Autonomous and Mobile Robotics: Introduction to Humanoid Robots

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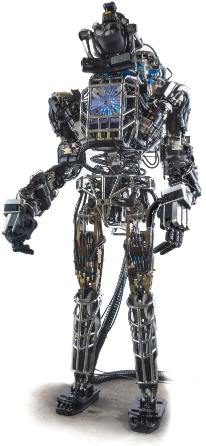
outline

- introduction to humanoid robots
- some history
- full dynamic model
- centroidal dynamics
- single-rigid-body dynamics
- zero moment point
- linear inverted pendulum



what is a humanoid robot?

- a humanoid robot is a robot with a **human-like morphology**
- usually has legs, but some robots on wheels can be considered partially humanoids (e.g., Aldebaran Pepper)
- most humanoid robots are also equipped with arms, either with grippers or with human-like hands

why humanoid robots?

- **practical** reasons: in many cases, humanoid robots are the most sensible choice, especially in environments structured for humans

- wheeled robots have difficulty moving in cluttered environments, or climbing stairs, etc...

- quadrupeds usually do not have arms, as they are not designed for locomanipulation

why humanoid robots?

- **interaction**: human-like motion is easier to interpret and predict
- **psychological** reasons: many people are fascinated by humanoid robots
- humanoid robots have a long history in literature, cinema and TV shows (Asimov's law of robotics, Star Wars, Futurama, ...)
- people usually find it easier to **empathize** with a robot with human-like features

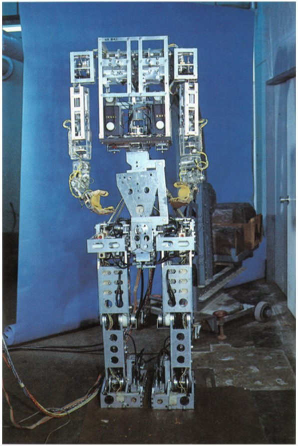
history of humanoid robots

- Waseda WABOT-1
- Honda ASIMO
- Kawada HRP-4
- Aldebaran NAO
- IIT iCub
- Boston Dynamics ATLAS
- Unitree G1
- Boston Dynamics “new ATLAS”

WABOT-1

- WABOT-1 is possibly the first full-scale humanoid robot ever constructed
- it was built by the Humanoid Research Laboratory at Waseda University (Tokyo, Japan)
- video

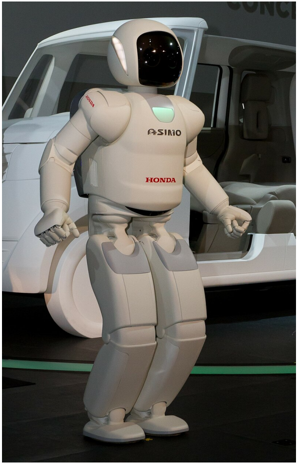
Year	1973
Height	130 cm
Weight	160 kg
DOFs	50



ASIMO

- ASIMO can be considered the pinnacle of Honda's research in humanoid robotics
- it can walk, run, jump, as well as perform bimanual manipulation (e.g., serve a drink)
- video

Year	2000
Height	130 cm
Weight	48 kg
DOFs	34



Aldebaran NAO

- NAO is a small humanoid robot developed by Aldebaran Robotics (now SoftBank Robotics)
- widely used in research and education, and for the RoboCup (5v5 football with robots)
- video

Year	2008
Height	58 cm
Weight	4.5 kg
DOFs	25

iCub

- iCub is a humanoid robot developed by the Italian Institute of Technology
- video

Year	2009
Height	104 cm
Weight	22 kg
DOFs	53

HRP-4

- HRP-4 is one of the humanoids built by Kawada Robotics
- it was developed in collaboration with the Japanese Institute of Advanced Industrial Science and Technology (AIST)
- video

Year	2010
Height	151 cm
Weight	39 kg
DOFs	34

Boston Dynamics Atlas

- Boston Dynamics Atlas is capable of performing backflips and parkour maneuvers using model predictive control
- it uses hydraulic actuation, but a new version using electric actuation was announced recently
- video

