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REST-TO-REST MOTION FOR PLANAR MULTI-LINK FLEXIBLE MANIPULATOR

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Abstract: In this work we consider the problem of rest-to-rest motion for planar flexible multi-link manipulators. We introduce a simple idea permitting the cancellation of end-effector residual vibration when reaching a desired angular equilibrium position, in a fixed travelling time. No internal elastic damping effect is considered and the structure of the control is simply the sum of feedforward and *joint feedback terms*. The new approach concerns the computation of the feedforward control, which is based on backward integration of the elastic dynamics, starting from a rest position of the flexible arms. For fast rest-to-rest motion, the feedback compensator fails to drive the system states along the desired trajectories, due to the relatively large initial elastic error. To overcome this limitation, proper joint motion is planned between the desired initial and final positions through optimization techniques. This scheme is validated via numerical tests on a two-link planar manipulator. Copyright © 2003 IFAC

Keywords: Backward integration, optimal trajectory planning.

1. INTRODUCTION

For flexible multi-link manipulators, three interesting control problems hold. The first one concerns the point to point motion in fixed time problem, where the final joint position should be reached in a desired finite time both with cancellation of residual tip oscillations (this differs from the regulation problem, where the time motion is not constrained). The second one concerns the tracking of a desired angular trajectory, together with the constraints of eliminating or at least minimizing the residual elastic deflection of the end-effector. And the last problem is more challenging and concerns the tracking of desired end-effector operational trajectories. In this paper we concentrate on the first problem. Many strategies exist for the position control of such systems. Firstly, the regulation problem has been treated by number of authors who have used simplified, linear models, for a one-link flexible arm. For example, in (Meckl, 1994), a method based on control input shaping is presented on spring-mass oscillating systems. In (Chan and Stelson, 1995), the oscillating system of spring-mass is also considered to present optimal shaped trajectories approach. The desired trajectories are shaped such that minimal elastic velocity, acceleration and jerk holds. We also cite (Singer and Seering, 1990), where it is shown that an open-loop strategy based on shaped-input filters is a simple method to reduce the end-effector vibrations and ensure good performance even when the actual system frequencies are imprecisely known. In (Zuo *et al.*, 1995), the authors treat the regulation problem, for the class of multi-link manipulators with one flexible link, using dominant elastic vibration frequencies filters added to simple rigid controllers. The rest-to-rest problem in fixed desired time is considered in (Chalhoub and Ulsoy, 1987), where a *full state* based controller is used. The dynamics of the system are linearized around an equilibrium point and a full state regulator is then computed. Good results are obtained on a two-link manipulator with flexible forearm. However, a full state feedback needs measurement or estimation of the elastic states, which is not straightforward for practical implementation. Finally, we cite the work (De Luca and Di Giovanni, 2001) where is studied the particular case of a two-link robot with a flexible forearm, when considering one elastic vibration mode to model the arm flexibility. The authors searched for a set of two outputs with respect to which the system has no zero dynamics, leading then to a closed-form solution for the rest to rest motion problem.

In this work we present a new solution to the rest-to-rest problem for multi-link flexible planar manipulators. The idea is based on backward integration of the nonlinear elastic dynamics, starting from a desired rest position. Then, the elastic states obtained are used to compute a feedforward control, which is added to simple joint state feedback (ensuring *local* exponential stability of the error dynamics equilibrium point). The whole controller is able to drive the system along the desired states, solving directly the rest-to-rest problem when dealing with relatively slow joints motions. However, for faster joint positioning, the initial desired elastic states obtained through backward integration become large. Thus, the local controller fails to stabilize the system dynamics along the desired states. To overcome this limitation, we propose to shape the joint trajectories, through an optimal planning technique, the goal being the minimization of the initial elastic states. The nice behavior of this approach is illustrated by the numerical simulation of a two-link manipulator.

