

Chapter 4

Motion and force control

In this chapter we deal with the motion control problem for situations in which the robot manipulator end effector is in *contact with the environment*. Many robotic tasks involve intentional interaction between the manipulator and the environment. Usually, the end effector is required to follow in a stable way the edge or the surface of a workpiece while applying prescribed forces and torques. The specific feature of robotic problems such as polishing, deburring, or assembly, demands control also of the exchanged forces at the contact. These forces may be explicitly set under control or just kept limited in an indirect way, by controlling the end-effector position. In any case, *force* specification is often complemented with a requirement concerning the end-effector *motion*, so that the control problem has in general *hybrid* (mixed) objectives.

In setting up the proper framework for analysis, an essential role is played by the model of the environment. The predicted performance of the overall system will depend not only on the robot manipulator dynamics but also on the assumptions made for the interaction between the manipulator and environment. On one hand, the environment may behave as a simple mechanical system undergoing small but finite deformations in response to applied forces. When contact occurs, the arising forces will be dictated by the *dynamic balancing* of two coupled systems, the robot manipulator and the environment. On the other hand, if the environment is stiff enough and the manipulator is continuously in contact, part of its degrees of freedom will be actually lost, the motion being locally constrained to given directions. Contact forces are then viewed as an attempt to violate the imposed *kinematic constraints*.

Three control strategies are discussed; namely, *impedance control*, *parallel control*, and *hybrid force/motion control*. Impedance control tries

to assign desired dynamic characteristics to the interaction with rather unmodelled objects in the workspace. Parallel control provides the additional feature of regulating the contact force to a desired value. Hybrid force/motion control exploits the partition of the task space into directions of admissible motion and of reaction forces, both arising from the existence of a rigid constraint.

All strategies can incorporate the best available model of the environment; however, the achieved performance will suffer anyway from uncertainty in the location and geometric characteristics (orientation, curvature) of the contact surfaces. Moreover, for impedance and hybrid force/motion control, the measure of contact forces may or may not be needed in nominal conditions. Thus, the main differences between these two strategies rely in the control objectives rather than in the implementation requirements of a specific control law. On the other hand, parallel control is aimed at closing a force control loop around a pre-existing position (impedance) control loop and, as such, it naturally makes use of contact force measurements.

We will investigate these three frameworks, each allowing the design of different types of controllers. A stability analysis of the most interesting ones will be provided.

4.1 Impedance control

The underlying idea of impedance control is to assign a prescribed dynamic behaviour for a robot manipulator while its end effector is interacting with the environment. The desired performance is specified by a *generalized dynamic impedance*, i.e., by a complete set of linear or nonlinear second-order differential equations representing a mass-spring-damper system. However, the way in which the environment generates forces in reaction to deformation is hardly modelled.

In general, impedance control is suitable for those tasks where contact forces must be kept small, typically to avoid jamming among parts in assembly or insertion operations, while accurate regulation of forces is not required. In fact, an explicit force error loop is absent in this approach, so that it is often stated that “force is controlled by controlling position”. Using programmable stiffness and damping matrices in the impedance model, a compromise is reached between contact force and position accuracy as a result of unexpected interactions with the environment.

In the following, we will first introduce the convenient format of the manipulator dynamic model in task space coordinates. The imposition of a desired impedance at the end-effector level will then be obtained by an inverse dynamics scheme designed in the proper task space coordinates.

Simplified control laws are then recovered from this general scheme.

4.1.1 Task space dynamic model

When additional external forces act from the environment on the manipulator end effector, the dynamic model (in the absence of friction forces) becomes

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u + J^T(q)f, \quad (4.1)$$

with the usual notation for the dynamic terms on the left-hand side, and where

$$f = \begin{pmatrix} \gamma \\ \mu \end{pmatrix} \quad (4.2)$$

is the $(m \times 1)$ vector of generalized forces expressed in the base frame; respectively, linear forces γ and angular moments μ . Notice that the forces $J^T f$ on the right-hand side of (4.1) have been taken with the positive sign, since they are exerted from the environment on the end effector.

In order to focus on the main aspects of the control problem, we will consider only the case $m = n$ and, without loss of generality, $n = 6$. Moreover, in the following developments it is assumed that *no kinematic singularities* are encountered. The square matrix $J(q)$ in (4.1) is the *geometric Jacobian* of the manipulator relating end-effector velocity to joint velocity

$$v = \begin{pmatrix} \dot{p} \\ \omega \end{pmatrix} = J(q)\dot{q}. \quad (4.3)$$

Since the interaction between the manipulator and the environment occurs at the task level, it is useful to rewrite the dynamic model directly in the task space. With reference to the *analytical Jacobian*, the differential kinematics equation can be written in the form

$$\dot{x} = J_a(q)\dot{q} = T_a^{-1}(\phi_e)J(q)\dot{q}, \quad (4.4)$$

where we have assumed that no representation singularities of ϕ_e occur.

The two sets of generalized forces f and f_a performing work on v and $\dot{x}$, respectively, are related by the virtual work principle, i.e.,

$$J^T(q)f = J^T(q)T_a^{-T}(\phi_e)f_a = J_a^T(q)f_a. \quad (4.5)$$

Then, the model (4.1) can be rewritten as

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u + J_a^T(q)f_a, \quad (4.6)$$

with an alternate format for the external force term. By further differentiation of (4.4)

$$\ddot{x} = J_a(q)\ddot{q} + \dot{J}_a(q)\dot{q} \quad (4.7)$$

and substitution into (4.6), it is easy to see that the dynamic model in the task space becomes

$$H_x(q)\ddot{x} + C_x(q, \dot{q})\dot{x} + g_x(q) = J_a^{-T}(q)u + f_a, \quad (4.8)$$

where

$$\begin{aligned} H_x(q) &= J_a^{-T}(q)H(q)J_a^{-1}(q) \\ C_x(q, \dot{q}) &= J_a^{-T}(q)C(q, \dot{q})J_a^{-1}(q) - H_x(q)\dot{J}_a(q)J_a^{-1}(q) \\ g_x(q) &= J_a^{-T}(q)g(q). \end{aligned} \quad (4.9)$$

Indeed, similar relationships are obtained if we desire that the vector f appears on the right-hand side of (4.8) —just drop the subscripts a .

Note that the functional dependence of nonlinear terms in (4.8) is still on q , $\dot{q}$, and not on the new state variables x , $\dot{x}$. This substitution is not essential and could be easily performed by using inverse kinematics relationship. From the computational point of view, e.g., for simulation purposes, it is more advantageous to keep the explicit dependence on joint variables.

As for the joint space dynamic model, analogous structural properties hold for (4.8). The following ones are of particular interest hereafter.

Property 4.1 The inertia matrix H_x is a symmetric positive definite matrix, provided that J_a is full-rank, which verifies

$$\lambda_{mx}I \leq H_x(q) \leq \lambda_{Mx}I \quad (4.10)$$

where λ_{mx} ($\lambda_{Mx} < \infty$) denotes the strictly positive minimum (maximum) eigenvalue of H_x for all configurations q . □

Property 4.2 The matrix $\dot{H}_x - 2C_x$ obtained via (4.9) is skew-symmetric, provided that the matrix $\dot{H} - 2C$ is skew-symmetric. □

Property 4.3 The matrix C_x verifies

$$\|C_x\| \leq k_{Cx}\|\dot{x}\| \quad (4.11)$$

for some bounded constant $k_{Cx} > 0$. □

4.1.2 Inverse dynamics control

To achieve a desired dynamic characteristic for the interaction between the manipulator and the environment at the contact, the design of the control input u in (4.8) (or (4.6)) is carried out in two steps.

The first step decouples and linearizes the closed-loop dynamics in task space coordinates so as to obtain

$$\ddot{x} = u_0, \quad (4.12)$$

where u_0 is an external auxiliary input still available for control. Based on (4.8), this is achieved by choosing

$$u = J_a^T(q)(H_x(q)u_0 + C_x(q, \dot{q})\dot{x} + g_x(q) - f_a) \quad (4.13)$$

or, in terms of the original model components,

$$u = H(q)J_a^{-1}(q)(u_0 - \dot{J}_a(q)\dot{q}) + C(q, \dot{q})\dot{q} + g(q) - J_a^T(q)f_a. \quad (4.14)$$

This *inverse dynamics control* law is formally equivalent to the direct task space control law presented in the previous chapter, with the addition of *force feedback*.

