From Verification to Synthesis

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Verification

Model Checking:

- *Given*: Program P, Specification φ.
- *Task*: Check that $P \models \varphi$

Success:

- *Algorithmic methods*: temporal specifications and finite-state programs.
- *Also*: Certain classes of infinite-state programs
- *Tools*: SMV, SPIN, SLAM, etc.
- *Impact* on industrial design practices is increasing.

Problems:

- Designing P is hard and expensive.
- Redesigning P when $P \not\models \varphi$ is hard and expensive.

Automated Design

Basic Idea:

- Start from spec φ, design P such that $P \models \varphi$.

  Advantage:

  – No verification
  – No re-design

- Derive P from φ algorithmically.

  Advantage:

  – No design

In essenece: Declarative programming taken to the limit.

Program Synthesis

The Basic Idea: Mechanical translation of human-understandable task specifications to a program that is known to meet the specifications.

Deductive Approach (Green, 1969, Waldinger and Lee, 1969, Manna and Waldinger, 1980)

- Prove *realizability* of function,
  e.g., $(\forall x)(\exists y)(Pre(x) \rightarrow Post(x, y))$
- Extract *program* from realizability proof.

Classical vs. Temporal Synthesis:

- *Classical*: Synthesize transformational programs
- *Temporal*: Synthesize programs for ongoing computations (protocols, operating systems, controllers, etc.)

Temporal Logic

Linear Temporal logic (LTL): logic of temporal sequences (Pnueli, 1977)

Main feature: time is implicit

- *next* ϕ: ϕ holds in the next state.
- *eventually* ϕ: ϕ holds eventually
- *always* ϕ: ϕ holds from now on
- ϕ *until* ψ: ϕ holds until ψ holds.

Semantics

- $\pi, w \models$ *next* φ if w $\bullet \longrightarrow \underset{\varphi}{\bullet} \longrightarrow \quad \bullet \longrightarrow \bullet \longrightarrow \bullet \ldots$

- $\pi, w \models \varphi$ *until* ψ if w $\underset{\varphi}{\bullet} \longrightarrow \underset{\varphi}{\bullet} \longrightarrow \quad \underset{\varphi}{\bullet} \longrightarrow \underset{\psi}{\bullet} \longrightarrow \bullet \ldots$

Examples

- always not (CS_1 and CS_2): mutual exclusion (safety)

- always (Request implies eventually Grant): liveness

- always (Request implies (Request until Grant)): liveness

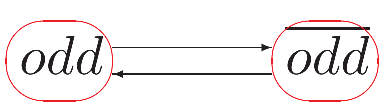
Synthesis of Ongoing Programs

Specs: Temporal logic formulas

Early 1980s: Satisfiability approach
(Wolper, Clarke+Emerson, 1981)

- *Given*: φ
- *Satisfiability*: Construct $M \models \varphi$
- *Synthesis*: Extract P from M.

Example: $always\ \ (odd \rightarrow next\ \neg odd) \wedge$
$always\ \ (\neg odd \rightarrow next\ odd)$



Reactive Systems

Reactivity: Ongoing interaction with environment (Harel+Pnueli, 1985), e.g., hardware, operating systems, communication protocols, etc. (also, *open systems*).

Example: Printer specification –
J_i - job i submitted, P_i - job i printed.

- *Safety*: two jobs are not printed together
  $always\ \neg(P_1 \wedge P_2)$
- *Liveness*: every jobs is eventually printed
  $always\ \bigwedge_{j=1}^{2}(J_i \rightarrow eventually\ P_i)$

Satisfiability and Synthesis

Specification Satisfiable? Yes!

Model M: A single state where J_1, J_2, P_1, and P_2 are all false.

Extract program from M? No!

Why? Because M handles only one input sequence.

- J_1, J_2: input variables, controlled by environment
- P_1, P_2: output variables, controlled by system

Desired: a system that handles *all* input sequences.

Conclusion: Satisfiability is inadequate for synthesis.