The paper is organized as follows. In section 2 the proposed scheme is presented, after a short reminder of the dynamic model, for notations purposes. Section 3 is devoted to the numerical results obtained in the two-link case. Finally, conclusions are driven out in section 4.

2. THE CONTROL SCHEME

2.1 *Dynamic model*

We refer to (De Luca and Siciliano, 1991) for the assumed mode modelling approach. The dynamic model is thus obtained through a modal decomposition of the elastic deformation and a lagrangian formulation leading finally to separate equations for the rigid and the elastic parts respectively, that write:

$$\begin{cases} M_{rr}\ddot{\Theta} + M_{re}\ddot{Q} + h_r(\Theta, Q, \dot{\Theta}, \dot{Q}) = u \\ M_{re}^T\ddot{\Theta} + M_{ee}\ddot{Q} + h_e(\Theta, Q, \dot{\Theta}, \dot{Q}) + K_{ee}Q = 0 \end{cases} \tag{1}$$

where $\Theta = (\theta_1, ..., \theta_n)^T$ is the vector of rigid coordinates and $Q = (q_{11}, ..., q_{1m_1}, ..., q_{n1}, ..., q_{nm_n})^T$ the vector of elastic coordinates, consistently with the modal decomposition of the elastic deformation of n links the i^{th} link has m_i basis shape functions) and u is the vector of joint input torques.

2.2 *rest-to-rest motion in fixed time problem statement*

Given a set of desired initial and final angular positions:

$$\begin{aligned} \Theta_d(t_0) = (\theta_{d_1}(t_0) \dots \theta_{d_n}(t_0))^T \\ \Theta_d(t_f) = (\theta_{d_1}(t_f) \dots \theta_{d_n}(t_f))^T \end{aligned} \tag{2}$$

where t_0, t_f are respectively the fixed initial and final motion times, find a control law such that:

$$\begin{cases} \Theta(t_0) = \Theta_d(t_0), \;\; \Theta_d(t) = \Theta_d(t_f), \;\; for\ t \geq t_f \\ Q(t_0) = 0_{n_e\times 1}, \;\; Q(t) = 0_{n_e\times 1}, \;\; for\ t \geq t_f \\ \dot{Q}(t_0) = 0_{n_e\times 1}, \;\; \dot{Q}(t) = 0_{n_e\times 1}, \;\; for\ t \geq t_f \end{cases} \tag{3}$$

2.3 *Proposed solution for slow motion*

The control law used here is basically a feedforward plus feedback term based on the joint states. The main contribution of this work concerns the construction of the feedforward term. Considering a desired set of initial-final angular positions given by equation (2), these set can be interpolated through polynomial trajectories satisfying the following classical velocity and acceleration constraints:

$$\begin{cases} \Theta_d^{(1)}(t_0) = \Theta_d^{(1)}(t_f) = 0_{n\times 1} \\ \Theta_d^{(2)}(t_0) = \Theta_d^{(2)}(t_f) = 0_{n\times 1} \end{cases} \tag{4}$$

Let:

$$\Theta_d(t) = (\theta_{d_1}(t) \dots \theta_{d_n}(t))^T, \;\; t \in [t_0, \; t_f] \tag{5}$$

represents the vector of the obtained joint trajectories. Then, using (5) the feedforward control can be directly obtained from the first equation of (1), and writes: $u_{ff} = M_{rr}^d\ddot{\Theta}_d + M_{re}^d\ddot{Q}_d + h_{rd}(\Theta_d, Q_d, \dot{\Theta}_d, \dot{Q}_d)$ where: $M_{rr}^d = M_{rr}(\Theta_d, Q_d)$, $M_{re}^d = M_{re}(\Theta_d, Q_d)$ and Q_d represents the desired elastic vector associated to the desired joint trajectory and satisfies the following equation:

$$(M_{re}^d)^T\ddot{\Theta}_d + M_{ee}^d\ddot{Q}_d + h_{ed}(\Theta_d, Q_d, \dot{\Theta}_d, \dot{Q}_d) + K_{ee}Q_d = 0 \tag{6}$$

