz, **no** feedback



p_x, p_y actual and desired

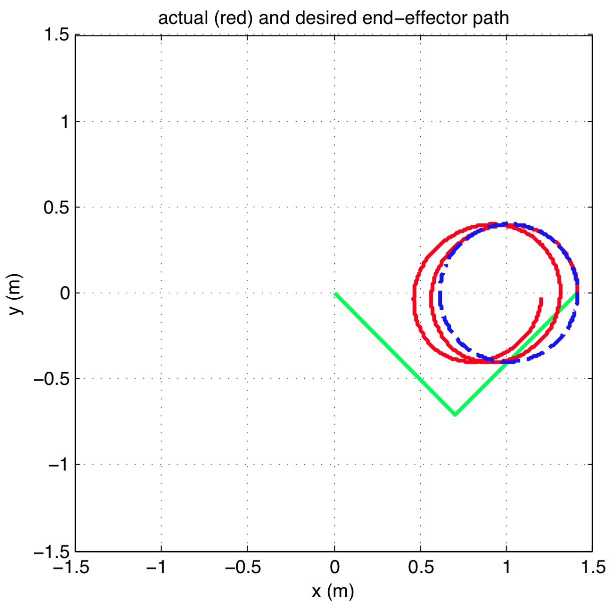


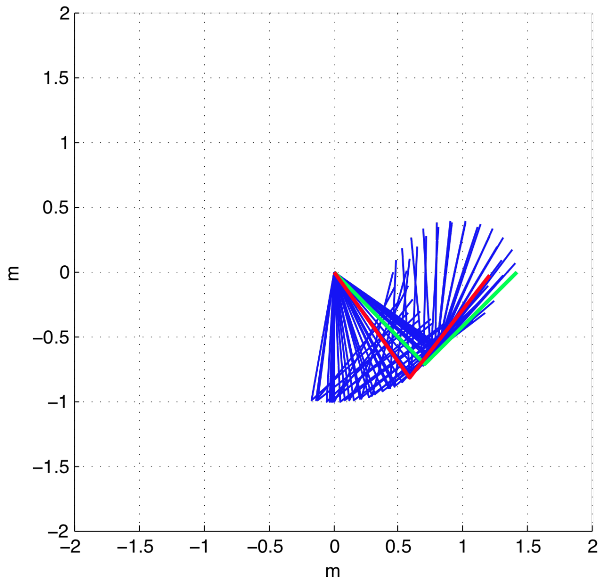
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$



joint variables q_1, q_2

a drift occurs along the path due to the "linearization error" along the path tangent

