

END-EFFECTOR REGULATION OF ROBOTS WITH ELASTIC ELEMENTS BY AN ITERATIVE SCHEME

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SUMMARY

The regulation of motor variable positions in robots with elastic elements has been solved using PD linear controllers in the absence of gravity and with the addition of a model-based feedforward term when gravity is present. When the mass of the links is not known, an iterative learning scheme can be derived to obtain the same result for both joint elasticity and link flexibility. An extension to end-effector regulation with a similar two-stage and hence time-consuming scheme has been proposed. In this paper we show the feasibility of a new iterative one-stage scheme able to directly regulate the end-effector position in robots with joint elasticity and in robots whose distributed flexibility is limited for each link to the plane orthogonal to the associated motor axis. Experimental results are included to show the improved rate of convergence of the proposed scheme applied to a two-link flexible robot available in our laboratory.

KEY WORDS robot control; regulation; elastic joints; flexible manipulators; iterative learning

1. INTRODUCTION

We consider the end-point regulation problem under gravity for general flexible manipulators, i.e. with link flexibility or with joint elasticity.

For rigid manipulators under gravity the most direct approach to set-point regulation is to globally cancel the gravity terms and apply a simple proportional–derivative (PD) feedback at the joint (motor) error level with positive definite gains. This leads to a non-linear feedback control law yielding at least asymptotic stability.¹ Under a mild condition on the proportional gain this scheme can be simplified to a constant gravity compensation as evaluated at the desired configuration.² A purely linear feedback law with a feedforward action is then obtained (denoted PD+), but in this case the proportional gain should be chosen so as to dominate the Jacobian of the gravity force over the whole robot workspace.

When flexible components are present in the robotic structure, a similar strategy based on PD plus a constant gravity feedforward has been shown to asymptotically stabilize also robots with elastic joints³ and with flexible links.⁴ For the elastic joint case feedback is closed only around the motor variables, while for the flexible link case only the joint (rigid) variables are used for control. This control strategy works under a further structural assumption on the joint or on the link stiffness respectively.

Moreover, in both the rigid and the flexible case an exact knowledge of the gravity vector is assumed, which may be difficult to achieve, e.g. for a robot picking up multiple unknown payloads. Furthermore, for robots with flexible components it is well known that controlling the

end-effector is quite a different problem from controlling the motion of the arm joints. In this case the source of steady state end-effector error under pure joint PD control is twofold: first, a displacement is present at the motor level just as in the fully rigid case; second, a further displacement is introduced by the arm (joint and/or link) deflection.

On the other hand, to compensate for gravity effects, a standard model-independent remedy is the addition of an integral term to the linear PD law. However, several problems arise with the design of an efficient PID control, partly owing to the non-linear nature of the robot dynamics. The presence of flexible dynamics introduces even more complications in the tuning process.

A different approach to the set-point regulation of both rigid and flexible arms has been proposed in Reference 5 based on an iterative learning scheme. The required compensation at the final configuration can be built up from a very limited knowledge of the robot gravity terms. A PD-based control law is applied iteratively at the motor level, while a constant gravity feedforward is learned at discrete instants, without the explicit introduction of an integral error term nor the use of high-gain feedback. Sufficient conditions are derived guaranteeing the global asymptotic convergence of the scheme to zero steady state error under the mild assumption that arm stiffness dominates gravity effects.

In the case of flexible arms the scheme was intended to achieve a desired arm configuration without directly facing the problem of end-effector regulation. An extension of the same approach was then presented in Reference 6, where a two-stage iterative scheme was asked to first regulate the flexible arm to a given configuration and then update this reference arm configuration so that the end-effector iteratively approaches the desired goal. The drawback of this method is clearly in its slower convergence.

In this paper we further exploit the same lines of thought to derive a one-stage iterative scheme that directly regulates the end-effector position of a flexible arm. The two cases of joint elasticity and link flexibility are handled in a unique framework. The sufficient conditions for convergence in our main result are very easy to satisfy in practice.

The experimental results reported for the two-link lightweight manipulator with a flexible forearm available in the Robotics Laboratory of our Department⁷ show the obtained faster convergence of the proposed scheme.

2. MODEL PROPERTIES

In this section the most relevant model properties of robots with flexible links or elastic joints will be reviewed, with particular emphasis on those concerning our control approach.

