Applicazioni dell'Automatica

Introduction to mobile robotics
Kinematics and modeling of WMRs

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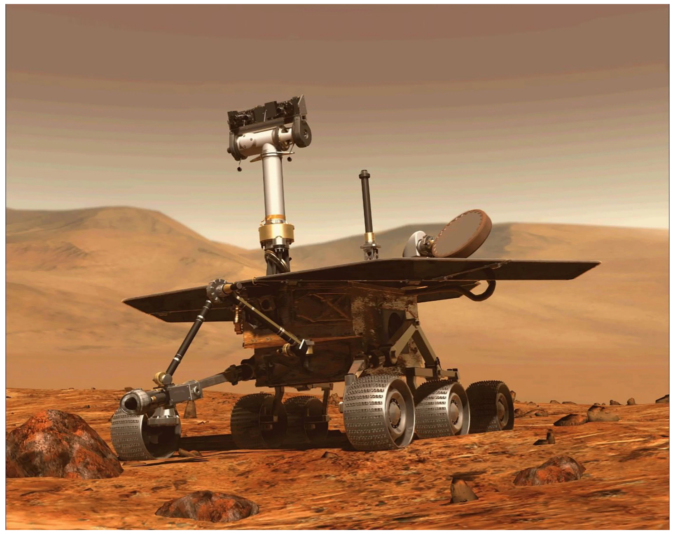
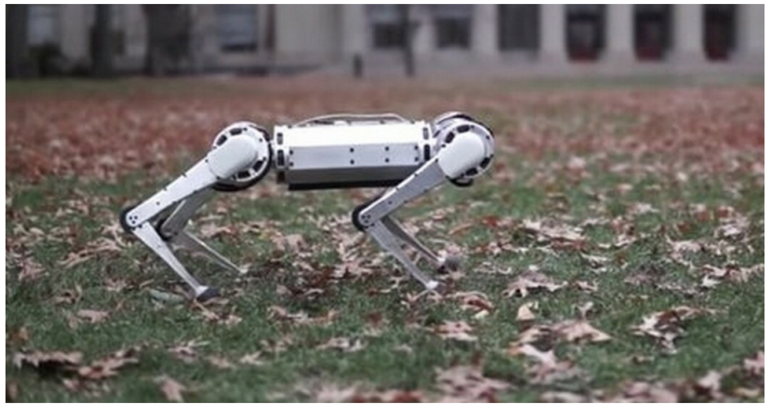
outline of this lecture

- ground locomotion and balance
- wheels
- kinematic structures
- unicycle robot and its kinematic model
- nonholonomic constraints
- unicycle: equivalent vehicles
- car-like robot and its kinematic model
- unicycle vs. car-like model: mobility
- controllability
- odometric localization

ground locomotion

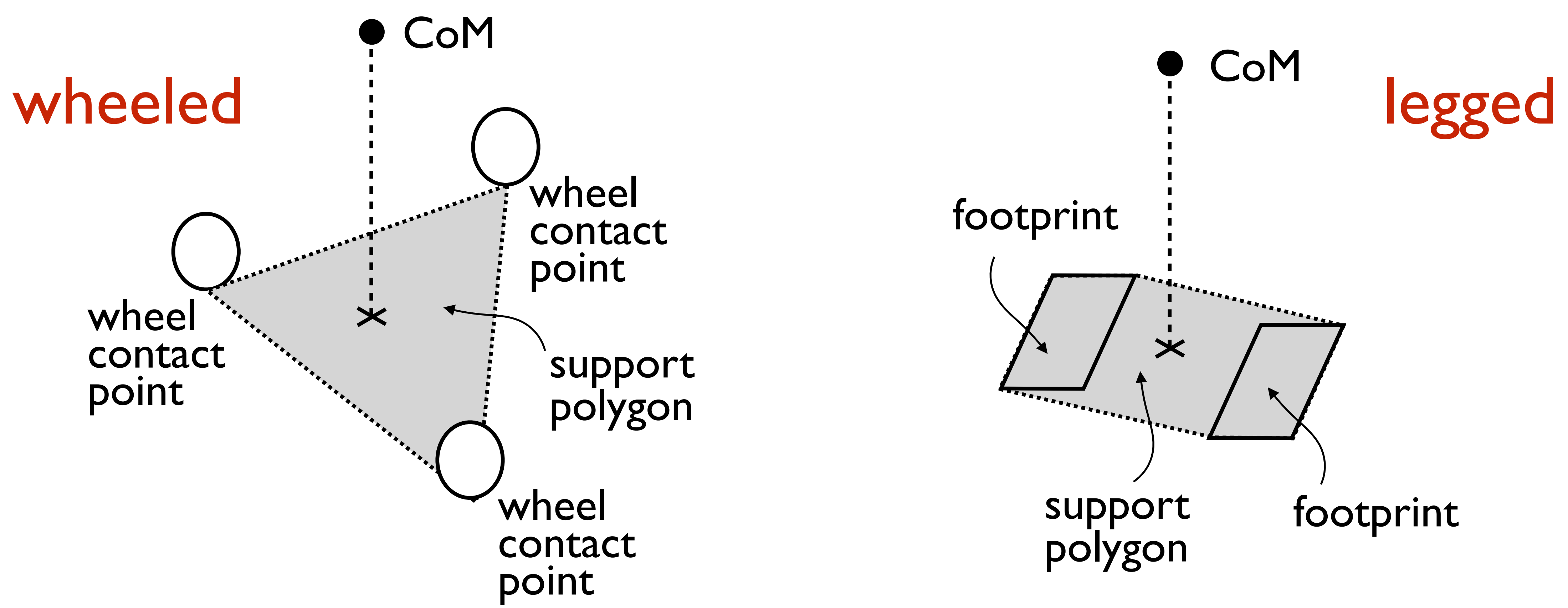
- requires contact via
  - wheels: wheeled mobile robots (WMRs), typically consisting of a rigid body (base or chassis) + wheels
  - feet: legged robots, typically consisting of several rigid bodies, articulated through joints
- some mobile robots can achieve locomotion on the ground without wheels or feet: e.g., snake robots





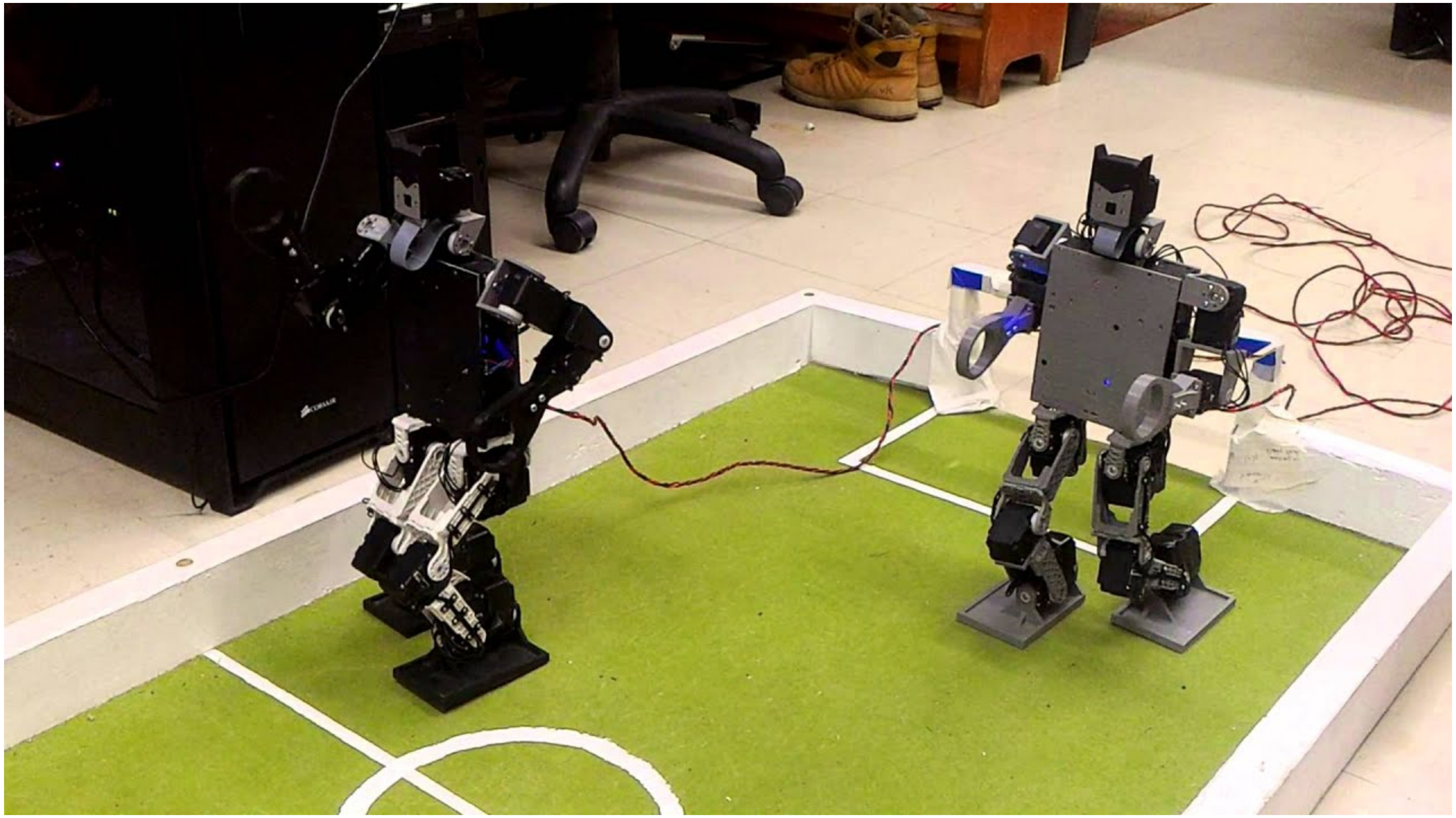
balance (not falling)

- statical balance is achieved when the projection of the robot Center of Mass (CoM) falls inside the support polygon; in the case of WMRs, one needs 3 wheels!