with, $M_{ee}^d = M_{ee}(\Theta_d, Q_d)$. The idea we introduce here is the following: since the goal of control is joint motion between two equilibrium points set with elimination of the elastic vibration at the end of the desired motion, we propose to proceed through backward integration of the elastic dynamics (6), starting from the desired rest configuration, namely: $Q_d(t_f) = 0_{n_e \times 1}$, $\dot{Q}_d(t_f) = 0_{n_e \times 1}$. This way, if we are able to drive the system dynamics along these nominal elastic trajectories, we can force the arm to reach the final rest position at the desired final time. The corresponding final condition differential equation system is then solved to obtain the desired elastic trajectory $Q_d(t)$. It is expected then that $Q_d(t)$ verifies a non zero initial condition $Q_d(t_0)$, $\dot{Q}_d(t_0)$, leading then to initial state error. However, the actual trajectory will be driven to reach the computed desired one, using an angular feedback regulator. The final control writes as $u_{cl} = u_{ff} + \tilde{K}(t)e(t)$, where: $e(t) = (\Theta_d(t) - \Theta(t), \dot{\Theta}_d(t) - \dot{\Theta}(t))^T$ and $\tilde{K}(t)$ is a time dependent diagonal gain matrix, which is computed such as to ensure local stability of the linearized system along the desired state trajectories (this is possible due to the local controllability property of planar flexible arms (Lopez-Linares *et al.*, 1994)). When this holds and under sufficiently slow motion, local uniform exponential stability is ensured for the nonlinear system along the desired states (Khalil, 1996). The solution proposed above is appropriate when dealing with slow joint motions. The problem remains for fast motion. In fact, in this case the system vibrations are expected to be more important and thus the elastic position and velocity $Q_d(t_0)$, $\dot{Q}_d(t_0)$ to have large values, since the joint feedback ensures only local stability of the error dynamics equilibrium point, the actual system states could then drift off from the desired trajectory. To overcome this problem we propose to search for a proper angular trajectory such that it minimizes the initial elastic position and velocity $Q_d(t_0)$, $\dot{Q}_d(t_0)$.

2.4 *Optimal joint trajectory generation for fast rest-to-rest motion*

We are searching for joint trajectories satisfying the constraints (2),(4) and minimizing the cost J:

$$J = \frac{1}{2}(Q_d(t_0)^T.K_1.Q_d(t_0) + \dot{Q}_d(t_0)^T.K_2.\dot{Q}_d(t_0)) \tag{7}$$

Where K_1, K_2 are two positive definite matrices. To solve this problem we define a general polynomial form for the joint trajectories

$$\theta_{d_k}(t) = \sum_{i=0}^{i=6} a_i.(t/t_f)^i + \sum_{j=1}^{j=m} b_j.(t/t_f)^{(j+6)} \tag{8}$$
$$k \in \{1...n\},\ m \in N^*$$

The $\theta_{dk}(t)$ can be expressed as function of the (b_i) coefficents by substituting equation (8) into (2),(4) and solving for the (a_i) vector. Using the obtained joint trajectories $\theta_{d_k}(t, B)$ (where $B = (b_1 \dots b_m)$), together with equation (6) we can formulate the optimization problem presented above as a Pontryagin optimal control problem (Benosman, 2002), leading to a direct solution existence analysis. When the optimal trajectories have been obtained, one can proceed through backward integration as explained in section (2.3). Since the trajectories have been designed such as to minimize the initial elastic states, a simple joint feedback will be enough to drive the system from the desired point to the goal through the optimal trajectories.

In the following section we present the numerical results obtained for a two-link flexible arm.