Realizability

I: input variables
O: output variables

Game:

- *System*: choose from 2^O
- *Env*: choose from 2^I

Infinite Play:
$i_0, i_1, i_2, \ldots$
$0_0, 0_1, 0_2, \ldots$

Infinite Behavior: $i_0 \cup o_0, i_1 \cup o_1, i_2 \cup o_2, \ldots$

Win: behavior $\models$ spec

Specifications: LTL formula on $I \cup O$

Strategy: Function $f \ : \ (2^I)^* \rightarrow 2^O$

Realizability: Abadi+Lamport+Wolper, 1989
Pnueli+Rosner, 1989
Existence of winning strategy for specification.

Church's Problem

Church, 1957: Realizability problem wrt specification expressed in MSO (monadic second-order theory of one successor function)

Büchi+Landweber, 1969:

- Realizability is decidable.
- If a winning strategy exists, then a *finite-state* winning strategy exists.
- Realizability algorithm *produces* finite-state strategy.

Rabin, 1972: Simpler solution via Rabin tree automata.

> **Question**: LTL is subsumed by MSO, so what did Pnueli and Rosner do?
> **Answer**: better algorithms!

Strategy Trees

Infinite Tree: D^* (D - directions)

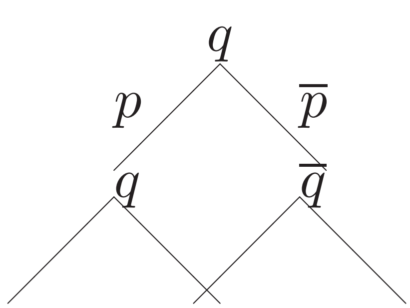
- *Root*: ε
- *Children*: xd, $x \in D^*$, $d \in D$

Labeled Infinite Tree: $\tau : D^* \rightarrow \Sigma$

Strategy: $f \ : \ (2^I)^* \rightarrow 2^O$

> *Rabin's insight*: A strategy is a labeled tree with directions $D = 2^I$ and alphabet $\Sigma = 2^O$.

Example: $I = \{p\}$, $O = \{q\}$



Winning: Every branch satisfies spec.

Rabin Automata on Infinite k-ary Trees

$A = (\Sigma, S, S_0, \rho, \alpha)$

- Σ: finite alphabet
- S: finite state set
- $S_0 \subseteq S$: initial state set
- ρ: transition function
  - $\rho \ : \ S \times \Sigma \to 2^{S^k}$
- α: acceptance condition
  - $\alpha = \{(G_1, B_1), \ldots, (G_l, B_l)\}$, $G_i, B_i \subseteq S$
  - *Acceptance*: along every branch, for some $(G_i, B_i) \in \alpha$, G_i is visited infinitely often, and B_i is visited finitely often.

Emptiness of Tree Automata

Emptiness: $L(A) = \emptyset$

Emptiness of Automata on Finite Trees: PTIME test (Doner, 1965)

Emptiness of Automata on Infinite Trees: Difficult

- Rabin, 1969: non-elementary
- Hossley+Rackoff, 1972: 2EXPTIME
- Rabin, 1972: EXPTIME
- Emerson, V.+Stockmeyer, 1985: In NP
- Emerson+Jutla, 1991: NP-complete

Rabin's Realizability Algorithm

REAL(φ):

- Construct Rabin tree automaton A_φ that accepts all winning strategy trees for spec φ.
- Check non-emptiness of A_φ.
- If nonempty, then we have realizability; extract strategy from non-emptiness witness.

Complexity: non-elementary

Reason: A_φ is of non-elementary size for spec φ in MSO.

Post-1972 Developments

- Pnueli, 1977: Use LTL rather than MSO as spec language.

- V.+Wolper, 1983: Elementary (exponential) translation from LTL to automata.

- Safra, 1988: Doubly exponential construction of tree automata for strategy trees wrt LTL spec (using V.+Wolper).

- Rosner+Pnueli, 1989: 2EXPTIME realizability algorithm wrt LTL spec (using Safra).

- Rosner, 1990: Realizability is 2EXPTIME-complete.

Standard Critique

Impractical! 2EXPTIME is a horrible complexity.

Response:

- 2EXPTIME is just worst-case complexity.
- 2EXPTIME lower bound implies a doubly exponential bound on the size of the smallest strategy; thus, hand design cannot do better in the worst case.

Real Critique

- Algorithmics not ready for practical implementation.
- Complete specification is difficult.

Response: More research needed!

