

A Swarm Aggregation Algorithm based on Local Interaction for Multi-Robot Systems with Actuator Saturations

Andrea Gasparri¹, Giuseppe Oriolo², Attilio Priolo¹ and Giovanni Ulivi¹

Abstract— We propose a swarm aggregation algorithm based on local interactions in the presence of saturations on the robot actuators. This assumption allows to better model the physical limitations of actual mobile robotic platforms. In our framework, robot-to-robot interactions are limited to the visibility neighborhood, i.e., to robots that are within the range of visibility of each other. A theoretical analysis of the convergence properties is presented for the proposed swarm aggregation algorithm. Extensive simulations have been performed to corroborate the theoretical results. In addition, experiments with a team of low-cost mobile robots have been carried out to show the effectiveness of the proposed approach.

I. INTRODUCTION

The coordination problem in multi-robot systems has been widely investigated in the last decade, e.g., see [1], [2], [3]. In particular, swarm robotics focuses on systems consisting of a large number of units, in which a collective behavior is expected to arise from the local interaction among robots and between the robots and the environment. The interests in this research field is motivated by the number of possible applications, ranging from exploration tasks [4], [5], to search and rescue operations [6], [7], environmental monitoring [8] and agricultural foraging tasks [9], [10]. The main advantages of robotic swarms are the increased robustness and flexibility achieved by using many simple units rather than a single complex robot. For an overview of swarm robotics, see [11].

Several aggregation algorithms can be found in the literature [12], [13], [14], [15]. In [12] a decentralized continuous-time model for finite-time swarm aggregation is proposed, also providing bounds on the swarm size. In [13], a robust control strategy based on artificial potential functions and sliding mode control is designed. An adaptive velocity swarm model which extends the well-known Vicsek model is proposed in [14]. In [15], a decentralized formation control scheme is presented that guarantees collision-free motion for a team of rovers which includes a leader.

Some works consider the presence of actuator saturations, an inevitable limitation of actual mobile platforms. In [16], a behavior-based approach to formation maneuvers for groups

of mobile robots is proposed that works under this assumption. In [17], a flocking algorithm is proposed for a multi-agent system with bounded control inputs; the agents are able to achieve all the same velocity under the assumption of the connectivity of the underlying communication graph. A formation control scheme for multiple unicycle systems with saturated inputs is described in [18].

In this work we propose a novel swarm control algorithm, which represents an extension of the aggregation methods propose in [1]. In particular, the local interaction among neighboring robots and the presence of actuator saturations are explicitly considered. A detailed theoretical analysis of the resulting aggregation dynamics is carried out. Both simulation and experimental results are provided to corroborate the theoretical findings.

The paper is organized as follows. In Section II, some basic results concerning graph theory for multi-agent systems are reviewed. Section III describes and analyzes in detail the proposed swarm aggregation method, providing convergence results and discussing the integration of a mechanism for avoiding environment obstacles. In Section IV we present the results of simulations performed in Player/Stage and experiments on a small team of mobile robots.

II. THEORETICAL BACKGROUND

Consider a swarm of n mobile robots whose interaction is represented by an undirected time-varying *proximity graph* $\mathcal{G}(t) = \{V, E(t)\}$ where $V = \{1, \dots, n\}$ are the vertices (robots) and $E(t) = \{e_{ij}(t)\}$ the edges. An edge $e_{ij}(t)$ exists between two robots i and j if they are within their range of visibility; i.e., we have $e_{ij}(t) = 1$ if $\|x_i(t) - x_j(t)\| \leq r$, and $e_{ij}(t) = 0$ otherwise, where $x_i(t) \in \mathbb{R}^d$ is the location of the i -th agent and $\|\cdot\|$ is the Euclidean norm.

Let $\mathcal{A}(\mathcal{G}(t))$ the $n \times n$ be *adjacency matrix* of $\mathcal{G}(t)$, with entries $a_{ij}(t) = 1$ if $e_{ij}(t) = 1$ with $i \neq j$, $a_{ij}(t) = 0$ otherwise. Denoting by $\mathcal{N}_i(t) = \{j \in V \setminus \{i\} : e_{ij}(t) = 1\}$ the set of *neighboring robots* for the i -th robot, define the $n \times n$ *degree matrix* of $\mathcal{G}(t)$ as $\mathcal{D}(\mathcal{G}(t)) = \text{diag}(D_1(t), \dots, D_n(t))$, where $D_i(t) = |\mathcal{N}_i(t)|$ is the i -th *agent degree*. Finally, define $\mathcal{L}(\mathcal{G}(t)) = \mathcal{D}(\mathcal{G}(t)) - \mathcal{A}(\mathcal{G}(t))$ as the $n \times n$ *Laplacian matrix* of $\mathcal{G}(t)$. In the following, the short notations $\mathcal{L}(t)$, $\mathcal{A}(t)$, $\mathcal{D}(t)$ are used.

