Situation calculus for controller synthesis in manufacturing systems with first-order state representation

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ABSTRACT

Manufacturing is transitioning from a mass production model to a service model in which facilities ‘bid’ to produce products. To decide whether to bid for a complex, previously unseen product, a facility must be able to synthesize, on the fly, a process plan controller that delegates abstract manufacturing tasks in a supplied process recipe to the available manufacturing resources. Often manufacturing processes depend on the data and objects (parts) they produce and consume. To formalize this aspect we need to adopt a first-order representation of the state of the processes. First-order representations of the state are commonly considered in reasoning about action in AI, and here we show that we can leverage the wide literature on the Situation Calculus and ConGolog programs to formalize this kind of manufacturing. With such a formalization available, we investigate how to synthesize process plan controllers in this first-order state setting. We also identify two important decidable cases—finite domains and bounded action theories—for which we provide techniques to actually synthesize the controller.

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1. Introduction

To be able to remain competitive in a global marketplace characterized by faster market response and demand for customization, modern manufacturing companies are transitioning from a traditional mass production model to more agile and cost-effective manufacturing networks and supply chains. Based on service-oriented principles, in the manufacturing as a service (Maas) paradigm, the manufacturing infrastructure is shared on-demand by potentially large numbers of different manufacturing processes, so that the products to be manufactured are not known in advance, batch sizes are often small, and a facility may produce items belonging to heterogeneous product families for several customers at the same time [1,2]. The cost of managing and maintaining the manufacturing infrastructure is thus distributed across all customers, enhancing resource utilization and reducing unit production costs, the lead-time is decreased and the sharing of knowledge and production processes makes better products. Different manufacturing models have been proposed in the literature to achieve

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the MaaS vision, with a recurring emphasis on flexibility, scalability, adaptability and customization, and on an increase in collaboration, automation, data and knowledge sharing through the entire supply chain. Examples include, among others, Agile Manufacturing, Virtual Manufacturing, Application Service Providers, Manufacturing Grid, and Cloud Manufacturing.

Among these, Cloud Manufacturing [3–5] is currently seeing the most advanced trends in MaaS. Enabled by an increasing development in information technology, IoT, embedded systems and cloud computing technologies, Cloud Manufacturing is proposing an advanced MaaS paradigm and business model in which manufacturing resources, such as Computer Numerical Control (CNC) machines and robots, are packaged as abstract descriptions of manufacturing capabilities, then advertised and made available to customers through a cloud platform. Likewise, the transformations that are required to manufacture a product are assumed to be specified as abstract, system-independent processes that need to be matched against the abstract capability descriptions that are offered by the facilities participating in the manufacturing cloud. By doing so, the MaaS paradigm, as envisioned by Cloud Manufacturing, allows the creation of dynamic production lines on-demand, by composing the pool of configurable manufacturing capabilities according to a pay-as-you-go business model. The objective is to connect customers to those providers who can best meet their product and process specifications and requirements, while limiting unit production cost and time. This model is also regarded as a more environmentally sustainable future for the manufacturing industry as a whole [6].

Critically, automation is key to achieve the MaaS vision: only minimal management effort or explicit customer-provider interaction can be assumed, so the fundamental requirement for MaaS is the ability of a facility participating in the cloud to autonomously assess, in real time, whether a given product can be manufactured in that facility. If the product can be manufactured, the facility may bid for the product, taking into account the overheads in terms of logistics. In mass production, the process planning phase, which transforms a process specification in concrete and practical production schedules for the resources on the shop floor, i.e., the process plan, is carried out by manufacturing engineers, and is largely a manual activity. In a MaaS approach, however, manual creation of process plans is clearly uneconomic for small batch sizes, and the time required to produce a plan is too great to allow facilities to bid for products in real time. Instead, manufacturing facilities must be able to automatically synthesize process plans for novel products ‘on the fly’. To automatically establish that a facility can manufacture a product, the abstract manufacturing tasks in the process recipe—the specification of how the product is to be manufactured—must be ‘matched’ against the available manufacturing resources in the facility. The resulting process plan details the low-level tasks to be executed and their order, the manufacturing resources to be used, and how materials and parts move between resources [7]. The process plan controller, i.e., the control software that delegates the operations in the plan to the appropriate manufacturing resources, is then synthesized. In doing so, no sensitive information about the resources and internal processes of the selected facilities should be exposed on the cloud platform.

Research in Artificial Intelligence and Computer Science can be exploited to provide a mathematical foundation for these domain concepts, and to solve the core challenges implicit in the MaaS vision. This is confirmed by recent efforts in basing MaaS on fundamental ideas coming from the literature on service composition in CS and behavior composition [8] in AI, so as to formalize the requirements and techniques for the automated synthesis of process plan controllers in the context of manufacturing [9–13]. These approaches have proven fruitful for developing preliminary and ‘proof-of-concept’ approaches for MaaS beyond the disciplines in which they were developed [14,15].

However these approaches are based on a propositional description of the states of the devices, workpieces and processes, which is too idealized and insufficient to achieve a fully fledged solution in practice. Indeed, manufacturing processes depend, in general, on the objects and data they produce and consume, including the cases where an unbounded number of product items (e.g., each with a unique serial number) or basic parts (e.g., each with a unique bar code, RFID tag or MAC address) must be produced. The approaches above do not explicitly account for this dependency, as they employ a finite state representation which hides important relationships between processes and data and, more importantly, cannot deal with a potentially unbounded number of objects. While in some cases this limitation can be circumvented, the resulting discretization is unwieldy and unnatural. For instance, one needs to directly encode into the process itself all the relevant dynamic knowledge, such as the current state of a part (e.g., painted, defective, etc.) or the state of a shared resource (e.g., a conveyor belt).

Instead, a concrete and realistic approach for MaaS necessarily requires a rich, relational description of states, an information model, as well as advanced computational techniques that are able to manipulate such relational representations. Although some previous work exists which can be used as the basis of an unambiguous description of the manufacturing concepts [16], the scientific literature has been lacking until now.

Our work here addresses exactly this point, by offering a “data-aware” process formalization in which data and objects are treated as first-class citizens. More specifically, we propose a relational representation of the states by relying on the research on reasoning about actions in AI. We see the operations in manufacturing processes as described by an action theory in logic, and the processes as high-level programs over such action theories. In this way, we can leverage the first-order state representations of action formalisms and the second-order/fixpoint characterization of state-change as provided by programs. Critically, we do not rely on ad-hoc representations, but we choose to adopt one of the most well developed formalisms for representing and reasoning about dynamic systems in AI, namely the Situation Calculus, to encode information models and how they change as the result of actions. In fact, we deal with multiple Situation Calculus theories simultaneously, so as to model process recipes working over both an abstract information model and a concrete, facility-level information model. Process recipes and manufacturing resources, in turn, are modeled as high-level ConGoLog pro-
grams [17] (over the action theories). All together, this yields a principled, formal and declarative representation of the MaaS setting.

By exploiting this rich representation, we formally define what it means to realize a process recipe in a manufacturing facility and present techniques to automatically synthesize controllers that implement those realizations. We show that these techniques are actually effective, that is, that they correspond to algorithms for extracting the actual controllers, when the resulting Situation Calculus action theories are state-bounded [18].

In our context, state-boundedness means that, while the facility may process an infinite number of objects overall, an unbounded number of them is never “accumulated” in any given state the number of objects being processed does not exceed a given bound. Notice that this case is the natural one in practice: the number of objects handled at a given time by the facility is naturally bounded by the size and structure of the shop-floor.

We stress that, technically and independently of the particular manufacturing setting in which we are interested, we provide here the first decidability result for controller synthesis in a setting with a relational/first-order state representation. We also observe that while our specific technical development is based on the Situation Calculus, our results and constructions can also be applied in other frameworks for reasoning about actions in AI as well as data-aware/artifact-centric processes frameworks in databases [19–21].

2. Situation calculus and ConGolog

The Situation Calculus [22,23] is a sorted predicate logical language for representing and reasoning about dynamically changing worlds. Changes in the world are the result of actions, which are terms in the language, and world histories are represented by situation terms. In addition to actions and situations, the language also includes an object sort, used to model the other entities. We assume a unique countably infinite set \( \Delta \) of objects as the object sort. For all objects in \( \Delta \) we have constants denoting them, called standard names, together with unique name assumption and domain closure [24,25]. This will allow us to fix a single interpretation domain for models of situation calculus formulas and blur the distinction between such standard names and objects of the domain. This way of proceeding is common in databases [26] and is convenient in our case because it makes it possible to denote each object and piece of information that is relevant for the manufacturing application.

We assume a finite number of (simple) action types, each of which takes a tuple of objects as arguments. For example, \( \text{drill}(\text{part}, \text{dmt}, \text{speed}, x, y, z) \) represents the action of drilling a hole of a certain diameter, at a certain spindle speed, in a specific position of a given part. In the manufacturing domain, we are concerned with operations that may occur simultaneously [23,27–29], hence we adopt the concurrent, non-temporal variant of the Situation Calculus, where a concurrent or compound action \( a \) is a possibly infinite set of simple actions, like the one above, that execute simultaneously [23, Chapter 7]. For example, \((\text{rotate}(\text{part}, \text{speed}), \text{spray}(\text{part}, \text{substr}))\) represents the joint execution of rotating a part at a given speed while spraying it with some substance. As shorthand, we denote by \( \text{A}(x) \) the compound action of type \( \text{A} \) with a vector \( x \) of arguments (of the right size, and assuming a standard ordering of simple actions so that their order can be ignored here).

In the concurrent, non-temporal variant, situations denote histories that stem from performing sequences of compound actions. We denote by \( S^0 \) the initial situation, i.e., the situation where no action has been performed yet, and assume that we have complete information about \( S^0 \). The situation resulting from executing a compound action \( a \) in \( s \) is represented by the situation term \(\text{do}(a, s)\). Predicates whose value varies from situation to situation are called fluents and take arguments of sort object plus a situation term as their last argument. For example, we may write \(\text{painted}(\text{part}, s)\) to denote that a part is painted in situation \( s \). These are the only fluents we consider, that is we deal with relational fluents only.

A basic action theory (BAT) [30,23] is a collection of axioms \( D \) describing the initial situation, preconditions and effects (and non-effects) of actions on fluents, as well as axioms for unique name assumptions and domain closure (for the object sort), which we denote, respectively, as \( D_{\text{una}} \) and \( D_{\text{dc}} \). We denote by \( D^0 \) the (complete) initial situation description, i.e., the set of axioms of \( D \) that describe the initial configuration of the world (which is unique, under complete information). Such a configuration corresponds to the extension of all the fluents of the theory in the initial situation \( S^0 \). A special predicate \( \text{Poss}(a, s) \) is used to express that the simple action \( a \) is executable in situation \( s \), and a precondition axiom specifies when the action can be legally performed. Formally precondition axioms have the form: \(\text{Poss}(a, s) \equiv \varphi(a, s)\) where \(\varphi(a, s)\) is a uniform Situation Calculus formula, that is a formula referring to only one situation \( s \) (the current one). Predicate \(\text{Poss} \) is extended to compound actions by writing \(\text{Poss}(a, s)\). Typically we can assume that each \( a \in \text{A} \) also needs to be possible by itself in the situation \( s \) (i.e., that \(\text{Poss}(a, s) \equiv \text{Poss}(a, s)\)), although \(\text{Poss} \) can be arbitrarily restricted further (as we will do in our running example). We say that a situation \( s \) is executable, denoted by \(\text{Executable}(s)\), if every (simple or compound) action performed in reaching \( s \) is possible in the situation in which it occurs [23].

Finally, a successor state axiom is used to encode causal laws specifying how each fluent changes as the result of executing (simple or compound) actions in the domain, encoding causal laws. There axioms have the form: \( f(x, \text{do}(a, s)) \equiv \varphi(x, a, s) \) where \(\varphi(x, a, s)\) is again a uniform Situation Calculus formula over the current situation, which determines the value of the fluent \( f(x, s') \) in the next situation \( s' = \text{do}(a, s) \) resulting from executing \( a \). Examples of BATs are provided in Section 5.

Note that, having assumed complete information on \( S^0 \), BATs are categorical, i.e., they essentially admit a single model [23,25]. The adoption of standard names allows for using, in a BAT \( D \), object names as constants. We call active object constants all the object names explicitly mentioned in the initial situation description \( D^0 \) or in some precondition or successor-state axiom. In other words, these are all the constants mentioned in \( D \) but not in \( D_{\text{una}} \) or \( D_{\text{dc}} \). Obviously, being
standard names, active object constants are always interpreted as themselves. The set of active object constants in a BAT $D$ is denoted as $AC_D$, possibly without subscript if no ambiguity may arise.

Complex manufacturing processes are specified using high-level programs. They are “high-level” in that they comprise actions and tests that belong to the domain of concern (rather than based on classical variables and assignment), and they are meant to be executed against a theory of actions. In the Situation Calculus, several such languages have been developed, such as Golog [17], which includes the usual programming constructs as well as constructs for nondeterministic choices, ConGolog [31], which extends Golog to accommodate concurrency, and indiGolog [25], which provides means for interleaving planning and execution.

We specify programs in a variant of ConGolog without recursive procedures [31] and where the test construct yields no transition and is final when satisfied [32,33]. This results in a synchronous test construct that does not allow interleaving (every transition involves the execution of an action). These are the ConGolog constructs we consider:

\[
\begin{align*}
\text{a} & \quad \text{simple or compound action} \\
\phi & \quad \text{test for a condition} \\
\delta_1; \delta_2 & \quad \text{sequence} \\
\delta_1 \mid \delta_2 & \quad \text{nondeterministic branch} \\
\pi x.\delta & \quad \text{nondeterministic choice of argument} \\
\delta^* & \quad \text{nondeterministic iteration} \\
\text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2 \text{ endif} & \quad \text{conditional} \\
\text{while } \phi \text{ do } \delta \text{ endwhile} & \quad \text{while loop} \\
\delta_1 || \delta_2 & \quad \text{interleaved concurrency}
\end{align*}
\]

where \( a \) can be a simple action (as in [31]), but also a compound action (these are the atomic instructions we use most in our setting) and $\phi$ is a situation-suppressed (uniform) Situation Calculus formula, i.e., a formula in the language with all situation arguments in fluents suppressed. We denote by $\phi[s]$ the (uniform) Situation Calculus formula obtained from $\phi$ by restoring the situation argument $s$ into all fluents in $\phi$. We require that the variable $x$ in programs of the form $\pi x.\delta$ ranges over objects, and occurs in some action term in $\delta$, i.e., $\pi x.\delta$ acts as a construct for the nondeterministic choice of action parameters.

Programs are executed over a BAT $D$ (or Situation Calculus action theory, in general). This means that the fluents mentioned in tests and conditions must be those in the BAT $D$. Similarly, all constants mentioned in a program $\delta$ come from the set $AC_D$ of $D$’s active object constants.

The semantics of ConGolog is specified in terms of single-steps, using the following two predicates [31]: Final($\delta, s$), specifying that the program $\delta$ may terminate in situation $s$, and Trans($\delta, s, \delta', s'$), specifying that one step of program $\delta$ in situation $s$ may lead to situation $s'$ with $\delta'$ remaining to be executed. The definitions of Trans and Final for the standard ConGolog constructs are given by:

\[
\begin{align*}
\text{Final}(a, s) & = \text{False} \\
\text{Final}(\phi?, s) & = \phi[s] \\
\text{Final}(\delta_1; \delta_2, s) & = \text{Final}(\delta_1, s) \land \text{Final}(\delta_2, s) \\
\text{Final}(\delta_1 \mid \delta_2, s) & = \text{Final}(\delta_1, s) \lor \text{Final}(\delta_2, s) \\
\text{Final}(\pi x.\delta, s) & = \exists x.\text{Final}(\delta, s) \\
\text{Final}(\delta^*, s) & = \text{True} \\
\text{Final}(\delta_1 || \delta_2, s) & = \text{Final}(\delta_1, s) \land \text{Final}(\delta_2, s) \\
\text{Trans}(a, s, \delta', s') & = do(a, s) \land \text{Poss}(a, s) \land \delta' = \epsilon \\
\text{Trans}(\phi?, s, \delta', s') & = \phi[s] \\
\text{Trans}(\delta_1; \delta_2, s, \delta', s') & = \text{Trans}(\delta_1, s, \delta_1', s') \land \delta' = \delta_1'; \delta_2 \lor \text{Final}(\delta_1, s) \land \text{Trans}(\delta_2, s, \delta', s_1) \\
\text{Trans}(\pi x.\delta, s, \delta', s') & = \exists x.\text{Trans}(\delta, s, \delta', s') \\
\text{Trans}(\delta^*, s, \delta', s') & = \text{Trans}(\delta, s, \delta', s') \land \delta' = \delta''; \delta^* \\
\text{Trans}(\delta_1 || \delta_2, s, \delta', s') & = \text{Trans}(\delta_1, s, \delta_1', s') \land \delta' = \delta_1'; \delta_2 \lor \text{Trans}(\delta_2, s, \delta_2', s') \land \delta' = \delta_1 || \delta_2'
\end{align*}
\]

Above, we use $\epsilon$ to denote the empty program. In fact $\epsilon$ is simply an abbreviation for $\text{True}?$. Note that the conditional and while-loop constructs are definable: if $\phi$ then $\delta_1$ else $\delta_2$ endif = $\phi?; \delta_1; \neg \phi?; \delta_2$ and while $\phi$ do $\delta$ endwhile = ($\phi?; \delta^*; \neg \phi?$). Also, we denote by loop $\delta$ the program $\delta^*$ to make the nondeterministic iteration more evident.

A configuration is a pair $\langle \delta, s \rangle$ with program $\delta$ and situation $s$, hence Trans denotes one-step transitions between configurations and Final denotes when a configuration is final. We use Trans* to denote the transitive closure of Trans, i.e., Trans*($\delta, s, \delta', s'$) means that there exists a sequence of one-step transitions evolving the configuration $\langle \delta, s \rangle$ into the configuration $\langle \delta', s' \rangle$. By using Trans* we can define $\text{Do}(\delta, s, s')$ as an abbreviation for $\exists \delta'. \text{Trans}^*(\delta, s, \delta', s') \land \text{Final}(\delta', s')$.
that the complete execution of the program \( \delta \) from \( s \) results in the new situation \( s' \) \[17,31\]. Both \( \text{Trans}^* \) and \( \text{Do} \) can be easily defined in second-order logic by using \( \text{Trans} \) and \( \text{Final} \).

It is important to notice that when we have complete information on the initial situation in a BAT, \( \text{Trans} \), \( \text{Final} \), \( \text{Trans}^* \) and \( \text{Do} \) are unequivocally defined by the model of the BAT. That is, the BAT, together with the definition of \( \text{Trans} \), \( \text{Final} \), \( \text{Trans}^* \) and \( \text{Do} \) is, again, categorical \[31\]. Moreover, since in our case the object domain consists of the standard names \( \Delta \), we have a syntactic denotation of each object, situation and program (and thus configuration). This allows us to blur the distinction between the semantic and syntactic objects of our formalization and switch back and forth seamlessly between the semantic and syntactic notions of configurations. Observe that, although unique, the BAT model has an infinite object domain, as well as infinite situations and programs. This makes working with such models nontrivial and substantially different from the way they are dealt with in model checking \[34,18,35\].

3. A variant of ConGolog for manufacturing

The standard constructs of ConGolog are not sufficient to express ‘simultaneous’ execution of processes in manufacturing. For this reason, we extend ConGolog with a new construct to model the simultaneous operation of two (or more) manufacturing resources: the synchronized concurrency operator \( \delta_1||\delta_2 \) is used to represent the synchronized concurrent execution of programs \( \delta_1 \) and \( \delta_2 \)—their next actions take place in the same transition step.