- Better algorithms
- Incremental algorithms – write spec incrementally

Classical AI Planning

Deterministic Finite Automaton (DFA)
$A = (\Sigma, S, s_0, \rho, F)$

- *Alphabet*: Σ
- *States*: S
- *Initial state*: $s_0 \in S$
- *Transition function*: $\rho : S \times \Sigma \to S$
- *Accepting states*: $F \subseteq S$

Input word: $a_0, a_1, \ldots, a_{n-1}$ **Run**: $s_0, s_1, \ldots, s_n$

- $s_{i+1} = \rho(s_i, a_i)$ for $i \geq 0$

Acceptance: $s_n \in F$.

Planning Problem: Find word leading from s_0 to F.

- *Realizability*: $L(A) \neq \emptyset$
- *Program*: $w \in L(A)$

Dealing with Nondeterminism

Nondeterministic Finite Automaton (NFA)
$A = (\Sigma, S, s_0, \rho, F)$
- *Alphabet*: Σ
- *States*: S
- *Initial state*: $s_0 \in S$
- *Transition function*: $\rho : S \times \Sigma \to 2^S$
- *Accepting states*: $F \subseteq S$

Input word: $a_0, a_1, \ldots, a_{n-1}$ **Run**: $s_0, s_1, \ldots, s_n$

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Acceptance: $s_n \in F$.

Planning Problem: Find word leading from s_0 to F.

- *Realizability*: $L(A) \neq \emptyset$
- *Program*: $w \in L(A)$

Automata on Infinite Words

Nondeterministic Büchi Automaton (NBW)
$A = (\Sigma, S, s_0, \rho, F)$
- *Alphabet*: Σ
- *States*: S
- *Initial state*: $s_0 \in S$
- *Transition function*: $\rho : S \times \Sigma \to 2^S$
- *Accepting states*: $F \subseteq S$

Input word: $a_0, a_1, \ldots$

Run: $s_0, s_1, \ldots$

- $s_{i+1} \in \rho(s_i, a_i)$ for $i \geq 0$

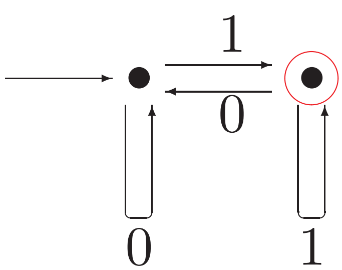
Acceptance: F visited infinitely often

Motivation:

- characterizes ω-*regular* languages
- equally expressive to MSO (Büchi 1962)
- more expressive than LTL

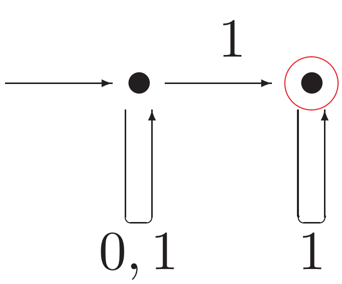
Examples

$((0+1)^*1)^\omega$:



– infinitely many 1's

$(0+1)^*1^\omega$:



– finitely many 0's

Infinitary Planning

Planning Problem: Given NBW $A = (\Sigma, S, s_0, \rho, F)$, find infinite word $w \in L(A)$

From Automata to Graphs: $G_A = (S, E_A)$,
$E_A = \{(s,t) : t \in \rho(s,a)$ for some $a \in \Sigma\}$.
Lemma: $L(A) \neq \emptyset$ iff there is a a state $f \in F$ such that G_A contains a path from s_0 to f and a cycle from f to itself.
Corollary: $L(A) \neq \emptyset$ iff there are finite words $u, v \in \Sigma^*$ such that $uv^\omega \in L(A)$.

Bonus: Finite-state program.

Synthesized Program: Do u and then repeatedly do v.

