Autonomous and Mobile Robotics
Midterm Class Test, 2019/2020

Problem 1

The kinematic model of a unicycle in polar coordinates is expressed as

$$\begin{aligned}\dot{\rho} &= -v\cos\gamma \\ \dot{\gamma} &= \frac{\sin\gamma}{\rho}v - \omega \\ \dot{\delta} &= \frac{\sin\gamma}{\rho}v\end{aligned}$$

where ρ, γ and δ are defined as in Fig. 1 and v, ω are the driving and steering velocity inputs.

1. Prove that the above kinematic model is controllable.
2. Assume that $v = \bar{v}$, with $\bar{v}$ constant and positive. Design a feedback control law for ω that will bring γ asymptotically to $\pi/2$ (*look at the second equation...*). What kind of Cartesian motion will the unicycle perform at steady state?

Problem 2

Using the bicycle equations, prove analytically that in this robot the velocity of the rear wheel (i.e., the velocity of the contact point of the rear wheel) is never larger than the velocity of the front wheel (i.e., the velocity of the contact point of the front wheel), and give a geometric interpretation of this fact.

Problem 3

Consider the unicycle robot of Fig. 2 moving in a corridor of width a (note the world frame). The robot uses a digital control scheme where the inputs are the driving and steering *accelerations*, respectively a_v and a_ω. The sensing equipment includes (1) a range finder located at the center of the robot that measures the distance d to the upper wall (2) an IMU that measures the orientation of the robot and its velocity along the x axis.

1. Write the kinematic model of the system with a_v and a_ω as control inputs (*it should be a 5-dimensional system...*).
2. Derive a discrete-time model of the system that can be used for odometric localization under the assumption that a_v and a_ω are known.
3. Build an EKF for estimating the complete *state* of the robot. Provide the filter equations and a block scheme showing all the signals involved in the process.

[2 h 50 min; see back for Figures 1-2]

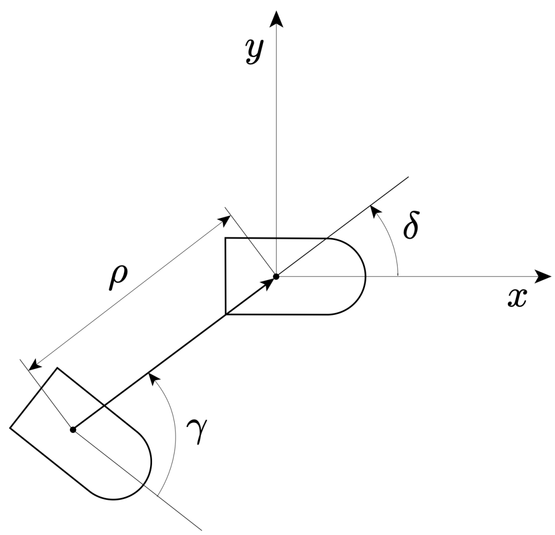

Figure 1: Polar coordinates for Problem 2

Figure 2: The geometric setting for Problem 3