Moreover, we want our theories to be able to allow the synchronous execution of a set of actions even if they are not executable by themselves. The underlying assumption here is that a number of sub-systems (in our domain, manufacturing resources) can legally perform a joint step only if this is explicitly deemed possible by a “global” BAT for compound actions. This gives us complete control when modeling manufacturing facilities, and allows to capture arbitrary constraints on the usage of the resources available. For instance, to lift a heavy object we may necessarily need two robots to perform a lifting action at the same time on the same object, while the individual action of lifting an object may not be allowed by the individual theories of these robots when they are considered in isolation.

To capture this, the semantics of \( ||| \) is defined as follows:

\[
\text{Final}(\delta_1||\delta_2, s) = \text{Final}(\delta_1, s) \land \text{Final}(\delta_2, s); \\
\text{Trans}(\delta_1||\delta_2, s, \delta', s') \equiv \text{Trans}'(\delta_1, s, \delta'_1, s'_1) \land s'_1 = \text{do}(a_1, s) \land \text{Trans}'(\delta_2, s, \delta'_2, s'_2) \land s'_2 = \text{do}(a_2, s) \land \text{Poss}(a_1 \cup a_2, s) \land s' = (\delta'_1||\delta'_2) \land s' = \text{do}(a_1 \cup a_2, s),
\]

where \( \text{Trans}' \) has analogous axioms as \( \text{Trans} \) except for two modifications:

1. For simple and compound actions, \( \text{Trans}'(a, s, \delta', s') \equiv s' = \text{do}(a, s) \land \delta' = \epsilon \).
2. \( \text{Trans}'(\delta_1||\delta_2, s, \delta', s') \) does not require that \( \text{Poss}(a_1 \cup a_2, s) \) be true.

This allows us to capture executability of compound actions without necessarily requiring the executability of (subsets of) their component simple actions: in the above, we impose \( \text{Poss}(a_1 \cup a_2, s) \) without requiring also \( \text{Poss}(a_1, s) \) and \( \text{Poss}(a_2, s) \), although this can be explicitly required in the BAT (above, we implicitly extended the union operator to simple actions).

Observe that \( \delta_1||\delta_2 \) requires both \( \delta_1 \) and \( \delta_2 \) to execute one (possibly compound) action at each step. In case one program requires more steps than the other to reach a final configuration, they cannot be executed in a synchronized, concurrent fashion. However, a program may include “no-op” actions to explicitly model when the resource can remain idle.

Finally, note that synchronized concurrency is distinguished from interleaved concurrency. For example, in the program \( \delta_1||\delta_2 \), it is legal to execute either \( \delta_1 \) or \( \delta_2 \) completely before even starting the other, and it also legal to switch back and forth after each of their primitive actions. This is not possible with the synchronized concurrency program \( \delta_1||\delta_2 \), under which both \( \delta_1 \) and \( \delta_2 \) must execute an action at each step.

4. Manufacturing as a service

In this section we describe, in abstract terms, the MaaS model which we will formally capture through the logical framework illustrated in the next section. Specifically, we consider a type of MaaS setting as those recently envisioned by Cloud Manufacturing approaches, as discussed in Section 1. In this model, we consider a set of manufacturing facilities, each consisting of a set of configurable manufacturing resources that can be rapidly provisioned and released with minimal management effort or provider interaction. Examples of resources are CNC machines, robots and tools in flexible production lines. Facilities can join a manufacturing cloud to offer their production capabilities to cloud users (i.e., product designers) who wish to have their products manufactured. In turn, a user can submit to the cloud system a product model, i.e., the (process) recipe, that is a representation of the activities that are to be executed in order to complete an instance of the product. For simplicity, we assume that a product is entirely described by a single process recipe, although one could consider a hierarchical subdivision in multiple sub-assemblies and, thus, process recipes.

The main objective is to assess whether the product is manufacturable, that is, it can be manufactured through the cloud and, if so, to compute exactly how. The product can be manufactured if the recipe can be manufactured in a facility,
that is, if an implementation of all the possible sequences of resource-independent operations, therein prescribed, can be delegated at each step to the resources in the facility, also taking care of all the low-level additional operations that may be required (which, being dependent on the facility and its resources, are not included in the recipe). Indeed, a process recipe is a resource-independent process, designed by the product designer without any specific knowledge or assumption on the manufacturing system that will be selected for its implementation, and it thus assumes an information model that is common throughout the cloud. A MaaS model must then be able to bridge the gap between such abstract representation and the description of the physical manufacturing processes that each facility can execute, according to the resources that are available in the facility.

When the product is manufacturable in a facility, the module responsible for delegating actions in the recipe to resources (we refer to such delegation as either realization or orchestration) is called a process plan controller, or simply a controller. In contrast to mass production, where process planning is carried out by manufacturing engineers and is largely a manual activity, this setting requires facilities to be able to bid for products in real time, that is, to be able to automatically check whether a novel product is manufacturable and, if so, synthesize a controller ‘on the fly’ prior to submitting a bid.

Following these observations, we can provide a specific MaaS framework whose basic components match the core elements of a MaaS paradigm. This is illustrated in Fig. 1, where two distinct sorts are considered:

1. An information model $\mathcal{D}$ describes the data and the physical objects manipulated by processes, as well as the operations used to manipulate them.
2. A process $\delta$ describes the sequencing of the operations and captures the specific capabilities and the dynamic behavior of a component.

The various components that form our MaaS framework can be described as follows. An example will be given in the next section, where these concepts are presented in detail. For now, we focus on highlighting the role that each of these components has, their internal structure, and their relationship. For simplicity, as reflected in Fig. 1, we restrict ourselves to describe a single facility within the manufacturing cloud:

1. **Resources**: these are the physical manufacturing resources on the shop floor of a facility. Each resource is a couple $(\mathcal{D}_i, \delta_i)$, with $i \in \{1, \ldots, n\}$, that has its own information model $\mathcal{D}_i$ and its own resource process $\delta_i$, as these resources are typically sold by different companies. The actions and possible data in each $\mathcal{D}_i$ typically include low-level details.
2. **Facility information model**: it is the information model $\mathcal{D}_F$ for the facility, obtained by combining the information models $\mathcal{D}_i$ of each resource on the shop floor. It is the responsibility of the system integrator of the process to suitably combine the information models of the resources.
3. **Facility process**: it is the process $\delta_F^0$ resulting from the synchronous, concurrent execution of the process $\delta_i$ of each resource. Synchronous concurrency is needed to capture the fact that resources can (and may be required to) execute actions at the same time and on the same objects.
4. **Cloud information model**: it is the information model $\mathcal{D}_R$, common throughout the cloud, that is assumed by any recipe that can be designed. It is the core component of the framework whose existence precedes any other component. It represents a resource-independent information model of the data and objects that the recipes manipulate, and through which they are modeled. $\mathcal{D}_R$ is developed by the manager of the cloud infrastructure.
5. **Mappings**: a set of mappings $\text{Maps}$ represents the mechanism for relating the cloud information model $\mathcal{D}_R$, which is resource-independent, to the facility information model $\mathcal{D}_F$, that is resource-dependent. $\text{Maps}$ relates the possible abstract executions described by the process recipe $\delta_F^0$ (see below) with the concrete executions of the facility process $\delta_F^0$. In fact we need two forms of mappings: the first relates the data and objects in $\mathcal{D}_R$ to those in $\mathcal{D}_F$; the second relates operations in $\mathcal{D}_F$ to possibly complex sub-processes that are required to implement them in the facility, according to $\mathcal{D}_F$. Operationally, these mappings are specified only once, namely when a facility joins the manufacturing cloud: the system
integrator of the facility, who has access to both $D_F$ and $D_R$, is responsible for engineering them, although automation is also possible. These mappings are essential for the MaaS framework to be able to check the manufacturability of the product in a facility (and compute the controller) without exposing any internal detail, such as the number and type of resources or their processes. As depicted in Fig. 1, the controller component is not at the cloud level but at the level of the facility; privacy is a critical requirement in MaaS.

6. **Facility**: by putting together some of the components above, a manufacturing facility is captured as a tuple $Fac = (D_R, D_F, δ^0_F, Maps)$, where $D_R$ is the cloud information model (which is fixed), $D_F$ is the facility information model, $δ^0_F$ is the facility program and Maps is the set of mappings as above.

7. **Recipes**: a recipe is a resource-independent process $δ^0_R$ designed by a product designer without any specific knowledge or assumption on the manufacturing facility that will be selected for its realization. The information model describing the operations that compose the recipe is the cloud information model $D_R$. The product designer can access $D_R$ to design the recipe but cannot alter it.

8. **Controller**: when a controller exists for a given recipe and facility, i.e., when the product is manufacturable, this component is responsible for realizing the recipe by orchestrating the resources in the facility. It can be informally understood as a function relating each possible execution of the recipe $δ^0_R$ to one execution of the process $δ^0_F$ of the facility. It is not a cloud-level entity, and it is never exposed outside of the organization that owns the facility.

Given a facility $Fac = (D_R, D_F, δ^0_F, Maps)$ and a recipe $δ^0_R$, the manufacturability problem amounts to establishing whether there exists a controller to orchestrate the resources in the facility $Fac$ to realize the recipe $δ^0_R$. The synthesis task is to automatically build the controller responsible for implementing the orchestration.

5. **Manufacturing as a service in the situation calculus**

In this section we detail how the framework above is formally captured in our approach. We see the actions in manufacturing processes as described by an action theory in logic, and processes as high-level programs over such action theories. In this way, we can leverage the first-order state representations of action formalisms and the second-order/fixpoint characterization of state-change as provided by programs. We now formalize each framework component and provide examples.

5.1. **Resources**

As shown in Fig. 1, a facility is composed of $n$ manufacturing resources $(D_i, δ_i)$, each identified by an index $i \in [1, \ldots, n]$. We formalize the information model $D_i$ of each resource $i$ as a BAT $D_i$ (we overload symbols used in the MaaS framework itself), which specifies the resource’s initial configuration and the actions it can execute (as a specification of fluent’s dynamics through precondition and successor-state axioms). For convenience, we include the resource index $i$ (a constant) as the last argument of all actions, as this will be useful when combining all these theories together. The resource process $δ_i$ is formalized as a CongGolog program $δ_i$ over the BAT $D_i$.

**Example 1** (Resources – based on the cell described in [36]). Consider a manufacturing cell consisting of five resources. Resource $(D_1, δ_1)$ is a robot that is able to perform different operations on parts by (automatically) equipping the appropriate end effector $ee$, from a nearby rack, using the action $\text{equip}(ee, z)$. An end effector is a device or tool connected to the end of a robot arm, the nature of which depends on the intended task: by equipping a drill it can drill parts; by equipping a rivet gun it can apply rivets. The BAT $D_1$ specifies when this simple action is possible for this resource:

$$\text{Pass(equip(ee, z), s) \equiv has\_effector(ee, z, s) \land \neg} \exists e. \text{equipd(e, z, s)}$$

that is, when the end effector is available on the rack and there is no end effector already equipped by the arm.

The $\text{Poss}$ predicates for other actions are specified in the same way. The drilling operation is modeled as the action $\text{robot\_drill}$ with arguments $\text{part, bit, dmtr, speed, feed, x, y, z}$ for the ID of the part (i.e. the workpiece), the drilling bit ID, the diameter, the spindle speed, the feed rate, and hole position. For instance, a fully specified action is $\text{robot\_drill(p, bit1, \cdot, 7, 215, \cdot, 2, 123, 87, 12, z)}$. The riveting action is analogous, and we represent it as $\text{rivet(part, rivet\_type, x, y, z, i)}$. Additional operations are $\text{start\_compressor(z)}$ for charging the compressor of the rivet gun (always possible); $\text{unequip(1)}$ for unequipping the current end effector and place it back on the rack; $\text{set\_bit(bit, bit\_type, dmtr, 1)}$ for changing the drilling bit (for simplicity, we assume a rack of drill bit holders which can be changed autonomously, but it is also possible to use this action to model the fact that the robot returns to a default position to allow an operator to perform the tool change). For instance, we consider:
Fig. 2. The resource program $\delta_1$, representing a robot arm with two end effectors. The resource cannot equip an end effector and then not use it, nor attempt to equip it if already mounted on the arm. Similarly, the robot cannot release the part if in the process of drilling or riveting it. Note that these constraints are not aimed at capturing preconditions of actions, which are instead specified in the BAT $D_1$ via $\text{Poss}$, but rather the control logic governing the robot, which is dictated by the resource's own design. The actual executions that are possible will depend on the interplay of the program, the BAT and the current situation, at each step.

$$\text{Poss}\left(\text{ROBOT\_DRILL}(\text{part}, \text{bit}, \text{dmtr}, \text{speed}, \text{feed}, x, y, z, 1), s\right) \equiv \\
\text{equipd}(\text{driller}, 1, s) \land \text{material}(\text{part}, m) \land \text{suitable}(\text{bit}, \text{dmtr}, m) \land \\
(\text{at}(\text{part}, 1, s) \lor \text{within\_reach}(\text{part}, 1, s)) \land \text{part}(\text{part}, s) \land \text{safe}(1, s)$$

$$\text{Poss}\left(\text{RIVET}(\text{part}, \text{rivet\_type}, x, y, z, 1), s\right) \equiv \\
\text{equipd}(\text{rivet\_gun}, 1, s) \land \\
\text{drilled}(\text{part}, \text{hole}(x, y, z), s) \land \text{charged}(\text{compressor}, 1, s) \land \\
(\text{at}(\text{part}, 1, s) \lor \text{within\_reach}(\text{part}, 1, s)) \land \text{part}(\text{part}, s) \land \text{safe}(1, s)$$

$$\text{Poss}\left(\text{SET\_RRT}(\text{bit}, \text{bit\_type}, \text{dmtr}, 1), s\right) \equiv \text{tool\_bits}(\text{bit}, \text{bit\_type}, \text{dmtr})$$

where $\text{at}(\text{part}, 1, i, s)$ is a fluent used to specify that a part is currently allocated to the resource $i$, and which we assume to be included in the BAT of every resource. Similarly, $\text{within\_reach}(\text{part}, 1, i, s)$ specifies that the work piece $\text{part}$ is within the envelope of the resource, so that it can either be moved on the working area, or operations can be performed on $\text{part}$ from a distance. Static relation $\text{tool\_bits}(\text{bit}, \text{bit\_type}, \text{dmtr})$ represents the catalogue of tool bits available in this facility. We also use situation dependent or independent fluents, which for simplicity we assume to be available in each BAT of each resource $i$, to denote that a part exists in the cell, that a hole has been drilled in a part by a resource, that a part has a certain material, that it is safe for the robot to move the arm, etc. We will give examples of their successor-state axioms later.

Moreover, parts are moved between resources by a part-handling system, which is typically a collection of conveyor belts or similar pieces of equipment, although it is also possible that parts are explicitly exchanged by the resources themselves (e.g., robots). We model this by using special actions $\text{IN}()$ and $\text{OUT}()$ for each resource $i$, denoting that a part is moved into or out of the work area of the resource, respectively. This allows us to model either the physical movement of the part from/to the part-handling system or the acquisition/release of the exclusive access lock to the part. This approach makes sure that at most one part is allocated a part (although other resources may be required to assist in complex machining or assembly operations, as we will show later). For the $\text{IN}$ and $\text{OUT}$ actions, we assume the following axioms to impose that each resource can hold at most one part at a time (larger bounds can be modeled in a similar way):

$$\text{Poss}(\text{IN}(\text{part}, i, s) \equiv \text{part}(\text{part}, s) \land \text{within\_reach}(\text{part}, i, s) \land \exists p. \text{at}(p, i, s)$$

$$\text{Poss}(\text{OUT}(\text{part}, i, s) \equiv \text{part}(\text{part}, s) \land \text{at}(\text{part}, i, s)$$

Finally, a special action $\text{nop}$, also common to all resources, can be used to keep the resource idle. The ConGoLog program $\delta_1$ (capturing the resource process) is shown in Fig. 2. Note that all free parameters are implicitly existential (i.e., in the scope of a choice $\pi$).

We model the other resources in a similar way. $(D_2, \delta_2)$ is a fixture that can perform an action $\text{HOLD\_IN\_PLACE}(\text{part}, \text{force}, 2)$, with a given clamping force. Resource $(D_3, \delta_3)$ is a robot that can move parts into and out of the cell from an external conveyor or pallet, position a part at a given location relative to another part (creating a single composite part as output), and, by equipping either a flat or hollow end effector, apply pressure to a part that is being worked on by another resource (hollow for drilling or milling, flat for riveting). These operations correspond to the actions $\text{IN\_CELL}(\text{part}, \text{weight}, \text{material}, \text{dimx}, \text{dimy}, \text{dimz}, 3)$, $\text{OUT\_CELL}(\text{part}, \text{code}, 3)$, $\text{POSITION}(\text{part1}, \text{part2}, \text{part}, x, y, z, 3)$, $\text{PRESSURE}(\text{part}, \text{force}, 3)$. These actions for $(D_2, \delta_2)$ and $(D_3, \delta_3)$ have the following preconditions:
loop:
  NOP | (IN(part, 2) ; (NOP | HOLD_IN_PLACE(part, force, 2))*) ; OUT(part, 2))

loop:
  IN(part, 3)  
  | (EQUIP(pressure_hollow, 3) | EQUIP(pressure_flat, 3)) ;  
  NOP ; PRESSURE(part, force_type, 3)* ; UNQUIP(3)  
  | EQUIP(gripper, 3) ; POSITION(part1, part2, part, x, y, z, 3)  
  | IN_CELL(part, weight, material, dimx, dimy, dimz, 3)  
  | OUT_CELL(part, code, 3)  
  | NOP

Fig. 3. Resources programs $\delta_2$ and $\delta_4$.

$\text{Poss}(\text{hold_in_place}(part, \text{force}, 2), s) \equiv \text{part}(part, s) \land \text{at}(part, 2, s)$

$\text{Poss}(\text{in_cell}(part, \text{weight, material, dimx, dimy, dimz, 3}), s) \equiv \text{part}(part, s) \land \text{on_site}(part, s) \land \text{safe}(3, s)$

$\text{Poss}(\text{out_cell}(part, \text{code, 3}), s) \equiv \text{part}(part, s) \land \text{at}(part, 3, s) \land \text{status}(part, \text{code, 3}) \land \text{safe}(3, s)$

$\text{Poss}(\text{position}(part1, part2, part, x, y, z, 3), s) \equiv \neg \exists p. \text{holding}(p, 3, s) \land \neg \text{part}(part, s) \land \text{part}(part1, s) \land \text{part}(part2, s) \land \text{at}(part, 3, s) \land \text{within_reach}(part2, 3, s) \land \text{safe}(3, s)$

$\text{Poss}(\text{pressure}(part, \text{force_type, 3}, 3), s) \equiv \text{part}(part, s) \land \text{at}(part, 3, s) \lor \text{within_reach}(part, 3, s)) \land ((\text{equip}(\text{pressure_flat}, 3, s) \land \text{type} = \text{flat}) \lor (\text{equip}(\text{pressure_hollow}, 3, s) \land \text{type} = \text{hollow})) \land \text{safe}(3, s)$

For instance, the first specifies that $(D_2, \delta_2)$ can hold in place parts that are physically brought to the fixture. The fourth one specifies that $(D_3, \delta_3)$ can position a part part1 onto a part part2 to create a composite part part if the resource is not holding anything (with its arm), part is not yet a physical part, part1 is allocated to the resource whereas part2 is within reach (e.g., held by another resource), and finally that the resource is in a safe condition to operate. The last one specifies that $(D_3, \delta_3)$ can apply pressure to a part part that is either allocated to the resource or is within reach, provided that the right type of pressure applicator is already equipped and that the resource is in a safe condition to operate. The processes for these resources are represented by the ConGolog programs $\delta_2$ and $\delta_3$ in Fig. 3.