Catching Bugs with A Lasso

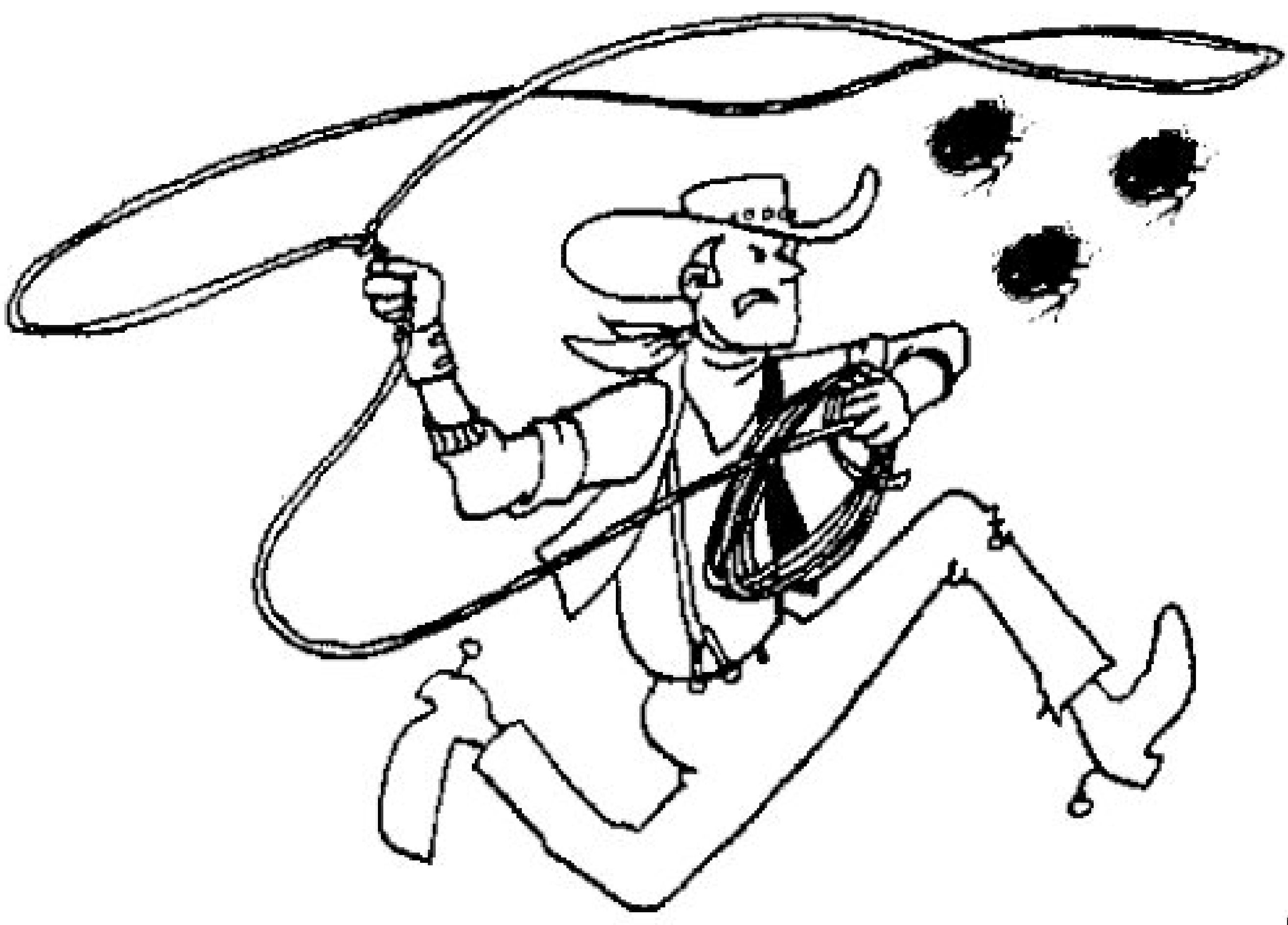


Figure 1: Ashutosh's blog, November 23, 2005

Dealing with Negative Specifications

Deterministic Automata:

- *Input* DFA $A = (\Sigma, S, s_0, \rho, F)$
- *Planning Problem*: Find word $w \notin L(A)$.
- *Realizability*: $L(A) \neq \Sigma^*$
- *Solution*: Solve classical planning with complementary DFA $A^c = (\Sigma, S, s_0, \rho, S - F)$.

Nondeterministic Automata:

- *Input* NFA $A = (\Sigma, S, s_0, \rho, F)$
- *Planning Problem*: Find word $w \notin L(A)$.
- *Realizability*: $L(A) \neq \Sigma^*$
- *Solution*: Solve classical planning with complementary DFA $A^c = (\Sigma, 2^S, \{s_0\}, \rho^c, F^c)$.