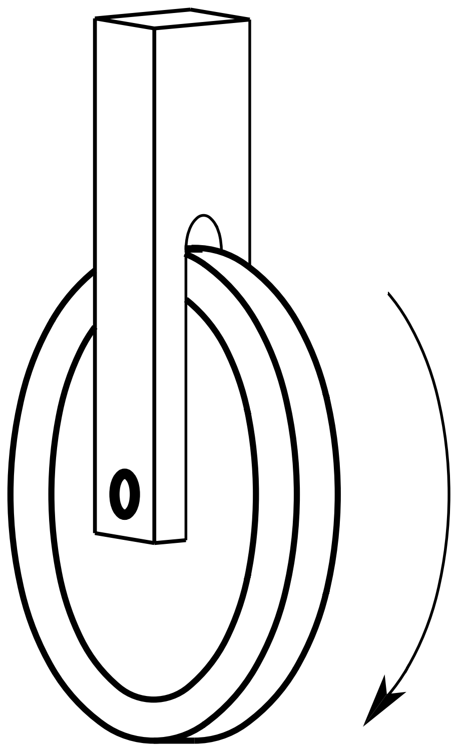
- dynamical balance is a different type of balance in which the CoM is replaced by the Zero Moment Point (ZMP)

balance (not falling)



dynamic walking vs static walking

wheels: 3 basic types

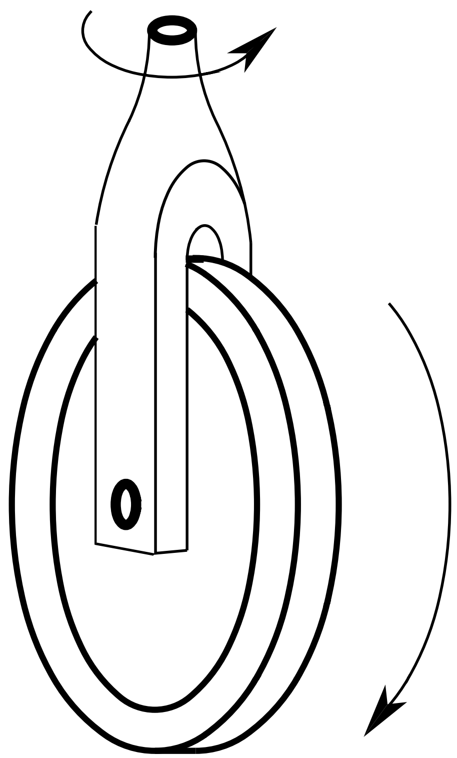


icon

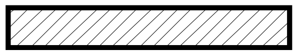
fixed wheel

- fixed orientation w.r.t. the chassis
- may be active (used for driving) or passive (used for balance)

wheels: 3 basic types



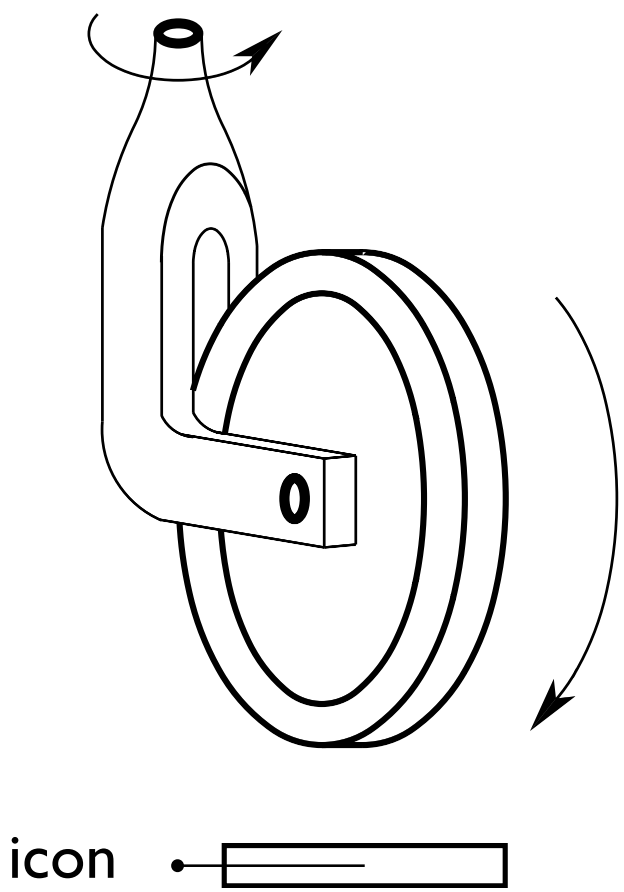
icon



orientable (steerable) wheel

- variable orientation w.r.t. the chassis
- typically active (used for steering)

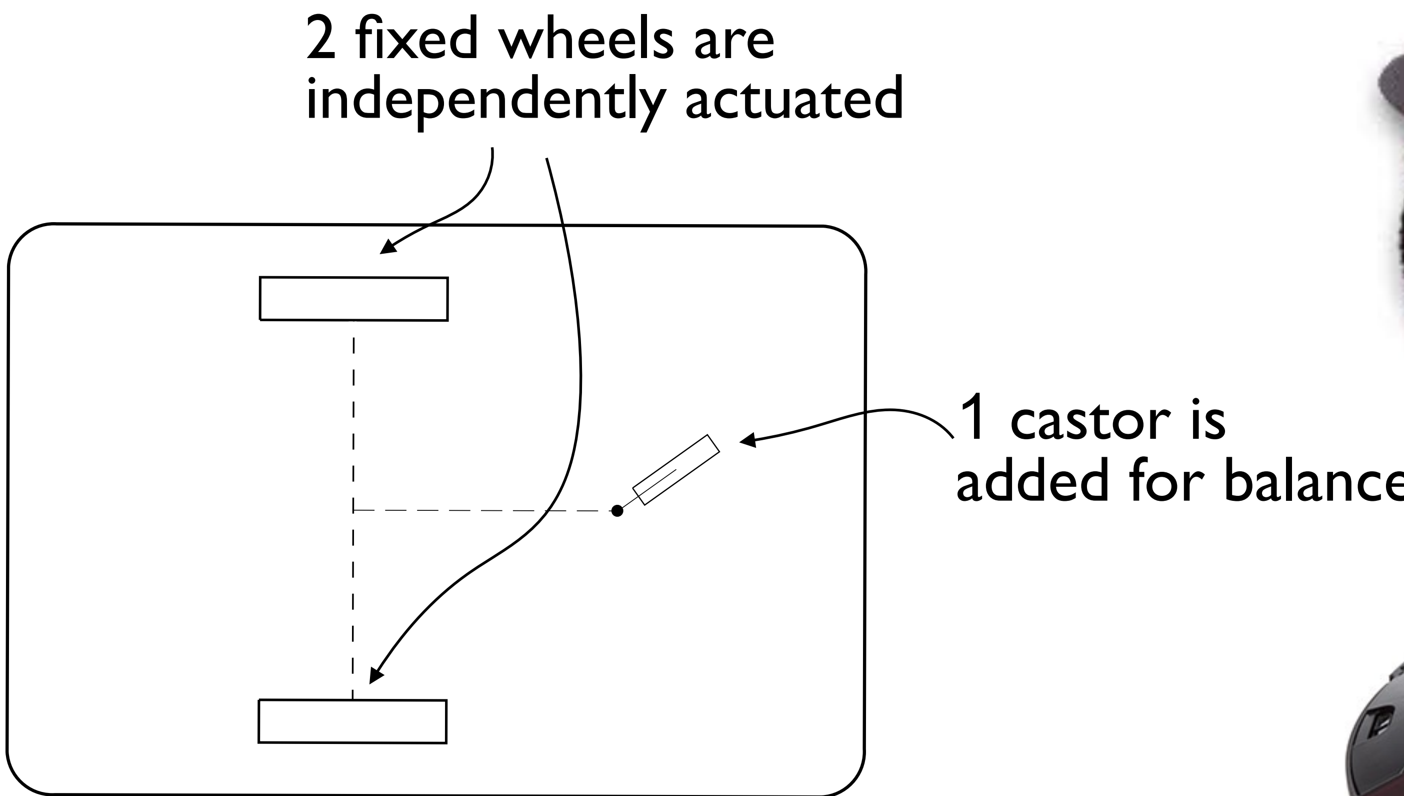
wheels: 3 basic types



caster wheel

- variable orientation w.r.t. the chassis
- automatically aligns with the direction of motion
- typically passive (used for balance)