Note that the information about the weight, material and size of parts loaded into the cell is made available by the arguments of in_cell, so that new fluents are added to the extension to represent information about fresh workpieces (see successor-state axioms, further below). As a restriction, one is not allowed to use operators that are not axiomatized in the theories themselves (e.g., to write $\text{dimx}(part, x) \land \geq(x, 30)$ to check that a part is wider than 30 inches), but we can assume discretized intervals for each numeric parameter, and consider fluents such as $\text{dimx}(part, \text{wide})$.

Resource $(D_4, \delta_4)$ is an upright drilling machine for drilling parts with high precision. Finally, $(D_5, \delta_5)$ is a human operator, who can operate the drilling machine by executing the action $\text{operate_machine}(4, s)$, can physically enter/exit the cell with $\text{enter}(s)$ and $\text{exit}(s)$, can bring small parts into the cell with $\text{in_cell}(\text{part, \ldots, 5})$, and can apply glue to parts with $\text{spray_gluue}(\text{part, glue_type, 5})$. Hence, in $D_4$ (drilling machine) we have the precondition axiom:

$\text{Poss}(\text{machine_drill}(\text{part, bit, dmtr, speed, feed, x, y, z, 4}), s) \equiv \text{material}(\text{part, m}) \land \text{suitable}(\text{bit, dmtr, m}) \land \text{at}(\text{part, 4, s})$

and in $D_5$ (human operator):

$\text{Poss}(\text{in_cell}(\text{part, weight, material, \ldots}), s) \equiv \text{on_site}(\text{part, s}) \land \text{size}(\text{part, small})$

$\text{Poss}(\text{spray_gluue}(\text{part, glue_type, s}), s) \equiv \text{within_reach}(\text{part, 5, s}) \land \text{avail}(\text{glue_type})$

$\text{Poss}(\text{safety_switch}(5), s) \equiv \text{true}$

while the processes for these resources are represented by the ConGolog programs $\delta_4$ and $\delta_5$ in Fig. 4.

According to $\delta_5$, the human operator essentially has two distinct sets of available operations when they are inside and outside the cell (the fluent $\text{entered}(s)$ is of course affected by the execution of $\text{enter}(s)$ and $\text{exit}(s)$). Note how the operator is required (by safety regulations) to always operate a safety switch immediately before and after entering or exiting the cell. Nonetheless, the BATs for the robots are “unaware” of the existence of such an action: the Poss axioms for the robots mention a fluent $\text{safe}(i, s)$ that must be tested for machining or assembly actions (as the robot arm may need to move within its spatial envelope), but there is still no relation between the value of this fluent and the execution of $\text{safety_switch}$, as these belong to different BATs. An analogous consideration applies to the fluent $\text{within_reach}(\text{part, i, s})$, as this would require to talk about other resources nearby. This will be covered in the next section, when we will show how the BATs for individual resources are merged in a semi-automated fashion into a single $D_T$, namely the facility information model, as depicted in Fig. 1. For now, we can assume:
loop:
  in(part, 4) | machine_drill(part, bit, dmtr, speed, feed, x, y, z, 4) | out(part, 4) | NOP

loop:
  if entered(s) then
    in(part, s) | spray_glue(part, glue_type, s) | operate_machine(j, s) |
    in_cell(part, ..., s) | out(part, s) | (exit(s) ; safety_switch(off, s)) | NOP
  else (safety_switch(on, s) ; enter(s)) | NOP endif

Fig. 4. The remaining resource programs $\delta_d$ and $\delta_h$.

Finally, examples of successor-state axioms for (some of) the fluents mentioned in the BATs above are listed below (they are assumed to be the same in each BAT). For conciseness, and whenever needed, we denote by $x, y$ and $z$ the parameters that appear as dots in the fluents within the scope of quantifiers.

\[
equipd(e, i, do(a, s)) \iff a = equip(e, i) \lor (equipd(e, i, s) \land a \neq unequip(i))
\]

\[
part(part, do(a, s)) \iff \exists y_a. (a = in_cell(part, ...) \lor a = position(\ldots, part, \ldots) \lor (part(part, s) \land \exists x. a = out_cell(part, \ldots)) \lor (part(part, i, s) \lor at(part2, part, \ldots) \lor (\exists a. a = in_cell(part, \ldots) \land (a \neq out(part, i) \land \not\exists z. a = out_cell(part, \ldots)))
\]

\[
material(part, material, do(a, s)) \iff \exists y_a. (a = in_cell(part, \ldots) \lor (part(part, material, s) \land \exists x_a. a = out_cell(part, \ldots)) \lor (drilled(hole(part, x, y, z, do(a, s)) \equiv \exists y_a. (a = robot_drill(part, \ldots, x, y, z) \lor (drilled(hole(part, x, y, z), s) \land \not\exists x_a. a = out_cell(part, \ldots)))
\]

For instance, the first states that an end effector $e$ is equipped by a resource $i$ either if the resource executes action equip(e, i) or $e$ was already equipped by $i$ and it is not removed. Similarly, a part exists after it is loaded into the cell if it is a composite part resulting from a positioning action or if already in the cell and not moved out of the cell. Analogously, the third one states that a part is allocated to resource $i$ if it was moved to $i$ or it is a composite part obtained by positioning another part on one that is in $i$.

The remaining predicates are situation-independent (such as tool_bits/3, avail/1 or suitable/3), that is, their extensions are included in the theories, but are not affected by actions as they capture instead domain knowledge (on materials, manufacturing transformations, etc.). $\square$

5.2. Facility information model

The facility information model $D_F$ for the manufacturing facility is also formalized as a Situation Calculus BAT. It is in this model that a system integrator will combine, in a semi-automated manner, all information models $D_1, \ldots, D_n$ of the resources, e.g., by taking into account knowledge about which resources are connected by the part-handling system, which subsets of resources can work on the same parts, which compound actions are meaningful, etc.

To formalize the facility information model as a Situation Calculus BAT $D_F$ from the BATs $D_1, \ldots, D_n$ of the resources we need to follow several steps:

1. Define the initial situation description for situation $S^0_F$ in the BAT $D_F$ as the union of the descriptions $D_i^0$ of the BAT of the resources, $D_F^0 = D_1^0 \cup \ldots \cup D_n^0$.
2. Define the successor-state axioms of fluents for compound actions. This is done by trivially extending the successor-state axioms for simple actions that are in the BATs $D_1, \ldots, D_n$.
3. Define the Poss predicate for all and only the exact compound actions that are deemed meaningful, and establish whether each of these compound actions also requires the executability of its constituent simple actions as defined in $D_1, \ldots, D_n$. Recall that Poss(a, s) does not imply, in general, Poss(a, s) for every $a \in a$ (see Section 3, including the definition of Trans($\delta_i||\delta_j, s, s', s''$)).
4. In doing the above, possibly introduce situation-independent fluents (with their definitions) that may be needed to relate the value of fluents in the various resources and to facilitate the writing of the axioms in the theory.

Note that the set of fluents in $D_F$ is $F_F = \bigcup_i F_{D_i}$, since we do not need to include the situation-independent fluents mentioned above (which can be seen as abbreviations). The set of active object constants mentioned in $D_F$ is $AC_F = \bigcup_i AC_{D_i}$. Also, we denote by $A_F$ the set of action types in $D_F$. 
The resulting BAT $D_F$ allows for capturing any possible execution of the facility that is composed of meaningful compound actions. Note however that this does not capture any knowledge about the manufacturing transformations yet, i.e., the designed (sub)programs which lead to an increase of value of workpieces (such as a set of possibly complex sequences of actions that collectively implement the drilling, polishing, painting, etc. of a workpiece, and which typically include various steps and also auxiliary compound actions and tests). This notion will be modeled in Section 5.5.

Example 2 (Facility information model). Consider the manufacturing cell from the previous example. To be able to integrate all resources together, one needs to address various aspects, of which we consider only a few. First, successor state axioms have to consider compound actions. For instance, the first successor state axiom listed at the end of Example 1, in the previous section, trivially becomes:

$$
equip(e, i, do(a, s)) \equiv \text{EQUIP}(e, i) \in a \lor (\text{equip}(e, i, s) \land \text{UNEQUIP}(i) \notin a)$$

Second, we have to take into account the layout of the shop floor, i.e., specify which resources can collaborate for executing a compound action (while the rest remain idle). We do so by a special situation-independent predicate $\text{coopMatrix}(i, j)$ specifying that resource $(D_1, \delta_i)$ can cooperate with resource $(D_j, \delta_j)$, that is, it can perform an action on a part that is currently allocated to the other (i.e. such that $at(part, j, s)$ holds). Hence we axiomatize in $D_F$ the within reach fluent (so far assumed to be such that within reach$(part, i, s) \equiv at(part, i, s)$ – see Example 1 in the previous section) as follows:

$$\text{within reach}(part, i, s) \equiv \exists j. j \neq i \land at(part, j, s) \land \text{coopMatrix}(i, j)$$

We also connect the action of the operator with the two robots in the cell by the successor-state axiom:

$$\text{safe}(i, do(a, s)) \equiv (\text{safe}(i, s) \land \neg \exists j. \text{SAFETY\_SWITCH}(\text{off}, j) \in a) \lor \exists j. \text{SAFETY\_SWITCH}(\text{on}, j) \in a$$

Third, once this step is completed (for all predicates, as needed), one needs to define the $\text{Poss}$ predicate of compound actions. $D_F$ includes arbitrary axioms for this purpose, however the following two are typically needed:

$$\text{Poss}(a \cup \text{nop}, s) \equiv \text{Poss}(a, s)$$

$$\text{Poss}(\text{in}(part, i), \text{out}(part, j), s) \equiv \text{Poss}(\text{in}(part, i), s) \land \text{Poss}(\text{out}(part, j), s) \land \text{partHandling}(j, i)$$

The first establishes that if an action $a$ is executable, so is any compound action that is equal to $a$ but to which no $\text{nop}$ actions are added (note that no $\text{nop}$ actions can be executed by resources only when their program allows it). The second specifies the possible passing of parts between resources, as allowed by the part-handling system. It assumes a situation independent predicate partHandling/2 so that partHandling$(j, i)$ specifies that, according to the layout of the shop floor, it is possible to move a part from resource $i$ to resource $j$ (typically by using fixed conveyors). This predicate can be easily replaced with a situation-dependent fluent.

Further, it remains to explicitly specify the $\text{Poss}$ for the remaining compound actions that are meaningful in the facility although, in principle, it is also possible to automatically determine the preconditions for any compound action resulting from any possible combination of simple actions. For instance, in the running example we want to allow the compound action in which resource 1 (the first robot arm) performs a drilling operation on a part that is currently positioned on the fixture (resource 2), while resource 3 (the second robot arm) applies opposing pressure on the part (with a hollow pressure applicator). In this case we do not impose additional constraints, but we require the executability of these actions individually, according to the $\text{Poss}$ of individual theories:

$$\text{Poss}((\text{ROBOT\_DRILL}(part, \cdots, 2), \text{HOLD\_IN\_PLACE}(part, \cdots, 2), \text{PRESSURE}(part, \cdots, 3)), s) \equiv$$

$$\text{Poss}((\text{ROBOT\_DRILL}(part, \cdots, 1), \text{HOLD\_IN\_PLACE}(part, \cdots, 2), \text{PRESSURE}(part, \cdots, 3)), s) \land \text{Poss}((\text{ROBOT\_DRILL}(part, \cdots, 1), \text{HOLD\_IN\_PLACE}(part, \cdots, 2), s) \land \text{Poss}((\text{PRESSURE}(part, \cdots, 3)))$$

In turn, according to $D_1 \cdots D_4$, this requires that the part is held on the fixture, that it is within the envelope of both robots, that the right drilling bit is in the end effector, etc. (see the Poss of these simple actions in Example 1).

According to the specification of Trans, by denoting the compound action above as $[a_1, a_2, a_3]$ for short (the order is irrelevant), in our example with five resources we have that if $\text{Trans}(a_1 ||| a_2 ||| a_3 ||| \text{nop} ||| \text{nop}, s, \delta', s')$ then it must be that $\text{Poss}(a_1, a_2, a_3, \text{nop}, \text{nop}, s)$ and therefore $\text{Poss}(a_1, a_2, a_3, s)$, as a result of $\text{Poss}(a \cup \text{nop}, s) \equiv \text{Poss}(a, s)$. Given the axiom above, this also implies $\text{Poss}(a_i)$ for $i \in \{1, 2, 3\}$, but this is not true in general (it is not required by Trans) and it can be determined case-by-case when generating $D_F$. Also, it is not always the case that $\text{Poss}(a_1, a_2, a_3, \text{nop}, \text{nop}, s)$ implies $\text{Poss}(a_1, a_2, a_3, a_4, a_5, s)$ for any action $a_4, a_5$.

This gives the system integrator great freedom and flexibility, and it allows to encode arbitrary knowledge about the possible meaningful executions of a set of resources on the shop floor. \qed

5.3. Facility process

Given a set of $n$ processes of manufacturing resources, each modeled as a ConGolog program $\delta_i$ over $D_1$, $i \in \{1, \ldots, n\}$, the resulting facility process $\delta_F^0$ is defined as the ConGolog program $\delta_F^0 := \delta_1 ||| \cdots ||| \delta_n$ over the BAT $D_F$, which models
the facility information model. Observe the use of the new synchronized concurrency operator introduced in Section 3. This operator executes all programs $\delta$ synchronously, generating a compound action from the actions in the various resources. In doing so, as explained in the previous section, it requires the executability of compound actions as defined in $D_r$, without necessarily relying on the executability of their component actions as defined in the resources’ BATs $D_1, \ldots, D_n$.

5.4. Cloud information model

As explained in Section 4, the cloud information model $D_R$ is the information model common throughout the cloud to be used by every recipe. It is the core component of our MaaS framework whose existence precedes any other component. It represents a resource-independent information model of data and objects the recipes manipulate, and on which they are expressed. The information model is specified by the manager of the cloud infrastructure.

We formalize $D_R$ as an action theory $D_R$, which is a variant of a Situation Calculus BAT. Importantly, $D_R$ is specified without knowing the possible facilities in the cloud nor the resources available in each facility. For this reason, fluents in $D_R$ are abstract, and they are typically affected by the resource-independent actions used in recipes. The evolution of such fluents can be specified through the initial situation description $D_R^0$ of an initial situation $S_R^0$ of $D_R$, and successor-state axioms in $D_R$.

However, when specifying a recipe, we also want to use additional fluents whose interpretation is determined at runtime during the execution on an actual facility, i.e., during production. For instance, in a recipe we may need to prescribe a runtime test in order to determine how the manufacturing process should continue. We call such fluents observations and denote the set of observations by $\text{Obs}$. Note that the fluents in $\text{Obs}$ are not determined by the initial situation description $D_R^0$ and they do not have successor-state axioms. Their evolution depends on how the cloud information model is coupled with the facility information model through the mappings, as described in the next section.

From the Situation Calculus point of view, the resulting action theory $D_R$ is a variant of a BAT (free-fluent BATs [37]), which omits successor-state axioms for the fluents in $\text{Obs}$. We denote such variants BAT’ to stress this distinction. Observe that, even under complete information about the initial situation, a BAT’ admits, in general, many models: without mappings relating observations to the actual facility, these fluents are free to take any extension at each situation. Nonetheless, as we formalize next, as we are capable of inspecting the facility to determine the extension of the fluent in $\text{Obs}$ through the mappings, the resulting theory still admits a unique model. We will provide an example in the next section.

We denote by $\mathcal{F}_R$ the set of fluents in $D_R$, with $\text{Obs} \subseteq \mathcal{F}_R$, and by $\mathcal{AC}_{D_R}$ the set of active object constants of $D_R$. Finally, we denote by $A_R$ the set of action types in $D_R$.

5.5. Mappings

In order to establish the manufacturability of a product by a facility, we need to define formal mappings between the abstract, resource-independent BAT’ $D_R$ and the BAT $D_F$. These mappings are computed for each facility when it joins the manufacturing cloud, either automatically or by hand [36]. Following the ideas in [38], we define such mappings as follows:

- For each action type $A \in A_R$ with parameters $x$, the action $A(x)$ is mapped to a (arbitrarily complex) program $\delta_A(x)$ in $D_F$ composed of physical (compound) actions of the available resources, augmented with, e.g., the passing of parts through the part-handling equipment, the supposing of end effectors, etc.
- For each fluent $f \in \mathcal{F}_R$ with parameters $x$, the atomic formula $f(x, s_R)$ in the current situation $s_R$ is mapped to a (uniform) Situation Calculus formula $\psi_f(x, s_F)$ over the fluents in $\mathcal{F}_F$ of the facility information model. The mapping establishes that $f(x, s_R)$ in the current situation $s_R$ of the BAT $D_R$ on the recipe has the same extension as $\psi_f(x, s_F)$ in the current situation $s_F$ on the BAT $D_F$ of the facility.

If $f$ is an observation in $\text{Obs}$ (i.e., without successor-state axiom, such as precision in Example 4) the mapping gives the extension to the observation $f(x, s_R)$. If instead $f$ is not an observation in $\text{Obs}$, then the mapping imposes a consistency requirement between the two theories as the fluent $f$ is constrained by the initial situation description and its successor-state axiom in $D_R$.

Summarizing, the mappings consist of a set Maps of mapping rules of two forms:

- Mapping rules for $D_R$’s actions of the form:
  $$A(x) \leftrightarrow \delta_A(x) \quad (\text{for each } A \in A_R)$$
  where $A(x)$ is an action type in $A_R$ with parameters $x$, and $\delta_A(x)$ is a program over $D_F$ with parameters $x$;
- Mapping rules for $D_R$’s fluents of the form:
  $$f(x) \leftrightarrow \psi_f(x) \quad (\text{for each } f \in \mathcal{F}_R)$$
  where $f(x)$ is a fluent of $D_R$ with the situation argument suppressed and $\psi_f(x)$ is a situation-suppressed (uniform) Situation Calculus formula over $D_F$. 

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We impose two requirements on $\varphi(x)$ as well as on formulas occurring in tests of $\delta_A(x)$. First of all, the active object constants mentioned in some mapping come from $A_{\mathcal{D}_A} \cup A_{\mathcal{D}_B}$. Second, these formulas must be domain-independent [26]. In our context a (uniform) Situation Calculus formula $\varphi$ is domain-independent if its evaluation depends only on the objects appearing in the extension of $A_{\mathcal{D}_A}$’s fluent in the current situation, i.e., the active domain of the current situation, and not by other standard names in the domain $\Delta$. One way to obtain domain-independence is to disallow negation $\neg \beta$ and instead use logical difference $\alpha \land \neg \beta$. Note that domain-independence is a standard requirement in databases that allows focusing on the active domain only, without loss of generality [26].

**Example 3 (Mappings).** To map the resource-independent drill action to a program specifying the possible ways in which a drilling operation can be performed in the facility of the running example, we specify:

\[
\delta_1^1 = \pi \text{ bit, feed, force, } i, j, k. (\text{pressure}(\text{part, force, hollow, } i)) \]

\[
\text{hold_in_place(\text{part, } \exists k, j) } \]

\[
\text{robot_drill(\text{part, bit, dmtr, speed, feed, } x, y, z, k)}
\]

\[
\delta_2^1 = \pi \text{ bit, feed, } i, j. (\text{operate_machine}(i, j) )\]

\[
\text{machine_drill(p, bit, dmtr, speed, feed, } x, y, z, j)\]

where each $A_i$ stands for $\pi x, a_{i,1}(x) | \cdots | a_{i,|\varphi|}(x)$: i.e., each resource may perform preparatory sequences of actions before the specified compound actions. Intuitively, the rule states that, in this facility, large parts can only be drilled by using three actions for clamping (with a fixed force), drilling and applying pressure with a hollow pressure applicator (for counterbalancing the drilling pressure), whereas small parts may also be drilled by manually operating a drilling machine.