Year	2017
Height	150 cm
Weight	89 kg
DOFs	28

Unitree G1

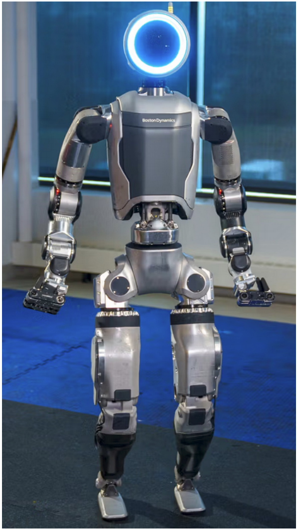
- Unitree G1 is a bipedal humanoid robot developed by the Chinese company Unitree Robotics
- it combines great agility and robustness at a relatively low price
- video

Year	2023
Height	165 cm
Weight	44 kg
DOFs	32

Boston Dynamics new Atlas

- the new Atlas is the latest version of the Boston Dynamics humanoids, with electric actuation
- video

Year	2024
Height	? cm
Weight	? kg
DOFs	?

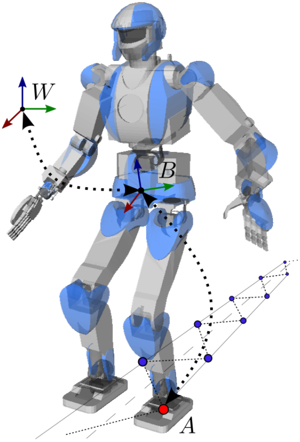


models of humanoid robots

- modeling the dynamics of a humanoid robot can be approached in many different ways
- a **full dynamic model** gives us the most complete description of the system, and we need this model for **whole-body** control
- for **predictive control**, however, the full model may be too complex: we need a **simplified model**

full dynamic model

- we know how to express the dynamics of a manipulator using **Lagrangian dynamics**
- we can do the same for a legged robot, but we must carefully choose the configuration variables
- the **joint configuration** is **not sufficient**! we must also express the position and orientation of the robot in the world



full dynamic model

- the **floating base** is a particular robot link (usually the torso), whose coordinates $(p_0, r_0) \in SE(3)$ are chosen to represent the absolute position and orientation of the robot

- typically, this is described as a **position vector**, and a **quaternion** for the orientation

$$\boldsymbol{p}_0 = (x_0, y_0, z_0), \quad \boldsymbol{r}_0 = a_0 + b_0\mathrm{i} + c_0\mathrm{j} + d_0\mathrm{k}$$

- the linear and angular **velocities** of the floating base are

$$\dot{\boldsymbol{p}}_0 = (\dot{x}_0, \dot{y}_0, \dot{z}_0), \quad \boldsymbol{\omega}_0 = (\omega_0^x, \omega_0^y, \omega_0^z)$$

full dynamic model

- the **configuration vector** $\boldsymbol{q}$, therefore, is composed of the position $\boldsymbol{p}_0$ and orientation $\boldsymbol{r}_0$ of the floating base, and the joint coordinates $\boldsymbol{q}_j$

$$\boldsymbol{q} = \begin{pmatrix} \boldsymbol{p}_0 \\ \boldsymbol{r}_0 \\ \boldsymbol{q}_j \end{pmatrix}$$

- the **pseudovelocities** $\boldsymbol{\nu}$ are given by the stack of the linear velocity $\dot{\boldsymbol{p}}_0$ and angular velocity $\boldsymbol{\omega}_0$ of the floating base, and the joint velocities $\boldsymbol{q}_j$

$$\boldsymbol{\nu} = \begin{pmatrix} \dot{\boldsymbol{p}}_0 \\ \boldsymbol{\omega}_0 \\ \dot{\boldsymbol{q}}_j \end{pmatrix}$$

full dynamic model

- the full dynamic model can be expressed in the familiar form

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + n(\boldsymbol{q}, \boldsymbol{\nu}) = \boldsymbol{S}\boldsymbol{\tau} + \sum_i \boldsymbol{J}_i^T \boldsymbol{f}_i$$

