ENERGY-BASED CONTROL OF THE BUTTERFLY ROBOT

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Abstract: We address the control problem for the Butterfly, an interesting example of 2-dof underactuated mechanical system. This robot consists of a butterfly-shaped rotational link on whose rim a ball rolls freely. The control objective is to stabilize the robot at a certain unstable equilibrium. To this end, exploiting the existence of heteroclinic trajectories, we extend a previously proposed energy-based technique. Simulation results show the effectiveness of the presented method.

Keywords: Energy-based control, underactuated systems, heteroclinic orbits

1. INTRODUCTION

In recent years, a remarkable effort has been devoted to the study of underactuated robots, i.e., mechanical systems with less control inputs than generalized coordinates. This class encompasses many interesting robotic devices, including minimalistic manipulation devices, elastic manipulators, underwater vehicles, and biped robots.

The Butterfly robot considered in this paper was introduced in (Lynch *et al.*, 1998) for the study of non-prehensile manipulation tasks. Other closely related systems are the Ball and Beam (Hauser *et al.*, 1992), the Acrobot (Spong, 1995) and the Pendubot (Spong and Block, 1995). Among the different available approaches, control techniques based on passivity and energy concepts have recently been extended to underactuated mechanical systems; see for example (Ortega *et al.*, 2002; Grizzle *et al.*, 2005).

With reference to the problem of stabilizing the Butterfly at a certain unstable equilibrium, we present in this paper the extension of an energy-based control technique introduced for the Pendubot in (Fantoni *et al.*, 2000) and subsequently refined in (Kolesnichenko and Shiriaev, 2002). The resulting controller combines a swing-up phase, aimed at approaching a heteroclinic trajectory, with a balancing phase designed on the linear approximation of the system.

The paper is organized as follows. Section 2 describes the Butterfly robot, while the existence of heteroclinic orbits is analyzed in Section 3. The proposed energy-based controller is presented in Section 4, and its stability properties are analyzed in Section 5. Simulation results for a laboratory prototype which is currently being instrumented are presented in Section 6.

2. THE BUTTERFLY

The Butterfly is a two-dof planar robotic mechanism, originally conceived to analyze the role of shape in dynamic (non-prehensile) manipulation tasks (Lynch *et al.*, 1998). As shown in Fig. 1, the system consists of a butterfly-shaped rotational link over whose rim a ball is free to roll. In par-

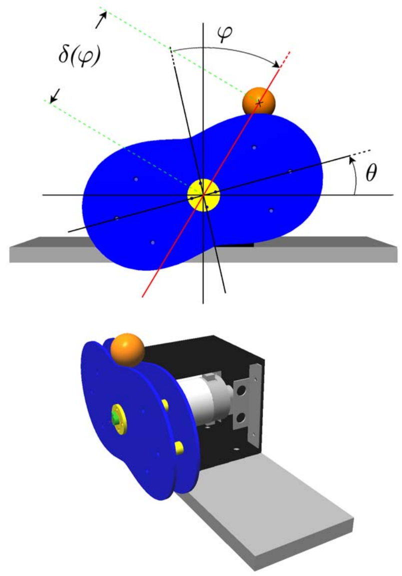

Fig. 1. The Butterfly system: front and side views

ticular, the rotational link is obtained by coupling together two thin metal surfaces so as to provide a track for the rolling ball.

The equations of motion can be derived following the Lagrangian approach. The generalized coordinate vector is $q = (\theta \quad \varphi)^T$, where θ is the angular position of the rotational link w.r.t. the horizontal axis and φ is the angular position of the ball w.r.t. the link vertical axis (see Fig. 1).

To simplify the analysis, we assume throughout the paper that the kinetic energy associated to the ball rolling around its center is negligible, i.e., that the ball can be considered as a point mass. Under this condition, the only relevant motion of the ball is its *sliding* along the rim. In the same spirit, we ignore the possibility that the ball loses contact with the first link, as well as friction effects.