Note that these rules do not specify which resources should be used for a particular operation, hence in a large facility with many pieces of equipment this allows a high degree of flexibility in the allocation of resources to these tasks. In our simple example, with only five resources, such allocation is obvious ($\delta_1^1$ can be executed by selecting $i=3$, $j=2$, $k=1$, and $\delta_2^1$ by selecting $i=5$ and $j=4$).

We can write similar mapping rules for the fluent $\text{precision} \in \text{Obs}$, e.g., specifying how to observe the precision of a hole that was drilled:

\[
\text{precision(hole(part, } x, y, z), \text{ precision}) \leftrightarrow \text{part(part)} \land \text{drilled(hole(part, } x, y, z), i) \land \text{prec_rating(precision, i)}
\]

This mapping captures the fact that the precision of drilled holes depends on the resource that was used for the drilling (in particular, on its precision rating), which is known only at runtime, i.e., during production. □

The mappings in Maps combined with the two theories $D_R$ and $D_F$ form a new theory, defined below. This is not a traditional Situation Calculus theory since it includes two situation sorts (instead of one), which are completely independent from each other, namely: $S_F$ for the facility information model $D_F$, with initial situation $S^F_0 \in S_F$, and $S_R$ for the cloud information system $D_R$, with initial situation $S^R_0 \in S_R$.

For sort $S_F$ we have that the value of fluents at each situation $s_F \in S_F$ is completely determined by $D_F$. Defining $S_R$, instead, needs an additional effort, since the value of the fluents in $\text{Obs}$, after an action has been performed, depends only on the mappings in Maps. To handle this, we extend all action types $A \in A_R$ with an extra parameter $s_F$ ranging over situations from $S_F$. Thus, every action $a(x)$ with parameters $x$ is turned into a corresponding action $a(x, s_F)$ with parameters $x$ and $s_F$. We then make the following changes to $D_R$:

- Every $D_R$’s precondition axiom $\text{Poss}(A(x), s_R) \equiv \Phi(x, s_R)$ is changed into
  
  $\text{Poss}(A(x, s_F), s_R) \equiv \Phi(x, s_F)$

  that is, we ignore the newly introduced extra parameter.

- Every $D_R$’s successor state axiom $f(x, a(A(y), s_R)) \equiv \Phi(x, y, s_R)$, for fluent $f \notin \text{Obs}$ instantiated on the action $A(y)$, is changed into
  
  $f(x, a(A(y, s_F), s_R)) \equiv \Phi(x, y, s_R)$

  that is, as before, we ignore the extra parameter.

- For each fluent $f \in \text{Obs}$ with mapping $f(x) \leftrightarrow \varphi_f(x)$, the following successor state axiom is defined:
  
  $f(x, a(A(y, s_F), s_R)) \equiv \varphi_f(x, s_F)$

  where the formula $\varphi_f(x, s_F)$ is situation-invariant w.r.t. $s_R$, i.e., its value depends only on the extra parameter $s_F$ added to action $A$.  

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• Finally, the initial situation description \( T_R^0 \) is changed by adding \( f(x, S_R^0) \equiv \psi_f(x, S_F^0) \) for each \( f \in \text{Obs} \). Using, again, the mapping \( f(x) \mapsto \psi_f(x) \).

We denote the resulting theory as \( T_R^{Maps} \). Note that since the initial situation description of \( T_R \) is complete, so is that of \( T_R^{Maps} \); hence, by distinguishing the two situation sorts \( S_R \) and \( S_F \) for the situations, it follows that \( T_R^{Maps} \) is categorical, i.e., admits essentially a unique model.

Among the situations of \( T_R^{Maps} \) in \( S_R \), we are interested only in those that correspond to actual executions in \( T_R \), according to the mappings \( Maps \). In other words, we need to single out the situations \( s_R \in S_R \) and \( s_F \in S_F \) that correspond to each other, i.e., which are synchronized. These can be defined by induction as the smallest predicate \( \text{Syn} \) such that:

- (Base case) \( \text{Syn}(S_R^0, S_F^0) \);
- (Inductive case) \( \text{Syn}(s_R, s_F) \land \bigwedge_{A \in \text{active}} \forall x. s_F \cdot \text{Do}(\delta_A(x), s_R, s_F) \land \bigwedge_{f \in \text{Obs}} f(x, \text{do}(A(x, s_F'), s_R)) \equiv \psi_f(x, s_F') \supset \text{Syn}(\text{do}(A(x, s_F'), s_R), s_R) \).

Intuitively, this definition says that \( S_R^0 \) and \( S_F^0 \) are synchronized, and that if \( s_R \) and \( s_F \) are synchronized and we do action \( A(x) \) in \( s_R \) and, correspondingly, we execute \( \delta_A(x) \) in \( s_F \), then the resulting situations \( \text{do}(A(x, s_F'), s_R) \) and \( s_F' \) are synchronized, as long as the successor-state axioms for non-observation fluents are satisfied. We are indeed interested in those situations \( s_R \in S_R \) such that there exists a \( s_F \) such that \( \text{Syn}(s_R, s_F) \), i.e., \( \{ s_R \in S_R | \exists s_F. \text{Syn}(s_R, s_F) \} \).

5.6. Facility

Given the cloud information model, a facility information model, a facility process and a set of mappings that are represented, respectively, as \( \text{BAT} \subseteq T_R \), a \( \text{BAC} \) of \( T_R \), a ConGolog program \( \delta_R^0 \) and mappings \( Maps \), a facility is formalized by the tuple \( \text{Fac} = (T_R, \text{BAC}, \delta_R^0, Maps) \). We denote by \( AC_{\text{Fac}} \) the set of active object constants of \( \text{Fac} \), that is either in \( T_R \) or in \( \text{BAC} \), i.e., \( AC_{\text{Fac}} = AC_{T_R} \cup AC_{\text{BAC}} \).

5.7. Recipes

A recipe specifies the possible way(s) in which a product can be manufactured and is captured by a ConGolog program \( \delta_R^0 \) for the action theory \( T_R \), i.e., the cloud information model. As such, \( \delta_R^0 \) can mention only constants from \( AC_{T_R} \). This is compliant with the MaaS paradigm, where recipes are resource independent [39], i.e., specified using actions \( A_R \) and fluents \( F_R \) common throughout the manufacturing cloud rather than by adopting the action theory of a specific facility.

Example 4 (Recipe). A simple example of a process recipe is given below. This recipe makes use of the observation precision already discussed and the actions \( \text{LOAD}(\text{part, weight, material, dimx, dimy, dimz}) \), \( \text{DRILL}(\text{part, dmtr, speed, x, y, z}) \), \( \text{APPLY\_GLUE}(\text{part, glue\_type}) \), \( \text{RIVET}(\text{part, x, y, z}) \), \( \text{PLACE}(\text{part1, part2, part, x, y, z}) \), \( \text{REAMING}(\text{part, dmtr, x, y, z}) \) and \( \text{STORE}(\text{part, code}) \), all in \( A_R \). As expected, these are not the same actions available in the facility (although the names and arguments are similar, for simplicity). We have commented in Example 3 on the mapping for \( \text{DRILL} \).

According to this recipe, two steel parts denoted by \( b \) and \( f \) are loaded, a hole is drilled in \( f \), then glue is applied to \( b \). This is then placed on \( f \), resulting in a composite part \( fb \). The loading of \( b \) and the drilling of \( f \) can occur in any order, but glue must be applied to \( f \) before \( b \) is placed. If the resource used for drilling is not high-precision (which is an observation – see Example 3), a reaming operation is performed on the hole. Finally a rivet is applied to the hole and \( fb \) is stored away, with an ok code.

\[
\text{LOAD}(f, 4, \text{steel}, 810, 756, 29) ;
(\text{LOAD}(b, 2, \text{steel}, 312, 23, 20) \parallel \text{DRILL}(f, .3, 200, 123, 89, 21)) ;
\text{APPLY\_GLUE}(b, \text{str\_adhesive}) ; \text{PLACE}(b, f, fb, 7, 201, 29) ;
\text{if} \quad \neg\text{precision}(\text{hole}(f, 123, 89, 21), \text{high}) \quad \text{do}
\quad \text{REAMING}(fb, .3, 123, 89, 21) \text{RIVET}(fb, 123, 89, 21) ; \text{STORE}(fb, \text{ok}) \; \square
\]

A recipe does not specify how the manufacturing operations should be executed because it is resource-independent. The specific implementation of these actions on a facility needs to be automatically synthesized.

6. Realizability

In this section we formally characterize the conditions under which we can say that a recipe is realizable by a facility. Intuitively, this requires that each abstract action executable by the recipe can be mapped to a (sequence of) executable
compound actions of the facility. In other words, no matter how the recipe may be progressed, the facility always has a way of replicating it. Moreover, whenever the recipe may be completed, the facility is in a final (i.e., safe) state.

We denote by $\langle \delta, s \rangle$ the configuration of an arbitrary program $\delta$ in situation $s$. We use $\langle \delta_R, s_R \rangle$ to denote the current program $\delta_R$ and situation $s_R \in S_R$ to which the facility process $\delta_R^0$ evolved from situation $S_R^0$. Similarly we use $\langle \delta_F, s_F \rangle$ to denote the current program $\delta_F$ and situation $s_F \in S_F$ to which the program $\delta_F^0$ evolved from situation $S_F^0$. Observe that $S_R$ is the set of situations of $D_R^{Maps}$, not of $D_R$; indeed we need to use the theory $D_R^{Maps}$ to evolve situations of the facility, so as to take into account the evolution of fluents $f \in Obs$. Initially, these configurations are $\langle \delta_F^0, S_F^0 \rangle$ and $\langle \delta_R^0, S_R^0 \rangle$.

We are ready to formally capture what it means for a facility to always be able to replicate the actions of a recipe, i.e., when the recipe can be realized by the facility. Formally, a recipe $\delta_R$ in situation $s_R$ can be realized by a facility $\delta_F$ in $s_F$ if:

**r0** for every non-observation fluent $f \notin Obs$ in $D_R$ the value in situation $s_R$ of $\langle \delta_F, s_F \rangle$ is compatible through the mapping $f(x) \mapsto \psi_f(x)$ with the value of $\psi_f(x)$ in situation $s_F$ of $\langle \delta_F, s_F \rangle$; note that for $f \in Obs$ this is already guaranteed by the definition of $D_R^{Maps}$;

**r1** Final($\delta_R, s_R$) implies Final($\delta_F, s_F$): if the recipe can be legally terminated, then all the resources can (safely) terminate their execution; and

**r2** for every possible executable abstract action $A(x)$ from $\langle \delta_R, s_R \rangle$ in the recipe, there exists a (arbitrarily complex) program $\delta_A(x)$, determined through the mapping $A(x) \mapsto \delta_A(x)$, that is executable from $\langle \delta_F, s_F \rangle$ to some $\langle \delta_F', s_F' \rangle$ and which represents the implementation of the action $A(x)$ in the facility. Crucially, for at least one such $\langle \delta_F', s_F' \rangle$, $\langle \delta_R, do(A(x, s_F'), s_R) \rangle$ is realized by $\langle \delta_F', s_F' \rangle$.

Note that in **r2**, the recipe situation $do(A(x, s_F'), s_R)$, resulting from the execution of the abstract action $A(x)$ in situation $s_R$, depends on the situation $s_F'$ reached by the facility after the execution of $\delta_A(x)$. This captures the synchronization of the recipe and the facility situations.

Formally, we define the notion of realizability by co-induction as the largest predicate $R$ between recipe and facility configurations such that:

$$\langle \delta_R, s_R \rangle R \langle \delta_F, s_F \rangle \iff$$

$$\begin{align*}
\textbf{r0} & \quad \forall f \in Obs \forall x : f(x, s_R) \equiv \psi_f(x, s_F) \land \\
\textbf{r1} & \quad \text{Final}(\delta_R, s_R) \lor \text{Final}(\delta_F, s_F) \\
\textbf{r2} & \quad \forall A \in \mathcal{A}_R \forall s_R' \in S_R. \exists \delta_R' \in S_R. \exists s_F' \in S_F : \text{Trans}(\delta_R, s_R', \delta_R', s_R') \land \\
& \quad \exists \delta_F', s_F' \in S_F : \text{Trans}(\delta_F, s_F, \delta_F', s_F') \land \text{Do}(\delta_A(x), s_F', s_F') \land (\delta_R', do(A(x, s_F'), s_R)) \land R(\delta_F', s_F')
\end{align*}$$

where $A(x) \mapsto \delta_A(x)$ and $f(x) \mapsto \psi_f(x)$ are the mappings in Maps. A relation $R$ satisfying the conditions above is called realizability. We say that a facility configuration $\langle \delta_F, s_F \rangle$ realizes a recipe configuration $\langle \delta_R, s_R \rangle$, written $\langle \delta_R, s_R \rangle \preceq \langle \delta_F, s_F \rangle$, if there exists a realizability relation $R$ such that $\langle \delta_R, s_R \rangle R \langle \delta_F, s_F \rangle$. It is easy to see that this is itself a realizability relation and, in fact, the largest one.

We observe that this definition bears some similarities with the definition of simulation in [40], however it takes into account that the value of some fluents in the cloud information model come from the facility and that these values are not controllable by the recipe itself.

To understand the above formula, observe that from the preconditions axioms in $D_R^{Maps}$, which ignore the facility situation arguments, we have that if $\text{Poss}(A(x, s_F'), s_R)$ for some $s_F'$ then $\text{Poss}(A(x, s_F'), s_R)$ for all $s_F'$. Now considering that $\text{Trans}(A(x, s_F'), s_R, \delta', s_R') \equiv s_R' = do(A(x, s_F'), s_R) \land \text{Poss}(A(x, s_F'), s_R) \land \delta' = \epsilon$ we can suppress the situation argument from $A(x, s_F')$ in programs $\delta_R$ and use simply $A(x)$ as action term in $\text{Trans}$, by defining:

$$\text{Trans}(A(x), s_R, \delta', s_R') \equiv \exists s_F' \in S_F. s_R' = do(A(x, s_F'), s_R) \land \text{Poss}(A(x)[s_F'], s_R) \land \delta' = \epsilon$$

Moreover if for some $s_F'$ we have $\text{Trans}(\delta_R, s_R, \delta', s_R')$ then for every $s_F''$ we have $\text{Trans}(\delta_R, s_R, \delta', s_R')$. Now considering that $\text{Trans}(A(x, s_F'), s_R, \delta', s_R') \equiv s_R' = do(A(x, s_F'), s_R) \land \text{Poss}(A(x, s_F'), s_R) \land \delta' = \epsilon$.

Based on these considerations, in $D_R^{Maps}$ we are allowed to use any situation term of $D_F$ (in particular, the situation constant $S_F^0$) as a placeholder for the situation of $D_R$, since such situation term does not affect the executability of $A(x)$ at $s_R$ nor the program $\delta_R'$ resulting after its execution.

**Definition 1 (Realizability).** We say that a recipe $\delta_R^0$ is realizable by a facility $Fac = \langle D_R, D_F, \delta_R^0, Maps \rangle$ iff $\langle \delta_R^0, S_R^0 \rangle \preceq \langle \delta_F^0, S_F^0 \rangle$.

When $\delta_R^0$ is realized by $\delta_F^0$ then, at every step, given a ground abstract action $A(x)$ selected by $\delta_R^0$, an execution of the corresponding program $\delta_A(x)$ exists that preserves the realizability relation $\preceq$, and at the end of which control is returned to the recipe for the selection of the next action. Notice that $\delta_A(x)$ is nondeterministic in general, as CondLog programs may include choices of arguments and nondeterministic branching. Nonetheless, the existence of the realizability relation guarantees that this is possible (similarly to [40], we assume that nondeterminism in $\delta_F^0$ is "angelic").
Note that once we have that \( \langle \delta_R^0, S^0_R \rangle \leq \langle \delta_F^0, S_F^0 \rangle \) we can evolve (in all possible ways) the two configurations \( \langle \delta_R^0, S^0_R \rangle \) and \( \langle \delta_F^0, S_F^0 \rangle \) in a synchronized way so as to maintain them in the relation \( \leq \). Formally, we can define by induction the smallest predicate \( \text{SynC} \) such that:

- **(Base case)** \( \text{SynC}(\delta_R^0, S^0_R, \delta_F^0, S_F^0) \);

- **(Inductive case)** \( \text{SynC}(\delta_R, S_R, \delta_F, S_F) \land \text{Trans}(\delta_R, S_R, \delta_R', S_R') \land \delta_R' = \langle \text{do}(A(x, s_F'), s_R) \rangle \land \text{Trans}^*(\delta_F, S_F, \delta_F', S_F') \land \langle \delta_R', S_R' \rangle \leq \langle \delta_F', S_F' \rangle \supset \text{SynC}(\delta_R', S_R', \delta_F', S_F'). \)

This predicate \( \text{SynC} \) can be seen as a refinement of the predicate \( \text{Syn} \) introduced above, in the precise sense given by the next proposition.

**Proposition 1** (Synchronization). Let \( \text{Fac} = (D_R, D_F, \delta_F^0, \text{Maps}) \) be a facility and \( \delta_F^0 \) a recipe realized by the facility, i.e., such that \( \langle \delta_R^0, S_R^0 \rangle \leq \langle \delta_F^0, S_F^0 \rangle \). Then for all pairs of configurations \( \langle \delta_R, S_R, \delta_F, S_F \rangle \) we have that \( \text{SynC}(\delta_R, S_R, \delta_F, S_F) \supset \text{Syn}(S_R, S_F) \).

**Proof.** By induction, immediately by the definitions. \( \square \)

This proposition guarantees that in the execution of recipe \( \delta_F^0 \) (from \( S_R^0 \)), as long as we follow the relation \( \leq \), we only generate values for the fluents in \( F \in \text{Obs} \) that come from corresponding concrete executions in the facility.

We define a controller as follows. Given the current configuration \( \langle \delta_R, S_R \rangle \) of the recipe, the current facility configuration \( \langle \delta_F, S_F \rangle \), and a new configuration \( \langle \delta_R', S_R' \rangle \) specifying the next recipe configuration that may result from the one-step execution of \( \delta_R \) through some action \( A(x) \), a controller returns a sequence of configurations \( \langle \delta_R^0, S_R^0 \rangle \ldots \langle \delta_R^m, S_R^m \rangle \) of length \( m \geq 0 \) representing the steps that the facility must execute in order to complete \( \delta_R(x) \). We require that the recipe and facility configurations are in the realizability relation before and after executing \( A(x) \) and \( \delta_R(x) \).

**Definition 2** (Controller). Given a facility \( \text{Fac} = (D_R, D_F, \delta_F^0, \text{Maps}) \) and a recipe \( \delta_F^0 \) realizable by \( \text{Fac} \), a controller for \( \delta_F^0 \) that realizes \( \delta_F^0 \) is a function \( \rho \) that, given two configurations \( \langle \delta_R, S_R \rangle \) and \( \langle \delta_F, S_F \rangle \) such that \( \langle \delta_R, S_R \rangle \leq \langle \delta_F, S_F \rangle \), an action \( A(x) \), and a program \( \delta_R' \) such that \( \text{Trans}(\delta_R, S_R, \delta_R', S_R') \) (recall that \( S_R \) is used as a placeholder, and does not affect \( \delta_R' \)), returns a sequence of facility configurations \( \langle \delta_R^0, S_R^0 \rangle \ldots \langle \delta_R^m, S_R^m \rangle \), such that:

- \( \text{Trans}(\delta_R^i, S_R^i, \delta_R^{i+1}, S_R^{i+1}) \) for \( i \in [0, m-1] \), and \( \delta_R^0 = \delta_R \) and \( S_R^0 = S_F \), that is, the sequence is executable in the facility;
- \( \text{Do}(\delta_R(x), S_R, S_R^m) \) that is, the situation \( S_R^m \) is the result of executing the concrete program \( \delta_R(x) \) corresponding to \( A(x) \) from \( S_R \);
- \( \langle \delta_R^m, \text{do}(A(x, S_R^m), S_R) \rangle \leq \langle \delta_F^m, S_F^m \rangle \), that is, realizability between the resulting programs is preserved. \( \blacksquare \)

In other words a controller is a function that, given a recipe and a facility configuration such that \( \langle \delta_R, S_R \rangle \leq \langle \delta_F, S_F \rangle \), an action \( A(x) \) and the remaining program \( \delta_R' \) after action execution, returns the witnesses for the realizability, i.e., a sequence of steps to progress to next recipe and facility configurations in such a way that \( \langle \delta_R^0, \text{do}(A(x, S_R^m), S_R) \rangle \leq \langle \delta_F^m, S_F^m \rangle \). As a result, given the initial recipe and facility configurations \( \langle \delta_R^0, S_R^0 \rangle \) and \( \langle \delta_F^0, S_F^0 \rangle \), for every possible evolution of \( \langle \delta_R^0, S_R^0 \rangle \) as determined by the choice of actions \( A(x) \) at each point, a controller \( \rho \) produces a corresponding evolution of \( \langle \delta_F^0, S_F^0 \rangle \) that fulfills the realizability requirements, and hence determines the sequence of concrete configurations that the facility must traverse in order to realize the recipe. Finally, observe that any evolution determined by any controller \( \rho \) defined as above is such that \( \text{SynC} \) (and hence \( \text{Syn} \)) holds.