Property	First Link	Second Link
Length. L (m)	1.005	0.52
Mass density. ρ (Kg/m)	2.032	0.706
Rigidity. EI (Nm^2)	47.25	1.749
Tip mass. m_{tip} (Kg)	7.66	0.51
Tip inertia. J_{tip} ($Kg.m^2$)	171e-3	623e-7
Hub inertia. J_h ($Kg.m^2$)	1.8e-3	225e-6
Arm thickness. d (mm)	3	1

Table 1. Mechanical properties of the arm

3. SIMULATION RESULTS

The arm is characterized by mechanical properties summarized in table 1. The dynamical model has been computed using two clamped modes for each arm. First we start by testing the proposed approach for relatively slow motion. We have chosen as desired angular equilibrium points $\Theta_0 = (0)_{(2,1)}rad$, $\Theta_f = (\pi/2, \pi/2)^T rad.$. To begin with, we test a 'slow' trajectory, fixing $t_f = 5s$. We pursue by testing a faster motion corresponding to the critical value $t_f = 3s$.

The procedure described in section 2.3 has been run to compute the elastic trajectories associated to the joint desired motion. Fig. 1 -Fig. 4 display the joint trajectories tracking obtained for a nominal simulation plan (without introducing elastic damping coefficients in the simulation model). It is clear that the angular final position is reached with small error. The tip elastic displacements are displayed in Fig. 5 and 6, where one can see that these deflection are damped out at the end of the desired rigid motion. Even if, the tip oscillations are not exactly zeroing at the the final instant, there residual amplitude are quite negligible.

The corresponding smooth closed-loop torques are given in Fig. 7 and Fig. 8. Fig. 9, displays the obtained time varying gain. It is clear that the local stability does not require high gain values, which is quite important for practical implementation. Consider now a faster motion case, namely the travelling time $t_f = 3s$ (with the same five order

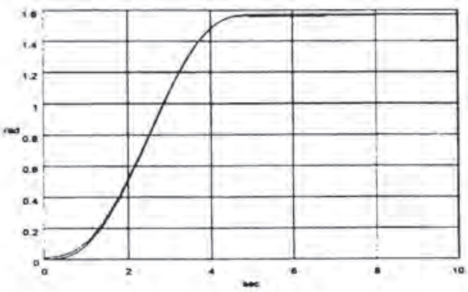

Fig. 1. First joint motion, desired (continuous line)and actual (dashed line)motion, nominal plan

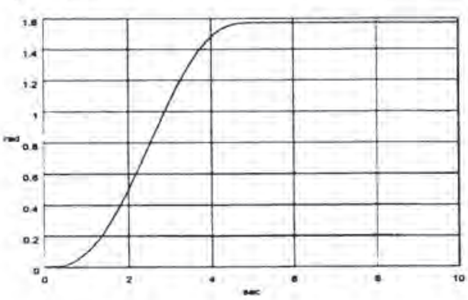

Fig. 2. Second joint motion, desired (continuous line)and actual (dashed line)motion, nominal plan

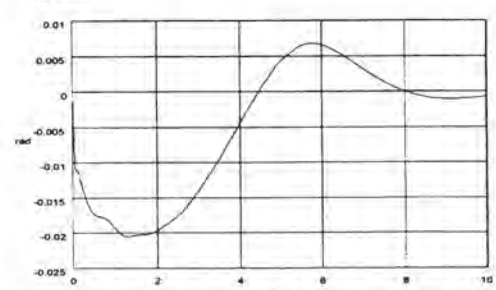

Fig. 3. First joint tracking error, nominal plan

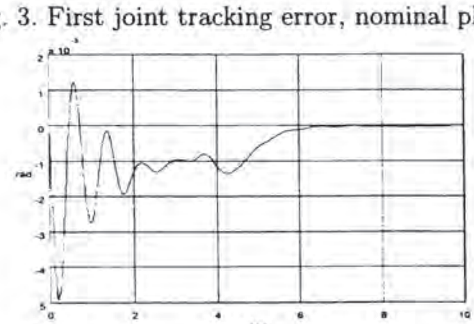

Fig. 4. Second joint tracking error, nominal plan

polynomial joint trajectories). To display the interest (and the necessity) of the optimal trajectory shaping, introduced in section 2.4, we first report the obtained results associated to a direct application of the backward integration approach (without trajectory planning). Fig. 10 shows the first angular motion. As expected (see section 2) the joint constraints of the rest-to-rest problem (3) is not achieved with good precision. Since the first