In the second step, the desired impedance model that dynamically balances contact forces f_a at the manipulator end effector is chosen as a linear second-order mechanical system described by

$$H_m(\ddot{x} - \ddot{x}_d) + D_m(\dot{x} - \dot{x}_d) + K_m(x - x_d) = f_a, \quad (4.15)$$

where H_m is the *apparent inertia matrix*, while D_m and K_m are the desired *damping and stiffness matrices*, respectively. The vector $x_d(t)$ specifies a reference trajectory which can be exactly executed only if $f_a = 0$, i.e., during free motion. When the manipulator is in contact, the automatic balance of dynamic forces will produce a different motion behaviour. If (4.15) has to represent a *physical impedance*, positive definite H_m and K_m matrices and a positive semi-definite D_m matrix should be chosen. All matrices are symmetric —typically diagonal. Notice also that the control objective of imposing the dynamic behaviour (4.15) to the original robotic system (4.8) can be recast in the general framework of *model matching* problems for nonlinear systems.

The desired mechanical impedance is then obtained by choosing u_0 in (4.14) as

$$u_0 = \ddot{x}_d + H_m^{-1}(D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x) + f_a), \quad (4.16)$$

so that the overall *impedance control* law becomes

$$\begin{aligned} u = J_a^T(q) & \left(H_x(q) \ddot{x}_d + C_x(q, \dot{q}) \dot{x} + g_x(q) \right. \\ & + H_x(q) H_m^{-1} (D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x)) \\ & \left. + (H_x(q) H_m^{-1} - I) f_a \right). \end{aligned} \quad (4.17)$$

The implementation of (4.17) requires feedback from the manipulator state $(q, \dot{q})$ and measure of the contact force f_a . However, no explicit force loop is imposed in this control law.

Some further considerations should be made about the choice of an impedance model. Although any choice for H_m , D_m , K_m that complies with physical characteristics is feasible, some limitations arise in practice. In particular, if the location where contact occurs is not exactly known, the elements of the usually diagonal matrices H_m and K_m should be chosen so as to avoid excessive impact forces.

As a suggestive representation, let us imagine we are searching for the electric switch panel on the wall of a dark room. During this guarded motion task, the human arm is “stiff” and moves fast only where no contact is expected (e.g., sideways) while it is rather “compliant” and slow in those directions where there may be an impact, based on the approximate knowledge of the distance from the wall. Moreover, the harder is the environment surface according to our prediction, the smoother and more careful will be the arm motion in the direction of approach. As a result, in this task we are neither controlling the contact force nor accurately positioning our dexterous end effector; we rather assign a selective dynamic behaviour to the arm in preview of the specific environment.

A robot manipulator under impedance control will mimic the human performance in the presence of environment uncertainty by choosing large inertial components H_{mi} and/or low stiffness coefficients K_{mi} along the anticipated directions of contact; vice-versa, large stiffness coefficients K_{mj} are imposed along those directions which are assumed to be free, thus closely realizing the associated motion command $x_{dj}(t)$. Instead, the choice of damping D_m is related only to the desired transient characteristics.

A relevant simplification occurs in (4.17) when a *nonlinear impedance* is prescribed. Replacing formally

$$H_m = H_x(q) \quad (4.18)$$

in (4.15) leads to

$$u = J_a^T(q) (H_x(q) \ddot{x}_d + C_x(q, \dot{q}) \dot{x} + g_x(q) + D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x)) \quad (4.19)$$

or, again in the original coordinates,

$$u = H(q)J_a^{-1}(q)(\ddot{x}_d - \dot{J}_a(q)\dot{q}) + C(q, \dot{q})\dot{q} + g(q) + J_a^T(q)(D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x)). \quad (4.20)$$

Remarkably, *no force feedback* is required in this case. In this respect, (4.20) can be seen as a pure motion controller, yet intended to keep limited the end-effector forces arising from contacts with the environment. The price to pay is that the effective inertia of the manipulator will be the natural one in the *task space*.

4.1.3 PD control

Provided that *quasi-static* assumptions are made ($\dot{x}_d = 0$ and $\dot{q} \approx 0$ in the nonlinear dynamic terms), the following controller is obtained from (4.20):

$$u = J_a^T(q)(K_m(x_d - x) - D_m\dot{x}) + g(q). \quad (4.21)$$

It can be recognized that this is nothing but the *task space PD control* law, with added gravity compensation, that was considered in the previous chapter. In (4.21), the equivalent task space elastic and viscous forces respectively due to the position error $x_d - x$ and to the motion $\dot{x}$, are transformed into joint space torques through the usual Jacobian transpose.

When a *constant* reference value x_d is considered in the absence of contact forces ($f_a = 0$), it has been shown in the previous chapter that this control law enforces asymptotic stability of x_d , provided that no kinematic singularities are encountered. In alternative, and with the aim of including in the analysis also contact forces, the following Lyapunov candidate could have been used

$$V_1 = \frac{1}{2}\dot{x}^T H_x(q)\dot{x} + \frac{1}{2}(x_d - x)^T K_m(x_d - x), \quad (4.22)$$

fully defined in terms of task space components. The time derivative of (4.22) along the trajectories of the closed-loop system (4.8) and (4.21) is

$$\begin{aligned} \dot{V}_1 &= \dot{x}^T H_x(q)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{H}_x(q)\dot{x} - \dot{x}^T K_m(x_d - x) \\ &= \dot{x}^T (-C_x(q, \dot{q})\dot{x} - g_x(q) + J_a^{-T}(q)g(q) + K_m(x_d - x) - D_m\dot{x}) \\ &\quad + \frac{1}{2}\dot{x}^T \dot{H}_x(q)\dot{x} - \dot{x}^T K_m(x_d - x) \\ &= -\dot{x}^T D_m\dot{x} \leq 0, \end{aligned} \quad (4.23)$$

where Property 4.2 and $g_x = J_a^{-T}g$ have been used. Asymptotic stability follows from La Salle's invariant set theorem, provided that the matrix $J_a^T(q)$ remains always of full rank.

When a contact force $f_a \neq 0$ arises during motion, then asymptotic stability of x_d is no longer ensured. In fact, a steady-state compromise between environment deformation and positional compliance of the controller, governed by the (inverse of) matrix K_m , will be reached. The stability analysis can be carried out assuming that the environment deformation is modelled as a generalized spring with symmetric *stiffness matrix* K_e and rest location x_e .

By a suitable selection of reference frames, a "simple" environment will generate reaction forces and torques if $x > x_e$, componentwise. As a result, when the manipulator is in full contact with the environment, a generalized force of the form

$$f_a = K_e(x_e - x) \quad (4.24)$$

is felt at the end-effector level. The (unique) equilibrium point x_E of the manipulator under the environment reaction force (4.24) and the control law (4.21) is found through the steady-state balance condition

$$K_m(x_d - x) + K_e(x_e - x) = 0, \quad (4.25)$$

leading to

$$x_E = (K_m + K_e)^{-1}(K_mx_d + K_ex_e) \quad (4.26)$$

and to the equilibrium force

$$f_E = K_e(x_e - x_E). \quad (4.27)$$

Lemma 4.1 *The equilibrium point (4.26) of the closed-loop system (4.8) and (4.21) is globally asymptotically stable.*

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Proof. Consider the modified Lyapunov candidate of the form

$$V_2 = V_1 + \frac{1}{2}(x_e - x)^T K_e(x_e - x) - V_2(x_E, 0) \quad (4.28)$$

where

$$V_2(x_E, 0) = \frac{1}{2}(x_d - x_E)^T K_m(x_d - x_E) + \frac{1}{2}(x_e - x_E)^T K_e(x_e - x_E) \geq 0. \quad (4.29)$$

Proceeding as with (4.22) and taking into account (4.24) gives

$$\dot{V}_2 = -\dot{x}^T D_m \dot{x} \leq 0. \quad (4.30)$$

From the structure of $V_2 \geq 0$, this implies that $\dot{x} \rightarrow 0$ as $t \rightarrow \infty$, and that both terms $(x_e - x)$ and $(x_d - x)$ are bounded. As a consequence, the location x_E is the asymptotically stable equilibrium point of the closed-loop system.

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Remarks

- It follows from (4.26) that when the environment is very stiff along a component i , $k_{ei} \gg k_{mi}$ and the equilibrium is $x_{Ei} \approx x_{ei}$. Vice-versa, if $k_{mj} \gg k_{ej}$ then $x_{Ej} \approx x_{dj}$, with a larger environment deformation.
- The control law (4.21) features a damping term in the task space. The presence of such a damping is crucial as revealed by both (4.23) and (4.30). Therefore, in case of redundant manipulators and/or occurrence of kinematic singularities, it may be advisable to operate the damping action in the joint space, i.e., by choosing the controller as

$$u = J_a^T(q)K_m(x_d - x) - D_{mq}\dot{q} + g(q). \quad (4.31)$$

with obvious meaning of D_{mq} .