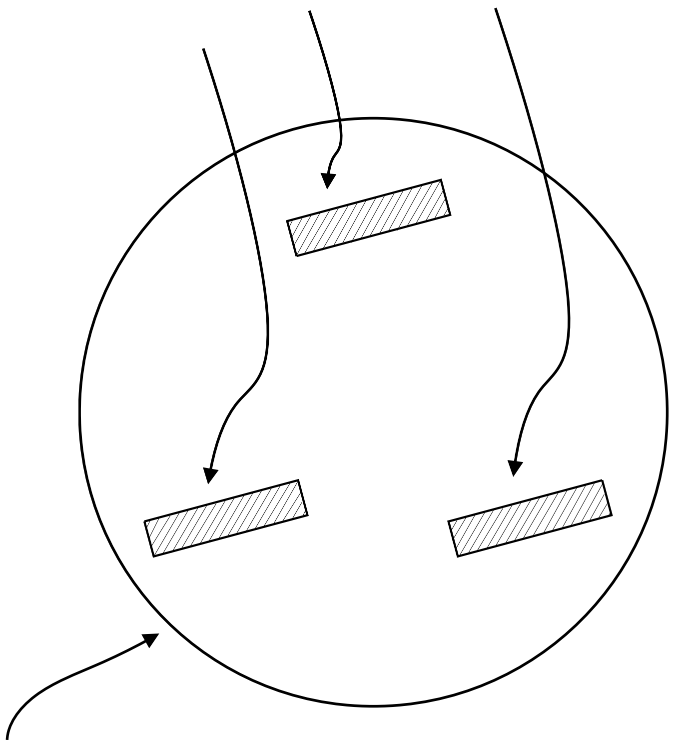
kinematic structures



differential-drive mobile robot

kinematic structures

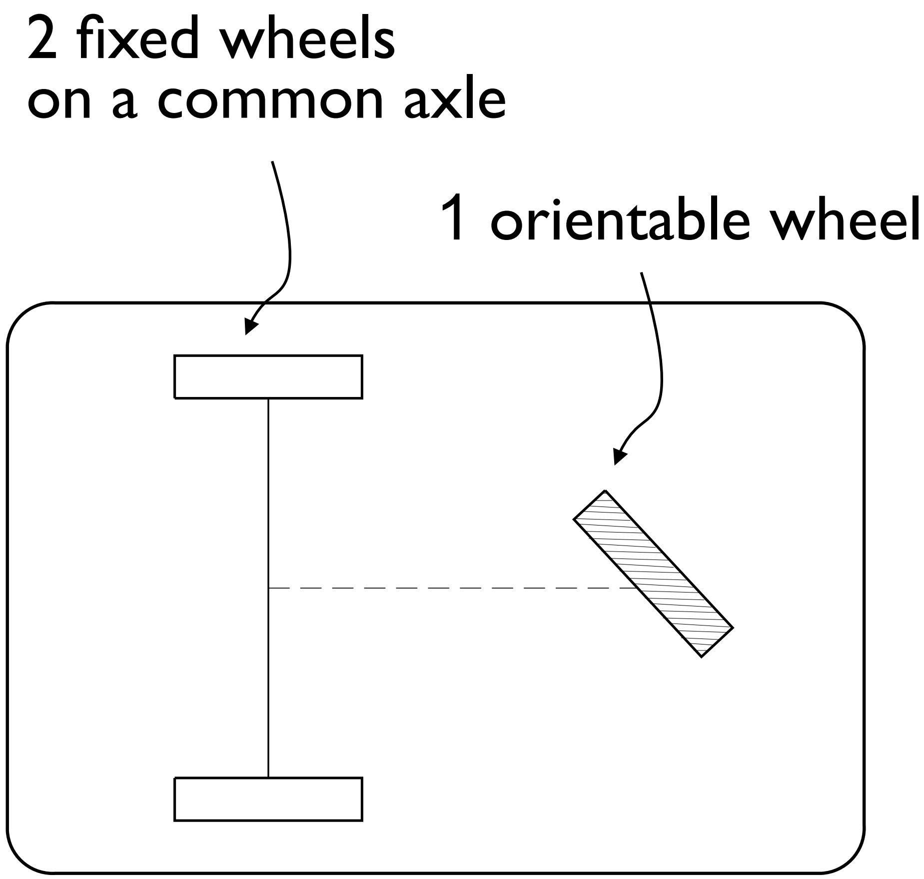
3 orientable wheels are
simultaneously actuated



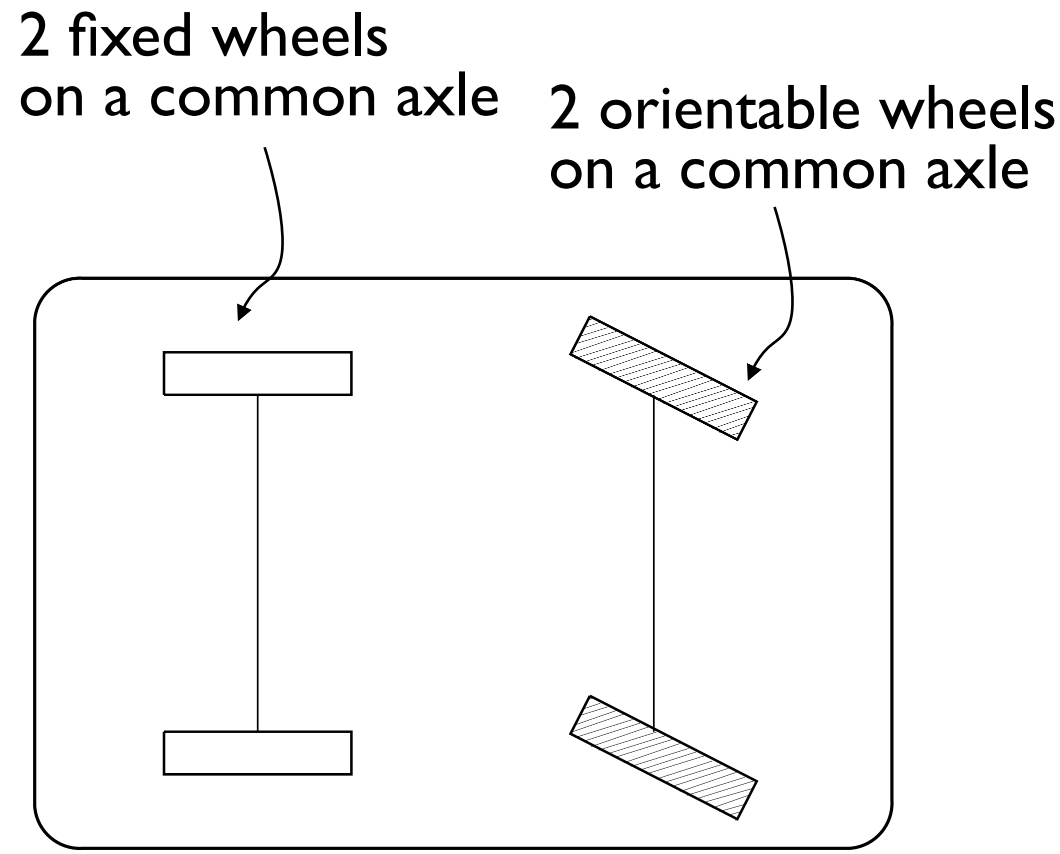
the orientation of the chassis
remains constant!

synchro-drive mobile robot

kinematic structures



tricycle

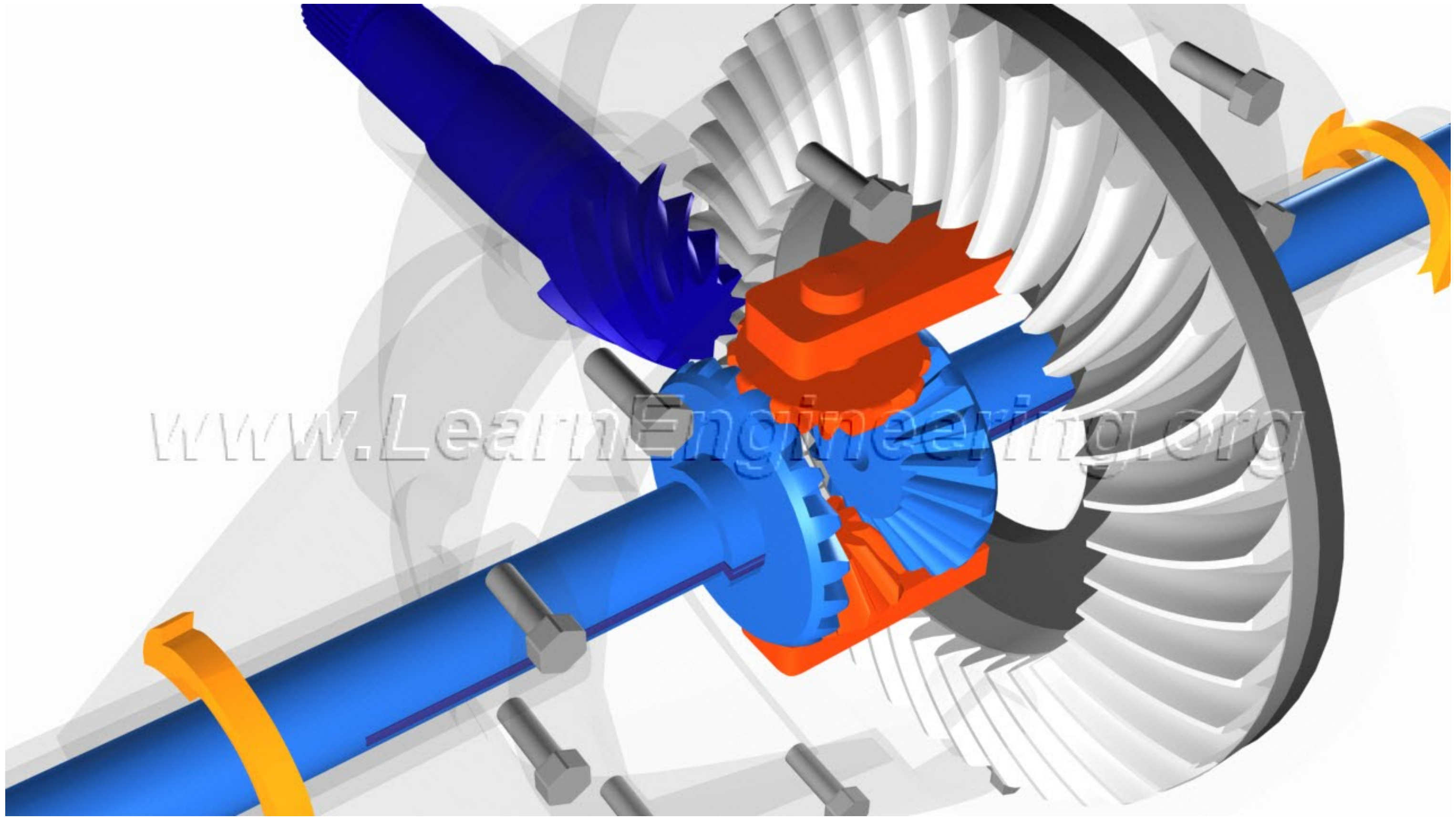


car-like

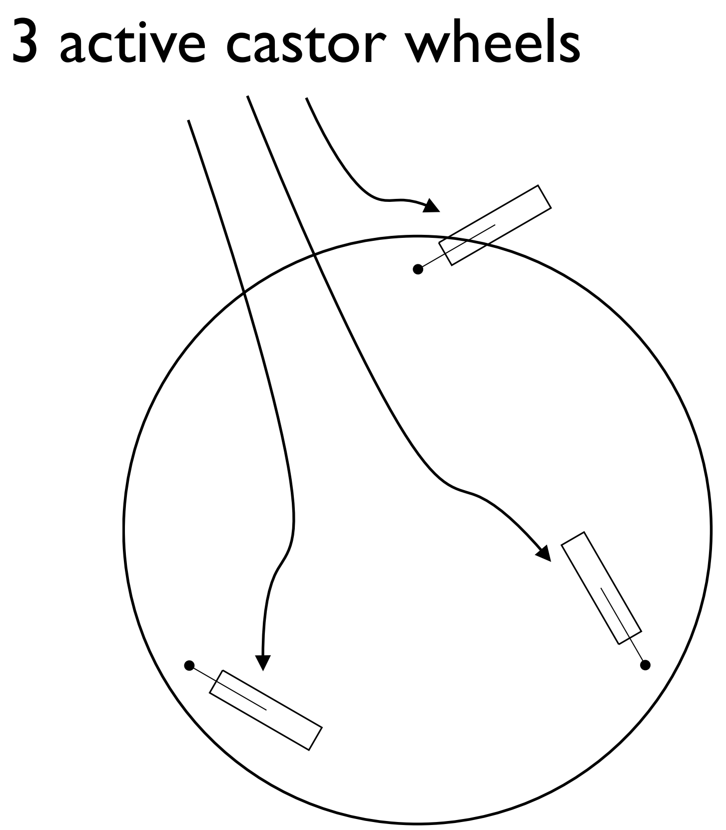
- both may be front-wheel drive or rear-wheel drive!