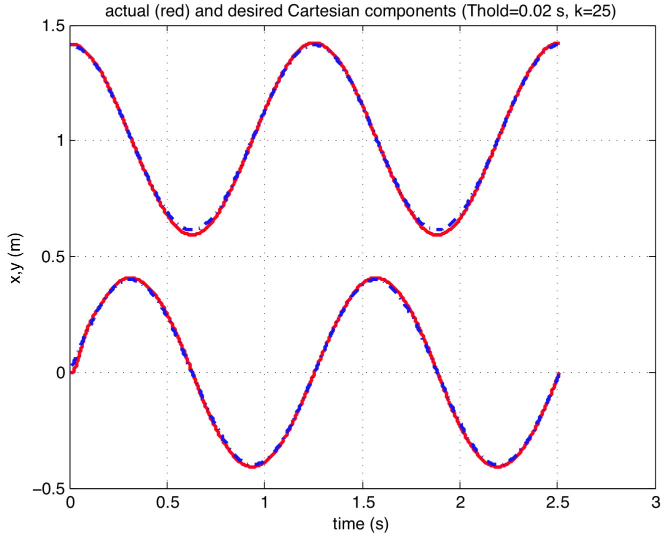




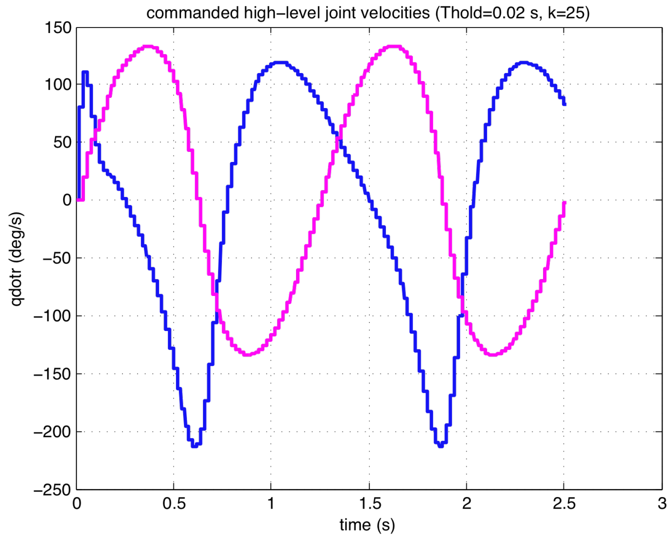
final configuration (after two rounds) differs from initial configuration

Results for task 2c

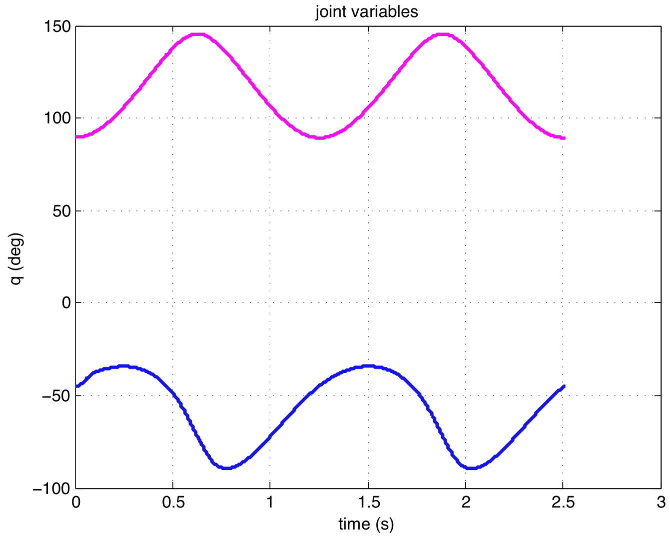
circular path: no initial error, **ZOH** at 50 Hz, **with** feedback



p_x, p_y actual and desired

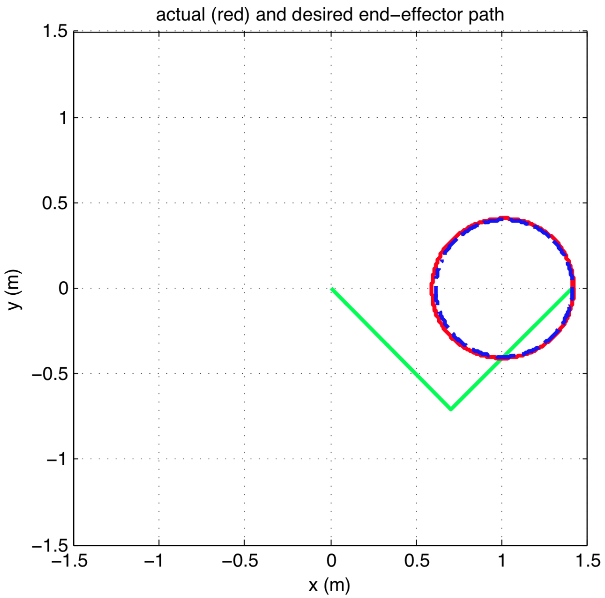


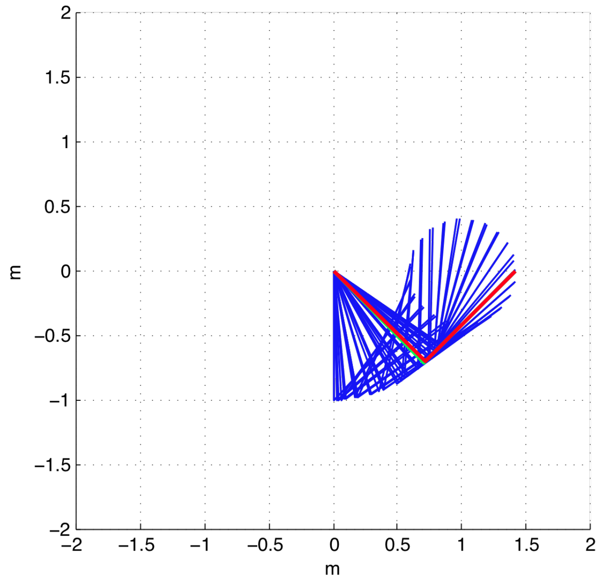
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$



joint variables q_1, q_2

(almost) same performance of the continuous case is recovered!!



