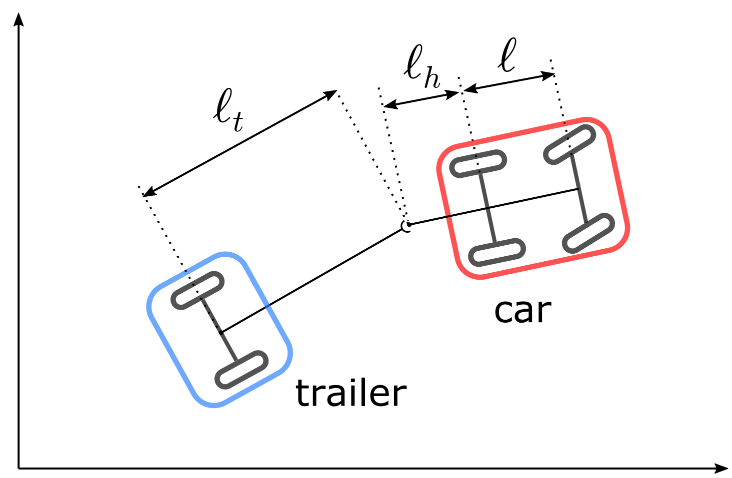
Autonomous and Mobile Robotics
Midterm Class Test, 2023/2024

Problem 1

Consider the vehicle shown in figure, consisting of a rear-wheel drive car towing a trailer. The trailer is a rigid body with an axle carrying two fixed wheels, and is connected to the car through a revolute joint located at a distance ℓ_h from the rear wheel axle.



1. Write the kinematic constraints to which the robot is subject, and derive a kinematic model for it.
2. In the special case $\ell_h = 0$, prove that the Cartesian coordinates of the midpoint of the trailer wheel axle are flat outputs for the system.

Problem 2

Consider a bicycle robot of length ℓ with rear-wheel drive.

1. Write a kinematic model of the robot in which the control inputs are the driving acceleration $a_v = \dot{v}$ and the steering acceleration $a_\omega = \dot{\omega}$.
2. Define as outputs the Cartesian coordinates $\boldsymbol{y} = (y_1, y_2)$ of a point P located at a distance b from the front wheel along the direction of the wheel itself. Design a feedback controller for driving $\boldsymbol{y}$ along a desired trajectory $\boldsymbol{y}_d(t)$ and provide the corresponding block scheme.

   Hints: (1) find an invertible mapping between the time derivative (of a suitable order) of the outputs and the control inputs (2) work as much as possible in matrix format.

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Problem 3

Consider again the bicycle robot of Problem 2, and assume that a digital control scheme is used, where the inputs a_v and a_ω are constant within each sampling interval of duration T_s. The sensing equipment includes (1) a range finder mounted on the front wheel, that measures range and bearing of a known landmark placed at (x_l, y_l) (2) an encoder that measures the rotation $\Delta\alpha$ of the *rear* wheel around the *horizontal* wheel axis during each sampling interval (3) an encoder that measures the rotation $\Delta\phi$ of the *front* wheel around the *vertical* wheel axis during each sampling interval.

Build a localization system for estimating in real time the complete *state* of the robot. Provide the filter equations (be sure to define all symbols), together with a block scheme including all the signals involved in the process and showing how each sensor is used.

Problem 4

Are the following claims *true* or *false*? Answer and provide a short explanation.

(a) Consider a quadruped robot with N_1 revolute joints carrying a manipulator with N_2 revolute joints. Its configuration space is $I\!R^3 \times (SO(2))^{N_1+N_2}$.

(b) A geometric constraint always implies a holonomic kinematic constraint.

(c) In a car-like robot, pure rolling implies that the two front wheels cannot have the exact same orientation in general, while the rear wheels can.

(d) Consider path planning based on flat outputs for a car-like robot. The initial and final values of the orientation θ generate two boundary conditions for the interpolation, whereas the initial and final values of the steering angle ϕ do not.

(e) A unicycle robot can follow arbitrary Cartesian trajectories, whereas a car-like robot cannot.

[3 h]