Stiffness control

A simplified control can be achieved if we assume that small deformations occur

$$(x_d - x) \approx J_a(q)(q_d - q). \quad (4.32)$$

If gravity terms are mechanically balanced, that is to say $g(q) = 0$, and no additional damping is included ($D_m = 0$), then (4.21) reduces to the so-called *stiffness control*:

$$u = J_a^T(q)K_m J_a(q)(q_d - q) = K_x(q)(q_d - q). \quad (4.33)$$

Here, a configuration dependent joint torque is generated in response to small joint position errors so as to correspond to a constant task space stiffness matrix K_m . This simple control scheme is the *active* counterpart of a passive compliant end-effector device (such as the Remote Center of Compliance) that possesses directional-dependent but only fixed accommodation characteristics.

4.2 Parallel control

The above impedance control schemes perform only indirect force control, through a closed-loop position controller, without explicit closure of a force feedback loop. In other words, it is not possible to specify a desired amount of contact force with an impedance controller, but only a satisfactory dynamic behaviour between end-effector force and displacement at the contact.

An effective strategy to embed the possibility of *force regulation* is given by the so-called *parallel control*. The key concept is to provide an impedance controller with the ability of controlling both position and force. This is obtained by closing an outer force control loop around the inner position control loop, which is typically available in a robot manipulator. By a suitable design of the force control action (typically an integral term), it is possible to regulate the contact force to a desired value. In order to provide motion control along the feasible task space directions, also a desired position is input to the inner loop. The result is two control actions, working in parallel; namely, a force control action and a position control action. In order to ensure force control along the task space directions where environment reaction forces exist, the force action is designed so as to dominate the position action.

Following the previous analysis of impedance control, the environment can be modelled as in (4.24). We consider here the case of $m = n = 3$ and contact with a *planar* surface. The choice of a planar surface is motivated by noticing that it is locally a good approximation to surfaces of regular curvature. Hence, K_e is a (3×3) constant symmetric stiffness matrix of rank 1, i.e., the contact force f_a is directed along the normal to the plane. Let

$$R_e = (n_e \quad s_e \quad a_e) \quad (4.34)$$

be the rotation matrix describing the orientation of a contact frame attached to the plane with respect to some base frame, where a_e is the unit vector along the normal and n_e, s_e lie in the plane. Then, the stiffness matrix can be written as

$$K_e = R_e \text{diag}\{0, 0, k_e\} R_e^T = k_e a_e a_e^T \quad (4.35)$$

where $k_e > 0$ is the stiffness coefficient.

The elastic contact model (4.24) and (4.35) suggests that a null force error can be obtained only if the desired contact force f_{ad} is aligned with a_e . On the other hand, null position errors can be obtained only on the contact plane, while the component of position x along a_e has to accommodate the force requirement specified by f_{ad} ; thus, the desired position x_d can

be freely reached only in the null space of K_e , i.e., where the environment provides no reaction forces.

4.2.1 Inverse dynamics control

With reference to the task space dynamic model (4.8), the design of the control input u can be carried out as for the above inverse dynamics controller in (4.14), where force measurements of f_a are assumed to be available. According to the parallel control approach, the new control input u_0 can be designed as the sum of a position control action and a force control action; namely, as

$$u_0 = u_{0x} + u_{0f} \quad (4.36)$$

with

$$u_{0x} = \ddot{x}_d + H_m^{-1}(D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x)) \quad (4.37)$$

$$u_{0f} = -H_m^{-1} \left(K_F(f_{ad} - f_a) + K_I \int_0^t (f_{ad} - f_a) d\tau \right) \quad (4.38)$$

where H_m, D_m, K_m assume the same meaning as for the above impedance controller, and K_F, K_I are suitable force feedback matrix gains characterizing a PI action on the force error. It is worth noticing that by setting $K_F = 0$ and $K_I = 0$ the original impedance control law (4.16) is recovered.

Substituting (4.36) with (4.37) and (4.38) in (4.12) yields

$$H_m(\ddot{x} - \ddot{x}_d) + D_m(\dot{x} - \dot{x}_d) + K_m(x - x_d) = K_F(f_a - f_{ad}) + K_I \int_0^t (f_a - f_{ad}) d\tau \quad (4.39)$$

which reveals that, thanks to the *integral* action, the force error ($f_{ad} - f_a$) is allowed to prevail over the position error ($x_d - x$) at steady state.

Assuming that the desired force f_{ad} is aligned with a_e and x_d has a constant component along a_e , the equilibrium trajectory of system (4.39) is characterized by

$$x_E = (I - a_e a_e^T) x_d + a_e a_e^T (x_e - k_e^{-1} f_{ad}) \quad (4.40)$$

$$f_E = k_e a_e a_e^T (x_e - x_E) = f_{ad}. \quad (4.41)$$

Stability of the closed-loop system can be developed according to classical linear systems theory. Choosing diagonal matrices in (4.39) as $H_m = h_m I$, $D_m = d_m I$, $K_m = k_m I$, $K_F = k_F I$, $K_I = k_I I$, and plugging (4.24) in (4.39) gives

$$h_m \ddot{x} + d_m \dot{x} + (k_m I + k_F k_e a_e a_e^T) x + k_I k_e a_e a_e^T \int_0^t x d\tau \quad (4.42)$$

$$= h_m \ddot{x}_d + d_m \dot{x}_d + k_m x_d - k_F (f_{ad} - k_e a_e a_e^T x_e) - k_I \int_0^t (f_{ad} - k_e a_e a_e^T x_e) d\tau$$

which represents a third-order linear system, whose stability can be analyzed by referring to the unforced system as long as the input is bounded. Therefore, setting to zero the right-hand side of (4.42), accounting for (4.34), and projecting the position vector on the contact frame as

$$R_e^T x = \begin{pmatrix} x_n \\ x_s \\ x_a \end{pmatrix} \quad (4.43)$$

leads to the system of three scalar decoupled equations

$$h_m \ddot{x}_n + d_m \dot{x}_n + k_m x_n = 0 \quad (4.44)$$

$$h_m \ddot{x}_s + d_m \dot{x}_s + k_m x_s = 0 \quad (4.45)$$

$$h_m \ddot{x}_a + d_m \dot{x}_a + (k_m + k_F k_e) x_a + k_I k_e \int_0^t x_a d\tau = 0, \quad (4.46)$$

revealing that a stable behaviour is ensured by a proper choice of the gains k_m, d_m, k_F, k_I for the third equation.

Remarks

- The parallel inverse dynamics control scheme ensures tracking of the desired position along the task space directions on the plane together with regulation of the desired force along the task space direction normal to the plane, but the control law is based on force and position measurements along all task space directions notwithstanding.
- The scheme requires that the desired force is correctly planned. If the end effector is about to make contact with the environment and no information about its geometry is available, i.e., the direction of a_e is unknown, it is convenient to set $f_{ad} = 0$ which is anyhow in the range space of any matrix K_e . Then, once contact is established and a rough estimate of a_e is available from the force measurements, we can switch to the non-zero value of desired force.
- If the desired force set point f_{ad} is not aligned with a_e , the equilibrium trajectory (4.40) and (4.41) is modified into

$$\bar{x}_E = x_E + v_E t \quad (4.47)$$

$$\bar{f}_E = a_e a_e^T f_{ad} \quad (4.48)$$

where x_E is as in (4.40) and

$$v_E = \frac{k_I}{k_m} (I - a_e a_e^T) f_{ad}, \quad (4.49)$$

showing that the misalignment of f_{ad} with a_e causes a drift motion due to the presence of the integral action ($k_I \neq 0$); notice, however, that choosing $k_m \gg k_I$ attenuates the magnitude of such a drift.

- In the case of a curved surface, a linear contact force model as in (4.24) still holds but K_e becomes a function of x_e which is in turn a function of x . The problem becomes more involved and we have to resort to hybrid force/motion control, as discussed in Section 4.3.

4.2.2 PID control

The parallel control scheme presented above requires complete knowledge of the manipulator dynamic model in order to ensure tracking of the desired end-effector position on the environment surface. Nevertheless, a computationally lighter control scheme can be devised when a *force and position regulation* task is considered. Such a scheme can be regarded as an extension of the PD controller based on (4.21).