Some properties of the Laplacian matrix \mathcal{L} of an undirected graph are now reviewed. First, \mathcal{L} is a weakly diagonal dominant symmetric matrix, i.e., the row sum and the column sum are both equal to zero. Thus, there is always at least a zero structural eigenvalue whose corresponding eigenvector is $\mathbf{1}$, the n -vector with all unit components. This implies

¹A. Gasparri, A. Priolo and G. Ulivi are with the Department of Computer Science and Automation, University of “Roma Tre”, Via della Vasca Navale, 79, Roma, 00146, Italy. Email: {gasparri, priolo, ulivi}@dia.uniroma3.it

²G. Oriolo is with the Dipartimento di Ingegneria Informatica, Automatica e Gestionale, Università di Roma La Sapienza, via Ariosto 25, 00185 Roma, Italy. Email: oriolo@dis.uniroma1.it

This work was partially supported by the Italian grant FIRB “Futuro in Ricerca”, project NECTAR “Networked Collaborative Team of Autonomous Robots”, code RBFR08QWUV, funded by the Italian Ministry of Research and Education (MIUR).

that $\mathcal{L}(\mathcal{G})\mathbf{1} = \mathbf{0}$ and $\mathbf{1}^T \mathcal{L}(\mathcal{G}) = \mathbf{0}^T$ for any \mathcal{G} . The number of zero eigenvalues corresponds to the number of connected components of \mathcal{G} and therefore $\text{Rank}(\mathcal{L}(\mathcal{G})) = n - c$, where c is the number of connected components of \mathcal{G} [19]. All the eigenvalues of the Laplacian are real and positive, and in particular belong to $[0, 2D_{max}(\mathcal{G})]$, where $D_{max}(\mathcal{G}) = \max_{i \in V} \{D_i(\mathcal{G})\}$ is the maximum degree among the nodes in the graph, as can be proved by applying the Gershgorin disc theorem [20]. Finally, its second smallest eigenvalue λ_2 , called *algebraic connectivity*, provides information about the connectedness of the graph. See [21] for further details.

III. SWARM AGGREGATION ALGORITHM

We first briefly review the swarm aggregation dynamics proposed in [1], which we took inspiration from.

A. Fully-Connected Swarm Aggregation Dynamics

Consider a swarm of n robots whose interaction is encoded by a *fully connected* proximity graph \mathcal{G} , and assume the following dynamics for the generic i -th robot:

$$\dot{x}_i = \sum_{j \neq i} g(x_i - x_j), \quad (1)$$

with the *interaction function* $g(\cdot)$ defined as

$$g(y) = -y (g_a(\|y\|) - g_r(\|y\|)), \quad y \in \mathbb{R}^d, \quad (2)$$

where $g_a(\cdot)$ and $g_r(\cdot)$ are the attractive and repulsive functions, respectively. The actual attraction and repulsion vectors generated by these functions are modulated by the distance between the two interacting agents and directed along the line connecting them, but in opposite directions. By construction the interaction function is odd, i.e., $g(y) = -g(-y)$.

The following assumptions are taken on g_a and g_r .

Assumption 1: There exists a unique distance δ at which $g_a(\delta) = g_r(\delta)$. Moreover, it is $g_a(\|y\|) \geq g_r(\|y\|)$ for $\|y\| \geq \delta$ and $g_r(\|y\|) \geq g_a(\|y\|)$ for $\|y\| < \delta$.

Assumption 2: There exist functions $J_a(\|y\|) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ and $J_r(\|y\|) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that $\nabla_y J_a(\|y\|) = y g_a(\|y\|)$ and $\nabla_y J_r(\|y\|) = y g_r(\|y\|)$.

Let $\bar{x}(t) = \frac{1}{n} \sum_{i=1}^n x_i(t)$ be the barycenter of the swarm. The main properties of model (1) are [1]:

- P1** The barycenter \bar{x} of the swarm is stationary over time.
- P2** The swarm converges to an equilibrium state.
- P3** The swarm converges to a bounded region.
- P4** The swarm reaches the bounded region in finite time.

B. Swarm Aggregation Dynamics based on Local Interaction under Actuator Saturations

Let us now go back to the general case of in Section II, in which the proximity graph $\mathcal{G}(t)$ is time-varying. Assume the following dynamics for each agent i :

$$\dot{x}_i = k \frac{\sum_{j \in \mathcal{N}_i(t)} g(x_i - x_j)}{1 + \left\| \sum_{j \in \mathcal{N}_i(t)} g(x_i - x_j) \right\|}, \quad (3)$$

where $k \geq 0$ is the saturated gain and the interaction function still given by (2), under Assumptions 1 and 2. Compared to

the original model (1), the aggregation dynamics (3) has the following characteristics:

- Robots only interact within the visibility range.
- Actuator saturations are considered.

A consequence of these is the loss of property P1, i.e., the swarm barycenter is no longer stationary. This is due to the fact that, while the mutual effects of interacting robots are always symmetric in the model (1), this is not true under limited visibility if saturation occurs. Nevertheless, it will be shown that model (3) still exhibits properties P2–P4.

In the rest of the paper, we will take the following assumption.