- $\boldsymbol{M}(\boldsymbol{q})$ is the inertia matrix
- $\boldsymbol{n}(\boldsymbol{q}, \boldsymbol{\nu})$ collects the Coriolis/centrifugal and gravity terms
- matrix $\boldsymbol{S}$ maps joint torques $\boldsymbol{\tau}$ to generalized forces performing work on generalized coordinates; the rows corresponding to the floating base coordinates are zero because the base is not actuated
- $\boldsymbol{J}_i$ is the Jacobian matrix of the i-th contact point; for example, if robot is walking it may be the ground reaction force on one foot

simplified models

- the full dynamic model is very complex and strongly nonlinear
- sometimes, it is better to adopt a simpler model, that captures only the essential aspects of the dynamics
- the simplified models can be derived from the full model, but it is much easier to start from scratch, by writing the **Newton-Euler equations**

Newton-Euler equations

- the **Newton-Euler equations** describe the dynamics of the robot as a whole, in terms of balance of forces and momenta
- **force balance**: the sum of all forces acting on the robot is equal to the total mass m times the acceleration of the **Center of Mass** (CoM) $\boldsymbol{p}_c = (p_c^x, p_c^y, p_c^z)$

$$m\ddot{\boldsymbol{p}}_c = m\boldsymbol{g} + \sum_i \boldsymbol{f}_i$$

- **moment balance**: the sum of all moments of forces is equal to the time derivative of the **angular momentum** $\boldsymbol{L}_c$ around the CoM

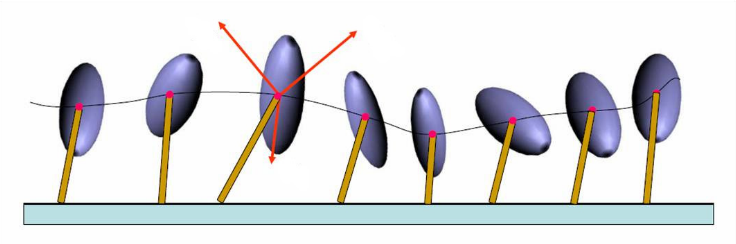
$$\dot{\boldsymbol{L}}_c = \cancel{(\boldsymbol{p}_c - \boldsymbol{p}_c) \times \boldsymbol{g}} + \sum_i (\boldsymbol{p}_i - \boldsymbol{p}_c) \times \boldsymbol{f}_i$$

centroidal dynamics

- the Newton-Euler equations describe the **centroidal dynamics** of the robot

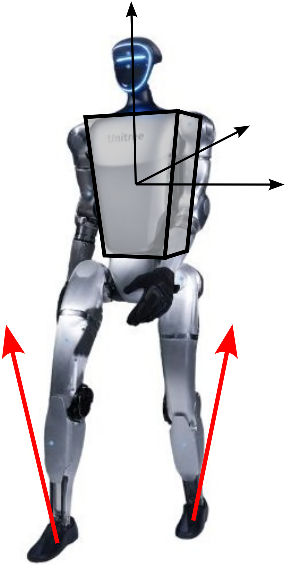
$$m\ddot{\boldsymbol{p}}_c = m\boldsymbol{g} + \sum_i \boldsymbol{f}_i, \qquad \dot{\boldsymbol{L}}_c = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}_c) \times \boldsymbol{f}_i$$

- these are the same equations that describe the motion of a body with mass m and a **variables inertia**, e.g., a deformable **ellipsoid** centered at the CoM



single rigid-body dynamics

- the centroidal angular momentum $\boldsymbol{L}_c$ varies with the configuration of the robot because the inertia varies with $\boldsymbol{q}_j$
- one way of simplifying the model is to assume that the angular momentum comes from the motion of a **single rigid body**
- for example, the orientation of this body could be identified with the orientation of the torso, or the entire robot upper body



single rigid-body dynamics

- velocity of a rigid body: **velocity** of its CoM and **angular velocity** (expressed in the **local frame**)

$$\dot{\boldsymbol{p}}_c = (\dot{p}_c^x, \dot{p}_c^y, \dot{p}_c^z), \qquad \boldsymbol{\omega} = (\omega^x, \omega^y, \omega^z),$$

- the variation of angular momentum of a rigid body is

$$\dot{\boldsymbol{L}}_c = \boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega})$$