– $\rho^c(P, a) = \bigcup_{s \in P} \rho(s, a)$

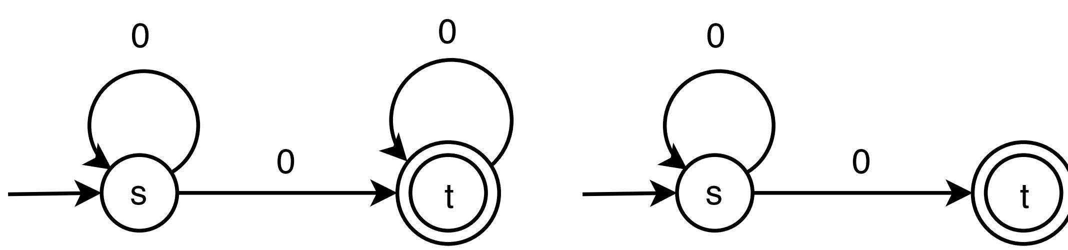
– $F^c = \{P : P \cap F = \emptyset\}$

Planning with Complemented Büchi Automata

- *Input*: NBW $A = (\Sigma, S, s_0, \rho, F)$
- *Planning Problem*: Find infinite word $w \notin L(A)$.
- *Realizability*: $L(A) \neq \Sigma^\omega$
- *Solution*: Solve infinitary planning with complementary NBW A^c:

– $L(A^c) = \Sigma^\omega - L(A)$

Problem: subset construction fails!



Büchi Complementation

History:

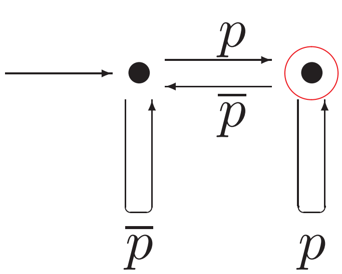
- Büchi, 1962: Doubly exponential complementation.
- Sistla-V.-Wolper 1985: Büchi's construction can be implemented with exponential blow-up (16^{n^2}).
- Safra, 1988: $n^{O(n)}$
- Michel, 1988: $n! \approx (n/e)^n$ lower bound.
- Kupferman-V., 1997: $(6n)^n$ upper bound.
- Friedgut-Kupferman-V., 2004: $(0.97n)^n$ upper bound.
- Yan, 2005: $(0.76n)^n$ lower bound.
- Schewe, 2009: $(0.76n)^n$ upper bound.

Temporal Logic vs. Büchi Automata

Paradigm: Compile high-level logical specifications into low-level finite-state language

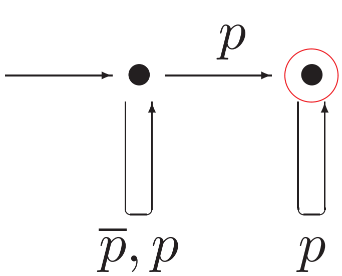
> **The Compilation Theorem**: V.-Wolper, 1983
>
> Given an LTL formula φ, one can construct an NBW A_φ such that a computation σ satisfies φ if and only if σ is accepted by A_φ. Furthermore, the size of A_φ is at most exponential in the length of φ.

always eventually p:



– infinitely many p's

eventually always p:



– finitely many $\overline{p}$'s

LTL Planning

Positive Direction:

- *Input* LTL formula φ
- *Planning Problem*: Find word $w \models \varphi$
- *Realizability*: φ is satisfiable.
- *Solution*: Solve infinitary planning with A_φ

Negative Direction:

- *Input* LTL formula φ
- *Planning Problem*: Find word $w \not\models \varphi$
- *Realizability*: $\neg\varphi$ is satisfiable.
- *Solution*: Solve infinitary planning with $A_{\neg\varphi}$

Synthesis of Reactive Systems

Game Semantics: view an open system S as playing a game with an adversarial environment E, with the specifications being the winning condition.

DFA Games:

- S choose output value $a \in \Sigma$
- E choose input value $b \in \Delta$
- *Round*: S and E set their values
- *Play*: word in $(\Sigma \times \Delta)^*$
- *Specification*: DFA A over the alphabet $\Sigma \times \Delta$
- S wins when play is accepted by by A.