Assumption 3: Graph $\mathcal{G}(t)$ remains connected at all times. In particular, it is $\lambda_2(\mathcal{L}_{h_a, \mathcal{G}}(t)) \geq \lambda_{2, \min} > 0$, with $\mathcal{L}_{h_a, \mathcal{G}}(t)$ the weighted Laplacian matrix related to the attractive term $h_a(\cdot) = g_a(\cdot)f(x_i)$, where $0 < f(x_i) = \frac{1}{1 + \left\| \sum_{j \in \mathcal{N}_i(t)} g(\epsilon_i - \epsilon_j) \right\|} \leq 1$.

In the following, denote by $\epsilon_i(t) = x_i(t) - \bar{x}(t)$ the vector distance of agent i from the barycenter $\bar{x}(t)$. Also, denote by $\chi(t) = [x_1(t) \dots x_n(t)]^T$ and $\epsilon(t) = [\epsilon_1(t) \dots \epsilon_n(t)]^T$, respectively, the collection of all the agents' locations and distances from the barycenter. We have the following result.

Theorem 1: Consider a swarm of robots whose dynamics is described by (3). Then the swarm converges to an equilibrium state for any initial condition.

Proof: Define $J(\|y\|) = J_a(\|y\|) - J_r(\|y\|)$, with $J_a(\cdot)$ and $J_r(\cdot)$ as in Assumption 2. Consider the following (generalized) Lyapunov candidate:

$$V(t) = \frac{1}{2} \sum_{(i,j) \in E(t)} J(\|x_i(t) - x_j(t)\|)$$

whose time derivative is

$$\dot{V}(t) = \sum_{i=1}^n (\nabla_{x_i} V(t))^T \dot{x}_i(t). \quad (4)$$

By construction, we have

$$\nabla_{x_i} V(t) = -f(x_i) \dot{x}_i(t) \quad (5)$$

where $f(x_i) = 1 + \left\| \sum_{j \in \mathcal{N}_i(t)} g(x_i(t) - x_j(t)) \right\|$. Therefore, substituting (5) in (4), we obtain:

$$\dot{V}(t) = - \sum_{i=1}^n f(x_i) \|\dot{x}_i\|^2 \leq 0.$$

Hence, using LaSalle's Invariance Principle, it follows that as $t \rightarrow \infty$ the state $\chi(t)$ converges towards the largest invariant subset of the set where $\dot{V}(t) = 0$, that is:

$$\Omega_e = \{\chi : \dot{\chi}(t) = 0\}.$$

Since Ω_e is made of equilibrium points, the thesis follows. ■

We now prove that the agents converge towards a bounded region. Indeed, this result is essential as the computation in closed form of the equilibrium points in Theorem 1 is

impractical. To this end, we add the following requirement (*bounded repulsion*):

Assumption 4: The norm of the total repulsive vector is bounded:

$$\|y\|g_r(\|y\|) \leq \beta. \quad (6)$$

Theorem 2: Consider a swarm of robots whose dynamics is described by (3). Then the swarm converges to the following bounded region:

$$\mathcal{B}_r = \left\{ x \in \mathbb{R}^d : \|x(t) - \bar{x}(t)\| \leq \frac{\beta(n-1)}{\lambda_{2,\min}} \right\}.$$

Proof: Consider the following Lyapunov candidate:

$$V(t) = \frac{1}{2} \sum_{i=1}^n \epsilon_i(t)^T \epsilon_i(t). \quad (7)$$

Its time derivative is (the bounds of the summation are omitted for compactness):

$$\dot{V}(t) = \sum_i \epsilon_i^T \dot{\epsilon}_i = \sum_i \epsilon_i^T (\dot{x}_i - \dot{\bar{x}}) = \underbrace{\sum_i \epsilon_i^T \dot{x}_i}_{\dot{V}_1} - \underbrace{\sum_i \epsilon_i^T \dot{\bar{x}}}_{\dot{V}_2}.$$

Let us first analyze the second term \dot{V}_2 :

$$\dot{V}_2 = \sum_i \epsilon_i^T \dot{\bar{x}} = \left(\sum_i \epsilon_i^T \right) \dot{\bar{x}} = \mathbf{0}^T \dot{\bar{x}} = 0. \quad (8)$$

This means that the motion of the barycenter does not affect the size of the region where the swarm is going to aggregate; indeed, it only affects the location of this region.

We now investigate the term \dot{V}_1 . Let $\bar{g}(\|\epsilon_i - \epsilon_j\|) = g_a(\|\epsilon_i - \epsilon_j\|) - g_r(\|\epsilon_i - \epsilon_j\|)$ and, without loss of generality, omit the constant k in (3). We have

$$\begin{aligned} \dot{V}_1 &= \sum_i \epsilon_i^T \frac{\sum_{j \in \mathcal{N}_i(t)} -(x_i - x_j) \bar{g}(\|x_i - x_j\|)}{1 + \|\sum_{j \in \mathcal{N}_i(t)} -(x_i - x_j) \bar{g}(\|x_i - x_j\|)\|} \\ &= \sum_i \epsilon_i^T \frac{\sum_{j \in \mathcal{N}_i(t)} -(\epsilon_i - \epsilon_j) \bar{g}(\|\epsilon_i - \epsilon_j\|)}{1 + \|\sum_{j \in \mathcal{N}_i(t)} -(\epsilon_i - \epsilon_j) \bar{g}(\|\epsilon_i - \epsilon_j\|)\|} \\ &= \sum_i f(x_i) \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} -(\epsilon_i - \epsilon_j) \bar{g}(\|\epsilon_i - \epsilon_j\|). \quad (9) \end{aligned}$$