Consider the *constant* set points x_d and f_{ad} , where f_{ad} is aligned with a_e , and the *PID control* law

$$u = J_a^T(q) \left(k_m(x_d - x) - d_m \dot{x} - (f_{ad} + k_F(f_{ad} - f_a) + k_I \int_0^t (f_{ad} - f_a) d\tau) \right) + g(q) \quad (4.50)$$

where $k_m, d_m, k_F, k_I > 0$. This controller corresponds to a position PD action + gravity compensation + desired force feedforward + a force PI action. In the case of perfect gravity compensation, the equilibrium for the closed-loop system is described by the same position in (4.40) and force in (4.41).

In order to study stability of system (4.8) under control (4.50) with the environment model (4.24) and (4.35), it is convenient to introduce a suitable state vector. Consider the end-effector deviation from the equilibrium position

$$\epsilon = x_e - x = x_d - x + d a_e \quad (4.51)$$

where

$$d = a_e a_e^T (k_e(x_e - x_d) - k_e^{-1} f_{ad}) \quad (4.52)$$

is a constant quantity taking into account the effects of the environment contact force and of the desired force set point along the normal to the plane.

Differentiating (4.51) with respect to time gives

$$\dot{\epsilon} = -\dot{x}. \quad (4.53)$$

From (4.51) and (4.52), the position and force errors can be respectively expressed as

$$x_d - x = \epsilon - da_e \quad (4.54)$$

$$f_{ad} - f_a = -k_e a_e a_e^T \epsilon. \quad (4.55)$$

Substituting (4.54) and (4.55) in (4.50) and using (4.53) leads to the closed-loop dynamics

$$H_x \ddot{\epsilon} + (C_x + d_m I) \dot{\epsilon} + (k_m I + k_F' k_e a_e a_e^T) \epsilon + k_I k_e w a_e = 0, \quad (4.56)$$

where $k_F' = 1 + k_F$ and

$$w = -k_e^{-1} a_e^T \left(\int_0^t (f_{ad} - f_a) d\tau + k_m k_I^{-1} d a_e \right). \quad (4.57)$$

Differentiating (4.57) with respect to time and using (4.55) gives

$$\dot{w} = a_e^T \epsilon. \quad (4.58)$$

Equations (4.56) and (4.58) provide a description of the closed-loop system in terms of the (7×1) state vector

$$z = (\dot{\epsilon}^T \quad \epsilon^T \quad w)^T \quad (4.59)$$

which can be written in the standard compact homogeneous form

$$\dot{z} = Fz \quad (4.60)$$

with

$$F = \begin{pmatrix} -H_x^{-1}(C_x + d_m I) & -H_x^{-1}(k_m I + k_F' k_e a_e a_e^T) & -k_I k_e H_x^{-1} a_e \\ I & 0 & 0 \\ 0 & a_e^T & 0 \end{pmatrix}. \quad (4.61)$$

Theorem 4.1 *There exists a choice of gains k_m, d_m, k_F, k_I that makes the origin of the state space of system (4.60) and (4.61) locally asymptotically stable.*

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Proof. Consider the Lyapunov function candidate

$$V = \frac{1}{2} z^T P z \quad (4.62)$$

where

$$P = \begin{pmatrix} H_x & \rho H_x & 0 \\ \rho H_x & (k_m + \rho d_m)I + k'_F k_e a_e a_e^T & k_I k_e a_e \\ 0 & k_I k_e a_e^T & \rho k_I k_e \end{pmatrix} \quad (4.63)$$

with $\rho > 0$. Computing the time derivative of V along the trajectories of system (4.60) and (4.61) gives

$$\dot{V} = -\dot{\epsilon}^T (d_m I - \rho H_x) \dot{\epsilon} + \rho \epsilon^T C_x^T \dot{\epsilon} - \epsilon^T (\rho k_m I + (\rho k'_F - k_I) k_e a_e a_e^T) \epsilon \quad (4.64)$$

where Property 4.2 has been conveniently exploited.

Consider the region of the state space

$$Z = \{z : \|\epsilon\| < \Phi\}. \quad (4.65)$$

The term $\rho \epsilon^T C_x^T \dot{\epsilon}$ in (4.64) can be upper bounded in the region (4.65) as

$$\rho \epsilon^T C_x^T \dot{\epsilon} \leq \rho \Phi k_{Cx} \|\dot{\epsilon}\|^2 \quad (4.66)$$

where Property 4.3 has been used.

From (4.66), the function (4.64) can be upper bounded as

$$\dot{V} \leq -(d_m - \rho \lambda_{Mx} - \rho \Phi k_{Cx}) \|\dot{\epsilon}\|^2 - \rho k_m \|\epsilon\|^2 - k_e (\rho k'_F - k_I) (a_e^T \epsilon)^2 \quad (4.67)$$

where Property 4.1 has been used.

On the other hand, the function candidate (4.62) and (4.63) can be lower bounded as

$$\begin{aligned} V \geq \frac{1}{2} (\dot{\epsilon}^T H_x \quad \epsilon^T) & \begin{pmatrix} (1/\lambda_{Mx})I & \rho I \\ \rho I & (k_m + \rho d_m)I \end{pmatrix} \begin{pmatrix} H_x \dot{\epsilon} \\ \epsilon \end{pmatrix} \\ & + \frac{k_e}{2} (a_e^T \epsilon \quad w) \begin{pmatrix} k'_F & k_I \\ k_I & \rho k_I \end{pmatrix} \begin{pmatrix} a_e^T \epsilon \\ w \end{pmatrix}. \end{aligned} \quad (4.68)$$

From (4.68) the function V is positive definite provided that

$$k_m + \rho d_m > \rho^2 \lambda_{Mx} \quad (4.69)$$

$$\rho(1 + k_F) > k_I, \quad (4.70)$$

whereas from (4.67) the function $\dot{V}$ is negative semi-definite provided that

$$d_m > \rho(\lambda_{Mx} + \Phi k_{Cx}) \quad (4.71)$$

and (4.70) again holds. Observing that condition (4.71) implies (4.69), it can be recognized that a choice of d_m , k_F and k_I exists such that conditions (4.70) and (4.71) are satisfied.

Since $\dot{V}$ is only negative semi-definite, the inequality $\dot{V} \leq 0$ must be further analyzed to prove asymptotic stability. In particular, $\dot{V} = 0$ implies $\dot{\epsilon} = 0$ and $\epsilon = 0$. From (4.60) and (4.61) local asymptotic stability around $z = 0$ follows from La Salle's invariant set theorem.

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Remarks

- The choice of the Lyapunov function (4.62) can be motivated by an energy-based argument. The diagonal terms in (4.63) express essentially the manipulator kinetic energy, the potential energy associated with the deviation from the equilibrium end-effector position, and the potential energy stored along the normal to the plane, respectively.
- The free parameter ρ is not used in the control law (4.50) and allows then an opportune choice of the gains d_m , k_F and k_I . Notice that k_m is not involved in the conditions (4.70) and (4.71) and thus is available to meet further design requirements during the non-contact phase of the task. Also, by increasing d_m a larger value of Φ can be tolerated which in turn determines the local nature of the stability proof.

4.3 Hybrid force/motion control

When the environment is quite rigid and the manipulator has to continuously keep its end effector in contact, a *constrained approach* may be more convenient to describe the system dynamics and to set up the control problem. This approach considers the generalized surface of the environment as an infinitely stiff, bilateral constraint with no friction.

In this ideal case, an algebraic vector equation restricts the feasible locations (positions and orientations) of the manipulator end effector and induces kinematic constraints on the robot manipulator motion, effectively reducing the number of generalized coordinates needed to describe the manipulator dynamics. In a dual fashion, this approach explicitly provides also the forces and torques occurring at the contact. In fact, these are obtained from the existing orthogonality relations between *admissible* generalized velocities v and generalized *reaction* forces f in the task space (Cartesian space).

The constrained formulation is best suited for cases where a direct control of arbitrary forces exerted against the environment is needed together with motion control along a reduced number of task space directions. Using the whole information about the environment geometry, *hybrid force/motion control* is then possible. When unavoidable modelling errors come into play, the constrained approach provides also a convenient setting for handling force and displacement error signals at the joint or at the end-effector level.

We start with the derivation of the constrained manipulator dynamic equations. These can be given two different formats: one of *full* dimension, with filtering of the applied torques so as to maintain consistency with the constraint; another of *reduced* dimension, eliminating as many variables as constraints.