Realizability and Synthesis:

- *Strategy* for S – $\tau : \Delta^* \to \Sigma$
- *Realizability* – exists *winning* strategy for S
- *Synthesis* – obtain such winning strategy.

Solving DFA Games

$A = (\Sigma \times \Delta, S, s_0, \rho, F)$

Define $win_i(A) \subseteq S$ inductively:

- $win_0(A) = F$
- $win_{i+1}(A) = win_i(A) \cup$

$\{s : (\exists a \in \Sigma)(\forall b \in \Delta)\rho(s, (a, b)) \in win_i(A)\}$

Lemma: S wins the A game iff $s_0 \in win_\infty(A)$.

Bottom Line: linear-time, least-fixpoint algorithm for DFA realizability. What about synthesis?

Transducers

Transducer: a finite-state representation of a strategy– deterministic automaton with output
$T = (\Delta, \Sigma, Q, q_0, \alpha, \beta)$

- Δ: input alphabet
- Σ: output alphabet
- Q: states
- q_0: initial state
- $\alpha : S \times \Delta \rightarrow S$: transition function
- $\beta : S \rightarrow \Sigma$: output function

Key Observation: A transducer representing a winning strategy can be extracted from
$win_0(A), win_1(A), \ldots$

Reachability Games

Game Graphs: $G = (V_0, V_1, E, v_s, W)$
- $E \subseteq (V_0 \times V_1) \cup (V_1 \times V_0)$
- v_s: start node
- $W \subseteq V_0 \cup V_1$: winning set
- Player 0 moves from V_0, Player 1 moves from V_1.
- Player 0 wins: reach W.

Fact: Reachability games can be solved in linear time –least fixpoint algorithm

Consequence: realizability and synthesis

NFA Games

NFA Games:

- S choose output value $a \in \Sigma$
- E choose input value $b \in \Delta$
- *Round*: S and E set their variables
- *Play*: word in $(\Sigma \times \Delta)^*$
- *Specification*: NFA A over the alphabet $\Sigma \times \Delta$
- S wins when play is accepted by by A.

Solving NFA Games: *Basic mismatch* between nondeterminism and strategic behavior.

- Nondeterministic automata have perfect foresight.
- Strategies have no foresight.

Conclusion: Determinize A and then solve.

NBW Games

NBW Games:

- S choose output value $a \in \Sigma$
- E choose input value $b \in \Delta$
- *Round*: S and E set their variables
- *Play*: infinite word in $(\Sigma \times \Delta)^\omega$
- *Specification*: NBW A over the alphabet $\Sigma \times \Delta$
- S wins when infinite play is accepted by by A.

Resolving the mismatch: Determinize A

LTL Games:

- *Specification*: LTL formula φ
- *Solution*: Construct A_φ and determinize.

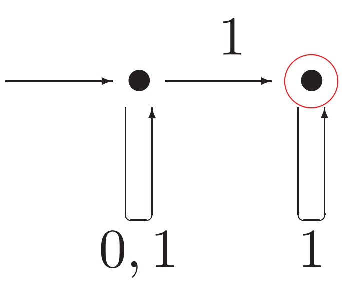
History:

- Church, 1957: problem posed (for MSO)
- Büchi-Landweber, 1969: decidability shown
- Rabin, 1972: solution via tree automata

Determinization

Key Fact (Landweber, 1969): Nondeterministic Büchi automata are more expressive than deterministic Büchi automata.

Example: $(0+1)^*1^\omega$:



– finitely many 0's

McNaughton, 1966: NBW can be determinized using more general acceptance condition – blow-up is *doubly exponential*.

Parity Automata

Deterministic Parity Automata (DPW)
$A = (\Sigma, S, s_0, \rho, \mathcal{F})$

- $\mathcal{F} = (F_1, F_2, \ldots, F_k)$ - partition of S.
- *Parity index*: k
- *Acceptance*: Least i such that F_i is visited infinitely often is even.