Therefore, eq. (9) can be rewritten as follows:

$$\begin{aligned} \dot{V}_1 &= \sum_i f(x_i) \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} -(\epsilon_i - \epsilon_j) \bar{g}(\|\epsilon_i - \epsilon_j\|) \\ &= \sum_i f(x_i) \left(-\epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) g_a(\|\epsilon_i - \epsilon_j\|) \right. \\ &\quad \left. + \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) g_r(\|\epsilon_i - \epsilon_j\|) \right) \\ &= \sum_i \left(-\epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_a(\|\epsilon_i - \epsilon_j\|) + \right. \\ &\quad \left. + \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_r(\|\epsilon_i - \epsilon_j\|) \right) \end{aligned}$$

or

$$\begin{aligned} \dot{V}_1 &= - \underbrace{\sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_a(\|\epsilon_i - \epsilon_j\|)}_{\dot{V}_a} \\ &\quad + \underbrace{\sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_r(\|\epsilon_i - \epsilon_j\|)}_{\dot{V}_r} \end{aligned}$$

where $h_a(\|\epsilon_i - \epsilon_j\|) = g_a(\|\epsilon_i - \epsilon_j\|)f(x_i)$ and $h_r(\|\epsilon_i - \epsilon_j\|) = g_r(\|\epsilon_i - \epsilon_j\|)f(x_i)$ are the attractive and repulsive terms, respectively.

We now further investigate the term \dot{V}_a . We have

$$\begin{aligned} \dot{V}_a &= - \sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_a(\|\epsilon_i - \epsilon_j\|) \\ &= -\epsilon^T \mathcal{L}_{h_a, \mathcal{G}}^d(t) \epsilon \leq -\lambda_2(\mathcal{L}_{h_a, \mathcal{G}}(t)) \|\epsilon\|^2 \quad (10) \end{aligned}$$

where: $\mathcal{L}_{h_a, \mathcal{G}}^d(t) = \mathcal{L}_{h_a, \mathcal{G}}(t) \otimes I_d$ with \otimes the Kronecker product and $\mathcal{L}_{h_a, \mathcal{G}}(t)$ the weighted Laplacian matrix related to the attractive term $h_a(\cdot)$:

$$\mathcal{L}_{h_a, \mathcal{G}}(t) = \mathcal{D}_{h_a, \mathcal{G}}(t) - \mathcal{A}_{h_a, \mathcal{G}}(t). \quad (11)$$

Here, $\mathcal{A}_{h_a, \mathcal{G}}(t)$ is the related adjacency matrix whose elements $\mathcal{A}_{h_a, \mathcal{G}}^{ij}(t)$ are defined as follows:

$$\mathcal{A}_{h_a, \mathcal{G}}^{ij}(t) = \begin{cases} h_a(\|\epsilon_i - \epsilon_j\|) & \text{if } (i, j) \in E(t), \\ 0 & \text{otherwise} \end{cases}$$

and $\mathcal{D}_{h_a, \mathcal{G}}(t)$ is the related degree matrix whose elements $\mathcal{D}_{h_a, \mathcal{G}}^{ij}(t)$ are defined as follows:

$$\mathcal{D}_{h_a, \mathcal{G}}^{ij}(t) = \begin{cases} \sum_{j \in \mathcal{N}_i(t)} h_a(\|\epsilon_i - \epsilon_j\|) & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Note that, although the network topology encoding the agent-to-agent interaction is described by a time varying undirected graph $\mathcal{G}(t)$, the Laplacian matrix $\mathcal{L}_{h_a, \mathcal{G}}(t)$ related to the attractive term is not doubly-stochastic but just row-stochastic as $f(x_i) \neq f(x_j)$ in general due to the saturations.

Recall that:

$$\|x_i - x_j\| g_r(\|x_i - x_j\|) \leq \beta.$$

or equivalently:

$$\|\epsilon_i - \epsilon_j\| g_r(\|\epsilon_i - \epsilon_j\|) \leq \beta.$$

Therefore, the following holds:

$$\begin{aligned} \dot{V}_r &= \sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_r(\|\epsilon_i - \epsilon_j\|) \\ &\leq \sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) f(x_i) \frac{\beta}{\|\epsilon_i - \epsilon_j\|} \\ &\leq \sum_i \|\epsilon_i\| \sum_{j \in \mathcal{N}_i(t)} \|\epsilon_i - \epsilon_j\| f(x_i) \frac{\beta}{\|\epsilon_i - \epsilon_j\|} \\ &\leq \sum_i \|\epsilon_i\| f(x_i) \beta |\mathcal{N}_i(t)| \leq \sum_i \|\epsilon_i\| f(x_i) \beta (n-1) \\ &\leq \sum_i \|\epsilon_i\| \beta (n-1) \leq \beta (n-1) \sqrt{n} \|\epsilon\|. \quad (12) \end{aligned}$$