7. Controller synthesis

To check whether a realizability relation exists and, if so, build a controller, we resort to model checking for a variant of the (modal) \( \mu \)-calculus in [41,35], which we call \( \mu_C \), interpreted over game arenas (GA), which are special (labeled) transition systems (TSs) capturing the rules of a turn-based game between two players, \( \text{ENVIRONMENT} \) and \( \text{CONTROLLER} \).

A (first-order) vocabulary is a pair \( \sigma = (F, AC) \), where \( F \) and \( AC \) are sets of, respectively, fluents and active (object) constants. Given an interpretation domain \( \Delta \) with standard names, such that \( AC \subseteq \Delta \), we denote by \( T^\Delta \) the set of all possible interpretations of \( F \) and \( AC \) over \( \Delta \) that interpret all constants from \( AC \) as themselves.

**Definition 3** (Game arena). Given a vocabulary \( \sigma = (F, AC) \), let:

- \( \text{turnCtrl} \) and \( \text{turnEnv} \) be special 0-ary fluents (i.e., propositions) not in \( F \);
- \( F_T = F \cup \{\text{turnCtrl}, \text{turnEnv}\} \);
- \( \sigma_T = (F_T, AC) \).
A game arena over $\sigma$ is a tuple $T = (\Delta_T, Q, q_0, \rightarrow, I)$, where:

- $\Delta_T$ is the GA object domain, with standard names, such that $AC \subseteq \Delta_T$;
- $Q$ is the set of GA states;
- $q_0 \in Q$ is the GA initial state;
- $\rightarrow : Q \times Q$ is the GA transition relation;
- $I : Q \mapsto I_{\Delta_T}$ is a labeling function associating each state $q \in Q$ with an interpretation $I(q) = (\Delta_T, I(q)) \in I_{\Delta_T}$, such that exactly one among turnCtrl and turnEnv is true (and all constants in $AC$ are interpreted as themselves).

We observe that the definition of game arena is consistent with the definition of labeled transition system in [35], thus all related results in [35] are applicable here (provided the respective hypotheses hold).

Given a facility $Fac = (D_R, D_f, \delta^0_R, Maps)$ and a recipe $\delta^0_R$, we next define the GA $T$ induced by $Fac$ and $\delta^0_R$. This GA essentially captures those asynchronous executions of $\delta^0_R$ over $D_f$ and $\delta^0_R$ over $D_R^{Maps}$ such that the fluents of $T_R^{Maps}$ are correctly synchronized with those of $D_R$, as per the mappings (over fluents, not actions) in $Maps$.

In defining $T$, it will be convenient to use a different, yet equivalent, representation of programs, to separate the assignments of “pick variables” to domain objects (resulting from the nondeterministic choice of arguments), i.e., the data, from the control flow, i.e., the program counter [42]. This will simplify proving our results. The new representation, which we call split representation, has the advantage of making explicit the structure of programs and, in particular, of isolating the data as the only source of infiniteness of a program’s closure. Indeed, since the ConGolog programs we consider here are not recursive, they yield, when executed, only finitely many program counters; on the other hand, for the finitely many program variables, there exist, in general, infinitely many possible assignments to distinct domain objects. Since programs are obtained by combining program counters with assignments, they are infinitely many only as a consequence of the infinite number of possible assignments to pick variables.

We represent a program $\delta^0$ as the pair $\langle \delta, x \rangle$, where $\delta$ denotes its current program counter (defined below) and $x = \langle x_1, \ldots, x_k \rangle$ is a tuple of object terms such that each $x_i$ is the domain object currently assigned to the $i$-th pick variable of $\delta^0$ (we assume an ordering of simple actions and parameters). We call $x$ the (current) environment. Note that the new representation is merely a syntactic variant: as shown in [42], we can reconstruct the original program $\delta^0$ by replacing the free pick variables of $\delta$ by the object terms assigned to the variables $x$. This is denoted by writing $\delta[x]$. Indeed, while the set of possible environments remains infinite along a computation, at each state the environment term, consisting of a single tuple of arity $k$, maintains only a bounded number of values (smaller than the size of the programs).

The set of possible program counters for a program is formalized as the program’s syntactic closure, defined by extending the definition in [42] to the new operator $||$.

Definition 4 (Syntactic closure of a program). The syntactic closure of a program $\delta^0$ is the set $\Gamma_{\delta^0}$ inductively defined as follows:

1. $\delta^0, e \in \Gamma_{\delta^0}$;
2. if $\delta_1; \delta_2 \in \Gamma_{\delta^0}$ and $\delta^* \in \Gamma_{\delta^0}$, then $\delta^*_1; \delta^*_2 \in \Gamma_{\delta^0}$ and $\Gamma_{\delta^0} \subseteq \Gamma_{\delta^0}$;
3. if $\delta_1 \mid \delta_2 \in \Gamma_{\delta^0}$ then $\Gamma_{\delta^0} \subseteq \Gamma_{\delta^0}$;
4. if $x \in \delta^0$ then $\Gamma_{\delta^0} \subseteq \Gamma_{\delta^0}$;
5. if $\delta^*_1 \in \Gamma_{\delta^0}$ then $\delta^*_1 \in \Gamma_{\delta^0}$;
6. if $\delta^*_1 \in \Gamma_{\delta^0}$ and $\delta^*_2 \in \Gamma_{\delta^0}$ then $\delta^*_1 \parallel \delta^*_2 \in \Gamma_{\delta^0}$;
7. if $\delta^*_1 || | \delta^*_2 \in \Gamma_{\delta^0}$ and $\delta^*_1 \in \Gamma_{\delta^0}$ and $\delta^*_2 \in \Gamma_{\delta^0}$ then $\delta^*_1 || | \delta^*_2 \in \Gamma_{\delta^0}$. ■

Observe that since the environment is separated from the program counter, the syntactic closure $\Gamma_{\delta^0}$ of a program $\delta^0$ is always a finite set. We stress that the split representation is just a syntactic variant of program representation, equivalent to the standard one, so we can freely switch between them without affecting the results.

The GA is a turn-based game between two players, called Controller (CTRL) and Environment (ENV, not to be confused with the variable environment used above for programs), who take care of progressing, respectively, the recipe and the factory. Turns are not strictly alternating. The components of the GA $T = (\Delta_T, Q, q_0, \rightarrow, I)$ induced by $Fac$ and $\delta^0_R$ are detailed below.

Vocabulary For the vocabulary $\sigma = \langle F, AC \rangle$, we define:

- $F = F_R \cup F_T \cup \{pR, envR, Act, xAct, pcF, envF, pcA, envA, finalEnv, finalCtrl, finalA\}$, where fluents in $F_R$ and $F_T$ have the situation argument suppressed, while the remaining fluents are added for book-keeping (their role will become clear when we consider the labeling function for states). Specifically, $pR, Act, pcF$, and $pcA$ are unary; the arity of $envF$ and $envR$ matches the number of pick variables in $\delta^0_R$ and $\delta^0_R$, respectively; the arity of $xAct$ is the maximum number $N_v$ of variables $x$ among all actions $A \in A_R$, and the arity of $envA$ is $N_v + N_p$, with $N_p$ the maximum number of pick variables among all programs $\delta_A(x) \in P$, with $P$ the set of programs $\delta_A(x)$ such that $A(x) \leftrightarrow \delta_A(x) \in Maps$, for some $A; finalEnv$, finalCtrl, and finalA are propositions;
• $AC = AC_{D_R} \cup AC_{D_A} \cup \mathcal{A}_R \cup \bigcup_{\delta} \Gamma_{D_R} \cup \bigcup_{\delta} \Gamma_{D_A} \cup \bigcup_{\delta} \Gamma_{\mathcal{A}_R}$, for $P$ as above.

Observe that actions from $D_R$ and program counters act as active constants (as well as objects). We require that $pcR, Act, pcF$, and $pA$ are sorted predicates that can take only objects from, respectively, $\Gamma_{D_R}, \mathcal{A}_R, \Gamma_{D_A}$, and $\bigcup_{\delta} \Gamma_{\mathcal{A}_R}$, which, thus, act as sorts. No other predicate can take objects from these sorts.

Object domain The GA object domain is $\Delta_T = \Delta \cup \Gamma_{D_R} \cup \bigcup_{\delta} \Gamma_{D_A} \cup \bigcup_{\delta} \Gamma_{\mathcal{A}_R} \cup \bigcup_{\delta} \Gamma_{\mathcal{A}_F} \cup \bigcup_{\delta} \Gamma_{\mathcal{P}_F} \cup \bigcup_{\delta} \Gamma_{\mathcal{S}_F}$). Recall that $\Delta_T$ has standard names, thus, in particular, program counters and actions act as both (active) constants and objects.

Set of states Let $C_{\delta,s}$ be the set of configurations to which an arbitrary program $\delta$ can evolve, starting from situation $s$. We let $C_{\delta,s} = C_{\delta,s}', C_{\delta,s} = C_{\delta,s}''$, and $C_F = \bigcup_{\delta \in \delta} C_{\delta,s}$, for $P$ as above (recall that these sets are defined over situations of $D_{\mathcal{R}}^\text{Maps}$). Notice that programs in configurations are actual programs, with actual parameters assigned to variables, i.e., they are not split into program counter and variable environment; consequently, the configuration spaces defined above are, in general, infinite. The set of states $Q \subseteq \{\text{Env}, Ctrl\} \times C_{\delta,s} \times C_{\delta,s} \times \bigcup_{\delta \in \delta} C_{\delta,s}$ is defined by mutual induction with the transition relation $\rightarrow$ (see transition relation below). Each state $q = (\vartheta, (\vartheta, s_R), (\vartheta, s_F), (\delta, s))$ is such that $s = s_F$. For notational convenience, we omit $s$ and avoid tuple nesting, using the following equivalent representation: $q = (\vartheta, (\vartheta, s_R), (\vartheta, s_F), (\delta, s))$. Each state captures a configuration of the GA, where: $\vartheta$ represents which player is next to move; $(\vartheta, s_R)$ stands for the current recipe configuration; $(\vartheta, s_F)$ is the current facility configuration; and $(\delta, s)$ is the current configuration of the program $\delta(x) \in P$ associated by Maps with the (most recent) action $\mathcal{A}$ executed by $\delta_R$. The definition of the transition relation specifies how these components evolve as the game proceeds. Notice that $Q$ is in general infinite, as so are the configuration spaces over which it is defined.

Initial state For the initial state, we let $q_0 = (\{\text{Env}, C_{\delta,s} \times C_{\delta,s} \times \bigcup_{\delta \in \delta} C_{\delta,s}\}, e)$. This captures the situation where: Environment is to move next; the recipe has not started yet (thus no action was executed); the factory program has not started yet; and no program from $P$ has been selected (as no action has been executed).

Transition relation The transition relation $\rightarrow \subseteq Q \times Q$ is defined by mutual induction with the set of states $Q$, as follows:

- $q_0 \in Q$;
- for $q = (\vartheta, (\vartheta, s_R), (\vartheta, s_F), (\delta, s))$ and $q' = (\vartheta', (\vartheta', s'_R), (\vartheta', s'_F), (\delta', s'))$, if
  - $\vartheta = \text{Env} \land \text{Trans}(\delta_R, s_R, (\delta'_R, s'_R)) \land \exists \mathcal{A}, x, s'_R = do(\mathcal{A}(x, s_R), s_R) \land \delta' = \delta_R \land \delta'_R = \delta_R \land s'_F = s_F \land \delta' = \text{Ctrl}$, or
  - $\vartheta = \text{Ctrl} \land (\delta'_R = \delta_R \land \exists \mathcal{A}, x, s'_R, s'_F, s_R = do(\mathcal{A}(x, s'_R), s'_F) \land s'_F = do(\mathcal{A}(x, s'_R), s'_F) \land
    \text{Trans}(\delta_R, s_R, (\delta'_R, s'_F)) \land (\vartheta' = \text{Env} \rightarrow \text{Final}(\delta', s'_F))$, then $q' \in Q$ and $q \rightarrow q'$.

Transitions differ based on which player moves. Environment selects an action $\mathcal{A}(x)$, together with the corresponding program $\delta_R(x)$, from those made available by the recipe $\delta_R$ (initially $\delta_R^0$) in the current configuration; Environment then advances the recipe configuration and the cloud situation $s_R$ of $D_{\mathcal{R}}^\text{Maps}$, consistently with the chosen action, and finally passes the turn to Controller. Notice that $s_R$ is advanced according to the initial facility situation, i.e., $s'_R = do(\mathcal{A}(x, s_R^0), s_R)$. Recall that $D_F$’s situation argument of $\mathcal{A}$ does not affect the interpretation of $D_R$’s non-observation fluents.

Controller chooses one among the actions that are currently legal for both $\delta_R$ (initially $\delta_R^0$) and $\delta$ in their current configuration. Observe that $\delta$ is assigned to $\delta_R(x)$ by Environment at its turn. After selecting the action, Controller advances $\delta, \delta_R,$ and $s_F$, consistently with the chosen action. This step corresponds to one execution step of $\delta$, i.e., one step of the implementation of action $\mathcal{A}(x, \cdot)$ previously selected by Environment, which is actually realized by the factory. In addition, Controller aligns the current cloud situation $s_R$ with the resulting factory situation $s'_R$ (recall that the interpretation of $D_R$’s non-observation fluents is not affected by the $D_F$’s situation argument in $\mathcal{A}$). The situation $s'_R$ where Environment chose the action $\mathcal{A}(x, \cdot)$ currently under execution is retrieved, and the cloud situation is assigned to $s'_R = do(\mathcal{A}(x, s'_F), s'_R)$. Notice that $s'_R$ is the situation resulting from the execution of $\mathcal{A}(x)$ at $s'_F$, given the factory situation $s'_F$. Controller can (but does not have to) return the turn to Environment only when $\delta$ has reached a final configuration.

Labeling function Let $q = (\vartheta, (\vartheta, s_R), (\vartheta, s_F), (\delta, s)) \in Q$ be a generic state of $T$:

- for every $f \in \mathcal{F}, f^{T(q)}(x) \iff D_{\mathcal{R}}^\text{Maps} = f(x, s_F)$;
- for every $f \in \mathcal{F}, f^{T(q)}(s_F) \iff D_{\mathcal{R}}^\text{Maps} = f(x, s_F)$;
- for every $f \in \mathcal{F}, f^{T(q)}(s_F) \iff D_{\mathcal{R}}^\text{Maps} = f(x, s_F)$;
- $pR^{T(q)} = s_R, \text{env}R^{T(q)} = [x]$, where $(\delta_R, x)$ is the split representation of $\delta_R$ into program counter and environment: $x$ is as $x$, possibly extended with trailing null values to match the arity of envR. For notational convenience we omit the $\sim$ symbol in the following, implicitly assuming that $x$ is suitably extended when needed; notice that both $pR$ and $\text{envR}$ are interpreted as singlets;
• for pcF and envF, and for pcA and envA, we proceed as above but considering, respectively, \( \delta_F \) and \( \delta \); the resulting interpretations are, again, singletons;
• \( Ac^{(q)} = [A] \) and \( xAct^{(q)} = [X] \) if \( \exists s_R, s_F, s_R = do(A(x, s'_F), s'_R) \), and \( Ac^{(q)} = [\emptyset] \), \( xAct^{(q)} = [\emptyset] \) otherwise;
• finally, we have that \( \text{finalEnv} = \text{Final}(\delta_F, s_R), \text{finalCtrl} = \text{Final}(\delta_F, s_F), \text{and finalA} = \text{Final}(\delta, s_F) \).

The labeling function provides the interpretation of the fluents in \( \mathcal{F} \), plus additional information about game turn, recipe action under execution, and configuration of the involved programs, associated with the current state of the game. This information is used to interpret \( \mu_{L_c} \) formulas on \( T \) (labelings retain all the relevant information).

Observe that \( T \) captures the moves available to \text{ENVIRONMENT} and \text{CONTROLLER}, but not the goal of the game. Such moves essentially correspond to: (i) execution steps (action executions) of the recipe process \( \delta_R^0 \) for \text{ENVIRONMENT}; (ii) execution steps of the factory process \( \delta_R^0 \) for \text{CONTROLLER}. \text{ENVIRONMENT} moves first, and when \text{CONTROLLER} returns the turn to \text{ENVIRONMENT}, a complete execution of the program associated with the last action selected by \text{CONTROLLER} has been correctly completed.

### 7.1. Controller synthesis from GAs

\( \mu_{L_c} \) model checking can be used both to compute a realizability relation between the recipe and the facility, and to synthesize the corresponding controller. Formulas of \( \mu_{L_c} \) have the following syntax:

\[
\Phi := \phi \mid \neg \Phi \mid \Phi_1 \wedge \Phi_2 \mid (\neg \Phi) \mid Z \mid \mu Z. \Phi \mid v Z. \Phi ,
\]

where: \( \phi \) is a FO sentence with predicates and (active) constants from a given vocabulary \( \sigma = (\mathcal{F}, AC) \); the modal operator \( (\neg \Phi) \) denotes the existence of a transition from the current state to a state where \( \Phi \) holds; we use the abbreviation \([\neg \Phi]\) for \( (\neg \Phi) \); \( Z \) is a second-order (SO) predicate variable over sets of states, and \( \mu Z. \Phi \) and \( v Z. \Phi \) denote the least and greatest fixpoints, respectively, with \( \Phi \) seen as a predicate transformer with respect to \( Z \). By the semantics below, one can see that the only interesting formulas are those closed w.r.t. to SO (in addition to FO) variables. In fact, SO variables are needed only for technical reasons, to make the fixpoint constructs available.

Note that \( \mu_{L_c} \) is a (strict) sub-language of the language \( \mu_{L_p} \) defined in [35]. Specifically, \( \mu_{L_c} \) disallows quantification across-states, i.e., the possibility of relating objects occurring in different states. As a consequence, all the results obtained for \( \mu_{L_p} \) are directly applicable to \( \mu_{L_c} \).