4.3.1 Constrained dynamics

In the following, we assume that the robot manipulator is subject to a *holonomic* constraint expressed directly in the *joint space*

$$\phi(q) = 0, \quad \phi : R^n \rightarrow R^m, \quad m < n. \quad (4.72)$$

This nonlinear equality constraint is the equivalent representation of a Cartesian rigid and frictionless environment surface on which the manipulator end effector “lies”. We consider only the time-invariant (*scleronomic*) case (4.72), but the extension to *rheonomic* constraints, $\phi(q, t) = 0$, is straightforward.

It is assumed that the vector constraint (4.72) is twice differentiable and that its m components are linearly independent. Differentiation of (4.72) yields then

$$\frac{\partial \phi}{\partial q} \dot{q} = J_\phi(q) \dot{q} = 0, \quad (4.73)$$

and

$$J_\phi(q) \ddot{q} + \dot{J}_\phi(q) \dot{q} = 0. \quad (4.74)$$

The assumed regularity condition implies that the rank of $J_\phi(q)$ will be equal to the number m of its rows, globally for q or at least locally in a neighbourhood of the operating point.

The manipulator dynamics subject to (4.72) is written as

$$H(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = u + u_f, \quad (4.75)$$

where the $(n \times 1)$ vector of generalized forces u_f arising from the presence of the constraint can be expressed in terms of an $(m \times 1)$ vector of multipliers

λ as

$$u_f = J_\phi^T(q)\lambda. \quad (4.76)$$

This characterization of the constraining forces associated with (4.72) follows from the virtual work principle.

The force multipliers λ can be eliminated by solving (4.75) for $\ddot{q}$ and substituting it into (4.74). We obtain

$$\lambda = \left(J_\phi(q)H^{-1}(q)J_\phi^T(q) \right)^{-1} \cdot \left(J_\phi(q)H^{-1}(q)(C(q, \dot{q})\dot{q} + g(q) - u) - \dot{J}_\phi^T(q)\dot{q} \right), \quad (4.77)$$

from which it follows that the value of the multipliers instantaneously depends also on the applied input torque u . Substituting (4.76) and (4.77) into (4.75) yields a (*full*) n -dimensional set of second-order differential equations whose solutions, when initialized on the constraint, automatically satisfy (4.72) for all times, i.e.,

$$H(q)\ddot{q} + J_\phi^T(q) \left(J_\phi(q)H^{-1}(q)J_\phi^T(q) \right)^{-1} J_\phi(q)\dot{q} = P(q)(u - C(q, \dot{q})\dot{q} - g(q)), \quad (4.78)$$

where

$$P(q) = I - J_\phi^T(q) \left(J_\phi(q)H^{-1}(q)J_\phi^T(q) \right)^{-1} J_\phi(q)H^{-1}(q). \quad (4.79)$$

Since $PJ_\phi^T = 0$ and $P^2 = P$ (idempotent), the $(n \times n)$ matrix $P(q)$ is a projection operator that filters out all joint torques lying in the range of the transpose of the constraint Jacobian. These correspond to generalized forces that tend to violate the imposed joint space constraint.

As opposed to the above description, a reduction procedure can be set up to obtain a more compact representation of the robotic system. Using the implicit function theorem, we can always partition the joint vector q in two vectors; namely, an $(m \times 1)$ vector q_1 and an $((n - m) \times 1)$ vector q_2 such that

$$\text{rank} \frac{\partial \phi}{\partial q_1} = m. \quad (4.80)$$

As a consequence, the constraint (4.72) can be expressed in terms of the variables q_2 only:

$$q_1 = \psi(q_2) \implies \phi(\psi(q_2), q_2) = 0. \quad (4.81)$$

The proper use of (4.81) eliminates from the n -dimensional constrained system the presence of the m variables q_1 together with all the constraint

equations, leading to a *reduced* $(n - m)$ -dimensional unconstrained dynamical system. For this, it is convenient to apply the change of coordinates

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} q_1 - \psi(q_2) \\ q_2 \end{pmatrix} = \Theta(q), \quad (4.82)$$

with inverse transformation

$$q = \begin{pmatrix} \theta_1 + \psi(\theta_2) \\ \theta_2 \end{pmatrix} = Q(\theta). \quad (4.83)$$

In the new coordinates, satisfaction of the constraint (4.72) implies $\theta_1 = 0$. We will also make use of the differential relation

$$\dot{q} = \begin{pmatrix} I & \frac{\partial \psi}{\partial \theta_2} \\ 0 & I \end{pmatrix} \dot{\theta} = T(\theta_2) \dot{\theta}. \quad (4.84)$$

Moreover, the input torques u_θ performing work on the new coordinates satisfy

$$u_\theta^T \dot{\theta} = u^T \dot{q} = u^T T(\theta_2) \dot{\theta}, \quad (4.85)$$

from which

$$u_\theta = T^T(\theta_2) u. \quad (4.86)$$

Differentiation of (4.84) gives

$$\ddot{q} = T(\theta_2) \ddot{\theta} + \dot{T}(\theta_2) \dot{\theta}, \quad (4.87)$$

and its substitution into (4.75) premultiplied by $T^T(\theta_2)$ yields

$$\begin{aligned} T^T(\theta_2) \Big(& H(Q(\theta)) T(\theta_2) \ddot{\theta} + H(Q(\theta)) \dot{T}(\theta_2) \dot{\theta} \\ & + C(Q(\theta), T(\theta_2) \dot{\theta}) T(\theta_2) \dot{\theta} + g(Q(\theta)) \Big) = T^T(\theta_2) (u + u_f). \end{aligned} \quad (4.88)$$

Defining

$$\begin{aligned} \hat{H}(\theta) &= T^T(\theta_2) H(Q(\theta)) T(\theta_2) \\ \hat{n}(\theta, \dot{\theta}) &= T^T(\theta_2) H(Q(\theta)) \dot{T}(\theta_2) \dot{\theta} \\ &\quad + T^T(\theta_2) C(Q(\theta), T(\theta_2) \dot{\theta}) T(\theta_2) \dot{\theta} + T^T(\theta_2) g(Q(\theta)) \end{aligned} \quad (4.89)$$

provides the more compact form

$$\hat{H}(\theta) \ddot{\theta} + \hat{n}(\theta, \dot{\theta}) = T^T(\theta_2) u + T^T(\theta_2) J_\phi^T(Q(\theta)) \lambda, \quad (4.90)$$

which is again the full manipulator dynamics, expressed in the θ coordinates.

Analyzing independently the first m and the last $n - m$ second-order differential equations in (4.90) will be useful for the subsequent control developments. Introduce the simple matrices

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = (E_1^T \quad E_2^T), \quad (4.91)$$

where the identity and null block matrices have dimensions so that E_1 is $(m \times n)$ and E_2 is $((n - m) \times n)$. Then, evaluate the dynamic terms in (4.90) on the constraint $\phi(q) = 0$, i.e., for $\theta_1(t) \equiv 0$ (thus, $\dot{\theta}_1 = \ddot{\theta}_1 = 0$). With a slight abuse of notation, we will write $\hat{H}(\theta_2)$ in place of $\hat{H}(0, \theta_2)$ and similarly for other restricted terms. This leads to

$$E_1 \hat{H}(\theta_2) E_2^T \ddot{\theta}_2 + E_1 \hat{n}(\theta_2, \dot{\theta}_2) = E_1 T^T(\theta_2) u + E_1 T^T(\theta_2) J_\phi^T(\theta_2) \lambda \quad (4.92)$$

and

$$E_2 \hat{H}(\theta_2) E_2^T \ddot{\theta}_2 + E_2 \hat{n}(\theta_2, \dot{\theta}_2) = E_2 T^T(\theta_2) u. \quad (4.93)$$

We remark that the term $E_2 T^T(\theta_2) J_\phi^T(\theta_2) \lambda$ was dropped from (4.93), being identically zero. To show this, differentiate $\phi(q) = 0$ with respect to time and use (4.81):

$$\begin{aligned} \dot{\phi}(\psi(q_2), q_2) &= \left(\frac{\partial \phi}{\partial q_1} \frac{\partial \psi}{\partial q_2} + \frac{\partial \phi}{\partial q_2} \right) \dot{q}_2 \\ &= \left(\frac{\partial \phi}{\partial q_1} \quad \frac{\partial \phi}{\partial q_2} \right) \begin{pmatrix} \frac{\partial \psi}{\partial q_2} \\ I \end{pmatrix} \dot{q}_2 = J_\phi(q) T(q_2) E_2^T \dot{q}_2 = 0. \end{aligned} \quad (4.94)$$

Since (4.94) holds for all $\dot{q}_2$, it follows that $E_2 T^T J_\phi^T = 0$.