The dynamic equations are then expressed in the classical form

$$B(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + e(q) = \begin{pmatrix} \tau \\ 0 \end{pmatrix} \qquad (1)$$

where $B(q)$ is the inertia matrix, $C(q,\dot{q})\dot{q}$ is the vector of centrifugal and Coriolis terms, $e(q)$ is the vector of gravitational terms, and τ the torque acting on the first link. In particular, we have

$$B(q) = \begin{pmatrix} J + m\,\delta^2 & -m\,\delta^2 \\ -m\,\delta^2 & m(\delta^2 + \delta'^2) \end{pmatrix}$$

$$C(q,\dot{q}) = m\,\delta' \begin{pmatrix} \delta\,\dot{\varphi} & \delta(\dot{\theta} - 2\dot{\varphi}) \\ -\delta\,\dot{\theta} & \dot{\varphi}(\delta + \delta'') \end{pmatrix}$$

$$e(q) = mg \begin{pmatrix} -\delta\sin(\theta - \varphi) \\ \delta\sin(\theta - \varphi) + \delta'\cos(\theta - \varphi) \end{pmatrix}$$

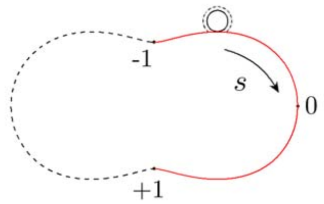

Fig. 2. The parametric description of the shape

Here, J is the centroidal moment of inertia of the rotational link, m is the ball mass, g is the gravity acceleration, and $\delta = \delta(\varphi)$ is the distance between the rotational joint axis and the center of the ball (see Fig. 1); also, we have set

$$\delta' = \frac{\partial\delta(\varphi)}{\partial\varphi} \qquad \delta'' = \frac{\partial^2\delta(\varphi)}{\partial\varphi^2}$$

If the rotational link is symmetrical w.r.t. its center of rotation, the above equations of motion are general; the particular shape of the chosen Butterfly is in fact embedded in the dependence of δ on φ.

Denote by (x_c, y_c) the coordinates of the point of contact between the ball and the link in the rotated frame centered at the joint and having the x axis aligned with the link horizontal axis. The Butterfly robot built in our laboratory is characterized by the following parametric equations

$$\begin{pmatrix} x_c(s) \\ y_c(s) \end{pmatrix} = \begin{pmatrix} 14.9 - 25s^2 + 10.1s^4 \\ -19.9s + 23.3s^3 - 10s^5 \end{pmatrix} \qquad (2)$$

which, for $s \in [-1, 1]$, give the right half of the profile shown in Fig. 2. From (2), neglecting the radius of the ball, one can write

$$\delta = \sqrt{x_c^2 + y_c^2} = \delta(x_c(s), y_c(s)) = \delta(s) \qquad (3)$$

$$\varphi = \mathrm{ATAN2}(y_c, x_c) = \varphi(x_c(s), y_c(s)) = \varphi(s) \qquad (4)$$

In principle, by inverting eq. (4) one could obtain $s = s(\varphi)$, which, substituted into eq. (3), would give the expression of δ as a function of φ.

3. HETEROCLINIC ORBITS

Recall that a *homoclinic point* of a dynamic system is a point where a stable and an unstable separatrix (invariant manifold) relative to the same saddle point intersect (Tabor, 1989). A *homoclinic orbit* is a trajectory connecting a saddle point to itself; this happens when the stable and unstable separatrix join up smoothly. A *heteroclinic orbit* is a trajectory connecting two distinct saddle points. In this section, we identify the particular heteroclinic orbits whose existence is exploited for designing the controller, and we show that they can be characterized by their energy level.

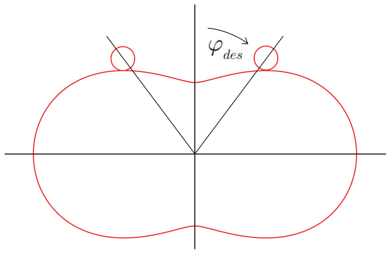

Fig. 3. The desired unstable equilibrium points

Consider the two equilibrium points

$$\begin{pmatrix} q_{des} \\ \dot{q}_{des} \end{pmatrix} = \begin{pmatrix} 0 \\ \pm\varphi_{des} \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

where $\pm\varphi_{des}$ are the symmetric angular positions of the ball shown in Fig. 3. Such equilibria are unstable and in particular they are saddle points.