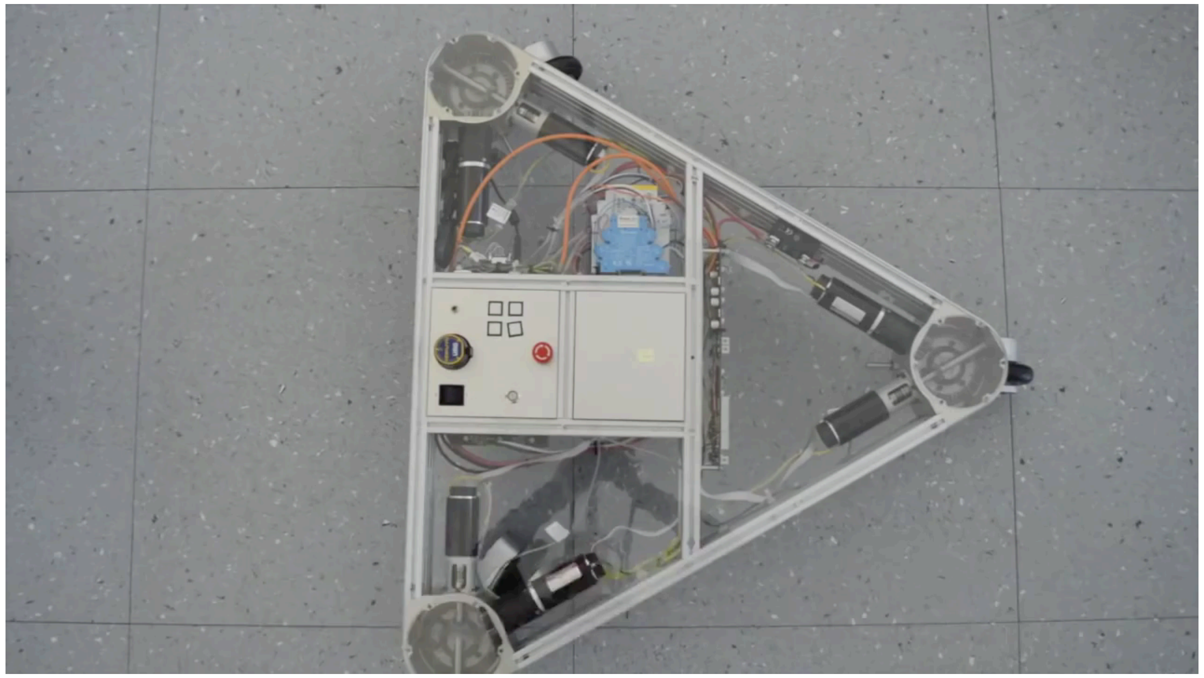
differential

- needed whenever two driving wheels are mounted on a common axle
- a mechanical device that allows the two wheels to move at different speeds



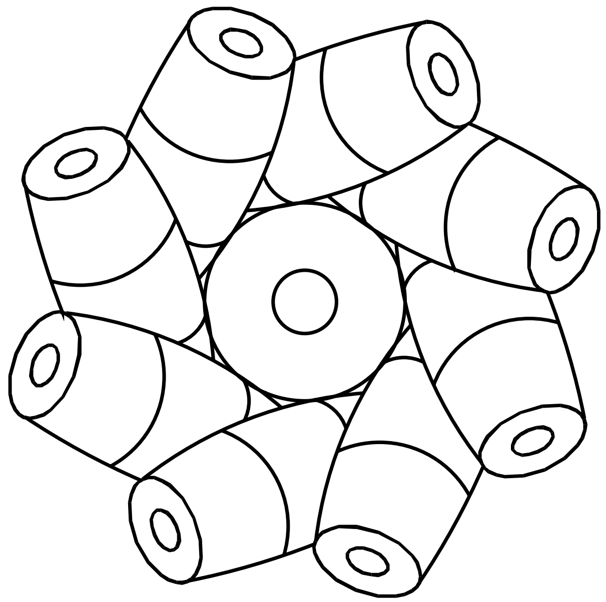
kinematic structures





omnidirectional mobile robot with
3 (actuated) caster wheels

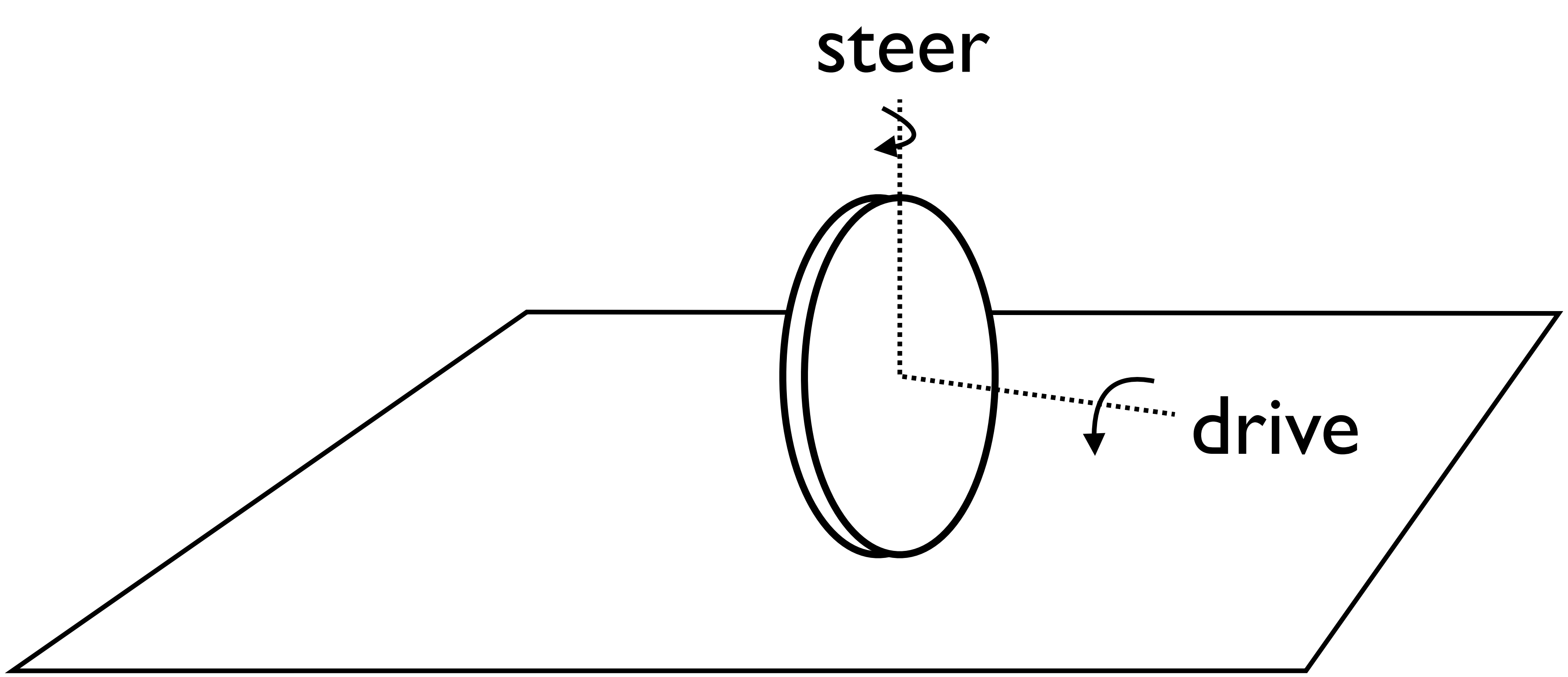
kinematic structures



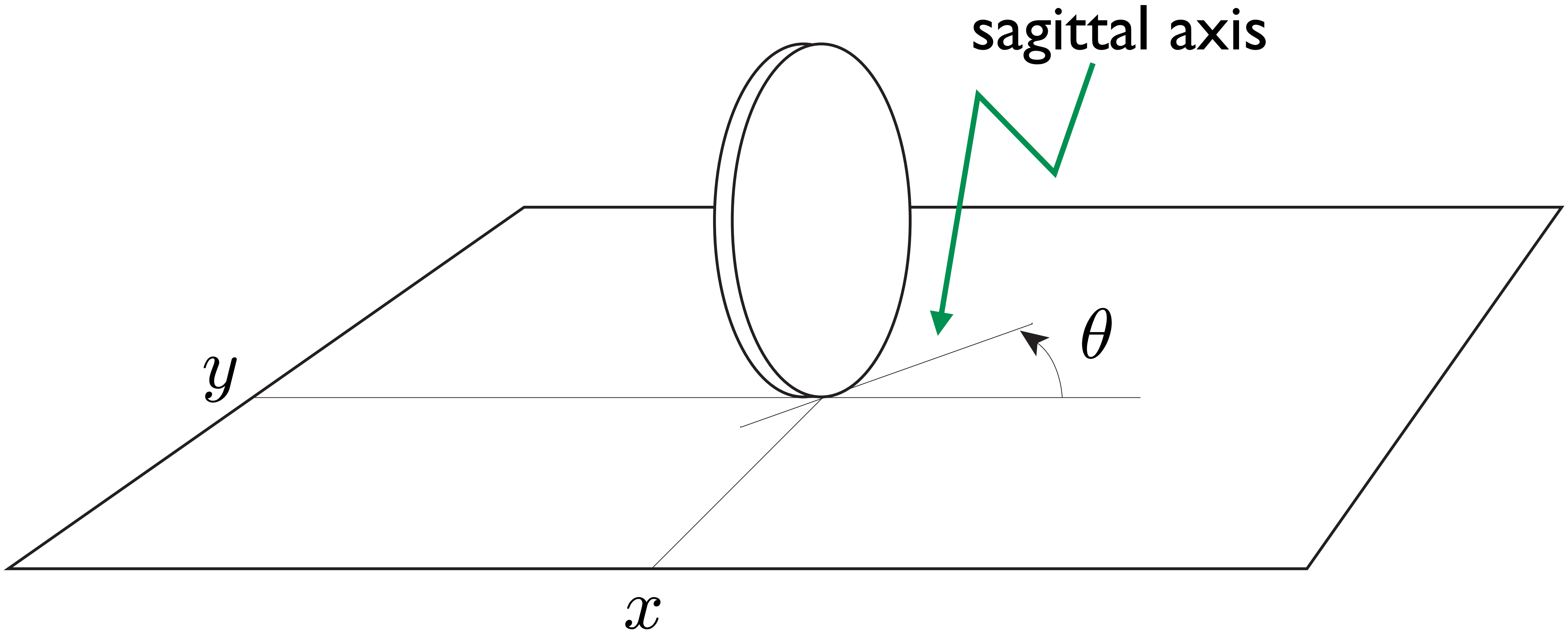
Mecanum (Swedish) wheels can be also used to build omnidirectional mobile robots