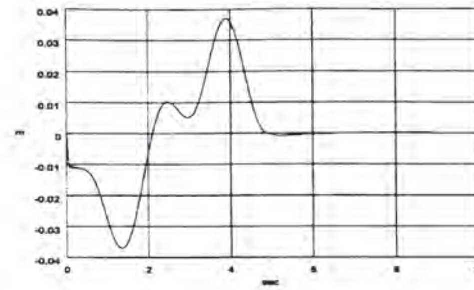

Fig. 5. First arm tip-deflection, nominal plan

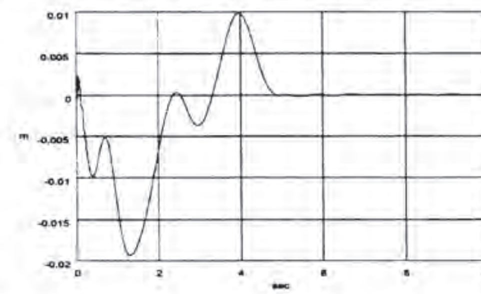

Fig. 6. Second arm tip-deflection, nominal plan

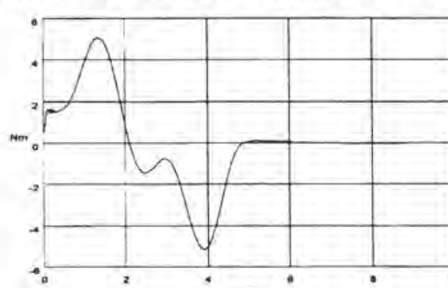

Fig. 7. First closed-loop torque, nominal plan

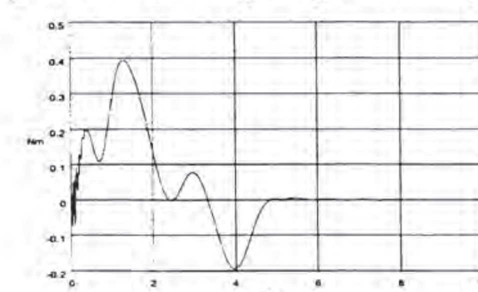

Fig. 8. Second closed-loop torque, nominal plan

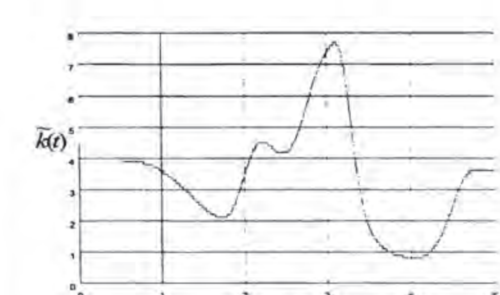

Fig. 9. Time scheduling gain

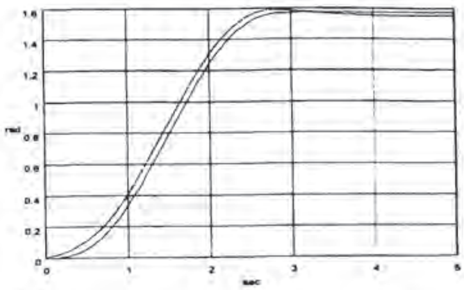

Fig. 10. Desired and actual (dashed line)motion

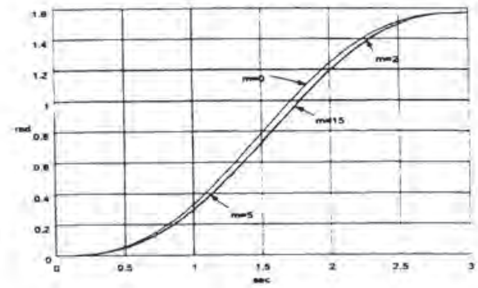

Fig. 11. Optimal trajectories for different m values

constraint of the control problem has not been satisfied we do not display the other results. We turn then to the application of the optimal planning scheme presented in section 2.4 and search for an optimal joint vector trajectories solving the problem defined by equation (7). To display the effect of the vector B dimension on the optimal trajectories, we have compared the trajectories obtained for different values. It is clear from Fig. 11 that changing the number of optimization parameter, does not affect significantly the optimal trajectory obtained. We have kept then the dimension $m = 5$ (which seems to yield a lower optimization time). The optimal problem has been solved using the Nelder-Mead simplex method, a nonlinear unconstrained minimization code, based on a direct search method. The obtained simulation results on the nominal plan are given in Fig. 12 to 19. Fig. 12 and 13 display the obtained joint motions. The small tracking errors are displayed through Fig. 14 and Fig.15. In fig. 16 and 17, we can see that the elastic displacement of the end-effector damped out at the end of the motion. The corresponding closed-loop control torques and varying gain are given in Fig.18, Fig.19 