Denoting by $K(q, \dot{q})$ the kinetic energy and by $U(q)$ the potential energy, the total energy of the system is given by

$$\begin{aligned} E(q, \dot{q}) &= K(q, \dot{q}) + U(q) \\ &= \frac{1}{2}\dot{q}^T B(q)\dot{q} + mg\delta\cos(\theta - \varphi) \end{aligned}$$

The above equilibrium points achieve the maximal potential energy compatible with the rotational link being horizontal. In particular, the energy at these points is

$$E_{des} = E(q_{des}, \dot{q}_{des}) = U(q_{des})$$

A fundamental observation[1] for our control design is that, if the Butterfly link is held horizontal (i.e., if $\theta(t) \equiv 0$), any trajectory starting at a point where $E = E_{des}$ will converge (depending on the sign of the initial $\dot{\varphi}$) to one of the equilibrium points (5) . This is clearly shown by the phase portrait for $\theta(t) \equiv 0$ reported in Fig. 4 (equivalently, we may say that this is the phase portrait for the zero dynamics corresponding to the output θ). Therefore, the heteroclinic orbits of interest are the two non-trivial system trajectories for which

$$\theta(t) \equiv 0 \quad \text{and} \quad E(q, \dot{q}) = E_{des} \tag{6}$$

Note that each heteroclinic orbit tends to one of the two unstable equilibria (see Fig. 4).

[1] This behavior is similar to that of the Pendubot when the first link is kept vertical and the system is initialized at an energy level corresponding to the upward unstable equilibrium (Fantoni *et al.*, 2000). In that case, the resulting trajectories are exactly the homoclinic orbits of a simple pendulum.

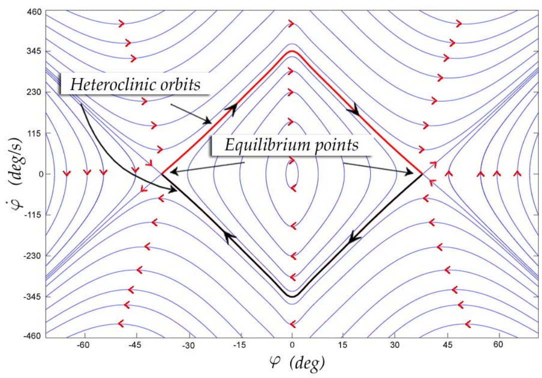

Fig. 4. The Butterfly phase portrait for $\theta(t) \equiv 0$

4. CONTROLLER DESIGN

In the sequel, an energy-based control approach is used to solve a set-point regulation problem. In particular, starting from any initial state, the Butterfly is to be stabilized at one of the two unstable equilibrium points (5).

As common in this kind of problems, a two-phase control approach is used. In the first phase (*swing-up*), the controller brings the system toward the two heteroclinic orbits implicitly defined by (6). Since the controller is energy-based, it does not distinguish between the two trajectories: indeed, the state will alternately approach both the desired equilibria (5) with increasing accuracy. In the second phase (*balancing*), the control switches to a locally stabilizing controller. For example, an LQR controller designed on the linearization of the system about the chosen desired equilibrium point can be used. In the following we focus on the design of the swing-up phase.