unicycle

- the simplest WMR we can conceive has a single wheel which can both drive (roll on the ground) and steer (rotate around the vertical axis)
- this rather abstract robot is called a unicycle
- a real unicycle would be unbalanced…



unicycle: kinematic model



- generalized coordinates $\boldsymbol{q} = [x \quad y \quad \theta]^T$

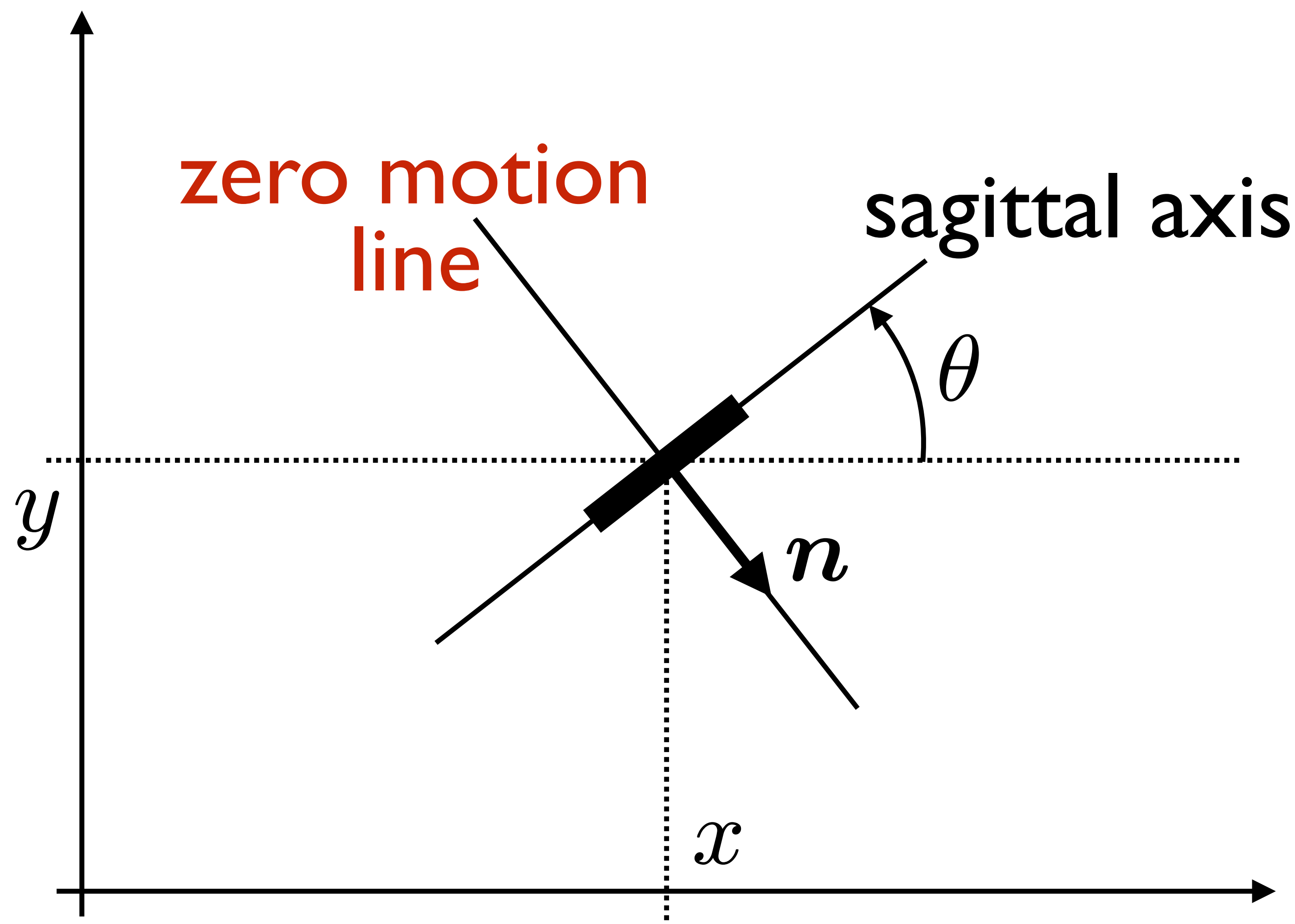
Cartesian or angular

robot position

robot orientation

- $\boldsymbol{q}$ is called configuration vector

unicycle: kinematic model



- if the wheel does not slip, we have

$$\begin{pmatrix} \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \boldsymbol{n}^T \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = 0$$

pure rolling constraint

no instantaneous motion along the zero motion line!

nonholonomic constraints

- the pure rolling constraint involves both generalized coordinates q and generalized velocities $\dot{q}$
- such constraints are called kinematic, and they may be integrable (holonomic) or not (nonholonomic)
- for example

  $x\,\dot{x} + y\,\dot{y} = 0$ can be integrated as $x^2 + y^2 = r^2$

  the robot must move on a circle around the origin!
- the pure rolling constraint is instead nonholonomic
- a system with a holonomic constraint cannot reach all configurations, while a nonholonomic constraint does not prevent this

unicycle: kinematic model

- how do we derive a model from this constraint?
- rewrite the constraint so that all coordinates appear

$$\begin{pmatrix} \sin\theta & -\cos\theta & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \boldsymbol{A}^T(\boldsymbol{q})\dot{\boldsymbol{q}} = 0$$

- then all admissible generalized velocities at $\boldsymbol{q}$ belong to the null space of $\boldsymbol{A}^T(\boldsymbol{q})$

$$\dot{\boldsymbol{q}} = v \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} + \omega \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

kinematic model

a basis of $\mathcal{N}(\boldsymbol{A}^T(\boldsymbol{q}))$

unicycle: kinematic model

- since

$$\dot{x}^2 + \dot{y}^2 = v^2 \quad \text{and} \quad \dot{\theta} = \omega$$

v is the driving velocity (modulus of Cartesian velocity) and ω is the steering velocity of the robot

- the kinematic model can be rewritten as

$$\dot{x} = v \cos\theta$$
$$\dot{y} = v \sin\theta$$
$$\dot{\theta} = \omega$$

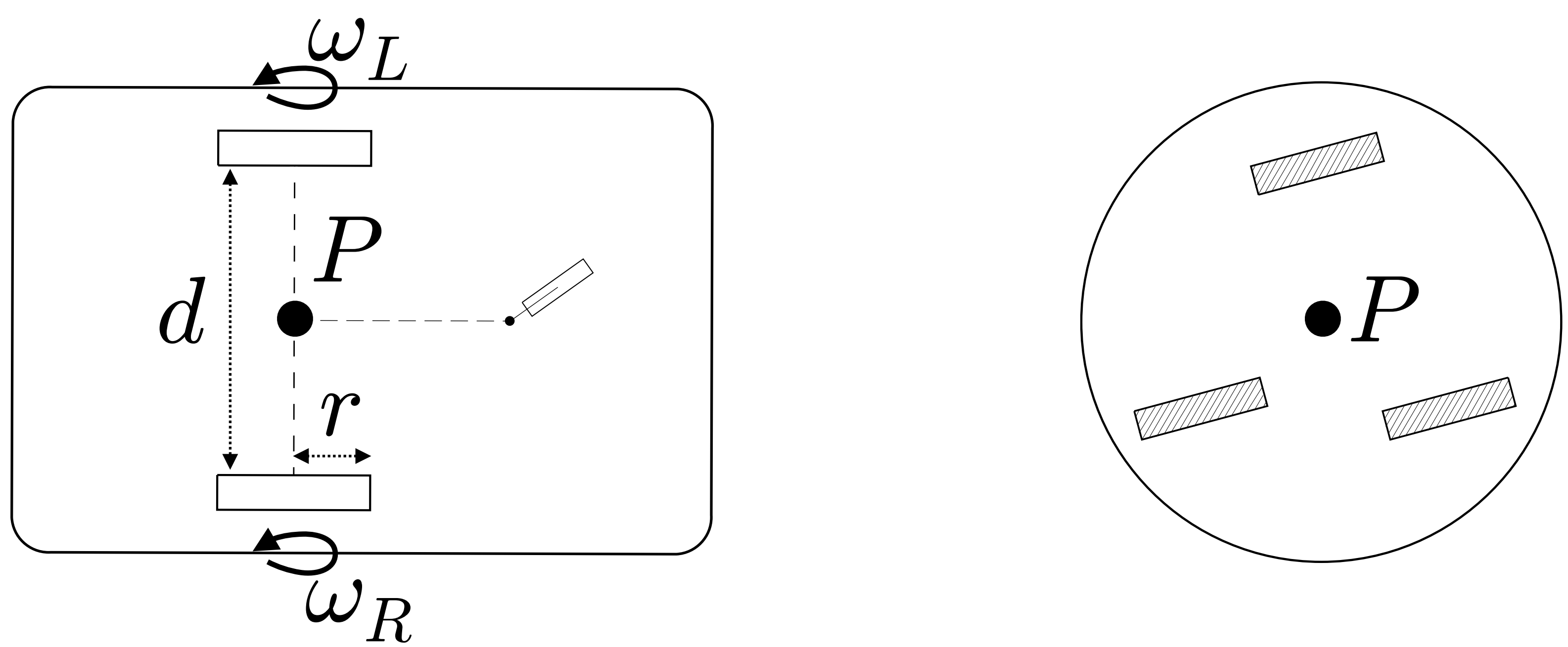
a dynamical system!