Now combine eqs. (10) and (12) to get:

$$\begin{aligned} \dot{V}_1 &= \dot{V}_a + \dot{V}_r \leq -\lambda_2(\mathcal{L}_{h_a, \mathcal{G}(t)}) \|\epsilon\|^2 + \beta(n-1)\sqrt{n}\|\epsilon\| \\ &\leq \|\epsilon\| \left(-\lambda_2(\mathcal{L}_{h_a, \mathcal{G}(t)}) \|\epsilon\| + \beta(n-1)\sqrt{n} \right) \end{aligned} \quad (13)$$

which is negative definite if

$$\|\epsilon\| \geq \frac{\beta(n-1)\sqrt{n}}{\lambda_2(\mathcal{L}_{h_a, \mathcal{G}(t)})} \quad (14)$$

Therefore, the solution of the system is bounded within the region:

$$\|\chi - \bar{\chi}\| \leq \frac{\beta(n-1)\sqrt{n}}{\lambda_{2,\min}} \quad (15)$$

with $\bar{\chi} = \mathbf{1} \otimes \bar{x}$. The bound given in eq. (15) can be made tighter by noticing that:

$$\|\epsilon\| \leq \sqrt{n} \max_{i \in \{1, \dots, n\}} \{\|\epsilon_i\|\}. \quad (16)$$

Recall that $\epsilon = [\epsilon_1, \dots, \epsilon_n]^T \in (\mathbb{R}^n)^d$, with n the number of agents, d the dimension of the space and $\epsilon_i \in \mathbb{R}^d$ the error of the i -th agent. This implies that the infinity norm $\|\cdot\|_\infty$ cannot be applied as it is. In particular, according to (16), eq. (14) can be rewritten as:

$$\begin{aligned} \|\epsilon\| &\geq \frac{\beta(n-1)\sqrt{n}}{\lambda_2(\mathcal{L}_{h_a, \mathcal{G}(t)})} \\ \max_{i \in V} \{\|\epsilon_i\|\} &\geq \frac{\beta(n-1)}{\lambda_2(\mathcal{L}_{h_a, \mathcal{G}(t)})} \end{aligned}$$

Therefore, the region the agents move towards and eventually remain within becomes:

$$\|x - \bar{x}\| \leq \frac{\beta(n-1)}{\lambda_{2,\min}} \quad (17)$$

In view of the convergence time analysis, let us now further detail the bound given in (17) under the following hypothesis (*linearly bounded below attraction*):

Assumption 5: The norm of the total attractive vector is bounded below by a linear function:

$$\|y\|g_a(\|y\|) \geq \alpha\|y\|. \quad (18)$$

Then, equation (10) becomes:

$$\begin{aligned} \dot{V}_a &= -\sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) h_a(\|\epsilon_i - \epsilon_j\|) \\ &\leq -\alpha \sum_i \epsilon_i^T \sum_{j \in \mathcal{N}_i(t)} (\epsilon_i - \epsilon_j) f(x_i) \\ &\leq -\alpha \epsilon^T \mathcal{L}_{f, \mathcal{G}(t)}^d \epsilon \\ &\leq -\alpha \lambda_2(\mathcal{L}_{f, \mathcal{G}(t)}) \|\epsilon\|^2 \end{aligned}$$

with

$$\mathcal{L}_{f, \mathcal{G}(t)}^d = \mathcal{L}_{f, \mathcal{G}} \otimes I_d(t)$$

and $\mathcal{L}_{f, \mathcal{G}(t)}$ the Laplacian matrix defined as in (11), where the elements $\mathcal{A}_{f, \mathcal{G}(t)}^{ij}$ and $\mathcal{D}_{f, \mathcal{G}(t)}^{ij}$ of the adjacency and degree matrices, respectively, are obtained by replacing the

terms $h_a(\|x_i - x_j\|)$ with the terms $f(x_i)$. This implies that the condition given in (14) can be rewritten as:

$$\|\epsilon\| \geq \frac{\beta(n-1)\sqrt{n}}{\alpha \lambda_2(\mathcal{L}_{f, \mathcal{G}(t)})} \quad (19)$$

Therefore the bound given in (15) becomes:

$$\|\chi - \bar{\chi}\| \leq \frac{\beta(n-1)\sqrt{n}}{\alpha \lambda_{2,\min}} \quad (20)$$

As before, this bound can be made tighter. In particular, according to (16), eq. (19) can be rewritten as:

$$\max_{i \in V} \{\|\epsilon_i\|\} \geq \frac{\beta(n-1)}{\alpha \lambda_2(\mathcal{L}_{f, \mathcal{G}(t)})}.$$