In order to bring the system trajectories to the heteroclinic orbits defined by eq. (6), the following positive semi-definite Lyapunov function is considered

$$V(q, \dot{q}) = \frac{1}{2}K_e(\Delta E)^2 + +\frac{1}{2}K_d\dot{\theta}^2 + \frac{1}{2}K_p\theta^2$$

with $\Delta E = E(q, \dot{q}) - E_{des}$ and K_e, K_d, K_p strictly positive constants. The time derivative of V is

$$\dot{V} = K_e\,\Delta E\,\dot{E} + K_d\,\dot{\theta}\,\ddot{\theta} + K_p\,\theta\,\dot{\theta} \tag{7}$$

Using the passivity property of the mechanism from τ to $\dot{\theta}$

$$\dot{E} = \dot{q}^T \begin{pmatrix} \tau \\ 0 \end{pmatrix} = \dot{\theta}\,\tau$$

eq. (7) can be rewritten as

$$\dot{V} = \dot{\theta}\left(K_e\,\Delta E\,\tau + K_d\,\ddot{\theta} + K_p\,\theta\right) \tag{8}$$

Using the dynamic equations (1) for $\ddot{\theta}$ and letting

$$\det B = m\left(J\delta^2 + J\dot{\delta}^2 + m\delta^2\dot{\delta}^2\right)$$

$$h = m^2\delta\delta'\Big[g(\delta' sin(\theta-\varphi) - \delta cos(\theta-\varphi)) + \\ +(\delta^2 + 2\delta'^2 - \delta\delta'')\dot{\varphi}^2 + \delta^2\dot{\theta}^2 - 2(\delta^2+\delta'^2)\dot{\theta}\dot{\varphi}\Big]$$

eq. (8) becomes

$$\dot{V} = \dot{\theta}\left[\left(K_e\Delta E + \frac{K_d b_{22}}{\det B}\right)\tau + K_p\theta + \frac{K_d h}{\det B}\right]$$

where $b_{22} = m(\delta^2 + \delta'^2)$ is the $(2,2)$ element of the inertia matrix B. Choosing the input as

$$\tau = -\frac{\det B}{\det B\, K_e\,\Delta E + K_d b_{22}}\cdot \left(K\dot{\theta} + K_p\theta + \frac{K_d h}{\det B}\right) \quad (9)$$

with $K > 0$, leads to a negative semi-definite $\dot{V}$

$$\dot{V} = -K\dot{\theta}^2 \quad (10)$$

To avoid singularities in the control law (9), it must be

$$\det B\, K_e\,\Delta E + K_d b_{22} \neq 0 \quad (11)$$

Since $\det B$ and b_{22} are positive, eq. (11) is satisfied if

$$|\Delta E| \neq \frac{K_d}{K_e}\frac{b_{22}}{\det B}$$

Defining

$$b_{22_{\min}} = \min_{\varphi} b_{22}(\varphi) \qquad \det B_{\max} = \max_{\varphi} \det B(\varphi)$$

eq. (11) is clearly satisfied if the following inequality is verified for some $\varepsilon > 0$

$$|\Delta E| \leq \frac{K_d}{K_e}\frac{b_{22_{\min}}}{\det B_{\max}} - \varepsilon \quad (12)$$

Under constraint (12), which in turn entails a condition involving the initial state and the gains K_d, K_e, the control law (9) is well defined.

5. STABILITY ANALYSIS

Since $\dot{V}$ is only negative semi-definite, the stability analysis is based upon LaSalle's invariance principle (Khalil, 1996). Being V nonincreasing, θ, $\dot{\theta}$ and $\dot{\varphi}$ are bounded. Moreover, the system trajectories are also bounded and remain inside a compact set Ω that depends on the initial state. Let Γ be the set of points in Ω where $\dot{V} = 0$, and M be the largest invariant set in Γ. LaSalle's invariance principle states that every solution starting in Ω approaches M as $t \to \infty$.

For the Butterfly, we have $\Gamma = \{(q,\dot{q}) \in \Omega : \dot{\theta} = 0\}$ and hence

$$M = \{(q,\dot{q}) \in \Gamma : \dot{\theta} = 0, \ddot{\theta} = 0\}$$

Comparing eqs. (8) and (10) the control torque on M satisfies

$$K_e\,\Delta E\;\tau + K_p\,\theta = 0 \quad (13)$$

Moreover, since on M it is $\dot{E}(q,\dot{q}) = \dot{\theta}\,\tau = 0$, it follows that $E(q,\dot{q}) =$ cost. We can have two different cases, $E(q,\dot{q}) = E_{des}$ and $E(q,\dot{q}) \neq E_{des}$.