For a robot with n flexible links a Lagrangian approach can be used to derive the dynamic equations of motion, describing with a finite set of space functions the deformation shape of each link, possibly modelled as Euler–Bernoulli beams.⁸ Let θ be the n -dimensional vector of joint motor coordinates and δ the m -dimensional vector of all generalized deformation modes. The following model can then be written:⁹

$$B(\theta, \delta) \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + h(\theta, \delta, \dot{\theta}, \dot{\delta}) + \begin{bmatrix} g_{\theta}(\theta, \delta) \\ g_{\delta}(\theta, \delta) \end{bmatrix} + \begin{bmatrix} 0 \\ K\delta + D\dot{\delta} \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (1)$$

The link deformation is expressed here in a frame *clamped* at the joint side so that the control input does not enter directly in the equations of the flexible part. B is the $(n+m) \times (n+m)$ positive definite inertia matrix, h is the $(n+m)$ -dimensional vector of Coriolis and centrifugal forces and D describes the modal damping of the links. K is the positive definite symmetric (and diagonal) stiffness matrix associated with the link elasticity. In particular, the matrix K is

derived from the potential energy associated with the elastic deformation of links under the hypothesis of small deformations. The gravity vector, defined by

$$g(\theta, \delta) = \partial U_g / \partial q \quad (2)$$

where U_g is the gravitational potential energy of the system, has been partitioned into two terms and contains only trigonometric functions of θ and linear/trigonometric functions of δ , where $q = (\theta, \delta)$ is the $n + m$ configuration vector. It has been shown⁶ that the inequalities

$$\left\| \frac{\partial g}{\partial q} \right\| \leq \alpha, \quad \left\| \frac{\partial g_\theta}{\partial q} \right\| \leq \alpha_\theta \leq \alpha, \quad \left\| \frac{\partial g_\delta}{\partial q} \right\| \leq \alpha_\delta \leq \alpha \quad (3)$$

are satisfied with $\alpha, \alpha_\theta, \alpha_\delta > 0$. The spectral norm of a matrix, defined as $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$, is used here. As a consequence of (3),

$$\|g(q_1) - g(q_2)\| \leq \alpha \|q_1 - q_2\|, \quad \forall q_1, q_2 \in \mathbb{R}^{n+m} \quad (4)$$

Since our goal is to regulate the end-effector of the flexible arm to a desired position, we need to define a direct kinematic function relating the robot configuration variables q to the pose of its end-effector (tip). In order to keep this description at a simple but still significant level, we assume that each arm deflects only in the plane of its rigid motion, i.e. the h th arm is constrained in the plane orthogonal to the h th joint axis. In this case the direct kinematics of the flexible arm has the same structure as the one of the equivalent rigid arm, where, however, the angles pointing from the motors to the end of each link replace the rigid joint angles.

To write the angular position y_h of the end-point of the h th link with respect to the previous one, observe that this is given by the sum of the joint angle variable θ_h and of a linear combination of the local deformation modes. In vector form we can set

$$y = \theta + \bar{\phi} \delta \quad (5)$$

where

$$\bar{\phi} = \text{block diag} \left[\begin{array}{ccc} \frac{\phi_{h1}(l_h)}{l_h} & \dots & \frac{\phi_{hm_h}(l_h)}{l_h} \end{array} \right] \quad (6)$$

with l_h the length of link h , ϕ_{hi} the i th mode shape of link h ,⁸ m_h the number of modes of link h and $\sum_{h=1}^n m_h = m$. As a result, the specification of a desired value y_d uniquely defines the end-effector pose via the usual direct kinematics. The end-effector regulation task can then be defined in terms of (5).

When considering robots with n elastic joints but rigid links, we need a set of n motor variables θ and a set of n link position variables θ_l to describe the state of the manipulator, as described in Reference 3. To keep a similar notation to the one adopted in the case of flexible robots, deflection variables $\delta = \theta_l - \theta$ are introduced. The resulting dynamic model can be written as:⁵

$$B(\theta + \delta) \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + h(\theta, \delta, \dot{\theta}, \dot{\delta}) + \begin{bmatrix} g_1(\theta + \delta) \\ g_1(\theta + \delta) \end{bmatrix} + \begin{bmatrix} 0 \\ K\delta + D\dot{\delta} \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (7)$$