Therefore, the region the agents move towards and eventually remain within becomes:

$$\|x - \bar{x}\| \leq \frac{\beta(n-1)}{\alpha \lambda_{2,\min}}. \quad (21)$$

C. Agent Dynamics Modeling Considerations

In our model, the agent dynamics have been designed by applying the following saturation policy:

$$\dot{x}_i = \frac{u_i}{1 + \|u_i\|}.$$

Since $\lim_{u_i \rightarrow 0} (1 + \|u_i\|) = 1$, we have

$$1 + \left\| \sum_{j \in \mathcal{N}_i(t)} g(\|x_i - x_j\|) \right\| \rightarrow 1 \text{ as } \sum_{j \in \mathcal{N}_i(t)} g(x_i - x_j) \rightarrow \mathbf{0},$$

that is, $f(x_i) \rightarrow 1$ for all i as the agents get close to the equilibrium within the bounded region. Note that Theorem 1 guarantees that an equilibrium is reached by the agents. Hence, the weighted (non-symmetric) Laplacian matrix $\mathcal{L}_{f, \mathcal{G}(t)}$ related to the attractive term tends to the (symmetric) Laplacian matrix $\mathcal{L}_{\mathcal{G}(t)}$ related to the graph $\mathcal{G}(t)$ encoding the network topology, that is:

$$\mathcal{L}_{f, \mathcal{G}(t)} \rightarrow \mathcal{L}_{\mathcal{G}(t)}.$$

In order to establish some interesting relationship with the fully-connected scenario proposed in [1], consider the behavior of the proposed interaction rule when approaching the equilibrium. Therefore, in the following the Laplacian matrix $\mathcal{L}_{f, \mathcal{G}}$ will be replaced with the Laplacian $\mathcal{L}_{\mathcal{G}}$ for the sake of the analysis. The time dependency is omitted in this analysis since we assume the graph to be fully connected when approaching the equilibrium. Consider that if the graph is fully connected it can be proven that $\lambda_2(\mathcal{L}_{\mathcal{G}}) = n$.

Let us now consider the bounds we have found under Assumption 5. As far as the tighter bound is concerned, in the case of a fully connected graph eq. (17) becomes:

$$\|x - \bar{x}\| \leq \frac{\beta(n-1)}{\alpha n} \leq \frac{\beta}{\alpha} \quad (22)$$

which is the same result obtained in [1]. Simulation results corroborate the bound given in (22).

D. Convergence time analysis

We now show that the convergence of the swarm dynamics (3) towards the bounded region occurs in finite time.

Theorem 3: Consider a swarm of robots whose dynamics is described by (3). Then, the swarm reaches the bounded region \mathcal{B}_r defined in Theorem 2 in a finite time t_f given by

$$t_f \leq -\frac{n}{\lambda_{2,\min}} \ln \left(\frac{\rho^2}{2V(0)} \right),$$

$$\text{with } \rho = \frac{2\beta(n-1)\sqrt{n}}{\lambda_{2,\min}}.$$

Proof: Let us consider the upper bound given in (13) for the Lyapunov function:

$$\begin{aligned} \dot{V}(t) &\leq -\lambda_2(\mathcal{L}_{h_a, \mathcal{G}}(t)) \|\epsilon\|^2 + \beta(n-1)\sqrt{n}\|\epsilon\| \\ &\leq -\lambda_{2,\min} \|\epsilon\|^2 + \beta(n-1)\sqrt{n}\|\epsilon\| \\ &\leq -\frac{\lambda_{2,\min}}{2} \|\epsilon\|^2 - \frac{\lambda_{2,\min}}{2} \|\epsilon\|^2 + \beta(n-1)\sqrt{n}\|\epsilon\| \\ &\leq -\frac{\lambda_{2,\min}}{2} \|\epsilon\|^2 - \|\epsilon\| \left[\frac{\lambda_{2,\min}}{2} \|\epsilon\| - \beta(n-1)\sqrt{n} \right] \end{aligned}$$

If the following condition holds:

$$\|\epsilon\| \geq \frac{2\beta(n-1)\sqrt{n}}{\lambda_{2,\min}} = \rho$$

the time derivative of the Lyapunov function can be bounded as follows:

$$\begin{aligned} \dot{V}(t) &\leq -\frac{1}{2} \lambda_{2,\min} \|\epsilon\|^2 \\ &\leq -\frac{\lambda_{2,\min}}{2n} \sum_{i=1}^n \|\epsilon_i\|^2 \\ &\leq -\frac{\lambda_{2,\min}}{n} V(t) \end{aligned}$$

where the fact that $\sqrt{n}\|\epsilon\| \geq \sum_{i=1}^n \|\epsilon_i\|$ has been used.

Thus, all the agents reach the bounded region given in (17) in finite time:

$$t_f \leq -\frac{n}{\lambda_{2,\min}} \ln \left(\frac{\rho^2}{2V(0)} \right). \quad \blacksquare$$

E. Obstacle Avoidance Integration

The swarm dynamics (3) does not take into account avoidance of environment obstacles. However, this is necessary to let the swarm navigate safely within the environment. A simple but effective solution is the following.

