

Autonomous and Mobile Robotics

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Wheeled Mobile Robots 5

Motion Control of WMRs: Regulation

DIPARTIMENTO DI INGEGNERIA INFORMATICA
AUTOMATICA E GESTIONALE ANTONIO RUBERTI



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regulation

- drive the unicycle to a desired **configuration** q_d
- the **obvious** approach (choose a path/trajectory that stops in q_d , then track it via feedback) **does not work**:
 - linear/nonlinear controllers based on the error dynamics require **persistent** trajectories
 - i/o linearization leads **point B** to the destination rather than the representative point of the unicycle
- being nonholonomic, WMRs (unlike manipulators) do **not** admit **universal controllers**, i.e., controllers that can stabilize arbitrary trajectories, **persistent or not**

cartesian regulation

- drive the unicycle to a given **cartesian position** (w.l.o.g., the **origin** $(0\ 0)^T$) with **any** orientation

- consider the control law

$$v = -k_1(x \cos \theta + y \sin \theta)$$

$$\omega = k_2(\text{Atan2}(y, x) - \theta + \pi)$$

- v is proportional to the **projection** of $e_p = (-x, -y)$ on the sagittal axis
- ω is proportional to the **pointing error** (the difference between the orientation of the unicycle and that of e_p)

- Lyapunov-like function

$$V = \frac{1}{2}(x^2 + y^2) \quad \text{positive semidefinite}$$

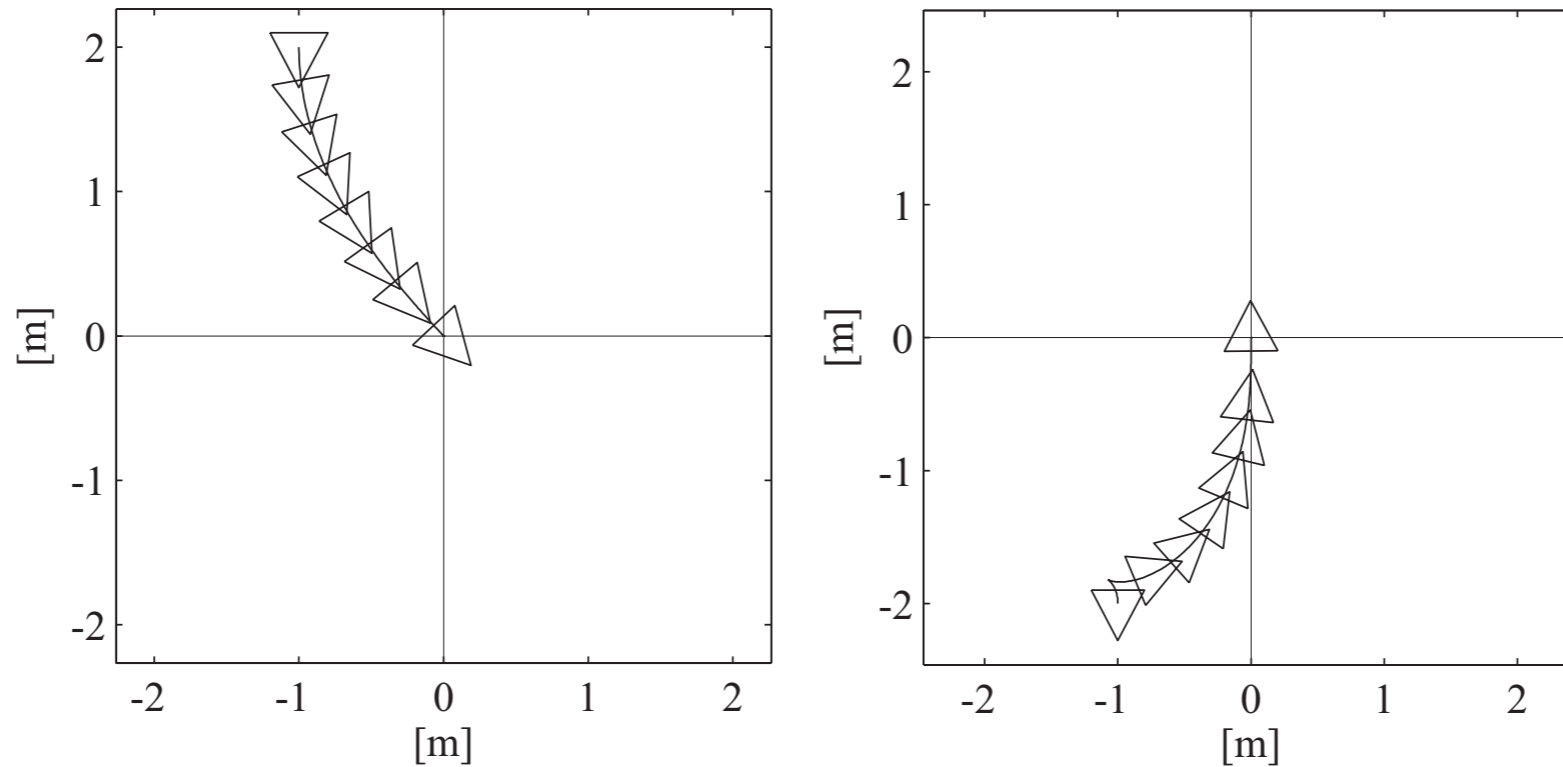
$$\dot{V} = -k_1(x \cos \theta + y \sin \theta)^2 \quad \text{negative semidefinite}$$