The first set of dynamic equations (4.92) can be discarded, since it provides only the actual value of the multipliers λ . In fact, solving for $\ddot{\theta}_2$ from (4.93) and substituting in (4.92) yields

$$\begin{aligned} E_1 \hat{H}(\theta_2) E_2^T (E_2 \hat{H}(\theta_2) E_2^T)^{-1} E_2 (T^T(\theta_2) u - \hat{n}(\theta_2, \dot{\theta}_2)) + E_1 \hat{n}(\theta_2, \dot{\theta}_2) \\ = E_1 T^T(\theta_2) u + E_1 T^T(\theta_2) J_\phi^T(\theta_2) \lambda, \end{aligned} \quad (4.95)$$

from which the force multipliers are obtained as

$$\begin{aligned} \lambda &= (E_1 T^T(\theta_2) J_\phi^T(\theta_2))^{-1} \cdot \\ &\quad \left(E_1 - \hat{H}_{12}(\theta_2) \hat{H}_{22}^{-1}(\theta_2) E_2 \right) \left(\hat{n}(\theta_2, \dot{\theta}_2) - T^T(\theta_2) u \right), \end{aligned} \quad (4.96)$$

being the matrix $E_1 T^T J_\phi^T = (\partial\phi/\partial q_1)^T$ square and nonsingular by assumption, and where block matrix notation has been used. The above expression for λ is equivalent to (4.77), but it is based on the reduced form of the dynamic equations. The overall dynamic behaviour will be driven only by (4.93) which, incidentally, can be rewritten simply as

$$\hat{H}_{22}(\theta_2)\ddot{\theta}_2 + \hat{n}_2(\theta_2, \dot{\theta}_2) = u_{\theta_2}. \quad (4.97)$$

This has to be completed by $\theta_1(t) \equiv 0$.

For illustration, the reduced equations of motion can be easily derived in the simple case of

$$\phi(q) = q_1 = 0, \quad (4.98)$$

a *linear* joint space constraint already set in explicit form (here, $\theta = q$). Since $J_\phi = (I \ 0)$, $T = I$, and then $\hat{H} = H$ and $\hat{n} = n = C\dot{q} + g$, eqs. (4.92) and (4.93) become

$$\begin{aligned} H_{12}(\theta_2)\ddot{\theta}_2 + n_1(\theta_2, \dot{\theta}_2) &= u_1 + \lambda \\ H_{22}(\theta_2)\ddot{\theta}_2 + n_2(\theta_2, \dot{\theta}_2) &= u_2 \end{aligned} \quad (4.99)$$

while the multipliers are given by

$$\lambda = n_1(\theta_2, \dot{\theta}_2) - u_1 - H_{12}(\theta_2)H_{22}^{-1}(\theta_2) \left(n_2(\theta_2, \dot{\theta}_2) - u_2 \right). \quad (4.100)$$

4.3.2 Inverse dynamics control

The control problem requires the asymptotic tracking of the following joint *motion* and contact *force* trajectories:

$$q = q_d(t) \quad u_f = u_{fd}(t) \quad t \geq 0, \quad (4.101)$$

on the assumption that these specifications are *consistent* with the model of the constraining surface. This implies that the $2n$ time functions in (4.101) are such that:

- $\phi(q_d(t)) = 0$ for all $t \geq 0$;
- for all $t \geq 0$, there exists an $(m \times 1)$ vector $\lambda_d(t)$ satisfying

$$u_{fd}(t) = J_\phi^T(q_d(t))\lambda_d(t). \quad (4.102)$$

Notice that the vector λ_d is a parameterization of the desired contact force in the constraint coordinates θ . In the full-rank hypothesis for J_ϕ , the unique value satisfying (4.102) is obtained through pseudoinversion

$$\lambda_d = (J_\phi(q_d)J_\phi^T(q_d))^{-1}J_\phi(q_d)u_{fd}, \quad (4.103)$$

for any given consistent choice of u_{fd} .

An *inverse dynamics* approach will be followed for the design of a control law that realizes the assigned task of motion in contact with the constraining surface with the prescribed exchange of forces. In doing so, the most convenient strategy will be to reconsider the task as specified by the $n - m$ motion assignments $\theta_{2d}(t)$ and by the m force multipliers $\lambda_d(t)$. In this way, a set of n *independent* time evolutions are assigned which can be realized by the suitable choice of the n control inputs u . Note also that from these assignments and from $\theta_{1d} = 0$, the whole motion vector q_d can be recovered using (4.83) while the n -dimensional joint space force is obtained through $u_{fd} = J_\phi^T(q_d)\lambda_d$. Vice-versa, (4.82) and (4.103) are used for moving the task specification from q_d and u_{fd} to θ_{2d} and λ_d .

The reduced form (4.92) and (4.93) of the system dynamic equations will be used for deriving an inversion control law. In particular, the control u will be found from the following m equations

$$\begin{aligned} E_1 T^T(\theta_2)u &= E_1 \hat{n}(\theta_2, \dot{\theta}_2) \\ &+ E_1 \hat{H}(\theta_2) E_2^T \left(\ddot{\theta}_{2d} + K_D(\dot{\theta}_{2d} - \dot{\theta}_2) + K_P(\theta_{2d} - \theta_2) \right) \\ &- E_1 T^T(\theta_2) J_\phi^T(\theta_2) \left(\lambda_d + K_\lambda(\lambda_d - \lambda) + K_I \int_0^t (\lambda_d - \lambda) d\tau \right), \end{aligned} \quad (4.104)$$

and the $n - m$ equations

$$\begin{aligned} E_2 T^T(\theta_2)u &= E_2 \hat{n}(\theta_2, \dot{\theta}_2) \\ &+ E_2 \hat{H}(\theta_2) E_2^T \left(\ddot{\theta}_{2d} + K_D(\dot{\theta}_{2d} - \dot{\theta}_2) + K_P(\theta_{2d} - \theta_2) \right). \end{aligned} \quad (4.105)$$

We note that a PD action on the motion error has been added to the feedforward acceleration $\ddot{\theta}_{2d}$, while a PI action was chosen on the force multiplier error, apart from the feedforward of λ_d .

Combining the above n equations and solving for u gives finally

$$\begin{aligned} u &= T^{-T}(\theta_2) \hat{H}(\theta_2) E_2^T \left(\ddot{\theta}_{2d} + K_D(\dot{\theta}_{2d} - \dot{\theta}_2) + K_P(\theta_{2d} - \theta_2) \right) \\ &+ T^{-T}(\theta_2) \hat{n}(\theta_2, \dot{\theta}_2) - T^{-T}(\theta_2) E_1^T E_1 T^T(\theta_2) J_\phi^T(\theta_2) \cdot \\ &\quad \left(\lambda_d + K_\lambda(\lambda_d - \lambda) + K_I \int_0^t (\lambda_d - \lambda) d\tau \right). \end{aligned} \quad (4.106)$$

The analysis of the characteristics of the closed-loop system is made in terms of the following errors:

$$\begin{aligned} e_1 &= \theta_1 = q_1 - \psi(q_2) \\ e_2 &= \theta_2 - \theta_{2d} = q_2 - q_{2d} \\ e_\lambda &= \lambda - \lambda_d. \end{aligned} \quad (4.107)$$

The closed-loop equations obtained from (4.92), (4.93), and (4.106) are

$$E_1 \hat{H}(\theta_2) E_2^T (\ddot{e}_2 + K_D \dot{e}_2 + K_P e_2) = E_1 T^T(\theta_2) J_\phi^T(\theta_2) \left(e_\lambda + K_\lambda e_\lambda + K_I \int_0^t e_\lambda d\tau \right), \quad (4.108)$$

and

$$E_2 \hat{H}(\theta_2) E_2^T (\ddot{e}_2 + K_D \dot{e}_2 + K_P e_2) = 0, \quad (4.109)$$

together with $e_1(t) \equiv 0$. From (4.109), since the block $\hat{H}_{22} = E_2 \hat{H} E_2^T$ on the diagonal of the inertia matrix is always invertible, it follows that

$$e_2(t) \rightarrow 0 \quad \implies \quad q_2(t) \rightarrow q_{2d}(t) \quad (4.110)$$

as $t \rightarrow \infty$, provided that $K_P > 0$ and $K_D > 0$. Then we have also

$$e_1(t) \equiv 0 \quad \implies \quad q_1(t) \rightarrow q_{1d}(t) = \psi(q_{2d}(t)). \quad (4.111)$$

Therefore, the study of eq. (4.108) reduces to the analysis of the asymptotic stability of the linear equation

$$e_\lambda + K_\lambda e_\lambda + K_I \int_0^t e_\lambda(\tau) d\tau = 0, \quad (4.112)$$

and then

$$e_\lambda(t) \rightarrow 0 \quad \implies \quad \lambda(t) \rightarrow \lambda_d(t) \quad (4.113)$$

as $t \rightarrow \infty$, provided that $K_\lambda \geq 0$ and $K_I \geq 0$. Note that both feedback gains in the force loop can be set to zero, without losing the asymptotic convergence to zero of the force error. In that case, the control law (4.106) would require *no force feedback* but only the feedforward of the desired force multipliers. This analysis, however, holds only in nominal conditions, i.e., when the environment surface is perfectly known and the only disturbance is a mismatch in the initial condition of motion *on the constraining surface*. When other uncertainties and disturbances are present, the benefits introduced by the force feedback terms become self-evident. In particular, the integral action is able to compensate for a constant bias disturbance.