- with state $\boldsymbol{q}$ and inputs v, ω
- nonlinear in the state
- driftless

- by "kinematic model" here we mean "a description of all admissible instantaneous motions of the vehicle"

unicycle: equivalent vehicles

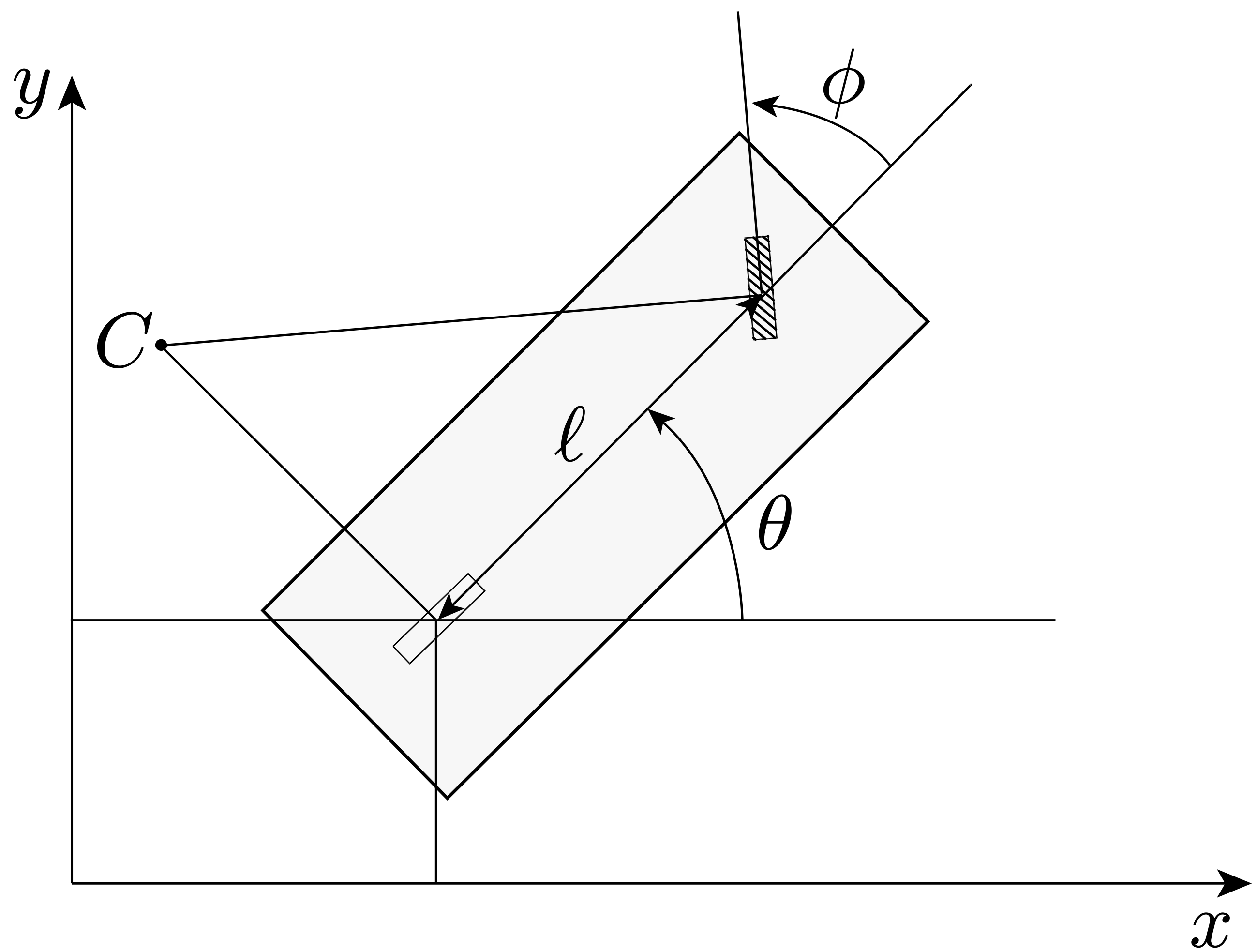
- the differential drive and the synchro drive robot are mechanically balanced versions of the unicycle



- if (x,y) denotes the position of P, their kinematic models are the same of the unicycle
- input transformation for the differential drive

$$v = \frac{r(\omega_R + \omega_L)}{2} \qquad \omega = \frac{r(\omega_R - \omega_L)}{d}$$

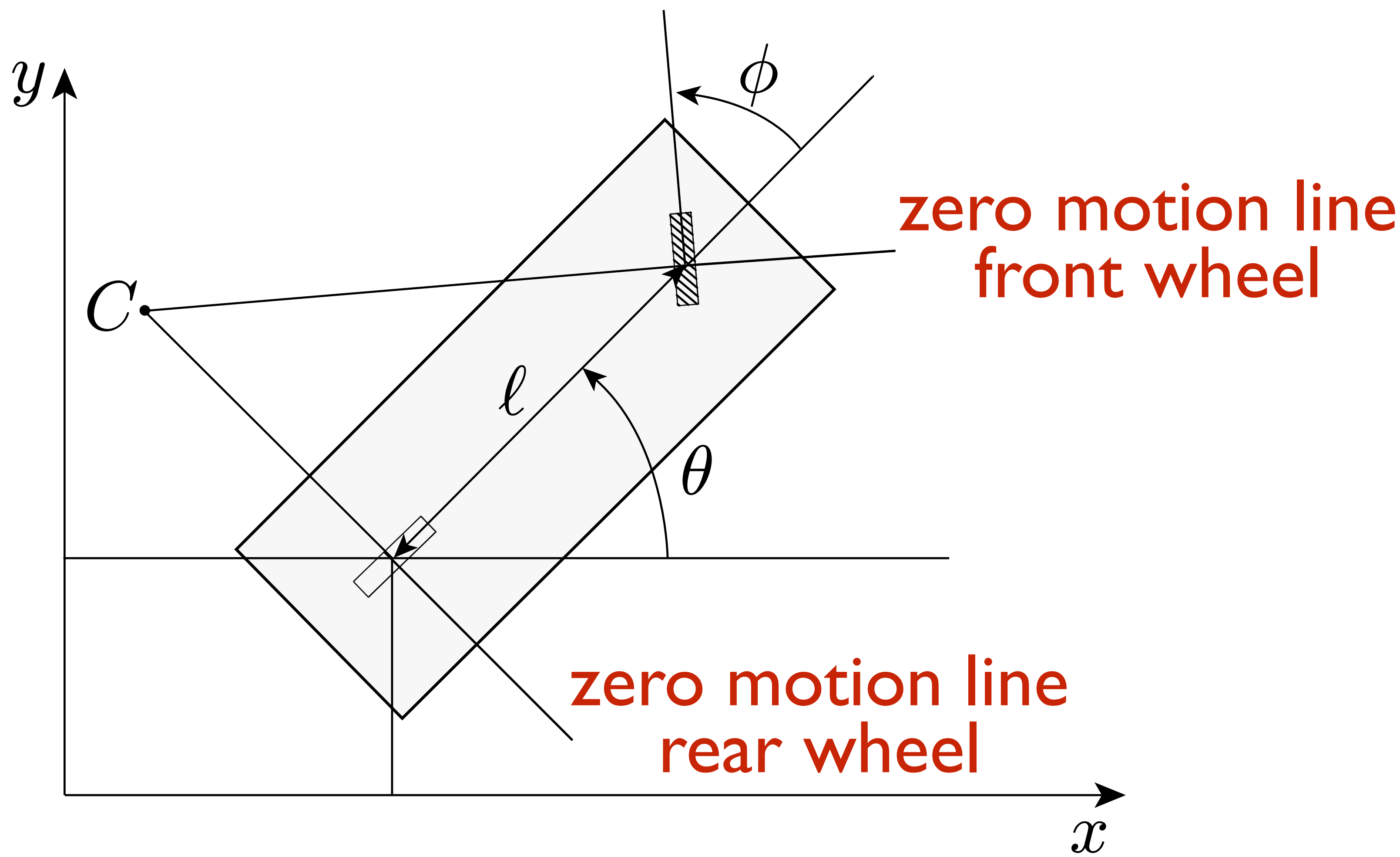
car-like robot: kinematic model



- collapse front wheels and rear wheels (bicycle model)
- generalized coordinates $\boldsymbol{q} = [x \;\; y \;\; \theta \;\; \phi]^T$

robot position (x, y)
robot orientation (θ)
steering angle (ϕ)

car-like robot: kinematic model



- two pure rolling constraints, one for each wheel
- $\boldsymbol{A}^T(\boldsymbol{q})$ is then a 2×4 matrix
- the two zero motion lines meet at a point C, called the instantaneous center of rotation (ICR)

car-like robot: kinematic model

- all admissible generalized velocities at $\boldsymbol{q}$ belong to the null space of $\boldsymbol{A}^T(\boldsymbol{q})$, which is now 2-dimensional
- this leads to the following kinematic model

$$\dot{\boldsymbol{q}} = v \begin{pmatrix} \cos\theta \\ \sin\theta \\ (\tan\phi)/\ell \\ 0 \end{pmatrix} + \omega \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