The key idea is to represent an obstacle as a set of virtual robots in the neighborhood of the detecting robot. The set of virtual agents are created on the boundary of the obstacle by projection, i.e. they are located on the boundary of the obstacle at the minimum distance from the detecting agent. In order to avoid an undesired attraction to the obstacle, it is sufficient to set δ as the activation threshold for the obstacle avoidance algorithm. Furthermore, the number of

virtual robots added in the neighborhood of the i -th agent has to be at least equal to the number of its current neighbors, i.e. $|\mathcal{N}_i(t)|$. This prevents any collision because the repulsion originated by the virtual agents will certainly counteract any possible influence by the current true neighbors. Therefore, the dynamics in (3) is modified as follows:

$$\dot{x}_i = k \frac{\sum_{j \in \mathcal{N}_i^+(t)} g(x_i - x_j) \sigma(i, j, \mathcal{N}_i^+(t))}{1 + \left\| \sum_{j \in \mathcal{N}_i^+(t)} g(x_i - x_j) \sigma(i, j, \mathcal{N}_i^+(t)) \right\|},$$

where $\mathcal{N}_i^+(t) = \mathcal{N}_i(t) \cup \mathcal{N}_i^{obs}(t)$ with $\mathcal{N}_i^{obs}(t)$ the indexes of the local set of obstacles detected by the i -th robot and $\sigma(i, j, \mathcal{N}_i^+(t)) = |\mathcal{N}_i(t)|$ if $j \in \mathcal{N}_i^{obs}(t)$, $\sigma(i, j, \mathcal{N}_i^+(t)) = 1$ otherwise. As a result, the dynamics are characterized by a switching topology where sets of virtual robots, i.e., obstacles, are appropriately added or removed. For such dynamics, the result given in Theorem 2 still holds.

IV. RESULTS

In this section, simulations and experiments which have been carried out to validate the proposed swarm aggregation dynamics are described. In particular, simulations have been conducted to investigate the scalability with respect to the number of robots, while experiments have been performed to analyze the efficacy in a real context against not-modeled factors such as noisy measurements. A bounded repulsive vector and a linear attraction vector were used in both the simulations and the experiments to model the agents interaction, i.e. $g_a(\|y\|) = \alpha$, $g_r(\|y\|) = -\beta/\|y\|$. The values of the parameters of the agents interaction function used along both the experiments and the simulations are: $\alpha = 1.5$, $\beta = 1$. For both the simulations and the experiments, a velocity drift is simultaneously given to each agent within the swarm by a central unit equipped with a gesture recognition system based on the Kinect hardware platform [22].

A. Simulations

Simulations were performed in the Player/Stage simulation environment [23]. An example of swarm aggregation behavior for a multi-robot system composed of 15 units moving from the left to the right within a cluttered environment is shown in the accompanying video. In particular, it can be noticed that the swarm of robots can fluidly move within the cluttered environment while showing a cohesive behavior.

B. Experiments

Experiments were performed using five mobile robots SAETTA developed at the Robotics and Sensor Fusion Lab of the Department of Computer Science and Automation at the University of Roma Tre. SAETTA, shown in Fig. 1, is a low-cost platform which features a complete sensorial system, a very accurate traction in indoor environments, and a wireless communication channel for multi-robot applications.

Each SAETTA unit was equipped with a compass to let the swarm share a common heading reference. This allows for a human supervisor to issue guidance commands to the swarm. The supervision was limited to high-level guidance



Fig. 1. The SAETTA robotic platform.

commands, e.g., “move to the left” referred to the swarm as a whole. Robots were responsible for the execution of the guidance commands while showing a cohesive behavior through local interaction. Furthermore, this central unit has been also equipped with a low-cost vision system to retrieve the relative distance among the robots. Communication between the central unit and the swarm has been realized by means of a wifi channel.

An example of the swarm aggregation behavior for the team of SAETTA robots is shown in the attached video. In particular, the team of robots was required to navigate within an environment with four obstacles while following the guidance commands issued by the central unit. As shown by the accompanying video, the robots are able to move smoothly according to the guidance commands while showing a cohesive behavior.

V. CONCLUSION

We have proposed a swarm aggregation algorithm based on local interaction in the presence of actuator saturations. The local interaction assumption accounts for realistic scenarios where interaction occurs only if the robots are within the range of visibility of each other, while the actuator saturation assumption reflects the physical limitation of actual robotic platforms. A complete theoretical analysis of the convergence properties of the proposed swarm aggregation algorithm has been carried out. In addition, simulations and experiments have been shown to corroborate the theoretical results and validate the effectiveness of the proposed swarm aggregation method in a real context.

Future work will focus on the connectivity analysis of the proposed swarm aggregation dynamics. This will allow to remove Assumption 3 and, in particular, to derive an analytic expression of the lower bound $\lambda_{2,\min}$ on the algebraic connectivity. It should also be noted that the inclusion of obstacle avoidance actions into the swarm dynamics may in principle cause disconnection. To this end, we shall study the possibility of integrating a connectivity maintenance control law within the proposed framework; suitable approaches are, e.g., [24], [25].