Given a GA \( T = (\Delta_T, Q, q_0, \tau, I) \) over a vocabulary \( \sigma = (\mathcal{F}, AC) \), \( \mu_{L_c} \) formulas over \( T \) are defined over the vocabulary \( \sigma_T \) (which, by Definition 3, is obtained from \( \sigma \) by extending \( \mathcal{F} \) with \( \text{turnCtrl} \) and \( \text{turnEnv} \)). The semantics of a \( \mu_{L_c} \) formula \( \Phi \) over \( T \) is inductively defined as follows, where \( v \) is an assignment from SO variables to sets of states:

\[
\begin{align*}
(\phi)^T &= [q \mid q \in Q \land T(q) \models \phi] \\
(\neg \Phi)^T &= Q \setminus (\Phi)^T \\
(\Phi_1 \wedge \Phi_2)^T &= (\Phi_1)^T \cap (\Phi_2)^T \\
(\neg \Phi_1)^T &= \{ q \mid \exists q', q \rightarrow q', q' \in (\Phi_1)^T \} \\
(Z)^T &= \nu(Z) \\
(\mu Z. \Phi)^T &= \{ \mathcal{E} \subseteq Q \mid (\Phi)^{T[Z/\mathcal{E}]} \subseteq \mathcal{E} \} \\
(v Z. \Phi)^T &= \{ \mathcal{E} \subseteq Q \mid \mathcal{E} \subseteq (\Phi)^{T[Z/\mathcal{E}]} \}
\end{align*}
\]

A state \( q \in Q \) is said to satisfy a \( \mu_{L_c} \) formula \( \Phi \) (under a SO assignment \( v \)), if \( q \in (\Phi)^T_v \). We say that \( T \) satisfies \( \Phi \) if \( q_0 \in (\Phi)^T \). Observe that when \( \Phi \) is closed w.r.t. SO variables, as are formulas of practical interest, \( v \) becomes irrelevant. When not needed, we omit \( v \) from \((\cdot)^T\), thus using \((\cdot)^T\).

As we will show below, the satisfaction of the following \( \mu_{L_c} \) formula by \( T \) implies the existence of a realizability relation between \( \delta_R^0 \) and \( \delta^0 \).

\[
\Phi_{\text{Real}} = \forall X . \mu Y . ((\phi_{\text{ok}} \land [\neg] X) \lor (\text{turnCtrl} \land (\neg Y))),
\]

where \( \phi_{\text{ok}} = \bigwedge_{f \in F_{\text{Ctrl}}} \forall X . \psi_f(x) \land \text{turnEnv} \land (\text{finalEnv} \supset \text{finalCtrl}) \), with \( \psi_f(x) \) being the situation-suppressed version of \( \psi_f(x, s_F) \). Notice that fluents in the labeling preserve the same names as in \( D_R \) and \( D_F \), thus the situation-suppressed version of \( \psi_f \) is defined over \( T \)’s vocabulary \( \sigma \). Intuitively, \( \phi_{\text{ok}} \) holds in those states \( q \) of \( T \) where: (i) the interpretation of every fluent \( f \in F_R \setminus \text{Obs} \) in the labeling of \( q \) matches the interpretation of the corresponding formula \( \phi_f \) over the same labeling; (ii) it is \text{ENVIRONMENT}’s turn; and (iii) if the recipe may terminate so can the facility. The formula \( \Phi_{\text{Real}} \) is true in all those states from which \text{CONTROLLER} can force the game to visit infinitely many times a state where \( \phi_{\text{ok}} \) holds, no matter how \text{ENVIRONMENT} moves in its turns. \( \Phi_{\text{Real}} \) also requires that \text{CONTROLLER} does not pass the turn until \( \phi_{\text{ok}} \) holds. The set \( \text{Win}(\Phi_{\text{Real}}) \) of winning states is the set of states where \( \Phi_{\text{Real}} \) holds.

**Theorem 1.** Given a facility \( \text{Fac} = (D_R, D_F, \delta_R^0, \text{Maps}) \), a recipe \( \delta_R^0 \) over \( D_R \) is realizable by the facility process \( \delta^0 \) iff \( q_0 \in \text{Win}(\Phi_{\text{Real}}) \).
**Proof.** For the If-part, assume that $q_0 = (\text{Env}, \delta_R, s_R, \delta_F, S_F, c) \in \text{Win} (\Phi_{\text{Real}})$. We prove that there exists a realizability relation $R$ such that $(\delta_R, s_R) R (\delta_F, S_F)$. Since $q_0 \in \text{Win} (\Phi_{\text{Real}})$, for each possible environment move $q_0 \rightarrow q_1$, Controller has a sequence of (legal) moves that take the GA $T'$ from $q_1$ to a state $q$ such that $q \models \phi_{\text{ok}}$, and from which, for all possible environment moves, Controller has a sequence of moves that take $T$ to a state $q'$ such that $q' \models \phi_{\text{ok}}$, and so on forever. In other words, whenever environment moves, Controller can force the game to achieve a state where $\phi_{\text{ok}}$ holds. Notice that $\phi_{\text{ok}}$ implies that Environment is to move. By considering all possible environment moves at each such state and, for each of them, a suitable Controller response sequence, we obtain a tree whose root is $q_0$, and where branches occur only at states where $\phi_{\text{ok}}$ holds, each corresponding to a possible environment move. These infinite paths $q_0 q_1 \cdots$ correspond to plays of $T$ and contain infinitely many states $q = (\text{Env}, \delta_R, s_R, \delta_F, s_F, \delta)$ such that $q \models \phi_{\text{ok}}$. For each path and each such state, we set $(\delta_R, s_R) R (\delta_F, s_F)$. Obviously, $(\delta'_R, s'_R) R (\delta'_F, s'_F)$.

To see that $R$ is a realizability relation, consider an arbitrary pair of configurations from $R$, $(\delta_R, s_R)$ and $(\delta_F, s_F)$. By the definition of $R$, there exists a path $q_0 \cdots q_i \cdots$ containing a state $q_i = (\text{Env}, \delta^i_R, s^i_F, \delta^i_F, \delta^i)$, such that $q_i \models \phi_{\text{ok}}$, and $s^i_R = \delta^i_R, s^i_F = \delta^i_F, \delta^i = \delta^i_R = \delta^i_F$. It is easy to see that the definition of $R$ above satisfies requirements $\textbf{r0}$ and $\textbf{r1}$ of the realizability relation (p. 15). Indeed, these are consequences of the fact that $q_i \models \phi_{\text{ok}}$, which, in particular, requires that the interpretation of each non-observation fluent $f$ of $T_R$ matches that of $\psi_f$ and that $\text{finalEnv}$ implies $\text{finalCtrl}$.

For requirement $\textbf{r2}$, consider an action $A(x)$ legal in $(\delta_R, s_R)$. By the definition of $T$, and in particular the transition relation, Environment has a move $q_i \rightarrow q_{i+1}$, with $q_{i+1} = (\text{Ctrl}, \delta^i_R, s^i_F, \delta^i_F, \delta^i, s^i_F, i, j, l)$, where: $(\delta^i_R, s^i_R) = \text{do} (A(x), s^i_R)$; $s^i_F = s^i_F$; and $(\delta^i, s^i_F) = \delta^i(x)$. Thus, the sequence of states $q_0 \cdots q_i$ is a path of the game and, since $q_0 \in \text{Win} (\Phi_{\text{Real}})$, Controller has a sequence of moves that extends the path with states $q_{i+2} \cdots q_{i+l}$, where $q_{i+l} \models \phi_{\text{ok}}$. Let $q_{i+j} = (\delta^i, s^i_R, \delta^i_F)$, $(\delta^i, \delta^i_F)$. Thus, the sequence of states $q_0 \cdots q_{i+j}$ is a path of the game, and, since $q_0 \in \text{Win} (\Phi_{\text{Real}})$, Controller has a sequence of moves that extends the path with states $q_{i+j} \cdots q_{i+l}$, where $q_{i+l} \models \phi_{\text{ok}}$. Let $q_{i+j} = (\delta^i, s^i_R, \delta^i_F)$), $(\delta^i, \delta^i_F)$.

Therefore, $q_0 \models \phi_{\text{ok}}$ and, by the definition of $R$, we have that $\text{Final} (\delta^i, \delta^i_F)$, $\text{possible}$. By the above, it follows that: (i) $\text{Trans} (\delta_R, s_R, \delta^i, \delta_F, s_F)$; (ii) $\text{Trans} (\delta^i, \delta_F, s_F, \delta^i_F)$; (iii) $\text{Do} (\delta^i_F, s_F, r_F)$; (iv) $\text{Trans} (\delta^i, \delta^i_F, s_F)$, $\text{Final} (\delta^i, \delta^i_F)$, $\text{Possible} (\delta^i, \delta^i_F)$, $\text{Trans} (\delta^i, \delta^i_F)$; (v) $\text{Do} (\delta^i_F, s_F, \delta^i_F)$; (vi) $\text{Trans} (\delta^i, \delta^i_F)$, $\text{Final} (\delta^i, \delta^i_F)$, $\text{Possible} (\delta^i, \delta^i_F)$, $\text{Trans} (\delta^i, \delta^i_F)$; (vii) $\text{Do} (\delta^i_F, s_F, \delta^i_F)$; (viii) $\text{Trans} (\delta^i, \delta^i_F)$, $\text{Final} (\delta^i, \delta^i_F)$, $\text{Possible} (\delta^i, \delta^i_F)$, $\text{Trans} (\delta^i, \delta^i_F)$. It is easy to check that the path so defined is indeed a path of $T$, as it starts in the initial state $q_0$ and is consistent, at every step, with $T$’s transition relation. The labeling of the states in the path can be obtained through the labeling function previously defined. With respect to this, by requirements $\textbf{r0}$ and $\textbf{r1}$ of $R$, it follows that $q_0 \models \phi_{\text{ok}}$.

We have thus shown that for every environment $q_0$, Controller has a way to force the game to reach a new state $q_i$ such that $q_i \models \phi_{\text{ok}}$. In proving this, we have used only the assumption that, for $q_0 = (\text{Env}, \delta_R, s_R, \delta_F, \delta_F, s_F) \in \text{Win} (\Phi_{\text{Real}})$. Then, since for $q_i = (\text{Env}, \delta_i, s_i, \delta_F, \delta_F, s_F)$ we have that $(\delta_i, s_i) R (\delta_i, s_i)$, we can generalize the argument above and prove that for every environment move $q_i \rightarrow q_{i+1}$, another state $q_{i+1}$ such that $q_{i+1} \models \phi_{\text{ok}}$ exists that can be reached after a suitable sequence of Controller moves, and so on forever. This ultimately proves that $q_0 \in \text{Win} (\Phi_{\text{Real}})$. □

Although a (possibly transfinite) fixpoint computation based on approximates provides a way to obtain $\text{Win} (\Phi_{\text{Real}})$, the number of approximates that we need to compute is bounded (by the size of the GA) only if $T$ is finite. Since, in our case, $T$ can be infinite, the fixpoint cannot be computed in general. Nonetheless, we show how a controller can be computed when the Controller player can win the game represented by $T$, that is, when it has a winning strategy.

Let $T = (\Delta_T, Q, q_0 \rightarrow \Delta)$ be a GA (over a vocabulary $\sigma$). A history of $T$ is a sequence $\tau = q_0 \cdots q_i \in Q^+$ such that, for $i \in [0, \ell - 1]$, $q_i \rightarrow q_{i+1}$, $H$ denotes the set of histories of a GA. A Controller strategy is a function $\zeta : H \rightarrow Q$ such that if $\zeta (q_0 \cdots q_i) = q$ then $q_i \models \text{turnCtrl}$ and $q_i \rightarrow q$. A history $\tau = q_0 \cdots q_i$ is induced by a strategy $\zeta$ if, for every $i \in [0, \ell - 1]$, whenever $q_i \models \text{turnCtrl}$, $q_{i+1} = \zeta (q_0 \cdots q_i)$. The strategies of interest are those, called winning, which enforce $\Phi_{\text{Real}}$.

**Definition 5** *(Winning strategy).* A strategy $\zeta$ for player Controller is said to be winning for $\Phi_{\text{Real}}$ if, for every history $\tau = q_0 \cdots q_i \cdot q_i$ induced by $\zeta$, $q_i \models \text{turnEnv}$ implies $q_i \models \phi_{\text{ok}}$; and $q_i \models \text{turnCtrl}$ implies that $\tau$ can be extended to a history $\tau' = q_0 \cdots q_i \cdot q_m$ induced by $\zeta$ s.t. $q_m \models \phi_{\text{ok}}$ and $q_i \models \text{turnCtrl}$, for $i = \ell, \ldots, m - 1$. 


Intuitively, a strategy for CONTROLLER is winning for $\Phi_{\text{Real}}$ if CONTROLLER has a way to play in its turns such that, no matter how ENVIRONMENT moves in its turns, the game will end up to a state $q_m$ where $\phi_{\text{ok}}$ holds, and from which this property is preserved.

While, in general, a strategy can prescribe different sequences of moves on histories that end in a same state, when a winning strategy exists, then there exists one that prescribes the same sequence on such histories. This is because we are essentially playing a model checking game over GA for a $\mu$-calculus objective, and as a result the winning states does not depend on the history [41]. In what follows, we therefore focus only on memoryless strategies, i.e., strategies that depend only on the last state of the history. For this reason, a memoryless strategy $\varsigma : H \mapsto Q$ can be represented as a function $\varsigma_m : Q \mapsto Q$, with $\varsigma(\tau) = \varsigma_m(\text{last}(\tau))$, where $\text{last}(\tau)$ denotes $\tau$’s last state.

**Theorem 2.** A controller $\rho$ for $\delta_R^0$ that realizes $\delta_R^0$ can be obtained from a (memoryless) winning strategy $\varsigma$ for $\Phi_{\text{Real}}$ and $\mathcal{T}$.

The controller $\rho$ can be obtained as follows. For every history $\tau = q_0 \cdots q_{\ell-1} q_\ell$ induced by $\varsigma$, such that $q_{\ell-1} \models \phi_{\text{ok}}$ (hence, by $\mathcal{T}$’s transition relation, $q_\ell \models \text{turnCtrl}$, $\delta^\ell_F = q_{\ell-1}^F$, and $s^\ell_F = s^\ell_F$), consider the sequence of states $q_{\ell+1}, \ldots, q_m$ obtained by iteratively applying $\varsigma$, starting from $\tau$, i.e., $q_{\ell+1} = \varsigma(q_0 \cdots q_\ell)$, $q_{\ell+2} = \varsigma(q_0 \cdots q_{\ell+1})$, and so on, until a state $q_m$ such that $q_m \models \phi_{\text{ok}}$ is obtained. Let $q_i = (\delta^i, q^i, s^i_F, s^i_R, s^i)$ and consider the action $A(x)$ associated with the concrete program $\delta^i$. We then define $\rho((\delta^i, q^i, s^i_F, s^i_R, s^i)) : A(x) \Rightarrow (q^i_{\ell+1} \cdots q^i_m, s^i)$.

It remains to establish how, and when, a memoryless winning strategy can be computed. Note that, as $Q$ may be infinite, this may not be possible in general.

**8. Bounded case: decidable synthesis**

In this section, we analyze the case where the facility $Fac = (\mathcal{D}_R, \mathcal{D}_F, \delta^0_R, \text{Maps})$ and the recipe $\delta^0_R$ induce a game arena that is “state-bounded”, showing that a controller that realizes $\delta^0_R$ can be effectively computed.

As a preliminary step, we study the problem of computing the winning set $\text{Win}(\Phi_{\text{Real}})$ and a corresponding strategy for CONTROLLER on a generic GA defined over the same vocabulary $\sigma$ as that of the induced GA, but with a finite set of states $Q$. This is a crucial step as we will reduce the problem of computing a controller for $\delta^0_R$ that realizes $\delta^0_R$ to that of computing $\text{Win}(\Phi_{\text{Real}})$ and a strategy for CONTROLLER on a particular finite-state GA over $\sigma$. Observe that, in general, the induced GA itself cannot be finite-state, even if the object domain $\Delta$ is finite: the transition relation essentially embeds, through $\mathcal{D}_R^\text{Maps}$, the infinitely many situations of $\mathcal{D}_R$ and $\mathcal{D}_F$, thus yielding an infinite set of states.

On a finite-state GA $\mathcal{T}$, it is well known that $\mu$-calculus fixpoints can be computed by iterative approximations (e.g., [41]). For a formula $\Phi = \nu X. \Psi(X)$, one starts with the initial approximate $X_0 = Q$ and then iteratively computes $X_i = (\Phi)_{X/X_i+1}^T$, until the fixpoint is reached, i.e., $X_n = X_{n+1}$. This is the desired fixpoint, i.e., $(\Phi)^T = X_n$ (the SO assignment $\nu$ is omitted as irrelevant). For formulas $\Phi = \mu X. \Psi(X)$, the same procedure can be used, but starting with the approximate $X_0 = \emptyset$. The winning set $\text{Win}(\Phi_{\text{Real}})$ can thus be obtained by applying such procedures to $\Phi_{\text{Real}} = \nu X. \mu Y.((\phi_{\text{ok}} \land [-]X) \lor (\text{turnCtrl} \land (\neg)Y))$. During the computation, the following approximates are produced, which are used to build the controller:

$X_i = X_{(i-1)n_{i-1}}$ (initially, $X_0 = Q$)
$X_i[0] = \emptyset$
$\ldots$
$Y_{ij} = ((\phi_{\text{ok}} \land [-]X) \lor (\text{turnCtrl} \land (\neg)Y))_{\nu[X/X_i, Y/Y_{ij-1}]}$
$\ldots$
$X_{i+1} = Y_{in_i}$ (for $n_i$ the smallest index such that $Y_{in_i} = Y_{i(n_i+1)}$)
$\ldots$
$X_k = Y_{kn_k}$ (for $k$ the smallest index such that $X_k = X_{k+1}$)

The greatest fixpoint $\text{Win}(\Phi_{\text{Real}}) = X_k = Y_{kn_k}$ is reached after computing a finite number $k$ of approximates $X_i$, each requiring, in turn, to compute a finite number $n_i$ of approximates $Y_{ij}$. Intuitively, $Y_{ij}$ contains all states $q \in Q$ such that either of these two properties holds:

1. $q \models \phi_{\text{ok}}$ and no matter how ENVIRONMENT moves from $q$ (remember $\phi_{\text{ok}}$ implies $\text{turnEnv}$), a state $q' \in X_i$ is reached after the move (first disjunct of $\Phi_{\text{Real}}$); or
2. $q \models \text{turnCtrl}$ and CONTROLLER can force the game to reach, in $m < j$ consecutive moves (and keeping the turn), a state $q' \in Q$ that satisfies Property 1 (second disjunct of $\Phi_{\text{Real}}$); in general, $q$ may belong to many approximates $Y_{kj}$, even with $j' < j$.

(Recall that $\text{turnCtrl}$ and $\text{turnEnv}$ are mutually exclusive in $\mathcal{T}$, thus so are the above properties.) Notice that, since $\text{Win}(\Phi_{\text{Real}}) = X_k = Y_{kn_k}$, every state of $\text{Win}(\Phi_{\text{Real}})$ fulfills either 1 or 2, for $i = k$ and $j = n_k$. In other words, for every state $q \in \text{Win}(\Phi_{\text{Real}})$, if it is ENVIRONMENT’s turn, no matter how ENVIRONMENT moves, CONTROLLER can force the game to reach, in a number $m < n_k$ of consecutive moves, a state $q' \in \text{Win}(\Phi_{\text{Real}})$ such that $\phi_{\text{ok}}$ holds and from which a new
state \( q'' \in \text{Win}(\Phi_{\text{Real}}) \) where the same holds can be forced again, and so on and so forth. It is then easy to see that if \( q_0 \in \text{Win}(\Phi_{\text{Real}}) \), Controller has a winning strategy for \( \Phi_{\text{Real}} \) on \( T \). We now discuss how one such strategy can be obtained.

Given a state \( q \in \text{Win}(\Phi_{\text{Real}}) \), we define \( \varsigma_m(q) \). By construction of \( \text{Win}(\Phi_{\text{Real}}) \), either 1 or 2 holds, for \( i = k \) and \( j = n_k \).