and Fig.20. The maximum torques values are clearly (and logically) larger than those obtained for the first motion. This can be avoided through a slight modification of the optimization cost, by introducing a time criterion and searching for a time optimal trajectories satisfying the maximum control values constraints. But this is not our goal here, and we report this for future works. Also, we refer the reader to (Benosman, 2002), for nice experimental results, obtained on a two-link flexible arm testbed.

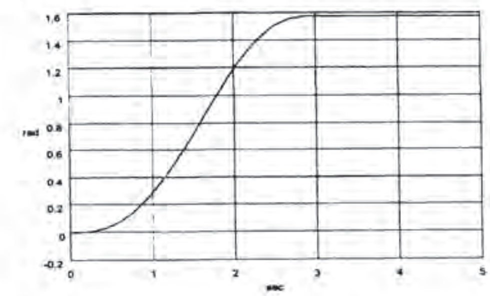

Fig. 12. First joint motion, desired (continuous line)and actual (dashed line)motion, nominal plan

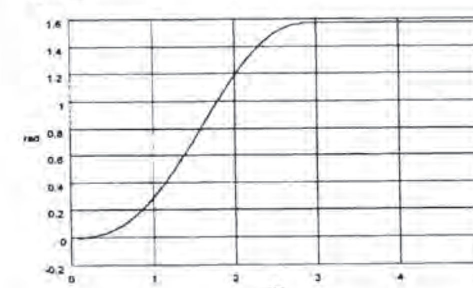

Fig. 13. Second joint motion, desired (continuous line)and actual (dashed line)motion, nominal plan

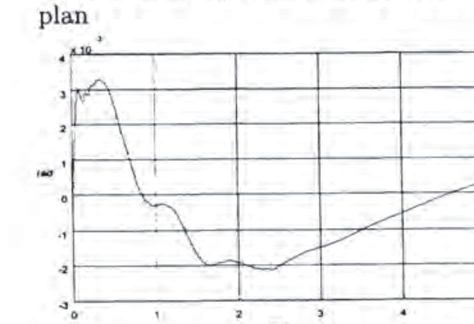

Fig. 14. First joint motion error, nominal plan

4. CONCLUSION

In this work we have proposed a simple idea to solve the problem of rest-to-rest motion for multi-link planar flexible manipulators. This problem amounts to achieve joint motion between two equilibrium points together with cancellation of the tips oscillations at the end of the desired joint motions. The solution we propose here achieves these two objectives and this, without considering any internal elastic damping in the simulation arm model. This scheme is based on backward integration of the elastic dynamics along the joint desired trajectory, starting from the desired rest position. Furthermore, *the unique required measurements (for feedback purposes) are the angular states.* The proposed scheme permits to directly achieve these goals, when dealing with slow motions. However, for fast joint trajectories, the direct application of the method did not show to be efficient. We have then proposed an op-