REFERENCES

- [1] V. Gazi and K. M. Passino, “A class of attractions/repulsion functions for stable swarm aggregations,” *International Journal of Control*, vol. 77, no. 18, pp. 1567–1579, 2004.
- [2] D. Wenjie and J. Farrell, “Cooperative control of multiple nonholonomic mobile agents,” *IEEE Transactions on Automatic Control*, vol. 53, no. 6, pp. 1434–1448, July 2008.
- [3] D. Dimarogonas and K. Kyriakopoulos, “Connectedness preserving distributed swarm aggregation for multiple kinematic robots,” *IEEE Transactions on Robotics*, vol. 24, no. 5, pp. 1213–1223, Oct. 2008.
- [4] W. Burgard, M. Moors, C. S. F.E., and Schneider, “Coordinated multi-robot exploration,” *IEEE Transactions on Robotics*, vol. 21, no. 3, pp. 376–386, June 2005.
- [5] P. Brass, F. Cabrera-Mora, A. Gasparri, and J. Xiao, “Multirobot tree and graph exploration,” *IEEE Transactions on Robotics*, vol. 27, no. 4, pp. 707–717, Aug. 2011.
- [6] X. Songdong and Z. Jianchao, “Sense limitedly, interact locally: the control strategy for swarm robots search,” in *IEEE International Conference on Networking, Sensing and Control (ICNSC)*, 2008.
- [7] A. Shia, F. Bastani, and Y. I-Ling, “A highly resilient framework for autonomous robotic swarm systems operating in unknown, hostile environments,” in *10th International Symposium on Autonomous Decentralized Systems (ISADS)*, 2011.
- [8] M. Yongguo, L. Yung-Hsiang, Y. Hu, and C. Lee, “Deployment of mobile robots with energy and timing constraints,” *IEEE Transactions on Robotics*, vol. 22, no. 3, pp. 507–522, June 2006.
- [9] L. Yanfei and K. Passino, “Stable social foraging swarms in a noisy environment,” *IEEE Transactions on Automatic Control*, vol. 49, no. 1, pp. 30–44, Jan. 2004.
- [10] N. Hoff, A. Sagoff, R. Wood, and R. Nagpal, “Two foraging algorithms for robot swarms using only local communication,” in *IEEE International Conference on Robotics and Biomimetics (ROBIO)*, 2010.
- [11] S. Erol and W. Alan, “Special issue on swarm robotics,” *Swarm Intelligence*, vol. 2, no. 2-4, pp. 69–72, 2008.
- [12] V. Gazi and K. Passino, “Stability analysis of swarms,” *IEEE Transactions on Automatic Control*, vol. 48, no. 4, pp. 692–697, April 2003.
- [13] V. Gazi, “Swarm aggregations using artificial potentials and sliding-mode control,” *IEEE Transactions on Robotics*, vol. 21, no. 6, pp. 1208–1214, Dec. 2005.
- [14] W. Li and X. Wang, “Adaptive velocity strategy for swarm aggregation,” *Phys. Rev. E*, vol. 75, p. 021917, Feb 2007.
- [15] A. Ray, P. Benavidez, L. Behera, and M. Jamshidi, “Decentralized motion coordination for a formation of rovers,” *IEEE Systems Journal*, vol. 3, no. 3, pp. 369–381, Sept. 2009.
- [16] J. Lawton, R. Beard, and B. Young, “A decentralized approach to formation maneuvers,” *IEEE Transactions on Robotics and Automation*, vol. 19, no. 6, pp. 933–941, Dec. 2003.
- [17] L. Bo and Y. Hai, “Flocking in multi-agent systems with a bounded control input,” in *International Workshop on Chaos-Fractals Theories and Applications. IWCF TA '09.*, Nov. 2009, pp. 130–134.
- [18] D. Kostic, S. Adinandra, J. Caarls, N. van de Wouw, and H. Nijmeijer, “Saturated control of time-varying formations and trajectory tracking for unicycle multi-agent systems,” in *2010 49th IEEE Conference on Decision and Control (CDC)*, Dec. 2010, pp. 4054–4059.
- [19] C. Godsil and G. Royle, *Algebraic Graph Theory*. Springer, Apr. 2001.
- [20] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.
- [21] M. Mesbahi and M. Egerstedt, *Graph Theoretic Methods in Multiagent Networks*. Princeton University Press, 2010.
- [22] A. Gasparri, A. Priolo, and G. Ulivi, “A swarm aggregation algorithm for multi-robot systems based on local interaction,” Università degli Studi Roma Tre, Dipartimento di Informatica ed Automazione, Tech. Rep. RT-DIA-193-2012, 2012.
- [23] R. Vaughan, “Massively multi-robot simulation in stage,” *Swarm Intelligence*, vol. 2, no. 2-4, pp. 189–208, 2008.
- [24] D. Dimarogonas and K. Johansson, “Decentralized connectivity maintenance in mobile networks with bounded inputs,” in *IEEE International Conference on Robotics and Automation, 2008. ICRA 2008.*, May 2008, pp. 1507–1512.
- [25] L. Sabattini, A. Gasparri, C. Secchi, and N. Chopra, “Enhanced connectivity maintenance for multi-robot systems,” in *10th International IFAC Symposium on Robot Control (SYROCO)*, 2012.