In the former case, we leave \( \varsigma_m(q) \) undefined, as \( q \models \text{turnEnv} \). In the latter case, instead, \( q \models \text{turnCtrl} \), thus \( \varsigma_m(q) \) must be defined. Since \( q \in \text{Win}(\Phi_{\text{Real}}) = Y_{\text{lin}} \), Controller can force the game, in \( m < n_k \) moves, to a state where 1 holds. Then, there must exist a transition \( q \rightarrow q' \) such that \( q' \in Y_{\text{lin}} \) (i.e., from \( q' \), Controller can force Property 1 in \( m - 1 \) moves). In other words, \( q' \) is one step “closer” to Property 1. Thus, by defining \( \varsigma_m(q) = q' \) for one such \( q' \), we guarantee progressing towards, and eventually achieve, Property 1.

It remains to show how the desired \( q' \) can be selected. Indeed, from \( q \), many possible Controller moves, i.e., successor states \( q' \) of \( q \), are available, but only some of them progress towards 1. The question is then how to select one of them. To address this, during the fixpoint computation, we label each state \( q \in \text{Win}(\Phi_{\text{Real}}) \) with its “distance from Property 1”, i.e., with the minimum number of moves Controller requires to force 1. To do this, we proceed as follows:

- when a new approximate \( Y_{i0} \) is initialized, all labels are removed;
- when an unlabeled state \( q \) enters an approximate \( Y_{ij} \) with \( j > 0 \), then we label \( q \) with \( j - 1 \).

Obviously, when the fixpoint is obtained, all states are left with the labeling defined during the last approximate computation. Property 1 and 2 guarantee that the so-obtained labeling corresponds to the distance of each state from Property 1.

Thus, given \( q \), we can define \( \varsigma_m(q) = q' \), for \( q' \) any state with minimal labeling among those such that \( q \rightarrow q' \). Thus, for a finite-state GA, we have a technique to actually build a winning controller strategy. This will be useful later on.

Now, we relax the state-finiteness assumption and consider an induced, infinite-state, GA \( T \). In this case, constructing a controller is not possible in general. We show, however, how this can be done when the information kept in each state of the GA is “bounded” ([42]). Informally, a GA is state-bounded if all of its states are labeled by interpretations containing only a bounded number of objects (or, equivalently, a bounded number of distinct tuples).

That is, in every state, the number of objects in the interpretation of all fluents is bounded by a given bound. We recall the corresponding formal definition below.

Given a FO interpretation \( I \), the active domain of \( I \) is the set \( \text{adom}(I) \) of all the objects that occur in the interpretation of some fluent in \( I \).

**Definition 6 (State-boundedness).** A GA \( T = (\Delta_T, Q, q_0, \rightarrow, I) \) is said to be state-bounded by \( b \in \mathbb{N} \) if \( |\text{adom}(I(q))| \leq b \), for every \( q \in Q \). \( T \) is said to be state-bounded if it is state-bounded by \( b \), for some \( b \).

We can show that the induced GA \( T \) is state-bounded whenever \( D_F \) and \( D_R \) are “bounded”, in the sense of the following definitions.

Following [18], for \( b \in \mathbb{N} \) and a fluent \( f \), we can write a FO formula \( \text{Bounded}_{f,b}(s) \), to express that fluent \( f \) contains fewer than \( b \) distinct tuples at situation \( s \). We then say that \( f \) is bounded by \( b \) in situation \( s \) (of a BAT \( D \)), if \( D \models \text{Bounded}_{f,b}(s) \).

**Definition 7 (Bounded BAT [18]).** Let \( \text{Bounded}_{b}(s) = \bigwedge_{f \in \mathcal{F}} \text{Bounded}_{f,b}(s) \). An action theory \( D \) is bounded if there exists \( b \in \mathbb{N} \) such that:

\[
D \models \forall s. \text{Executable}(s) \supset \text{Bounded}_{b}(s).
\]

Intuitively, the definition requires that the number of objects contained in the interpretation of the fluents at situation \( s \) be less than some \( b \in \mathbb{N} \).

We generalize the notion of boundedness to \( \text{BAT}^{-} \)s. In order to do this, we need to specifically handle observations, which are not constrained in any way in \( \text{BAT}^{-} \)s. The actual property of interest is whether a \( \text{BAT}^{-} \) is bounded “provided its observations are”. This assumption can be captured by the FO formula \( \text{Bounded}_{b}^{\text{Obs}}(s) = \bigwedge_{f \in \mathcal{F}} \text{Bounded}_{f,b}(s) \), which expresses that, at situation \( s \), the interpretations of all observations contain fewer than \( b \) distinct objects.

**Definition 8 (Bounded \( \text{BAT}^{-} \) modulo observations).** A \( \text{BAT}^{-} \) \( D \) is bounded modulo observations if there exist \( b, b' \in \mathbb{N} \) such that:

\[
D \models \forall s. (\text{Executable}(s) \land \text{Bounded}_{b'}^{\text{Obs}}(s)) \supset \text{Bounded}_{b}(s).
\]

Observe that Definition 8 generalizes 7: if \( D \) contains no observations, the former trivially reduces to the latter. For simplicity, when no ambiguity arises, we refer to \( \text{BAT}^{-} \)s which are bounded modulo observations simply as “bounded”.

We can now prove state-boundedness for every GA induced by a facility that includes a bounded \( D_F \) and a bounded \( D_R \).

**Theorem 3.** If a facility \( \text{Fac} = (D_R, D_F, S^0_F, \text{Maps}) \) is such that \( D_R \) is bounded (modulo observations) and \( D_F \) is bounded, then for any recipe \( \delta^0_R \) the induced GA \( T \) is state-bounded.
Proof. First observe that, by the definition of \( T \)'s transition relation, all \( D_R \) and \( D_F \) situations, respectively \( s_R \) and \( s_F \), occurring in each state \( q \) of \( T \) are executable. Thus, because \( D_F \) is bounded by hypothesis, so are all of its fluents at every such \( s_F \). Consider a fluent \( f \in F_T \). By the labeling function, we have that \( f^{I}(q) \) iff \( D_R^{\text{Maps}} := f(x, s_R) \), thus, by \( D_F \)'s boundedness, it follows that \( f^{I}(q) \) contains a bounded number of distinct tuples; to express this, we simply say that \( f^{I}(q) \) is bounded. Similarly, for fluents \( f \in \text{Obs} \), since \( f^{I}(q) \) iff \( D_R^{\text{Maps}} := \psi_f(x, s_F) \), because \( \psi_f(x, s_F) \) is domain-independent, \( D_F \) bounded, and \( s_F \) executable, \( f^{I}(q) \) is bounded.

For fluents \( f \in F_T \setminus \text{Obs} \), by the labeling function, we have that \( f^{I}(q) \) iff \( D_R^{\text{Maps}} := f(x, s_R) \). Boundedness of \( f^{I}(q) \) follows from boundedness modulo observations of the BAT \( D_R \) and executability of \( s_F \) and \( s_R \). Indeed, by state-boundedness of \( D_F \) and executability of \( s_F \), all observation fluents in \( D_R^{\text{Maps}} \) are bounded at situation \( s_F \) (regardless of \( s_R \)). But then, for such \( s_F \), by boundedness modulo observations of \( D_R \) and executability of \( s_R \), it follows that all non-observation fluents of \( D_R^{\text{Maps}} \) are bounded at situations \( s_F \) and \( s_R \).

Finally, it is easy to see that all other fluents in the labeling are bounded, as being either propositions or singletons. \( \square \)

Infinite, state-bounded GAs are the norm when BATs are used to model manufacturing facilities. Typically, fresh parts arrive continuously in a facility for processing. This yields, in general, an infinite number of distinct states (if we consider the parts currently processed as part of the state). However, when resources have bounded capacity, a recipe operates on finitely many parts at a time, and requires only finitely many operations, thus the number of objects processed at any point in time does not exceed the capacity bound. As an example of state-bounded BAT, consider the running example of previous sections. Note that fluents such as \text{part}(\text{part}, s) \), \text{at}(\text{part}, i, s) \), or \text{material}(\text{part}, m, s) \), disappear from the extension as soon as the part in question is stored away, i.e., it exits the cell through an action \text{OUT}. This reflects the fact that, once processed, a part is no longer involved in the process and all the corresponding information can be safely forgotten. In other words, the fluents carrying information about a given part eventually disappear and do not “accumulate”. Thus, since only a bounded number of parts are processed at a time and since the information about each of these is bounded, it follows that, at any situation, the corresponding interpretation contains only a bounded number of facts, i.e., a bounded number of objects. This implies a state-bounded induced GA.

Later, we will prove that the induced GA \( T \) is also “generic”, a notion which intuitively captures that \( T \)'s transitions do not depend on the actual objects contained in the labeling of the relevant states (apart from finitely many active constants) but only on the mutual relationships among objects. This notion is formally stated as follows.

Given two FO interpretations \( I, I' \in \mathcal{I}_\Delta^{F,K} \), an isomorphism \( h \) between \( I \) and \( I' \) is a function \( h : \Delta \rightarrow \Delta \) so that (i) for every fluent \( f \in F \), \( h(y) \in f^{I} \) if and only if \( y \in f^{I} \), and (ii) for every active constant \( k \in K \), \( k^{I} = h(k^{I'}) = k \). We say that \( I \) and \( I' \) are isomorphic (under \( h \)), written \( I \sim_h I' \), if there exists an isomorphism \( h \) between \( I \) and \( I' \). Intuitively, \( I \) and \( I' \) are isomorphic if they are the same interpretation modulo an object renaming that preserves the identity of active constants.

Definition 9 (Genericity). A GA \( T = (\Delta_T, Q, q_0, \rightarrow, I) \) is said to be generic if: for every \( q_1, q'_1, q_2 \in Q \) and every function \( h : \Delta_T \rightarrow \Delta_T \) such that \( I(q_1) \sim_h I(q_2) \), if \( q_1 \rightarrow q'_1 \), then there exists \( q'_2 \in Q \) such that \( q_2 \rightarrow q'_2 \) and \( I(q_1') \sim_h I(q_2') \).

Intuitively, this says that a GA \( T \) is generic if, whenever two states are isomorphic under \( h \), they yield the same transitions modulo the same object renaming prescribed by \( h \). Differently put, a GA is generic if states that are identical modulo object renaming are involved in exactly the same transitions (still modulo object renaming).

We thus have that \( T \) is generic (proven later), and that if \( D_R \) and \( D_F \) are bounded, then \( T \) is state-bounded. This allows the application of the results in \cite{42,35} which, in turn, allow us to prove the following central result:

Theorem 4. Given a facility \( \text{Fac} = (D_R, D_F, \delta_0^R, \text{Maps}) \) such that \( D_R \) and \( D_F \) are bounded, and a recipe \( \delta_0^R \) that is realizable by \( \text{Fac} \), there exists a controller for \( \delta_0^R \) that realizes \( \delta_0^R \) and is effectively computable.

In the next two sections we prove this theorem and show how to actually build and execute a controller.

9. Verifying realizability for the bounded case

In \cite{35} (see Theorems 5 and 6 therein), the problem of checking whether a Transition System that is state-bounded and generic satisfies a formula \( \Phi \) was proven decidable for \( \Phi \) belonging to \( \mu L_p \). In the context of this paper, a corresponding result can be formally stated as follows (recall that \( \mu L_c \) is a sub-language of \( \mu L_p \)):

Theorem 5. Given a generic, state-bounded GA \( T \), there exists a finite-state GA \( \hat{T} \) s.t., for every \( \mu L_c \) formula \( \Phi \), \( T \models \Phi \) iff \( \hat{T} \models \Phi \).

When it exists, we call such a \( \hat{T} \) a faithful abstraction of \( T \). The theorem says that if state-boundedness and genericity hold for \( T \) then one can check whether it satisfies \( \Phi \) by simply checking whether (one of) its faithful abstraction(s) \( \hat{T} \) satisfies it.
In this section we prove Theorem 4. To this end, we first prove in Section 9.1 that the induced GA $\mathcal{T}$ is generic. This, together with Theorem 3, enables the application of Theorem 5 to $\mathcal{T}$ (under the assumption that $D_\mathcal{T}$ is bounded), which, in turn, implies the existence of a faithful abstraction $\tilde{\mathcal{T}}$. Then, in Section 9.2 we prove that:

**Theorem 6.** There exists a strategy $\zeta$ on $\mathcal{T}$ for player Controller iff a winning strategy $\xi$ exists on a faithful abstraction $\tilde{\mathcal{T}}$ of $\mathcal{T}$.

Finally, in Section 10 we show how a controller for $\mathcal{T}$ can be extracted from $\xi$. Together, these results guarantee that (i) there exists a finite GA $\tilde{\mathcal{T}}$ that is a faithful abstraction of $\mathcal{T}$ and that (ii) a controller for $\mathcal{T}$ can be effectively extracted from $\tilde{\mathcal{T}}$. From these, Theorem 4 follows (as the results hold also for memoryless strategies).

9.1. Faithful abstraction

We start by proving the following result.

**Lemma 1.** The GA $\mathcal{T}$ induced by a facility $Fac = (D_R, D_F, \delta^0_R, \text{Maps})$ and a recipe $\delta^0_R$ is generic.

**Proof.** Consider three states $q_1, q'_1, q_2 \in Q$ of $\mathcal{T}$ such that $q_1 \rightarrow q'_1$ and $I(q_1) \sim_h I(q_2)$, for some isomorphism $h : \Delta_\mathcal{T} \rightarrow \Delta_\mathcal{T}$. Let $q_1 = (\delta^1, \delta^2, s^1_R, s^2_F, s^1_p, \delta^1)$ (possibly primed). Since $I(q_1) \sim_h I(q_2)$, the interpretations of $pcR$ at $q_1$ and $q_2$ match, i.e., $pcR^I(q_1) = pcR^I(q_2)$ (recall that program counters act as active constants, thus $h$ is the identity on them) and the interpretations of $envR$ at the same states are the same modulo the object renaming defined by $h$ (modulo $h$-renaming, for short). Thus, by considering the relationship established by the labeling function between state components and state labeling, we have that $\delta^1_R$ and $\delta^2_R$ have the same program counter but environments that are isomorphic under $h$. By the same argument, the above also holds for $pcF$, $envF$, $pcA$, and $envA$. Thus, $\delta^1_R$ and $\delta^2_R$ are the same program modulo $h$-renaming of their respective environments, and $\delta^1$ and $\delta^2$ are the same program modulo $h$-renaming of their respective environments. For the same reason, we have that: $Final(\delta^1_R, s^1_F)$ iff $Final(\delta^2_R, s^2_F)$, $Final(\delta^1_R, s^1_p)$ iff $Final(\delta^2_R, s^2_p)$, and $Final(\delta^2_R, s^2_p)$ iff $Final(\delta^1_R, s^1_p)$. In a similar way, we can prove that $\delta^1 = \delta^2$, i.e., the same player is to move at $q_1$ and $q_2$.

Now, observe that, by the definition of $T$’s labeling function, the interpretations of the fluents associated with a game state $q_1$ is essentially that given by $D_{R}^{\text{Maps}}$ at its (combined) situations $s^1_F$ and $s^2_F$. Therefore, an action $A(x, s_F)$ is legal for $(\delta^1_R, s^1_F)$ iff action $A(h(x), s_F)$ is legal for $(\delta^2_R, s^2_F)$, and, similarly, an action $A(x)$ is legal for $(\delta^1_R, s^1_F)$ and $(\delta^1, s^1_p)$ iff $A(h(x))$ is such for $(\delta^2_F, s^2_F)$ and $(\delta^2, s^2_p)$. Consequently, by the definition of $T$’s transition relation, since $q_1 \rightarrow q'_1$, there must exist a state $q'_2$ such that $q_2 \rightarrow q'_2$. The fact that $I(q'_1) \sim_h I(q'_2)$ is a consequence of the fact that: (i) $I(q_1) \sim_h I(q_2)$, i.e., $q_1$ and $q_2$ have the same labeling modulo $h$-renaming; (ii) the moves $q_1 \rightarrow q'_1$ and $q_2 \rightarrow q'_2$ are defined by the same action, modulo $h$-renaming, respectively $A(x, s_F)$ and $A(h(x), s_F)$, if $\delta^1 = \delta^2 = Env.$, and $A(x)$ and $A(h(x))$, if $\delta^1 = \delta^2 = Ctrl.$; (iii) Successor-state axioms are FO formulas, which are invariant to isomorphisms, modulo the object renaming defined by the isomorphism itself. Thus move $q_2 \rightarrow q'_2$ is the same as $q_1 \rightarrow q'_1$, modulo $h$-renaming, by which it follows that $I(q'_1) \sim_h I(q'_2)$. \hfill \Box

We now address the issue of computing a faithful, finite abstraction of $\mathcal{T}$.

**Lemma 2.** If a GA $\mathcal{T}$ is generic and state-bounded, then a finite, faithful abstraction $\tilde{\mathcal{T}}$ of $\mathcal{T}$ is effectively computable.

**Proof.** Existence of $\tilde{\mathcal{T}}$ is an immediate application to $\mathcal{T}$ of Theorem 17 in [35]. Effective computability and finiteness follow from the proof of the same theorem, once we observe that: (i) since $\mathcal{T}$ is state-bounded, the successor state $q'$ of a generic state $q$ is computable, hence we do not need to construct $\mathcal{T}$ explicitly to build its finite abstraction; (ii) state-boundedness of $\mathcal{T}$ implies that checking whether two interpretations are isomorphic is decidable; and (iii) there exist only finitely many equivalence classes of isomorphic interpretations, thus only finitely many equivalent states. Based on these observations the definitions used by the proof to build the abstraction are operational, thus enabling effective construction of $\tilde{\mathcal{T}}$. \hfill \Box

Intuitively, $\tilde{\mathcal{T}}$ is a GA obtained from $\mathcal{T}$ by collapsing the classes of states having isomorphic interpretations into one representative state, and by adding a transition from one class $q$ to another class $q'$ if there exists one transition between the chosen representatives in $\mathcal{T}$ (this, by generality, implies that all states in $q$ have a transition to some state in $q'$). In this paper we do not describe how to obtain this abstraction, but refer the reader to the procedure illustrated in [35]. The procedure is applicable to generic transition systems with first-order state representations, of which induced GAS are instances. In our setting, the procedure requires that the transition function of $\mathcal{T}$ is computable and that the existence of an isomorphism between states is decidable, which is indeed the case, as discussed in the proof of Lemma 2. Theorem 5 comes as a direct consequence of Lemma 2.

The resulting GA is $\tilde{\mathcal{T}} = (\tilde{\Delta}, \tilde{Q}, q_0 \rightarrow, \tilde{I})$, where $\tilde{\Delta}$, $\tilde{Q}$, and $\rightarrow$ are finite subsets of their counterparts in $\mathcal{T}$, and $\tilde{I}$ is the projection of $I \times Q$, with $\Delta$ replaced by $\tilde{\Delta}$. Notice that $\mathcal{T}$ and $\tilde{\mathcal{T}}$ share the same set $\mathcal{F}$ of fluents.
We conclude this section by characterizing the relationship between $\mathcal{T}$ and $\tilde{\mathcal{T}}$, which will be needed when computing executable strategies for $\mathcal{T}$ from those for $\tilde{\mathcal{T}}$. This result relies on a variant of the well-known notion of bisimulation, called persistence-preserving bisimulation, or $p$-bisimulation for short, defined in [35].

We adapt $p$-bisimulation to GAs. The notion is defined co-inductively over triples $(q_1, h, q_2)$, where $q_1$ and $q_2$ are states of two GAs and $h \in H$ is an isomorphism between their interpretations which, differently from the case of bisimulation, is restricted to the active domains [18].