- cannot use La Salle theorem, but **Barbalat lemma** implies that \dot{V} tends to zero, i.e.

$$\lim_{t \rightarrow \infty} (x \cos \theta + y \sin \theta) = 0$$

- under the proposed controller, this implies that the **cartesian error** goes to zero

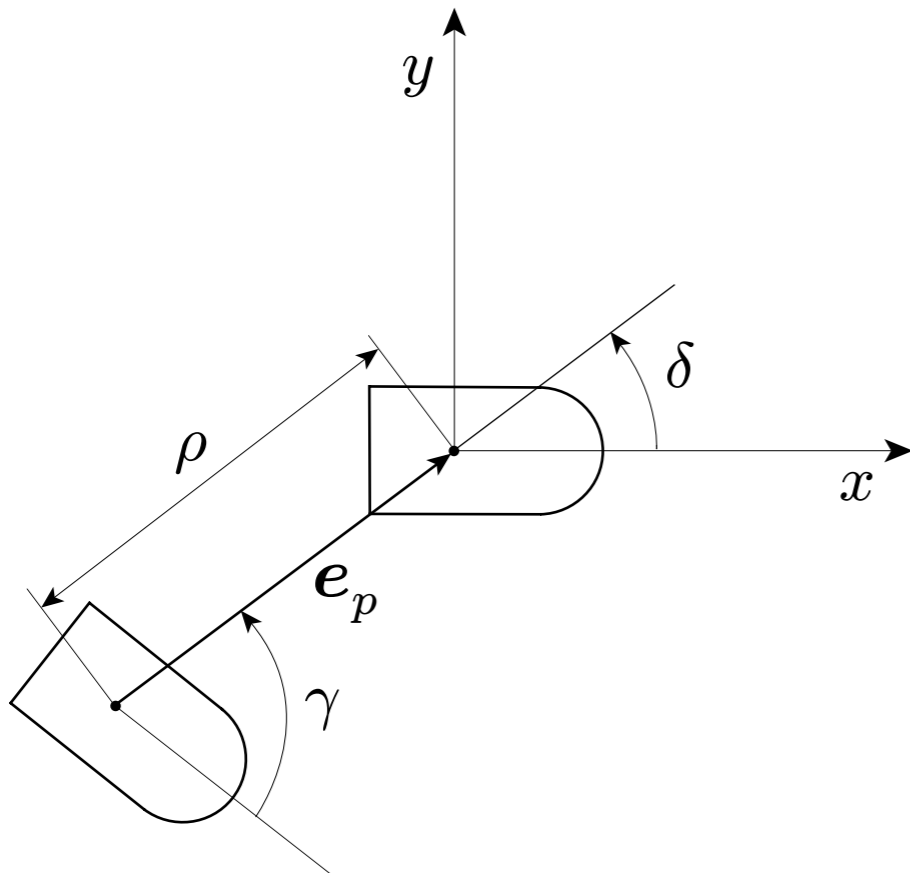
simulation



- final orientation is **not** controlled
- at most one **backup** maneuver

posture regulation

- drive the unicycle to a given **configuration** (w.l.o.g., the **origin** $(0 \ 0 \ 0)^T$)
- convert to **polar coordinates**



$$\rho = \sqrt{x^2 + y^2}$$

$$\gamma = \text{Atan2}(y, x) - \theta + \pi$$

$$\delta = \gamma + \theta$$

- kinematic model in polar coordinates

$$\dot{\rho} = -v \cos \gamma$$

$$\dot{\gamma} = \frac{\sin \gamma}{\rho} v - \omega$$

$$\dot{\delta} = \frac{\sin \gamma}{\rho} v$$

note the **singularity** at the origin

- consider the control law (compare with previous)

$$v = k_1 \rho \cos \gamma$$

$$\omega = k_2 \gamma + k_1 \frac{\sin \gamma \cos \gamma}{\gamma} (\gamma + k_3 \delta)$$

