Autonomous and Mobile Robotics
Midterm Class Test, 2021/2022

Problem 1

Consider a point robot moving in 3D space, with configuration $\boldsymbol{q} = (x, y, z)$. Now assume that the robot is constrained to move over the surface of a sphere centered at the origin and having radius r.

(a) Write the geometric constraint on the robot motion and the corresponding kinematic constraint.

(b) Discuss local and global mobility of the robot.

(c) Derive a kinematic model of the robot and study its controllability.

Problem 2

Consider a (2,3) chained form. It is easy to verify that the application of the control inputs

$$\begin{aligned} v_1 &= a \sin \omega t \\ v_2 &= b \cos \omega t \end{aligned}$$

over the time interval $[0, 2\pi/\omega]$ brings z_1, z_2 back to their initial values and causes a displacement of z_3 equal to $ab\,\pi/\omega^2$. Exploiting this fact (no need to prove it), design a 2-phase algorithm for planning a trajectory that moves the chained form system from any initial configuration $\boldsymbol{z}_i$ to any final configuration $\boldsymbol{z}_f$. Moreover:

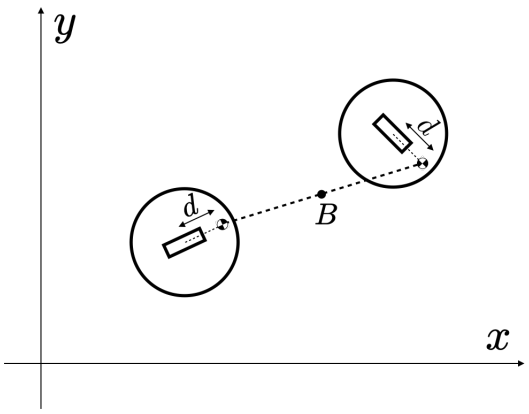
(a) provide a pseudocode description of the algorithm;

(b) discuss which are its free parameters and how to choose their values;

(c) assuming $\boldsymbol{z}_i = (0, 0, 0)$ and $\boldsymbol{z}_f = (1, 1, 1)$, compute the total time needed for reaching $\boldsymbol{z}_f$ and draw the trajectory of the system on the z_1, z_2 plane.

Problem 3

Consider a team composed by two identical unicycle robots. Assume that the barycenter of each robot is located along the sagittal axis at a distance d from the wheel center.



(a) Design a feedback control law that drives the barycenter $B = (x_b, y_b)$ of the team along a given trajectory $x_b^*(t), y_b^*(t)$.

(b) Draw a block scheme of the control system, and indicate which variables must be known for its implementation.

(c) Assume that each robot is equipped with *(i)* a wheel encoder and *(ii)* a range finder (located at the wheel center) that measures both the distance to the other robot and the distance to a known landmark. Design a localization algorithm for estimating the variables needed for implementing the proposed controller. Provide equations (be sure to define all symbols) and a block scheme.

[2 h 30 min]