Example: $(0+1)^* 1^\omega$

$$\begin{tikzcd} {} \arrow[r] & \ell \arrow[r, shift left, "1"] \arrow[loop below, "0"] & r \arrow[l, shift left, "0"] \arrow[loop below, "1"] \end{tikzcd}$$

– finitely many 0's

Parity condition: $(\{\ell\}, \{r\})$

Safra, 1988: NBW with n states can be translated to DPW with $n^{O(n)}$ states and index $O(n)$.

Parity Games

Game Graphs: $G = (V_0, V_1, E, v_s, \mathcal{W})$
- $E \subseteq (V_0 \times V_1) \cup (V_1 \times V_0)$
- v_s: start node
- $W \subseteq V_0 \cup V_1$: winning set
- Player 0 moves from V_0,
Player 1 moves from V_1.
- $\mathcal{W} = (W_1, W_2, \ldots, W_k)$ – partition of $V_0 \cup V_1$
- Play 0 wins: least i such that W_i is visited
infinitely often is even.

Solving Parity Games: complexity

- Jurdzinski, 1998: UP∩co-UP
- Jurdzinski, 2000: $n^{O(k)}$
- Jurdzinski+Petterson+Zwick, 2000: $n^{O(\sqrt{n})}$

Open Question: In PTIME?

LTL Synthesis

Algorithm for LTL Synthesis:
- Convert specification φ to NBW A_φ (exponential blow-up)
- Convert NBW A_φ to DPW A^d_φ (exponential blow-up)
- Solve parity game for A^d_φ (exponential)

Pnueli-Rosner, 1989: LTL realizability and synthesis is 2EXPTIME-complete.

- *Transducer*: finite-state program with doubly exponentially many states (exponentially many state variables)

Theory, Experiment, and Practice

Automata-Theoretic Approach in Practice:

- Mona: MSO on finite words
- Linear-Time Model Checking: LTL on infinite words

Experiments with Automata-Theoretic Approach:

- Symbolic decision procedure for CTL (Marrero 2005)
- Symbolic synthesis using NBT (Wallmeier-Hütten-Thomas 2003)

Why no implementation of LTL synthesis?

- *NBW determinization is hard in practice*: from 9-state NBW to 1,059,057-state DRW (Althoff-Thomas-Wallmeier 2005)
- *NBW determinization is hard in practice*: no symbolic algorithms
- lack of incremental algorithms

2EXPTIME: Should not be an insurmountable problem.

A Safraless Approach

Kupferman-V., 2005:

- Limit search to strategy trees that are generated by transducers of bounded size
  - Existence of bounded-size transducers follows from the Safraful approach
- Construct recurrence games that are generated by bounded-size transducers
- Solve recurrence games

Crux: focus on subset of strategies

- No determinization
- No parity games

Recurrence Games

Game Graphs: $G = (V_0, V_1, E, v_s, W)$
- $E \subseteq (V_0 \times V_1) \cup (V_1 \times V_0)$
- v_s: start node
- $W \subseteq V_0 \cup V_1$: winning set
- Player 0 moves from V_0,
Player 1 moves from V_1.
- Player 0 wins: *infinitely many* visits to W.

Fact: Recurence games can be solved in quadratic time– greatest fixpoint of reachability.

Consequence: reachability and synthesis.

Safraless vs. Safraful

Question: Is the new approach practical?

Answer: Experimentation needed!

Promise:

- Approach shown practical (after optimization) for Büchi complementation
- Symbolic approach possible
- First implementation report in FMCAD'06 (Jobstmann-Bloem)

Compositional Synthesis

Basic Weakness of Synthesis: full specifications required to get started – unrealistic!

- Specifications evolve!

Compositional Synthesis: Suppose we synthesized programs for specifications φ and ψ, can we get programs for $\varphi \wedge \psi$ *without* starting from scratch.

Kupferman-Piterman-V., 2006: Use realizability proofs for φ and ψ as starting point for realizability testing and synthesis for $\varphi \wedge \psi$.

Discussion

Question: Can we hope to reduce a 2EXPTIME-complete approach to practice?

Answer:

- Worst-case analysis is pessimistic.
  - Mona solves nonelementary problems.
  - SAT-solvers solve huge NP-complete problems.
  - Model checkers solve PSPACE-complete problems.
  - Doubly exponential lower bound for program size.
- We need algorithms that blow-up only on hard instances
- New approach is promising.
- Algorithmic engineering is needed.