Note that the $2n \times 2n$ positive definite inertia matrix B and the gravity vector g actually depend only on the link variables, while h, K and D are defined as in model (1). Similarly to (3), one has

$$\left\| \frac{\partial g}{\partial q} \right\| \leq \alpha, \quad \left\| \frac{\partial g_1}{\partial \theta_l} \right\| \leq \alpha_1 \leq \alpha \quad (8)$$

with α , $\alpha_1 > 0$.⁵ In this case the angular positions of the end-points of each link coincide with θ_i and we can write, in conformity with (5),

$$y = \theta_i = \theta + \delta \quad (9)$$

In the following, owing to the similarity of the two models, we will refer only to the flexible link case (1), observing that conclusions can be derived also for elastic joint robots by simply replacing

$$g_\theta(\theta, \delta) = g_1(\theta + \delta), \quad g_\delta(\theta, \delta) = g_1(\theta + \delta), \quad \bar{\phi} = \mathbf{I} \quad (10)$$

3. SET-POINT REGULATION WITH PD+ CONTROL

For a desired constant motor position θ_d under perfect knowledge of the gravity term the input torque u can be chosen as a *linear PD+ control*, i.e.

$$u = K_p(\theta_d - \theta) - K_d\dot{\theta} + g_\theta(\theta_d, \delta_d), \quad K_p > 0, \quad K_d > 0 \quad (11)$$

where from (1) the associated δ_d is implicitly defined as the solution of

$$g_\delta(\theta_d, \delta) + K\delta = 0 \quad (12)$$

This includes the case of elastic joints³ with positions (10).

For the dynamic model (1) it has been shown⁴ that under the assumption

$$\lambda_{\min} \begin{pmatrix} K_p & 0 \\ 0 & K \end{pmatrix} > \alpha \quad (13)$$

the state $q = q_d = (\theta_d, \delta_d)$, $\dot{q} = 0$ is the *unique equilibrium* of the closed loop system (1), (11), i.e. the unique solution to

$$g_\theta(\theta, \delta) = K_p(\theta_d, \theta) + g_\theta(\theta_d, \delta_d) \quad (14a)$$

$$g_\delta(\theta, \delta) = -K\delta \quad (14b)$$

Condition (13) can always be satisfied provided that an assumption on the structural flexibility holds:

$$\lambda_{\min}(K) = \min_{i=1, \dots, n} \{k_i\} > \alpha \quad (15)$$

with K diagonal. This is not restrictive in general and depends on the relative magnitude of link and/or joint stiffness versus gravity. As a result, by choosing the proportional control gain so that $\lambda_{\min}(K_p) > \alpha$, the equilibrium state $q = q_d$, $\dot{q} = 0$ of system (1) under control (11) is asymptotically stable.

We finally remark that the same condition (13) guarantees a unique equilibrium point also when an approximate constant gravity compensation \hat{g}_θ is used in (11) in place of $g_\theta(\theta_d, \delta_d)$. In this case the equilibrium point will indeed be a $\bar{q} = (\bar{\theta}, \bar{\delta}) \neq q_d$.

4. ITERATIVE CONTROL SCHEME

The iterative scheme introduced here is able to achieve set-point regulation of the robot end-effector. It is based on the idea of using a simple proportional-derivative control loop at the motor level and an update law for learning the correct gravity and deflection compensation at the set-point.

On one hand, the control effort applied at the end of each iteration is used to update the compensation term required at the final point, as in the scheme proposed in Reference 5. On the other hand, the correct set-point value for motor variables, required to keep the tip in the desired position, is iteratively obtained from the reading of link deflections (or link positions in the case of elastic joint robots). The updates are performed at the same time, resulting in a one-stage iterative scheme.

Let $g_\theta = (\theta_0, \delta_0)$ be the configuration at the initial point with $y_0 = \theta_0 + \bar{\phi}\delta_0$ ($\bar{\phi} = I$ for elastic joints). The control law during iteration i , with $i = 1, 2, \dots$, is defined as

$$u(t) = \frac{1}{\beta} K_p (\theta_{d,i-1} - \theta(t)) - K_d \dot{\theta} + u_{i-1} \quad (16)$$

with positive definite gain matrices K_p and K_d as in Section 3 and a constant feedforward u_{i-1} . The set-point value $\theta_{d,i-1}$ is the current estimate of the *a priori* unknown desired motor position. The value u_0 can be chosen simply as zero or as the best available estimate for the gravity vector $u_0 = \hat{g}_\theta(q_d)$. Any value can be chosen for $\theta_{d,0}$, but the most correct one is $\theta_d = y_d$, namely the desired tip position if the arm were fully rigid.