If $\Delta E = 0$ (i.e., $E(q,\dot{q}) = E_{des}$), it follows from eq. (13) that $\theta(t) \equiv 0$; therefore, the closed-loop system is evolving along one of the heteroclinic orbits defined by eq. (6).

In the second case we have $E(q,\dot{q}) = E^* \neq E_{des}$. From eq. (13), defining $\Delta E^* = E^* - E_{des}$, one has that $\tau = \tau^* = -k_p\theta^*/K_e\Delta E^*$. This implies that the ball must be also at rest, since otherwise a non-constant torque would be needed to keep the rigid body at rest. Therefore the only additional states which can potentially belong to M, are equilibrium points $(\theta^*\;\;\varphi^*\;\;0\;\;0)^T$ such that

$$e(\theta^*,\varphi^*) = \begin{pmatrix}\tau^*\\ 0\end{pmatrix}$$

or, more explicitly

$$\begin{cases} \dfrac{K_p\theta^*}{K_e\Delta E^*} = mg\delta(\varphi^*)\sin(\theta^* - \varphi^*) \\ \delta(\varphi^*)\sin(\theta^* - \varphi^*) + \delta'(\varphi^*)\cos(\theta^* - \varphi^*) = 0 \end{cases} \quad (14)$$

To find out if these additional equilibria exist, it is necessary to analyze (14). From the second equation, we get

$$\theta^* = \varphi^* + \arctan\left(-\frac{\delta'(\varphi^*)}{\delta(\varphi^*)}\right)$$

which, plugged into the first one, leads to a relationship of the form

$$K_p f_1(\varphi^*) = K_e f_2(\varphi^*) \quad (15)$$

Note that, up to now, the specific form of the profile $\delta = \delta(\varphi)$ has not been used. For our Butterfly prototype, no closed form is available for δ, and therefore the solutions of eq. (15) must be found numerically. Figure 5 shows in red (dashed lines) the family of curves $K_e f_2(\varphi)$ when K_e varies, and in blue (continuous lines) the family of curves $K_p f_1(\varphi)$ when K_p varies.

For any choice of K_p and K_e, as shown in Figure 5, there always exist at least three solutions, corresponding to the desired equilibrium states (5) and the origin. Note also that for any fixed value of K_e

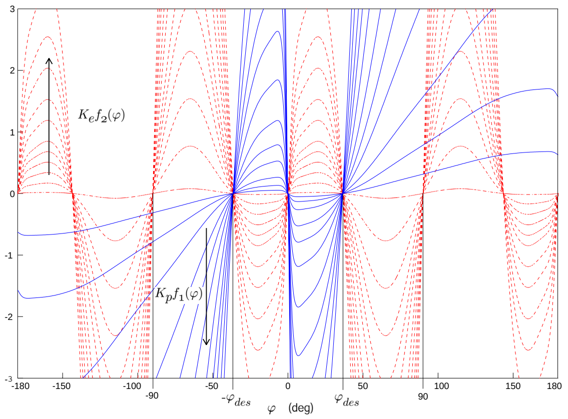

Fig. 5. Equation (15): curves representing the lhs and rhs when K_p and K_e vary.

(K_p) it is always possible to avoid other solutions by choosing K_p high enough (K_e low enough).

It is then necessary to seek conditions under which the equilibrium point at the origin is avoided. To this end, it is sufficient to require that at all time instants

$$|\Delta E| < \overline{\Delta E} = |E_{des} - E_0|$$

where E_0 is the energy of the Butterfly at the origin. Putting this together with the sufficient condition (12) for the control input to be well defined, we conclude that at all time instants ΔE must be such that

$$|\Delta E| < \gamma = \min\left(\overline{\Delta E}, \frac{K_d b_{22_{\min}}}{K_e \det B_{\max}} - \varepsilon\right) \quad (16)$$

Since V is nonincreasing, we can write

$$\frac{1}{2} K_e \, |\Delta E|^2 \le V(t) \le V(0)$$

so that eq. (16) is certainly satisfied provided that

$$V(0) \le K_e \frac{\gamma^2}{2} \quad (17)$$

In conclusion, if the initial conditions are chosen [2] so as to satisfy eq. (17), the closed-loop system trajectories will converge to the set M given by the heteroclinic orbits. In general, the Butterfly will perform growing oscillations, which will lead the system state to alternately approach both the equilibria (5) with increasing accuracy. In the particular case that the initial state belongs to one of the two heteroclinic orbits, no oscillation occurs and the system monotonically approaches the corresponding equilibrium point.