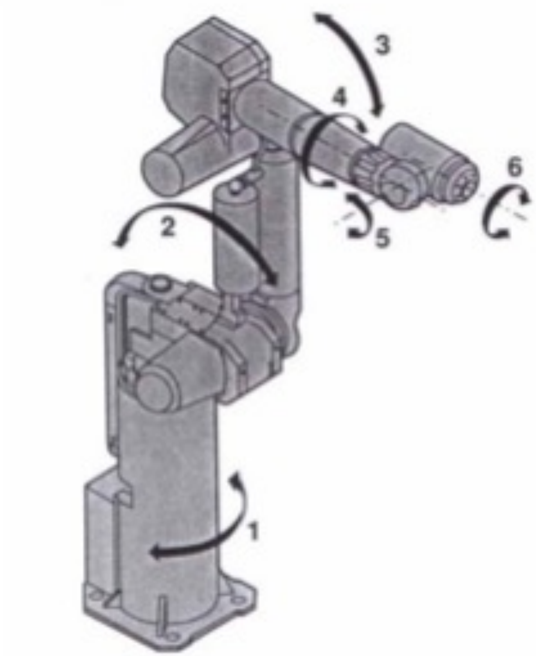


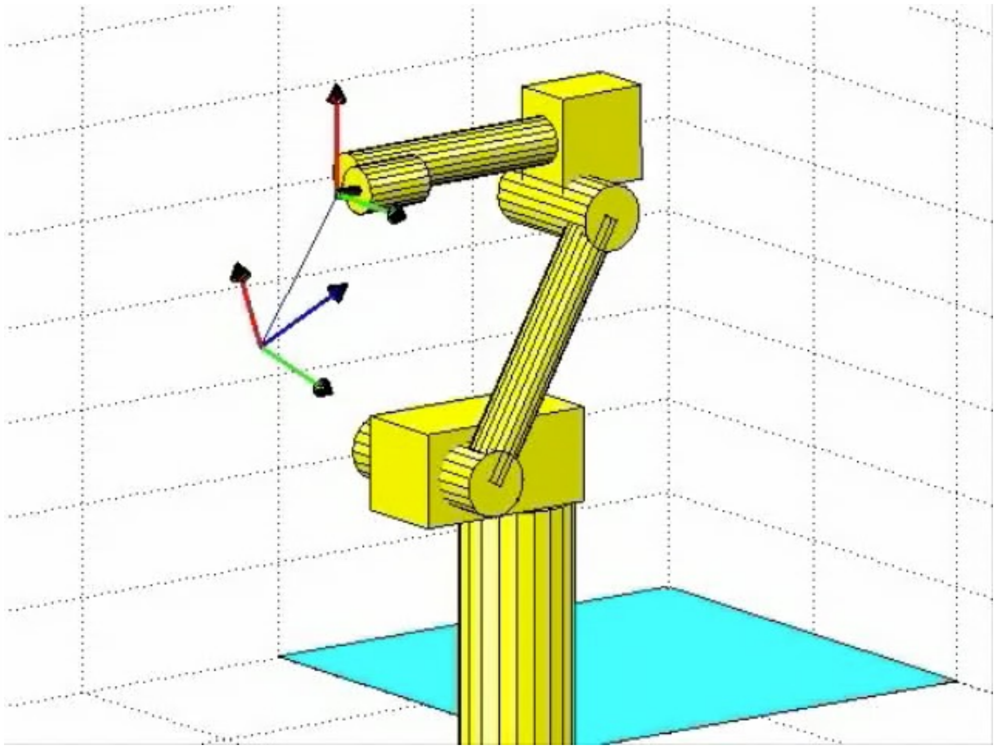
larger P gains will eventually lead to unstable behavior (stability limits for discrete-time implementations of a continuous control law)

3D simulation

why is final convergence so slow here?

video





kinematic control of Cartesian motion of Fanuc 6R (Arc Mate S-5) robot

simulation and visualization in Matlab