- then rotational part of the dynamics can be written as

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega}) = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}_c) \times \boldsymbol{f}_i$$

zero-moment point

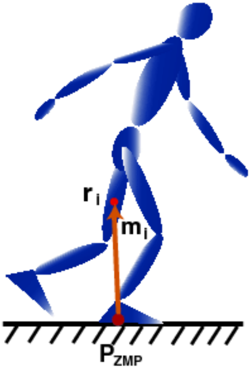
- the **zero-moment point** (ZMP) is an alternative way of encoding information on the contact forces
- it represents the point of application of the resultant **ground reaction force** (GRF)
- in statics, a body is balanced if the ground projection of the CoM is inside the **support polygon**: in dynamics, we do something similar with the ZMP

zero-moment point

- define the ZMP as the point $\boldsymbol{p}_z$ with respect to which the sum of the moments of contact forces is zero

$$\sum_i (\boldsymbol{p}_i - \boldsymbol{p}_z) \times \boldsymbol{f}_i = 0$$

- given a set of forces and their points of applications, we want to find the position of the ZMP



zero-moment point

- compute the vector products in the previous equation

$$\sum_i \begin{pmatrix} 0 & -(p_i^z - p_z^z) & p_i^y - p_z^y \\ p_i^z - p_z^z & 0 & -(p_i^x - p_z^x) \\ -(p_i^y - p_z^y) & p_i^x - p_z^x & 0 \end{pmatrix} \begin{pmatrix} f_i^x \\ f_i^y \\ f_i^z \end{pmatrix} = 0$$

- assume that the humanoid walks on **flat ground**, so that all contact points as well as the ZMP have zero component along z

$$\sum_i \begin{pmatrix} (p_i^y - p_z^y) f_i^z - (\cancel{p_i^z} - \cancel{p_z^z}) f_i^y \\ (\cancel{p_i^z} - \cancel{p_z^z}) f_i^x - (p_i^x - p_z^x) f_i^z \\ (p_i^x - p_z^x) f_i^y - (p_i^y - p_z^y) f_i^x \end{pmatrix} = 0$$

- the first two equations are

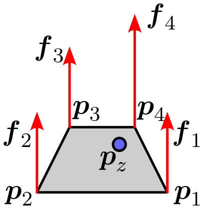
$$\sum_i (p_i^y - p_z^y) f_i^z = 0$$

$$\sum_i (p_i^x - p_z^x) f_i^z = 0$$

- we can write the first two equations compactly as

$$\boldsymbol{p}_z^{x/y} \sum_i f_i^z = \sum_i \boldsymbol{p}_i^{x/y} f_i^z$$

$$\boldsymbol{p}_z^{x/y} = \frac{\sum_i \boldsymbol{p}_i^{x/y} f_i^z}{\sum_i f_i^z}$$



- this is the **position of the ZMP** on flat ground

zero-moment point

- if we denote the total vertical force as $f_z = \sum_i f_i^z$, we can write the position of the ZMP as

$$\boxed{\boldsymbol{p}_z^{x/y} = \sum_i \boldsymbol{p}_i^{x/y} \frac{f_i^z}{f_z}}$$

  this is a **weighted sum** of the position of the contact points

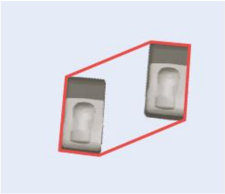
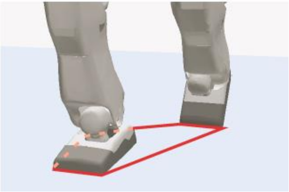
- the coefficients f_i/f_z are positive because forces point up

$$\frac{f_i^z}{f_z} \geq 0, \qquad \sum_i \frac{f_i^z}{f_z} = 1$$

- thus, the ZMP position is a **convex combination** of the contact points

- requiring that contact forces must be **unidirectional** is equivalent to requiring that the ZMP must be inside the **convex hull** of the contact surfaces

- this region is called the **support polygon**

ZMP dynamics

- go back to the **moment balance** equation

$$\dot{\boldsymbol{L}}_c = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}_c) \times \boldsymbol{f}_i$$