If one of the two equilibria (5) is specifically assigned as set-point, the balancing controller must be designed so as to capture (by proper switching) and stabilize the Butterfly around the chosen point. In view of the above discussion, initial states along the 'wrong' heteroclinic orbit must be avoided (in addition to the origin).

[2] Note that the origin is not an admissible initialization.

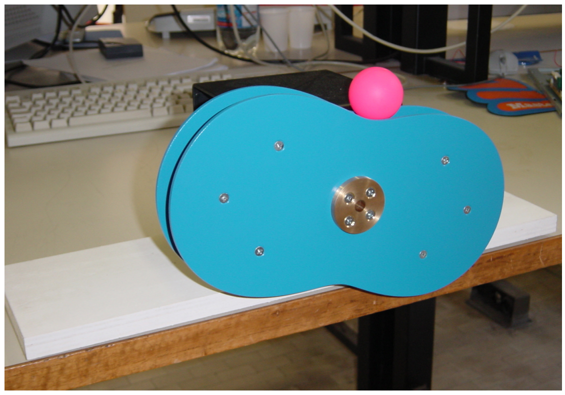

Fig. 6. The Butterfly experimental set-up

We mention that the proof of (Fantoni *et al.*, 2000), that has been closely followed in this paper, has been modified in (Kolesnichenko and Shiriaev, 2002) where less stringent conditions on the controller parameters have been presented. The lack of an explicit dependence of δ from φ prevents the use of such a result in this context.

6. SIMULATION RESULTS

We remind the reader that the model presented in Section 2 is valid only if the contact between the ball and the Butterfly is not lost. The numeric simulator used in our analysis includes a test on the contact conditions.

The system parameters, computed from a CAD model of the prototype built in our lab (see Fig. 6), are $J = 0.853\,10^{-2}$ kg m^2 and $m = 0.15$ kg. The controller gains are $K_p = 1000$, $K_d = 10$, $K_e = 10000$ and $K = 100$. The initial state is $(\,0\;\, 10^\circ\;\, 0\;\, 0\,)^T$; i.e., the Butterfly is horizontal and at rest while the ball starts at $\varphi(0) = 10^\circ$ with zero velocity. The desired set-point is $(\,0\;\, \varphi_{des}\;\, 0\;\, 0\,)^T$.

Figures 7 and 8 show, respectively, the evolution of $(\theta, \dot{\theta})$ and $(\varphi, \dot{\varphi})$. As a result of the stabilization to the energetic level of the heteroclinic orbits, the control input injects energy into the system so as to make the ball oscillate with increasing amplitude, until the trajectory enters the basin of attraction of the linear controller. Note how this is obtained with a very limited oscillation of the first link. Figure 9 shows how, during the swing-up phase, the energy asymptotically converges to the desired value while the Lyapunov function goes to zero.