**Definition 10 ($p$-bisimulation).** A relation $\beta \subseteq Q_1 \times H \times Q_2$ is a $p$-bisimulation between two GAs $\mathcal{T}_1$ and $\mathcal{T}_2$ if $(q_1, h, q_2) \in \beta$ implies that:

(i) $q_1$ and $q_2$ have isomorphic fluent extensions, according to $h : \text{adom}(I_1(q_1)) \mapsto \text{adom}(I_2(q_2))$ (objects not occurring in fluent extensions are neglected). We denote this by writing $\bar{I}_1(q_1) \sim_h \bar{I}_2(q_2)$;

(ii) for every successor $q'_1$ of $q_1$ there exists a successor $q'_2$ of $q_2$ and a bijection $b : \text{adom}(I_1(q_1)) \cup \text{adom}(I_1(q'_1)) \mapsto \text{adom}(I_2(q'_2))$ that extends $h$ to $\text{adom}(I_1(q'_1))$ such that, for the restriction $h'$ of $b$ to $\text{adom}(I_2(q'_2))$, $(q'_1, h', q'_2) \in \beta$;

(iii) the analogue of (ii) holds for every successor $q'_2$ of $q_2$.

This property intuitively captures the fact that two states of two GAs $\mathcal{T}_1$ and $\mathcal{T}_2$ are persistence-preserving bisimilar if there is an isomorphism between them that can be extended in successor states, while preserving bisimulation. In other words, the identity of objects is preserved as long as they persist in the active domain or if they have just disappeared from it. Two GAs are $p$-bisimilar if their respective initial states are in some $p$-bisimulation. Since from [35] we have that every transition system is $p$-bisimilar to its faithful abstractions, then we directly have that:

**Lemma 3.** $\mathcal{T}$ is $p$-bisimilar to $\tilde{\mathcal{T}}$.

The notion of $p$-bisimilarity will be essential to relate (winning) strategies for $\tilde{\mathcal{T}}$ to those of the infinite-state $\mathcal{T}$. This will allow us to prove Theorem 6, in the next section.

9.2. Strategy existence

In this section we address Theorem 6, namely showing that there exists a strategy $\varsigma$ on $\mathcal{T}$ for player controller iff a winning strategy $\varsigma'$ exists on $\tilde{\mathcal{T}}$, relying on the notion of $p$-bisimilarity introduced above. First, we need a way to relate the strategies of two different GAs, and in particular two that are $p$-bisimilar. This is done through the following definition, which we will then apply to $\mathcal{T}$ and $\tilde{\mathcal{T}}$.

**Definition 11 ($p$-bisimilar strategy transformation).** Consider two GAs $\mathcal{T}$ and $\mathcal{T}'$ that are $p$-bisimilar, and let $\varsigma$ be a controller strategy for $\mathcal{T}$. A strategy $\varsigma'$ for $\mathcal{T}'$ is said to be a $p$-bisimilar transformation of $\varsigma$ to $\mathcal{T}$, if there exists a $p$-bisimulation $\beta$ such that for every history $\tau = q_0 \cdots q_t$ of $\mathcal{T}$ induced by $\varsigma$, there exists a history $\tau' = q'_0 \cdots q'_{t'}$ of $\mathcal{T}'$ induced by $\varsigma'$ and a sequence of bijections $h_i : \text{adom}(I(q_i)) \mapsto \text{adom}(I(q'_i))$, with $i \in [0, t]$, such that for every $i$:

- $(q_i, h_i, q'_i) \in \beta$ and
- if $\bar{I}(q_i) \sim_{h_i} \bar{I} \bar{I}(q'_i)$ and $\bar{I}(q_{i+1}) \sim_{h_{i+1}} \bar{I}'(q'_{i+1})$ then there exists a bijection $b : \text{adom}(I(q_i)) \cup \text{adom}(I(q_{i+1})) \mapsto \text{adom}(I(q'_i)) \cup \text{adom}(I(q'_{i+1}))$ so that $h_{i+1} = b|_{\text{adom}(I(q_{i+1}))}$ and $h_{i+1} = b|_{\text{adom}(I(q_{i+1}))}$.\(^1\)

We call the isomorphism $h_{i+1}$ the update of $h_i$ with respect to $q_{i+1}$ and $q'_{i+1}$.

For $p$-bisimilar GAs, the following result holds.

**Theorem 7.** If two GAs $\mathcal{T}$ and $\mathcal{T}'$ are $p$-bisimilar then there exists a controller strategy $\varsigma$ on $\mathcal{T}$ iff there exists a controller strategy $\varsigma'$ on $\mathcal{T}'$ that is a $p$-bisimilar transformation of $\varsigma$.

**Proof.** By $p$-bisimilarity, there exists a bisimulation $\beta$ such that for every history $\tau = q_0 \cdots q_t$ of $\mathcal{T}$ induced by $\varsigma$, there exists a history $\tau' = q'_0 \cdots q'_{t'}$ of $\mathcal{T}'$ that fulfills the requirement of $\tau'$ in Definition 11. For the if-part, we define $\varsigma'$ as $\varsigma'(q'_0 \cdots q'_{t-1}) = q'_t$, for every history $q_0 \cdots q_{t-1}q_t$ of $\mathcal{T}$ induced by $\varsigma$, such that $q_{t-1} \models \text{turnCtrl}$. The only-if part is analogous. $\square$

In particular, the result holds for memoryless, winning strategies as well (as required by Theorem 2). Hence, by Lemma 3, Theorem 6 follows by applying the theorem above with $\mathcal{T}'$ replaced by $\tilde{\mathcal{T}}$.

\(^1\) The symbol | denotes projection.
10. Computing and executing the controller for the bounded case

Theorem 6 and Theorem 2 provide with a constructive way of transforming a memoryless winning strategy \( \pi_0 \) for \( \Phi_{\text{Real}} \) (cf. Section 7.1) and \( \tilde{T} \) into an actual controller for \( \delta_0^\tilde{T} \) that realizes \( \delta_0^T \).

Specifically, we follow the construction of Definition 11, thanks to Theorem 7. To do so, we need to relate the states of a history \( \tau \) of \( \tilde{T} \) to those of a history \( \tau \) of \( T \), by applying the isomorphisms that preserve the identity of the objects that persist and of those that have just disappeared from the active domain. While the existence of a \( \rho \)-bisimulation \( \rho \) between \( T \) and \( \tilde{T} \) is guaranteed by Lemma 3, we cannot represent it explicitly (\( T \) is infinite): the sequence of isomorphisms must be computed on the fly.

The procedure works as depicted in Fig. 5. Initially, both \( T \) and \( \tilde{T} \) are in their initial state (\( q_0 = \tilde{q}_0 \) and \( h_0 = q_0 \)) and the identity function. Then, since \( q_0 \models turnEnv \) and it is the turn of \( \text{Environment} \), for any state \( q_1 \) in \( T \) such that \( q_0 \rightarrow q_1 \) (for some action \( A(x) \) in the recipe), we select an isomorphic state \( \tilde{q}_1 \) in \( \tilde{T} \) such that \( q_1 \sim_\text{h} \tilde{q}_1 \), where \( h_1 \) is the update of \( h_0 \) with respect to \( q_1 \) and \( \tilde{q}_1 \) as in Definition 11. After this (it is now the turn of \( \text{Controller} \)), assume \( \tilde{q}_2 = \pi_{\text{m}}(\tilde{q}_1) \) is the state selected by the strategy on the abstraction \( \tilde{T} \). We then select a state \( q_2 \) such that \( q_2 \sim_\text{h} q_2 \), which gives us the move \( q_2 \rightarrow \pi_{\text{m}}(q_1) \) on \( T \). In the inductive step, assume that \( \tau = q_0 \ldots q_{\ell-1} \), with \( \tau = q_0 \ldots q_{\ell-1} \), is the bisimilar history on \( T \) computed so far, with \( \text{last}(\tau) = turnCtrl \). Let \( \tilde{q}_\ell = \pi_{\text{m}}(q_{\ell-1}) \). We proceed in the same way and obtain the move \( q_{\ell} = \pi_{\text{m}}(q_{\ell-1}) \) on \( \tilde{T} \), with \( q_{\ell} \sim_\text{h} \tilde{q}_\ell \), and so on. In Fig. 5, \( B(y) \) and \( C(z) \) are the two compound actions which constitute a possible, complete execution of the program \( \delta_0(x) \) to which the action \( A(x) \) is mapped. In \( q_3 \), the turn is given back to \( \text{Environment} \) since \( q_3 \models \text{finalEnv} \).

Following the same reasoning, we can define this as a procedure which constructively computes and executes on-the-fly a controller returning the sequence of moves in \( T \) which correspond to the implementation of the last move of \( \text{Environment} \), given a winning \( \text{Controller} \) strategy \( \pi \) for \( \tilde{T} \). The procedure is given as Algorithm 1, which is initially called for \( \tilde{q} = q = q_0 \) and with \( h \) initialized as the identity function. Note that the procedure computes the next compound action to execute on the fly, by restricting only to the execution of the GA \( T \) that is being determined at runtime.

**Algorithm 1 execute_controller(\( \tilde{T}, \pi_{\text{m}}, \tilde{q}, q, h \)).**

1. let \( q = (\delta_R, s_R, \delta_F, s_F, \cdot) \) – we use \( \cdot \) for don’t-care elements
2. if \( q \models \text{finalEnv} \) then
3. let \( q' = (\text{turnCtrl}, \delta_R, s_R, \delta_F, s_F, \cdot) \) be the state of \( T \) selected by the \( \text{Environment} \), with \( q \rightarrow q' \).
   - This corresponds to some recipe action \( A(x), S^2 \) such that \( s^2 = do(A(x), S^2), s_R \);
4. let \( \tilde{q}' \) be a state of \( \tilde{T} \), with \( \tilde{q} \rightarrow \tilde{q}' \), that is isomorphic to \( q' \), i.e., such that \( \tilde{q} \sim_\text{h} \tilde{q}' \), for \( h' \) the update of \( h \) w.r.t. \( q' \) and \( \tilde{q}' \);
5. else
6. let \( \tilde{q}' = \pi_{\text{m}}(\tilde{q}) \) be the state of \( \tilde{T} \) selected by the winning strategy;
7. let \( q' = (\delta_R, s_R, \delta_F, s_F, \cdot) \in T \) be such that \( q \rightarrow q' \) and \( \tilde{q} \sim_\text{h} \tilde{q}' \), for \( h' \) the update of \( h \) w.r.t. \( q' \) and \( \tilde{q}' \);
8. execute the action \( B(y) \) on \( T \), so that \( s'_R = do(B(y), s_R) \);
9. end if
10. if \( q' \not\models \text{finalEnv} \) then
11. execute_controller(\( \tilde{T}, \pi_{\text{m}}, \tilde{q}', q', h' \));
12. end if

For simplicity, differently from the formal definition of controller (but consistent with the notion of memoryless strategies), the procedure returns, at each step, a single compound action to be executed in the facility rather than returning a complete sequence of actions. Indeed, according to Definition 2, a controller for \( \delta_0^T \) that realizes \( \delta_0^\tilde{T} \) is a function \( \rho : C_0^\tilde{T} \times C_0^\tilde{T} \times \cdots \rightarrow C_0^\tilde{T} \) which, given the current configurations of the recipe and facility together with a new selected configuration for the recipe, returns a sequence of new configurations for the facility. This sequence, in turn, identifies the sequence of compound actions to execute. It is however possible to reconstruct the function \( \rho \) from the execution of Algorithm 1: given \( q \) at line 1, which specifies the current configurations \( \langle \delta_R, s_R \rangle \) and \( \langle \delta_F, s_F \rangle \) of the recipe and of the facility process, given then a new configuration \( \langle \delta'_R, s'_R \rangle \) for the recipe as in line 3, then \( \rho(\delta_R, s_R, \delta_F, s_F, \langle \delta'_R, s'_R \rangle) \) is defined as the sequence of configurations \( \langle \delta'_R, s'_F \rangle \) of the facility process as in line 7, computed by the iterative execution of the procedure, until a new state \( q' \) such that \( q' \not\models \text{finalEnv} \) is reached (i.e., then the turn is given back to the \( \text{Environment} \) player in the GA \( T \)).
Fig. 6. A fragment of a possible controller for $\delta_R^B$ that realizes $\delta_R^B$ as in the running example, represented as a control structure in which double-circled control states correspond to states $q$ of $T$ such that $q = \phi_{\omega_k}$, i.e., it is the turn of player ENVIRONMENT. Dashed transitions from these control states are labeled with the next action of the recipe, and the rest with compound actions for the facility process (we do not include nor actions for brevity). Each control state is associated with configurations of $\delta_R^A$ and $\delta_R^B$, indicating their current program and situation, but these are not shown for brevity.

**Example 5** (A controller for the running example). The controller for the facility process $\delta_R^B$ described in Section 5 that realizes the recipe $\delta_R^B$ in Example 4, corresponding to a possible winning strategy for player CONTROLLER, is depicted graphically in Fig. 6. For brevity, let us refer to $\{R_i, \delta_i\}$, with $i \in \{1, \ldots, n\}$, by $R_i$. We now show how to execute this controller: at the beginning, the recipe can only execute action $\text{LOAD}(f, 4, \text{steel, 810, 756, 29})$. In response, the controller prescribes the execution of two compound actions. With the first, resource $R_2$ equips an end effector in order to prepare the loading of the part b into the cell (recall that b denotes a part), while all other resources remain idle. The compound action is $\{\text{NOP, NOP, EQUIP(grasper, 3)}, \text{NOP, NOP}\}$. With the second, $R_3$ loads the part while other resources remain idle: the compound action is $\{\text{NOP, NOP, IN_CELL}(f, 4, \text{steel, 810, 756, 29}), \text{NOP, NOP}\}$. At this point, two alternatives are possible for the recipe: the next instruction is $\text{LOAD}(b, 2, \text{steel, 312, 23, 20}) \parallel \text{DRILL}(f, 3, 200, 123, 89, 21)$, that is, the recipe can either first load part b or first drill a hole in f. As shown in Fig. 6, in the former case this controller first makes it so that f is moved from $R_3$ to $R_2$, then $R_3$ detaches the gripper while $R_1$ equips a drilling end effector to prepare for the next recipe request, and finally $R_5$ (the operator) is instructed to fetch part b (without entering the cell). In the latter case, after the same preparatory steps, $R_3$ sets the correct drill bit in the drill while $R_2$ equips a hollow pressure applicator; then $R_1$ is instructed to perform the drilling of the part f which is now currently held on $R_2$ (the fixture), while $R_3$ is applying pressure to balance the drilling force. Note that there are some arguments of these actions that were determined by the procedure given as Algorithm 1: for instance, the drill bit $\{\text{bit, 7}\}$, the feed rate $\{12\}$, the glue $\{\text{glue, 36}\}$. These arguments corresponded to pick variables (i.e., nondeterministic choice of arguments) in the programs determined by the mappings Maps. Since the GA is generic, any alternative value which is equivalent modulo isomorphism could have been selected.

The interaction between the recipe, the controller and the facility program continues until the recipe is completed. A possible resulting execution of this controller, for a possible evolution of recipe $\delta_R^B$, is the following (NOP actions are omitted):

1. $\{\text{EQUIP(grasper, 3)}\}$
2. $\{\text{IN_CELL}(f, 4, \text{steel, 810, 756, 29}), \text{IN_CELL}(f, 4, \text{steel, 810, 756, 29})\}$
3. $\{\text{IN(f, 2), OUT(f, 3)}\}$
4. $\{\text{EQUIP(driller, 1), UNEQUIP(3)}\}$
5. $\{\text{IN_CELL(b, 2, steel, 312, 23, 20, 5)}\}$
6. $\{\text{EQUIP(bit, 7, drill, 3, 1), EQUIP(pressure_hollow, 3)}\}$
7. $\{\text{ROBOT_DRILL}(f, \text{bit, 7, 3, 200, 123, 89, 21, 2), \text{HOLD_IN_PLACE}(f, 2k, \text{flat, 3})\}$
8. $\{\text{SAFE_SWITCH(on, 5)}\}$
9. $\{\text{SHEET_SWITCH(off, 5)}\}$
10. $\{\text{SAFE_SWITCH(on, 5)}\}$
11. $\{\text{HOLD_IN_PLACE}(f, 3k, 2), \text{PRESSURE}(f, 2k, \text{hollow, 3})\}$
12. $\{\text{SHEET_SWITCH(off, 5)}\}$
13. $\{\text{HOLD_IN_PLACE}(f, 3k, 2), \text{PRESSURE}(f, 2k, \text{hollow, 3})\}$
14. $\{\text{UNEQUIP(3)}\}$
15. $\{\text{ROBOT_DRILL}(f, \text{bit, 22, 3, 123, 89, 21, 2), \text{HOLD_IN_PLACE}(f, 2k, \text{flat, 3})\}$
16. $\{\text{UNEQUIP(3)}\}$
17. $\{\text{EQUIP(rivet_gun, 1), EQUIP(pressure_flat, 3)}\}$
18. $\{\text{HOLD_IN_PLACE}(f, 2k, \text{flat, 3})\}$
19. $\{\text{SHEET_SWITCH(on, 5)}\}$
20. $\{\text{HOLD_IN_PLACE}(f, 2k, \text{flat, 3})\}$
21. $\{\text{EQUIP(grasper, 3)}\}$
22. $\{\text{OUT_CELL(b, ok, 3)}\}$
11. Conclusions

The ability of manufacturing providers to automatically assess the manufacturability of products and synthesize process plan controllers is essential to realize any real-world MaaS application. Research in Artificial Intelligence and Computer Science can be exploited to provide mathematical foundations for the manufacturing concepts and to solve the core challenges of MaaS, as shown by recent efforts in basing MaaS on fundamental ideas from CS and advancements in AI.

On the one hand, this has allowed previous approaches to formalize the requirements and techniques for the automated synthesis of process plan controllers [9,11–13,43,44] and offer a formal foundation to practical manufacturing approaches [14,15,45]. On the other hand, however, these previous approaches are based on a propositional description of the states of the devices, workpieces, and processes, and such representations are too idealized for implementing fully-fledged applications. While in some simpler scenarios a propositional approach may be sufficient, the resulting discretization is unwieldy and unnatural and, more importantly, cannot deal with potentially unbounded objects. Concrete and realistic approaches for MaaS necessarily require a rich, relational description of states, as well as advanced computational techniques that are able to manipulate this relational representation: real manufacturing processes depend on the objects and data they produce and consume, which are in general unbounded.

In this paper, we have addressed these shortcomings and proposed a formal logical framework that explicitly accounts for this dependency. Our approach offers a "data-aware" process formalization where data and objects are treated as first-class citizens, addressing relational representations of the states by relying on the research on reasoning about actions in AI. Critically, we did not rely on ad-hoc representations. Our framework uses Situation Calculus action theories for capturing actions in manufacturing processes, and high-level ConGolog programs over such action theories for capturing the processes defined over these actions. This makes a whole body of related work readily available to address a number of problems arising in the context of manufacturing systems [46–51]. In addition, we can leverage the first-order state representations of action formalisms and the second-order/fixpoint characterization of state-change provided by programs, giving a formal and declarative representation of the MaaS manufacturing setting. We have also shown that these techniques are actually effective, in that they correspond to actual algorithms that allow the extraction of actual controllers when the objects and data in the resulting Situation Calculus action theories are never "accumulated" (i.e., state-bounded theories [18]). This is the first decidability result for controller synthesis in a relational/first-order state reasoning about actions settings such as the Situation Calculus.

We have only scratched the surface of what KR formalisms like the Situation Calculus can bring to this new manufacturing paradigm. Our results and constructions can be applied in other frameworks for reasoning about actions in AI as well as data-aware/artifact-centric processes frameworks in databases [19–21]. Furthermore, it would be interesting to equip resources with autonomous deliberation capabilities [52], e.g., to react to exogenous events during execution, or to monitor streaming production data [53], to include an explicit treatment of time and other continuous value quantities [54], or to consider non-Markovian action theories [55] for manufacturing recipes. We plan to address these directions as future work.

With the theory and framework in place, a next step is to devise actual tools for the synthesis of manufacturing processes, that are based on a ConGolog formalization. In this respect, possible approaches to start from are, e.g., those of [36], based on predicate abstraction, or [57,58], where verification is addressed by resorting to First-Order BDDs, and [59], where (LTL) realizability is addressed by compilation into safety and reachability games.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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