a basis of $\mathcal{N}(\boldsymbol{A}^T(\boldsymbol{q}))$

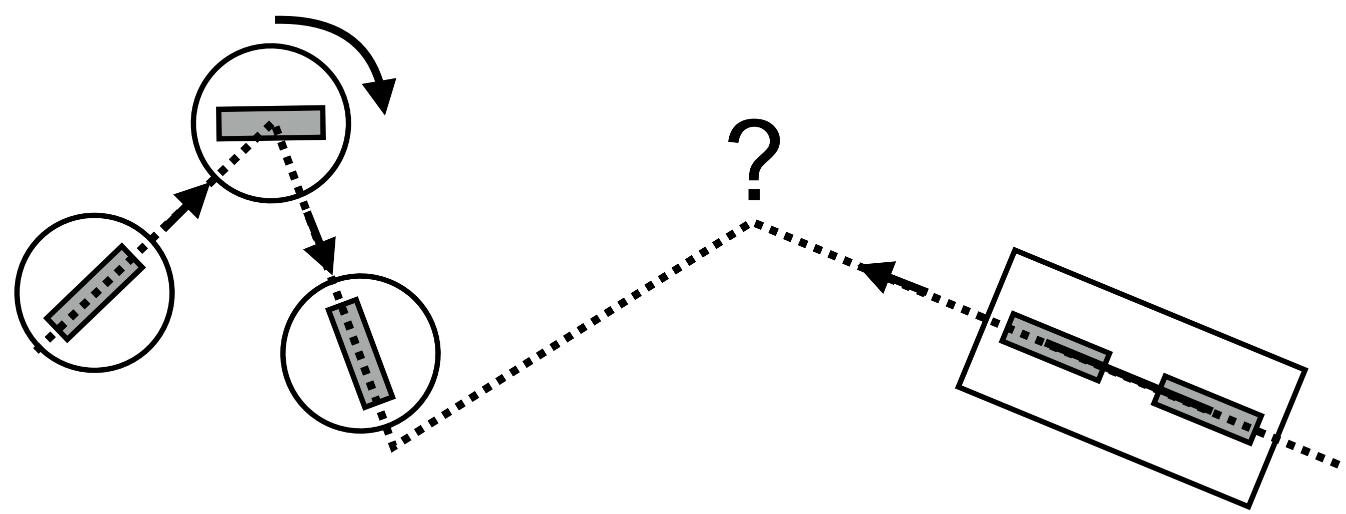
- since

$$\dot{x}^2 + \dot{y}^2 = v^2 \quad \text{and} \quad \dot{\phi} = \omega$$

v is the driving velocity (assuming rear-wheel drive) and ω is the steering velocity of the robot

unicycle vs car-like robot: mobility

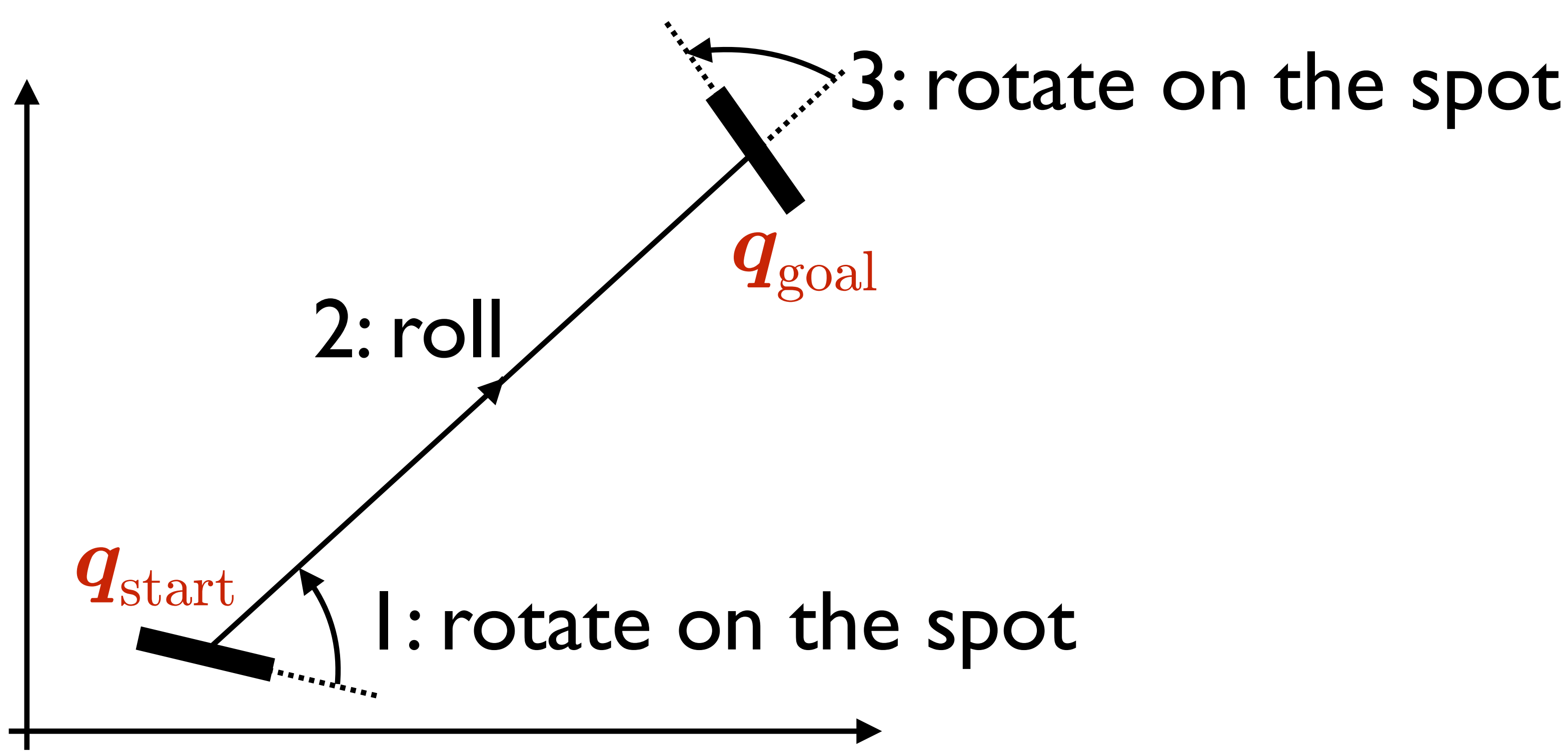
- the unicycle can follow Cartesian paths with corners, as it can rotate on the spot; the car-like robot cannot



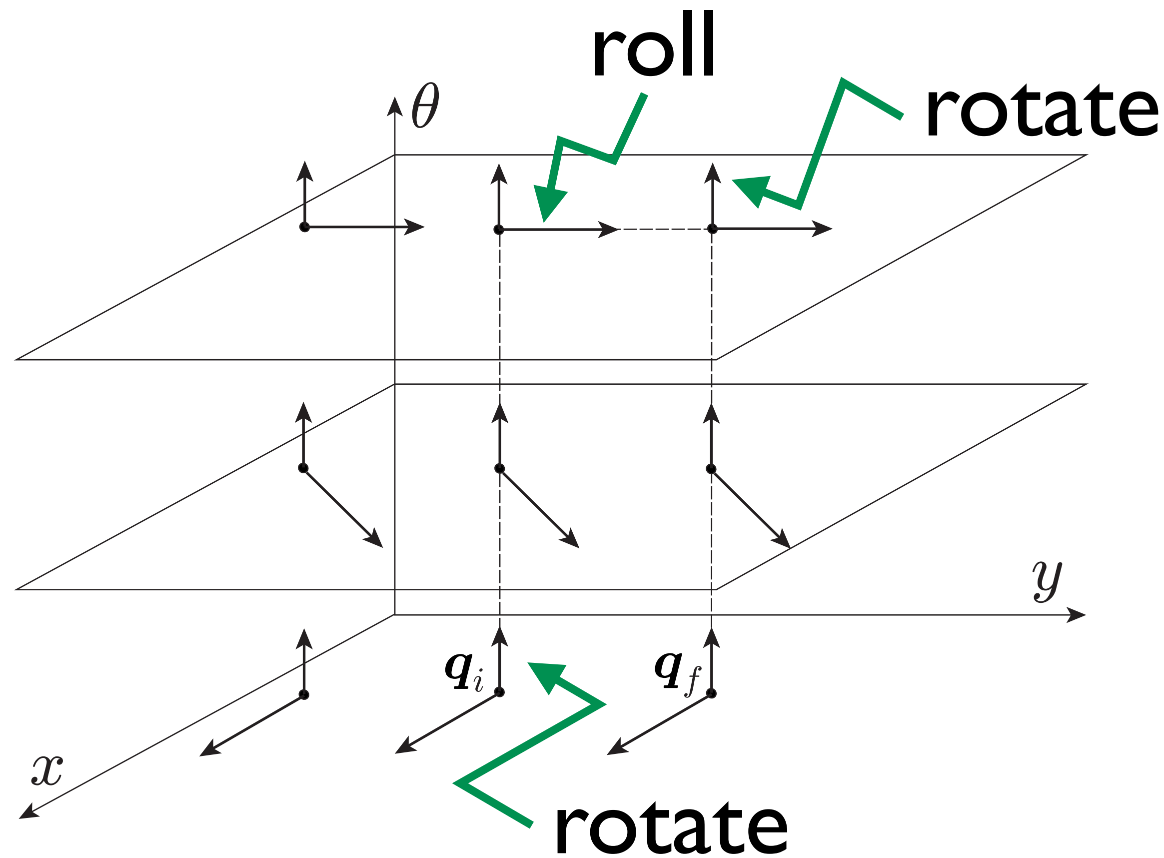
- the unicycle can handle curvature jumps; the car-like robot needs to stop and turn its steering wheel

controllability

- in spite of the different mobility, both robots are controllable: i.e., it is possible to drive from any configuration to any other configuration (parking)
- unicycle: rotate on the spot until the robot is pointing towards the final position; roll to it; rotate on the spot again to achieve the final orientation



- the same maneuver in the configuration space $\mathcal{C}$



- controllability holds also in the presence of obstacles, as long as the size of the robot allows it

- similar but more complicated maneuvers can be devised for the car-like robot

odometric localization

- localization is a procedure for estimating the robot configuration q, typically in real time (*where am I?*), for planning and control purposes
- in robot manipulators, joint encoders provide a direct measure of q
- WMRs are equipped with incremental encoders that measure only the rotation of the wheels, not the position and orientation of the vehicle
- a basic feature is odometric localization, in which the kinematic model is used to keep track of the motion of the motion

- consider a unicycle under constant velocity inputs v_k, ω_k in $[t_k, t_{k+1}]$, as in a digital control implementation; in each sampling interval, the robot moves along an arc of circle of radius v_k/ω_k (a line segment if $\omega_k = 0$)

- assume $\boldsymbol{q}_k$, v_k and ω_k are known; compute $\boldsymbol{q}_{k+1}$ by integration of the kinematic model over $[t_k, t_{k+1}]$

- first possibility: Euler integration

$$
\begin{aligned}
x_{k+1} &= x_k + v_k T_s \cos\theta_k \\
y_{k+1} &= y_k + v_k T_s \sin\theta_k \qquad\qquad T_s = t_{k+1} - t_k \\
\theta_{k+1} &= \theta_k + \omega_k T_s
\end{aligned}
$$