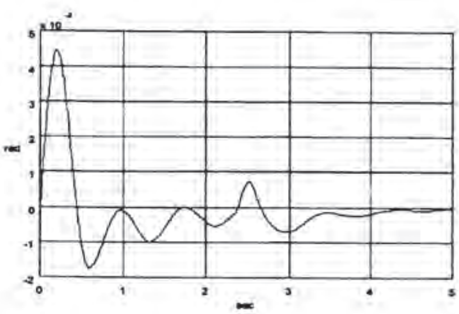

Fig. 15. Second joint motion error, nominal plan

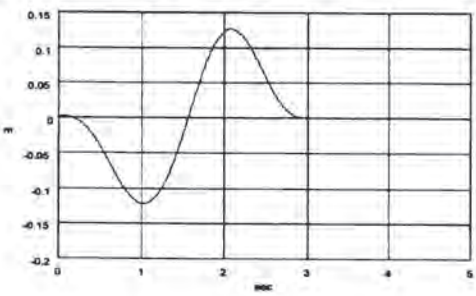

Fig. 16. First link tip deflection, nominal plan

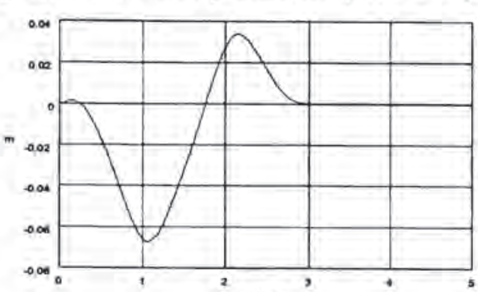

Fig. 17. Second link tip deflection, nominal plan

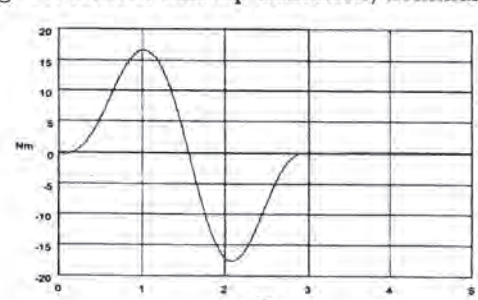

Fig. 18. First closed-loop torque, nominal plan

timal trajectory planning to overcome this limitation. The optimization planning statement has been formulated as a classical Pontryagin optimal control problem, yielding direct optimal solution existence. Numerical tests on a two-link flexible manipulator have shown the good behavior of the proposed ideas.

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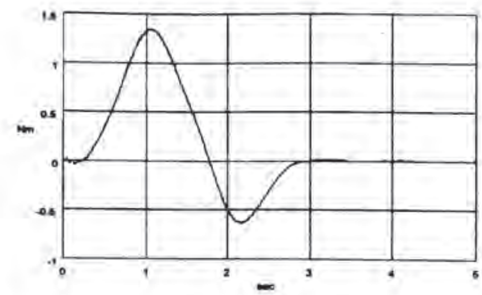

Fig. 19. Second closed-loop torque, nominal plan

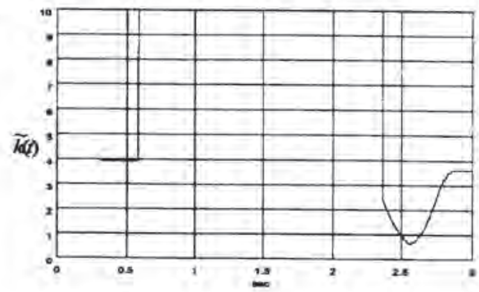

Fig. 20. Time scheduling gain

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