The inversion controller (4.106) can also be rewritten in terms of the original variables q , using

$$\theta_d = \Theta(q_d) \quad (4.114)$$

$$\dot{\theta}_d = T^{-1}(q_{2d}) \dot{q}_d \quad (4.115)$$

$$\ddot{\theta}_d = T^{-1}(q_{2d}) \left(\ddot{q}_d - \dot{T}(q_{2d}) T^{-1}(q_{2d}) \dot{q}_d \right), \quad (4.116)$$

where $q_d(t)$ is a specification consistent with the constraint $\phi(q) = 0$. After some manipulation, noting also that $\ddot{\theta}_{2d} = E_2 \ddot{\theta}_d$, we finally get

$$\begin{aligned} u = & H(q)T(q) \left(T^{-1}(q_d)\ddot{q}_d - T^{-1}(q_d)\dot{T}(q_d)T^{-1}(q_d)\dot{q}_d \right. \\ & + E_2^T K_P E_2 (\Theta(q_d) - \Theta(q)) + E_2^T K_D E_2 (T^{-1}(q_d)\dot{q}_d - T^{-1}(q)\dot{q}) \\ & + H(q)\dot{T}(q)T^{-1}(q)\dot{q} + C(q, \dot{q})\dot{q} + g(q) \\ & \left. - J_\phi^T(q) \left(\lambda_d + K_\lambda(\lambda_d - \lambda) + K_I \int_0^t (\lambda_d(\tau) - \lambda(\tau)) d\tau \right) \right). \quad (4.117) \end{aligned}$$

In the form (4.117), it is apparent that this control law performs a cancellation of the manipulator dynamic nonlinearities as well as of the inertial couplings due to constraint curvature ($\dot{T} \neq 0$). Furthermore, a desired linear error dynamics is imposed via the matrix gains K_P , K_D , K_λ and K_I , which are typically chosen diagonal so as to preserve the independent specification of desired force multipliers and of free admissible motion components. We recognize that these considerations are the spirit of an exact linearization and decoupling approach based on nonlinear state feedback.

A slightly different inversion controller can also be derived if we choose to react through u to force errors defined directly in terms of vector u_f in (4.75), i.e., to

$$\begin{aligned} e_f &= J_\phi^T(q)(\lambda - \lambda_d) \\ &= u_f - J_\phi^T(q) (J_\phi(q_d) J_\phi^T(q_d))^{-1} J_\phi(q_d) u_{fd}, \quad (4.118) \end{aligned}$$

where (4.76) and (4.103) have been used. The control law u will now be obtained by imposing the following alternate set of equations in place of (4.104) and (4.105):

$$\begin{aligned} E_1 T^T(\theta_2) u &= E_1 \hat{n}(\theta_2, \dot{\theta}_2) \\ &+ E_1 \hat{H}(\theta_2) E_2^T \left(\ddot{\theta}_{2d} + K_D(\dot{\theta}_{2d} - \dot{\theta}_2) + K_P(\theta_{2d} - \theta_2) \right) \\ &- E_1 T^T(\theta_2) J_\phi^T(\theta_2) \lambda_d - E_1 E_1^T K_F E_1 T^T(\theta_2) J_\phi^T(\theta_2) (\lambda_d - \lambda) \end{aligned} \quad (4.119)$$

and

$$\begin{aligned} E_2 T^T(\theta_2) u &= E_2 \hat{n}(\theta_2, \dot{\theta}_2) \\ &+ E_2 \hat{H}(\theta_2) E_2^T \left(\ddot{\theta}_{2d} + K_D(\dot{\theta}_{2d} - \dot{\theta}_2) + K_P(\theta_{2d} - \theta_2) \right) \\ &- E_2 T^T(\theta_2) J_\phi^T(\theta_2) \lambda_d - E_2 E_1^T K_F E_1 T^T(\theta_2) J_\phi^T(\theta_2) (\lambda_d - \lambda). \end{aligned} \quad (4.120)$$

We point out that the last two terms in (4.120) are zero because (4.94) holds and $E_2 E_1^T = 0$. Similarly, $E_1 E_1^T = I$ in the last term of (4.119) so

that it could be dropped. However, these terms are kept for the sake of symmetry between (4.119) and (4.120).

Combining the above two equations and solving for u gives now

$$\begin{aligned} u = & T^{-T}(\theta_2)\hat{H}(\theta_2)E_2^T \left(\ddot{\theta}_{2d} + K_D(\dot{\theta}_{2d} - \dot{\theta}_2) + K_P(\theta_{2d} - \theta_2) \right) \\ & + T^{-T}(\theta_2)\hat{n}(\theta_2, \dot{\theta}_2) - J_\phi^T(\theta_2)\lambda_d \\ & - T^{-T}(\theta_2)E_1^T K_F E_1 T^T(\theta_2)J_\phi^T(\theta_2)(\lambda_d - \lambda). \end{aligned} \quad (4.121)$$

In this case, the controller contains only a proportional action on the force error. It can be easily shown that $K_F \geq 0$ guarantees asymptotic stability of the force error ($e_f \rightarrow 0$). The inclusion of a robustifying integral term is more problematic in this context, due to the time-varying nature of the direction of the force error e_f —coming from the presence of $J_\phi^T(q)$ in (4.118).

As before, also the control law (4.121) can be rewritten in terms of the original coordinates q . This leads to

$$\begin{aligned} u = & H(q)T(q) \left(T^{-1}(q_d)\ddot{q}_d - T^{-1}(q_d)\dot{T}(q_d)T^{-1}(q_d)\dot{q}_d \right. \\ & + E_2^T K_P E_2 (\Theta(q_d) - \Theta(q)) + E_2^T K_D E_2 (T^{-1}(q_d)\dot{q}_d - T^{-1}(q)\dot{q}) \Big) \\ & + H(q)\dot{T}(q)T^{-1}(q)\dot{q} + C(q, \dot{q})\dot{q} + g(q) \\ & - J_\phi^T(q)(J_\phi(q_d)J_\phi^T(q_d))^{-1}J_\phi(q_d)u_{fd} \\ & - T^{-T}(q)E_1^T K_F E_1 T^T(q) \left(J_\phi^T(q)(J_\phi(q_d)J_\phi^T(q_d))^{-1}J_\phi(q_d)u_{fd} - u_f \right). \end{aligned} \quad (4.122)$$

Remarks

- In the control law (4.122), the matrices

$$\begin{aligned} \Sigma_F &= E_1^T K_F E_1 \\ \Sigma_P &= E_2^T K_P E_2 \\ \Sigma_D &= E_2^T K_D E_2, \end{aligned} \quad (4.123)$$

represent filtering matrices on the force, position, and velocity errors, respectively. This allows us to *selectively* react to error components depending on their specific directions. In this way it is guaranteed that no superposition of contrasting control actions results; there is one and only one effective linear controller for each force or complementary motion component, leading to m force loops and $n - m$ motion (position/velocity) loops.

- In practice, the whole approach starts from a p -dimensional ($p < n$) constraint task vector $r(x) = 0$ imposed on the end-effector location. We have then

$$r(x) = r(k(q)) = \phi(q) = 0, \quad (4.124)$$

while the developments remain the same (with $m = p$). However, in this form it becomes evident that both the characteristics of the constraint task in $J_r(x) = \partial r / \partial x$ and the analytical Jacobian $J_a(q) = \partial k / \partial q$ are crucial, being $J_\phi(q) = J_r(k(q))J_a(q)$.