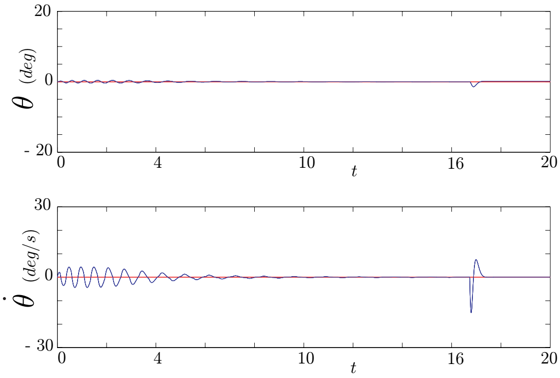

Fig. 7. Simulation: evolution of θ and $\dot{\theta}$

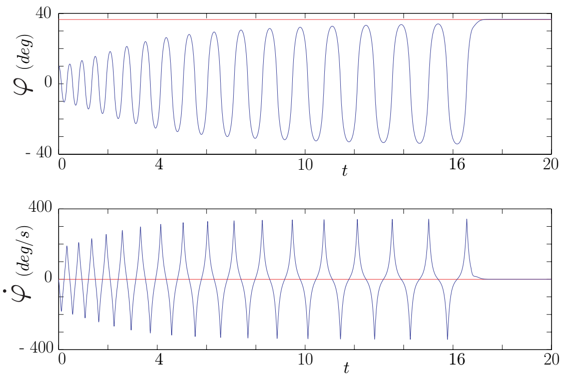

Fig. 8. Simulation: evolution of φ and $\dot{\varphi}$

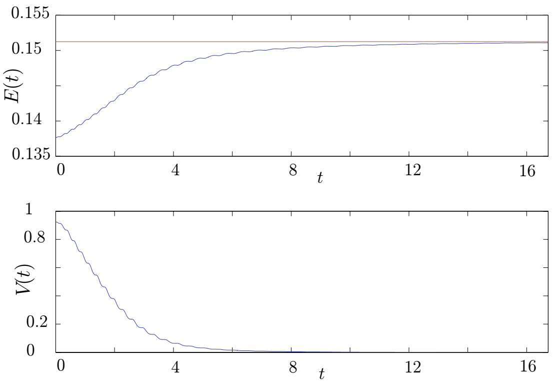

Fig. 9. Simulation: evolution of E and V

At $t = 16.7$ sec the state is sufficiently close to the desired set-point, so that the balancing control kicks in. Note the sudden change in θ at the switching instant, due to particular choice of gains in the LQR design.

7. CONCLUSIONS

We have studied a particular control problem for the Butterfly, an example of 2-dof underactuated mechanical system of interest in nonprehensile manipulation. The specific objective is to stabilize the robot at certain unstable equilibria. Relying on the existence of heteroclinic trajectories converging to such equilibria, an energy-based control technique has been proposed which extend previous results obtained for the Pendubot system . Simulation results have been presented to show the effectiveness of the method. Current work includes the vision-based implementation and experimental validation of our approach on a laboratory prototype.

REFERENCES

Fantoni, I., R. Lozano and M.W. Spong (2000). Energy based control of the Pendubot. *IEEE Transactions on Automatic Control* **45**, 725–729.

Grizzle, J.W., C.H. Moog and C. Chevallereau (2005). Nonlinear control of mechanical systems with an unactuated cyclic variable. *IEEE Transactions on Automatic Control* **50**, 559–576.

Hauser, J., S. Sastry and P. Kokotovic (1992). Nonlinear control via approximate input-output linearization: the ball and beam example. *IEEE Transactions on Automatic Control* **37**, 392–398.

Khalil, H.K. (1996). *Nonlinear Systems*. Prentice Hall.

Kolesnichenko, O. and A.S. Shiriaev (2002). Partial stabilization of underactuated euler-lagrange systems via a class of feedback transformations. *Systems & Control Letters* **45**, 121–132.

Lynch, K.M., N. Shiroma, H. Arai and K. Tanie (1998). The roles of shape and motion in dynamic manipulation: The butterfly example. In: *IEEE Int. Conf. on Robotics and Automation*. pp. 1958–1963.

Ortega, R., M.W. Spong, F. Gomez-Estern and G. Blankenstein (2002). Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment. *IEEE Transactions on Automatic Control* **47**, 1218–1233.

Spong, M.W. (1995). The swing up control problem for the Acrobot. *IEEE Control Systems Magazine* **1**, 49–55.

Spong, M.W. and D.J. Block (1995). The Pendubot: a mechatronic system for control research and education. In: *34th IEEE Conf. on Decision and Control*. pp. 555–556.

Tabor, M. (1989). *Chaos and integrability in nonlinear dynamics*. New York: Wiley.