- x_{k+1} and y_{k+1} are approximate; θ_{k+1} is exact

- second possibility: 2nd order Runge-Kutta integration

$$x_{k+1} = x_k + v_k T_s \cos\left(\theta_k + \frac{\omega_k T_s}{2}\right)$$
$$y_{k+1} = y_k + v_k T_s \sin\left(\theta_k + \frac{\omega_k T_s}{2}\right)$$
$$\theta_{k+1} = \theta_k + \omega_k T_s$$

- the average orientation during $[t_k, t_{k+1}]$ is used

- as a consequence, x_{k+1} and y_{k+1} are still approximate, but more accurate

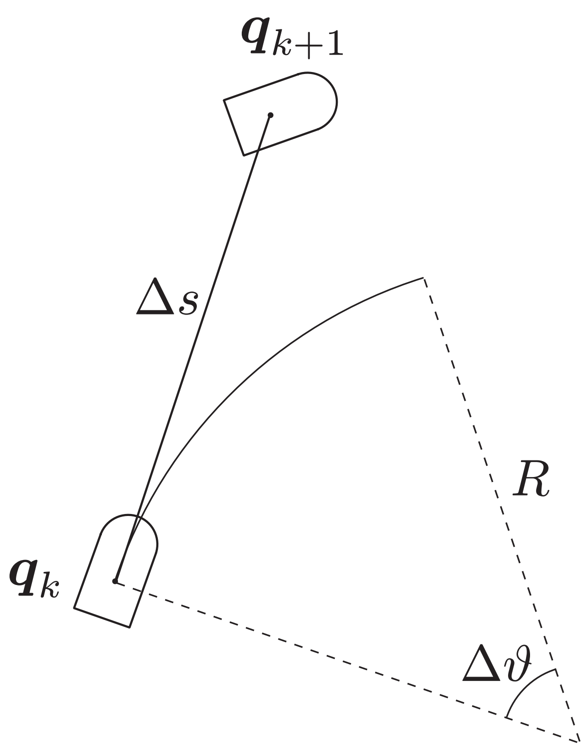
- third possibility: exact integration

$$x_{k+1} = x_k + \frac{v_k}{\omega_k}(\sin\theta_{k+1} - \sin\theta_k)$$
$$y_{k+1} = y_k - \frac{v_k}{\omega_k}(\cos\theta_{k+1} - \cos\theta_k)$$
$$\theta_{k+1} = \theta_k + \omega_k T_s$$

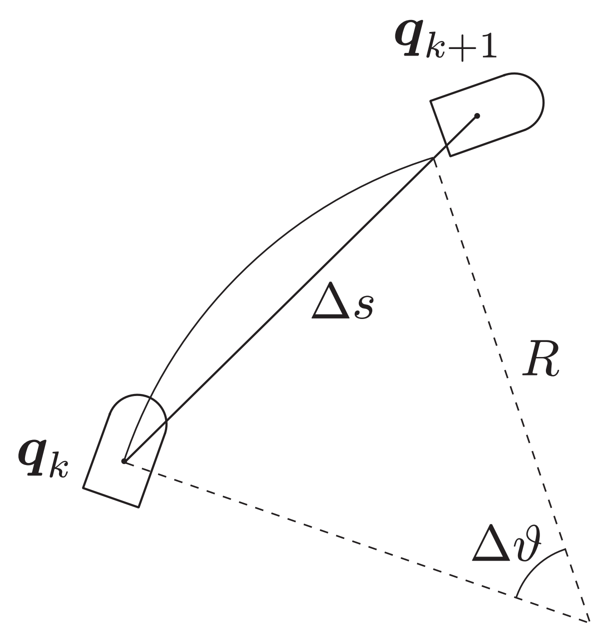
- for $\omega_k=0$, x_{k+1} and y_{k+1} are still defined and coincide with the solution by Euler and Runge-Kutta

- for $\omega_k\approx 0$, a conditional instruction may be used in the implementation

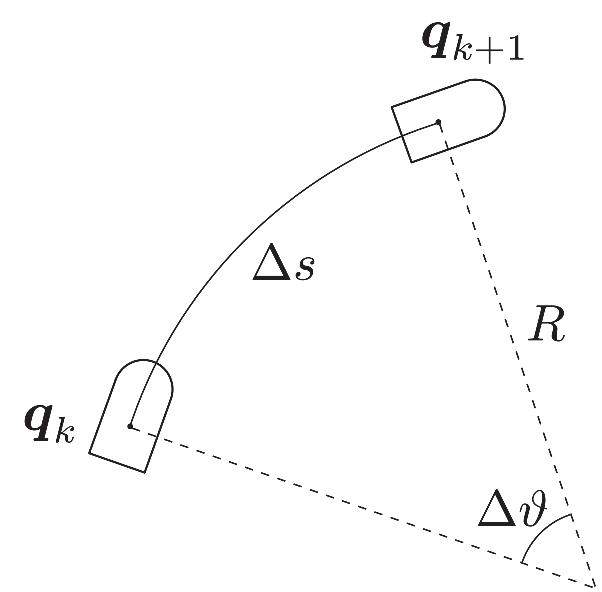
geometric comparison



Euler



Runge-Kutta



exact

- in practice, due to the non-ideality of any actuation system, the commanded inputs v_k and ω_k are not used
- instead, measure the effect of the actual inputs:

$$v_k T_s = \Delta s \qquad \omega_k T_s = \Delta\theta \qquad \frac{v_k}{\omega_k} = \frac{\Delta s}{\Delta\theta}$$

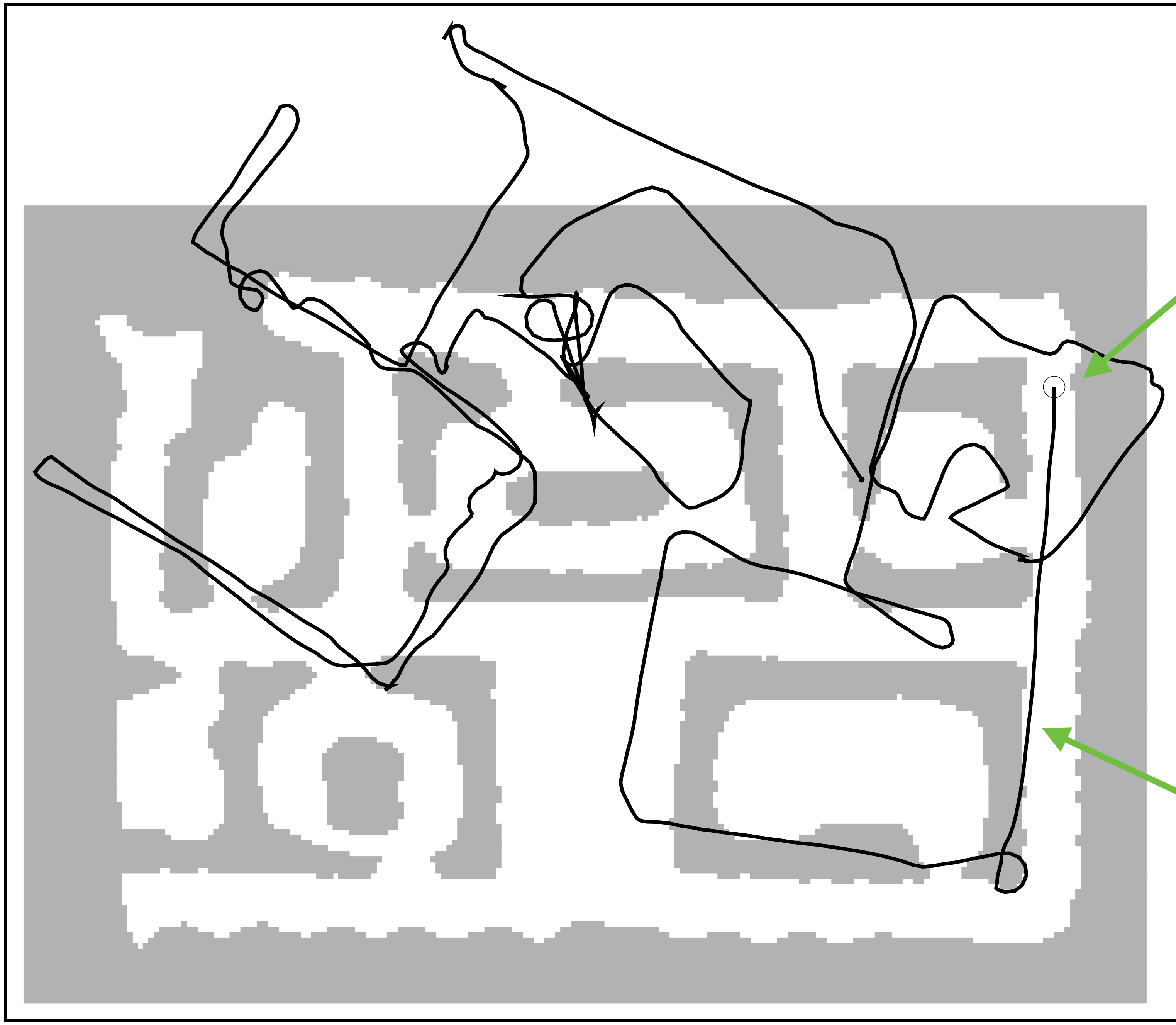
  Δs (traveled length) and $\Delta\theta$ (total orientation change) are reconstructed via proprioceptive sensors

- for example, for a differential-drive robot

$$\Delta s = \frac{r}{2}(\Delta\phi_R + \Delta\phi_L) \qquad \Delta\theta = \frac{r}{d}(\Delta\phi_R - \Delta\phi_L)$$

  where $\Delta\phi_R$ and $\Delta\phi_L$ are the total rotations measured by the wheel encoders

- odometric localization (also called dead reckoning) is subject to an error (odometric drift) that grows over time, and will typically become unacceptably large over sufficiently long paths
- causes include wheel slippage (model perturbation), inaccurate calibration of, e.g., wheel radius (model uncertainty) or numerical integration errors
- effective localization methods use proprioceptive as well as exteroceptive sensors
- the latter provide a local view of the environment which is continuously compared with what is known (the map) to correct the estimate



robot starts here

path reconstructed
by integration of kinematic model
using encoder measurements

a typical dead reckoning result