- When the kinematic constraint is time-varying, i.e.,

$$r(x, t) = 0 \quad r : R^m \times R^+ \rightarrow R^p \quad p < n, \quad (4.125)$$

further terms will appear in its first time derivative

$$\frac{\partial r}{\partial x} \dot{x} + \frac{\partial r}{\partial t} = J_r(x, t) \dot{x} + v_r(x, t) = 0 \quad (4.126)$$

as well as in the second time derivative

$$J_r(x, t) \ddot{x} + \dot{J}_r(x, t) \dot{x} + \dot{v}_r(x, t) = J_r(x, t) \ddot{x} + a_r(x, \dot{x}, t) = 0 \quad (4.127)$$

which takes the place of (4.74). By substituting

$$\begin{aligned} J_\phi(q) &\mapsto J_r(k(q), t) J_a(q) \\ \dot{J}_\phi(q) \dot{q} &\mapsto J_r(k(q), t) \dot{J}_a(q) \dot{q} + a_r(k(q), J_a(q) \dot{q}, t) \end{aligned} \quad (4.128)$$

in all subsequent equations, the rest follows accordingly.

4.3.3 Hybrid task specification and control

In the constrained approach we have assumed that the robot manipulator is always in contact with the environment, as represented by the vector equation (4.72). In practice, uncertainty in the geometric knowledge of the object surfaces and the nonideal characteristics (finite stiffness, friction) of the constraint, together with the presence of various disturbances, introduce small but systematic position and force errors. These errors can be handled by a relaxation of the constrained framework. For this reason, it is convenient to restate the main point of this approach: due to the presence of a constraint expressed in suitable coordinates, it is possible to characterize generalized directions of admissible velocity and of reaction forces. At this point, this idea can be used even *without* the explicit introduction of a constraint in the form (4.72).

For most tasks there exists a convenient space where the vectors of admissible velocity and of contact reaction force can be used to form an *orthogonal* and *complete* basis set. In particular, we can define a time-varying *task frame* with the following property; at every instant, in each coordinate direction either a force or a velocity can be independently assigned, the other quantity being naturally imposed by the task geometry. We notice that this orthogonality can be stated only between force and velocity (or displacement) generalized vectors, not between force and position, justifying the current terminology of *hybrid force/velocity control*. The key point in the control scheme is to use the task frame concept also in off-nominal conditions for “filtering out” errors that are not consistent with the model, thus reacting only to force or motion error components along “expected” directions.

The steps for deriving a hybrid controller follow closely the previous derivations, taking also into account the above final remarks. Relevant dynamic quantities (velocities and forces) are first transferred in the suitable *task space*, through kinematic transformations. They are next compared with the current desired values, generating directional errors and simply eliminating those that do not agree with the task model (e.g., considering only measured forces that are normal to an assumed frictionless plane on which the manipulator end effector is moving). This filtering action plays a role similar to that of the Σ matrices in (4.123). In response to these errors, commands are generated by a set of m scalar force controllers and $n - m$ scalar velocity controllers, each independently designed for one task coordinate. In particular, in the design of the force controllers we may take advantage of an assumed compliant contact model, similar to (4.24). Finally, these control commands, which are dimensionally homogeneous to accelerations, are brought back to the joint space and are used to produce the applied input torques —possibly, based on the manipulator dynamics if an inversion controller is desired.

The resulting hybrid force/velocity control law can be easily derived from the previously presented ones, e.g., from (4.122). The only difference is that all dynamic quantities are evaluated *at the current state*, not necessarily satisfying a constraining equation.

Contact with dynamic environments

As opposed to “completely static” (constrained) or “purely dynamic” (impedance) interactions, there is a whole class of tasks which are accurately modelled only by considering a more general dynamic behaviour for the environment and a more flexible description of the robot manipulator contact. In this respect, the manipulator end effector may exert *dynamic* forces, i.e.,

forces not compensated by a constraint reaction and producing active work on the environment geometry, while still being subject to further kinematic constraints. A paradigmatic example is the task of a manipulator turning a crank, when crank dynamics is relevant. Also, the dynamic world seen by a manipulator may be another manipulator itself, as in the case of cooperative robotic tasks. An effective modelling technique should be able to handle mixed situations in which the end effector is kinematically constrained and/or dynamically coupled with the external world. Finally, purely *compliant* interactions should also be considered, typically associated with the potential energy of contact deformation.

In these situations, the hybrid control cannot be applied as such. Orthogonality can be stated only between two reduced sets of task directions, one in which it is possible to control only motion parameters and one in which only reaction forces can be controlled. Instead, along dynamic directions *either* a force parameter *or* a motion parameter can be directly controlled by the hybrid scheme, the other quantity resulting from the dynamic balance of the interaction between the manipulator and environment.

4.4 Further reading

Early work on force control can be found in [52, 53], typically applied to assembly tasks. Stiffness control was proposed in [42] and is conceptually equivalent to compliance control [39]. The original idea of a mechanical impedance model used for controlling the interaction between manipulator and environment is due to [24], and a similar formulation is given in [28]. Achieving the expected impedance may be difficult due to uncertainties on the manipulator dynamic model. Linear [12] and nonlinear [29, 33] adaptive impedance control algorithms have been proposed to overcome this problem, while a robust scheme can be found in [34]. An approach based on impedance control where the reference position trajectory is adjusted in order to fulfill a force control objective has been recently proposed in [8]. A survey of all the above schemes in quasi-static operation can be found in [54].

The parallel approach to force/position control of robot manipulators was introduced in [10] using an inverse dynamics controller. As an alternative to the inverse dynamics scheme, a passivity-based approach to force regulation and motion control has been proposed in [44], which naturally allows derivation of an adaptive control scheme [46]. The PID parallel controller for force and position regulation was developed in [11], where exponential stability is proved in addition to asymptotic stability. More recently, the regulator has been extended in an output feedback setting to

avoid velocity measurements [45]. The case of imperfect gravity compensation is treated in [43] where an adaptive scheme is proposed to resume the original equilibrium. On the other hand, the original idea of closing an outer force control loop around an inner position control loop dates back to [18]. The use of an integral action for removing steady-state force errors was also proposed in [51], and robustness with respect to force measurement delays was investigated in [50, 55].

The original hybrid position/force control concept was introduced in [41], based on the natural and artificial constraint task formulation of [35]. The explicit inclusion of the manipulator dynamic model was presented in [30], and the problem of correct definition of a time-varying task frame was pointed out in [15]. The constrained task formulation with inverse dynamics controllers is treated in [57, 58] using a Cartesian constraint as well as in [36] using a joint space constraint. A projection technique of Cartesian constraints to the joint space was presented in [5]. A comparison of the benefits of adopting a reduced vs. a complete set of dynamic model equations can be found in [26]. The constrained approach was also used in [37] with a controller based on linearized equations, and in [59] with a dynamic state feedback controller. The hybrid task specification in connection with an impedance controller on the constrained directions was proposed in [3], as well as in [16] where a sliding mode controller is used for robustness purposes. On-line identification of the task frame orientation with hybrid control experiments was presented in [22]. Theoretical issues on orthogonality of generalized force and velocity directions are discussed in [31, 19]. Transposition of model-based schemes from unconstrained motion control to constrained cases was accomplished for adaptive control in [47, 32, 4], for robust control in [48], and for learning control in [1], respectively. On the other hand, adaptation with respect to the compliance characteristics of the environment has been proposed in [9, 56]. Modelling of interaction between a manipulator and a dynamic environment was presented in [14], while hybrid control schemes based on these models can be found in [13].

It has been generally recognized that force control may cause unstable behaviour during contact with environment. Dynamic models for explaining this phenomenon were introduced in [20, 21]. Experimental investigations can be found in [2] and [49]. Emphasis on the problems with a stiff environment is given in [27, 23], while stability of impedance control was analyzed in [25]. Moreover, control schemes are usually derived on the assumption that the manipulator end effector is in contact with the environment and this contact is not lost. Impact phenomena may occur which deserve careful consideration, and there is a need for global analysis of control schemes including the transition from non-contact into contact and vice-versa [38, 6].

The automatic specification of robotic tasks that involve multiple and time-varying contacts of the manipulator end-effector with the environment is a challenging issue. General formalisms addressing this problem were proposed in [17, 40, 7].

We conclude by pointing out that the task of controlling the end-effector velocity of a single manipulator and its exchanged forces with the environment is the simplest of a series of robotic problems where similar issues arise. Among them we wish to mention *telemanipulation*, where the use of force feedback enhances the human operator capabilities of remotely handling objects with a slave manipulator, and *cooperative robot systems*, where two or more manipulators (viz., the fingers of a dexterous robot hand) should monitor the exchanged forces so as to avoid “squeezing” of the commonly held object.

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