- make the ZMP appear

$$\begin{aligned}\dot{\boldsymbol{L}}_c &= \sum_i (\boldsymbol{p}_i - \boldsymbol{p}_c) \times \boldsymbol{f}_i + \sum_i (\boldsymbol{p}_c - \boldsymbol{p}_z) \times \boldsymbol{f}_i - \sum_i (\boldsymbol{p}_c - \boldsymbol{p}_z) \times \boldsymbol{f}_i \\ &= \cancel{\sum_i (\boldsymbol{p}_i - \boldsymbol{p}_z) \times \boldsymbol{f}_i} - (\boldsymbol{p}_c - \boldsymbol{p}_z) \times \sum_i \boldsymbol{f}_i\end{aligned}$$

the first term is zero because of the definition of ZMP, while in the second term the sum of contact forces appears

ZMP dynamics

- computing the sum of contact forces from the first of the Newton-Euler equations $m\ddot{\boldsymbol{p}}_c = m\boldsymbol{g} + \sum_i \boldsymbol{f}_i$ and plugging it in the last equation we get

$$\dot{\boldsymbol{L}}_c = -m(\boldsymbol{p}_c - \boldsymbol{p}_z) \times (\ddot{\boldsymbol{p}}_c - \boldsymbol{g})$$

- in particular, we are interested in the x and y components of this equation

$$\begin{aligned} \dot{L}_c^x &= -m(p_c^y - p_z^y)(\ddot{p}_c^z - g^z) + m(p_c^z - \cancel{p_z^z})(\ddot{p}_c^y - \cancel{g^y}) \\ \dot{L}_c^y &= m(p_c^x - p_z^x)(\ddot{p}_c^z - g^z) - m(p_c^z - \cancel{p_z^z})(\ddot{p}_c^x - \cancel{g^x}) \end{aligned}$$

ZMP dynamics

- from these equations, we can derive the dynamics of the CoM in terms of the position of the ZMP (g^z is $-g$, because it is pointing down)

$$\ddot{p}_c^y = \frac{\ddot{p}_c^z + g}{p_c^z}(p_c^y - p_z^y) + \frac{\dot{L}_c^x}{m p_c^z}$$
$$\ddot{p}_c^x = \frac{\ddot{p}_c^z + g}{p_c^z}(p_c^x - p_z^x) - \frac{\dot{L}_c^y}{m p_c^z}$$

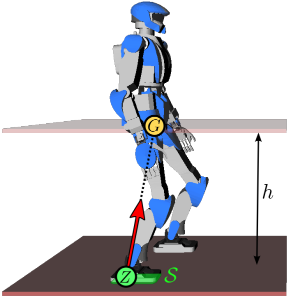
- this can be written compactly as

$$\ddot{\boldsymbol{p}}_c^{x/y} = \frac{\ddot{p}_c^z + g}{p_c^z}(\boldsymbol{p}_c^{x/y} - \boldsymbol{p}_z^{x/y}) + \frac{1}{m p_c^z}\boldsymbol{R}\dot{\boldsymbol{L}}_c^{x/y}, \quad \boldsymbol{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

where $\boldsymbol{R}$ is a $\pi/2$ rotation matrix

linear inverted pendulum

- let us now take two simplifying assumptions: the **CoM height** is constant, i.e., $z_c = h$, and the centroidal **angular momentum** derivative is zero, i.e., $\dot{L}_c = 0$
- these are usually reasonable approximations if the robot walks on flat ground at a normal pace



- the ZMP-CoM dynamics becomes

$$\ddot{\boldsymbol{p}}_c^{x/y} = \cancelto{g/h}{\frac{\ddot{\boldsymbol{p}}_c^z + g}{\boldsymbol{p}_c^z}} (\boldsymbol{p}_c^{x/y} - \boldsymbol{p}_z^{x/y}) + \cancel{\frac{1}{m p_c^z} \boldsymbol{R} \dot{\boldsymbol{L}}_c^{x/y}}$$

linear inverted pendulum

- the **Linear Inverted Pendulum** (LIP) dynamics is

$$\boxed{\ddot{\boldsymbol{p}}_c^{x/y} = \eta^2(\boldsymbol{p}_c^{x/y} - \boldsymbol{p}_z^{x/y})} \qquad \eta = \sqrt{\frac{g}{h}}$$

- it is an **unstable dynamics**: this makes sense because it represents the essence of the dynamics of balancing

- interpretation: the ZMP **pushes away** the CoM