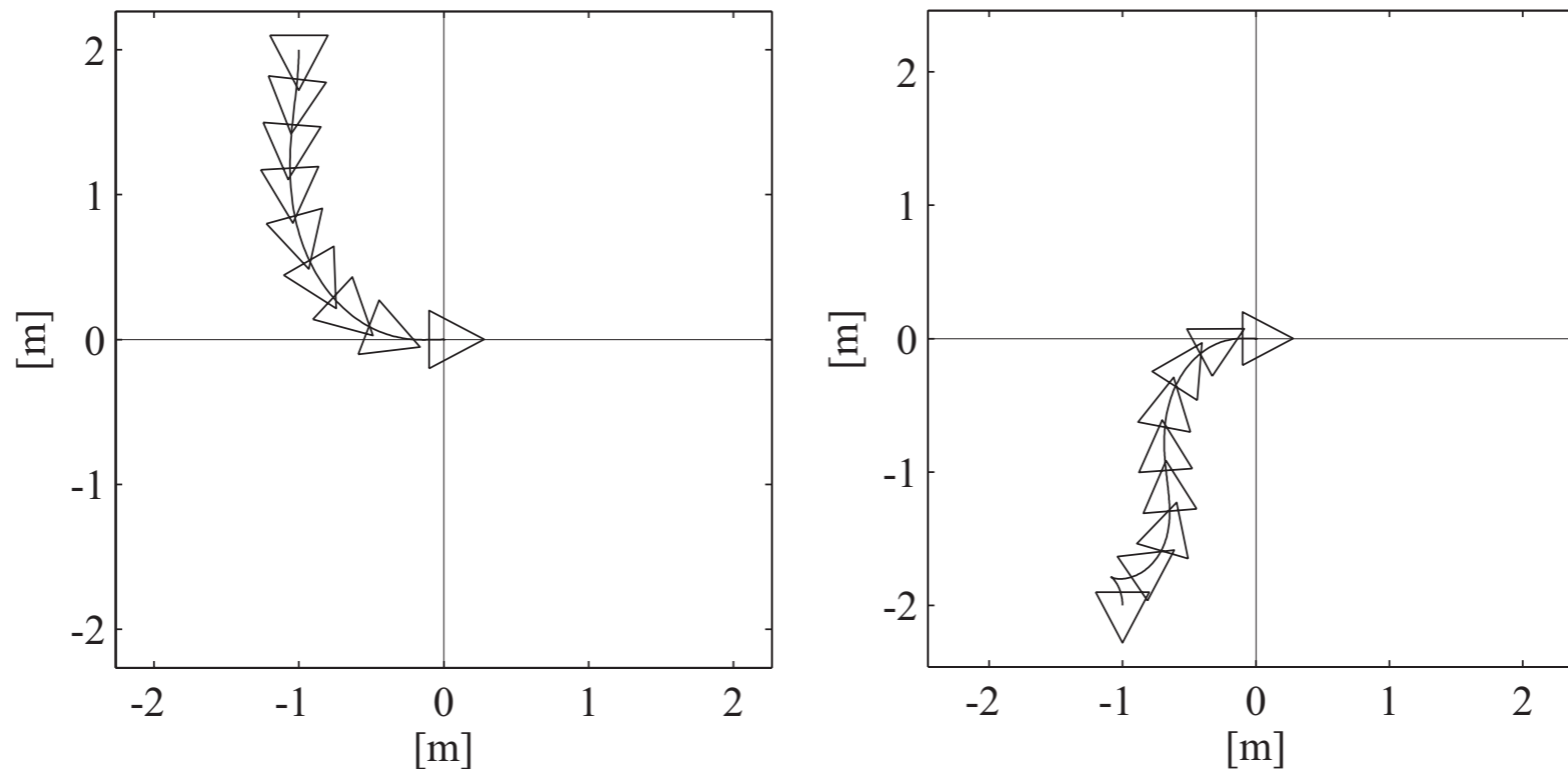
- Lyapunov candidate

$$V = \frac{1}{2} (\rho^2 + \gamma^2 + k_3 \delta^2) \quad \text{positive definite}$$

$$\dot{V} = -k_1 \cos^2 \gamma \rho^2 - k_2 \gamma^2 \quad \text{negative semidefinite}$$

- **Barbalat lemma** implies that ρ, γ and δ go to zero
- the above control law, once mapped back to the original coordinates, is **discontinuous** at the origin
- in fact, due to the nonholonomy, all posture stabilizers must be necessarily **discontinuous w.r.t. the state** or **time-varying**

simulation



- final orientation is **zeroed** as well
- at most one **backup** maneuver