At the end of the i th iteration, system (1) (or system (7)) under control (16) reaches the unique equilibrium point $q = q_i = (\theta_i, \delta_i)$, $\dot{q} = 0$, with end-point position at $y_i = \theta_i + \bar{\phi}\delta_i$, satisfying

$$g_\theta(\theta_i, \delta_i) = \frac{1}{\beta} K_p (\theta_{d,i-1} - \theta_i) + u_{i-1} \quad (17a)$$

$$g_\theta(\theta_i, \delta_i) = -K\delta_i \quad (17b)$$

It is worth noting that the control effort read at the controller output at the i th steady state is equal to the unknown gravity term $g_\theta(q_i)$.

We now use the gravity term learned in the configuration q_i as a better approximation of the one required at the final point and simultaneously use the link deflection (or the link position minus the motor position for elastic joints) measured at q_i as an estimate of the natural arm deflection when the tip is at the desired point. For the next iteration define the feedforward update as

$$u_i = \frac{1}{\beta} K_p (\theta_{d,i-1} - \theta_i) + u_{i-1} \quad (18)$$

and the new motor reference as

$$\theta_{d,i} = \theta_i + (y_d - y_i^*) \quad (19)$$

where y_i^* is the measured end-effector position at the end of iteration i . In the case of flexible link robots equation (19) can be rewritten as

$$\theta_{d,i} = y_d - \bar{\phi}\delta_i \quad (20)$$

obtained from (5) by substituting $y_i^* = \theta_i + \bar{\phi}\delta_i$. For elastic joint robots, using (9), equation (19) becomes

$$\theta_{d,i} = y_d - \delta_i \quad (21)$$

This scheme is shown in Figure 1, where the memory blocks are buffers activated at specified instants.

Define now the current error at the motor level as

$$e_i = \theta_{d,i-1} - \theta_i \quad (22)$$

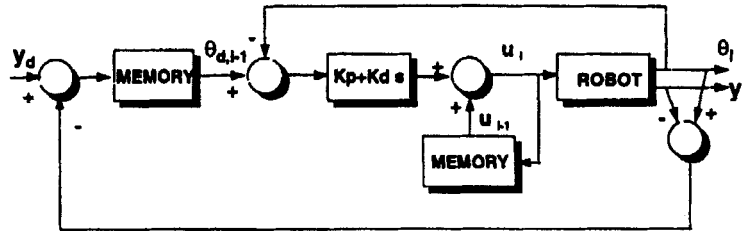


Figure 1. Iterative learning scheme

and the motor reference variation as

$$\Delta\theta_{d,i} = \theta_{d,i} - \theta_{d,i-1} \quad (23)$$

If the algorithm converges in the sense that

$$\|e_i\| \rightarrow 0 \quad \text{and} \quad \|\Delta\theta_{d,i}\| \rightarrow 0 \quad \text{as} \quad i \rightarrow \infty \quad (24)$$

then from the equations (17b) and (20) the limit values $\theta_\infty = \lim_{i \rightarrow \infty} \theta_i$ and $\delta_\infty = \lim_{i \rightarrow \infty} \delta_i$ satisfy

$$g_\delta(\theta_\infty, \delta_\infty) = -K\delta_\infty \quad (25)$$

$$\theta_\infty = y_d - \bar{\phi}\delta_\infty \quad (26)$$

Following the development of Section 3, equations (25) and (26) have the unique solution

$$\theta_\infty = \theta_d \quad (27)$$

$$\delta_\infty = \delta_d \quad (28)$$

In the next section we will prove the convergence of the proposed algorithm for the two considered classes of flexible robots.

5. CONVERGENCE CONDITIONS

Our main result is stated in the following.

Theorem 1

Under control (16), with updates (18) and (19), the sequence of end-effector positions $\{y_0, y_1, \dots\}$, with $y_i = \theta_i + \bar{\phi}\delta_i$ and θ_i and δ_i defined by (17), converges to y_d , starting from any initial robot configuration q_0 , provided that

- (a) $\lambda_{\min}(K) > \gamma\alpha$
- (b) $\lambda_{\min}(K_p) > \alpha$
- (c) $\gamma > 1 + \|\bar{\phi}\|$
- (d) $0 < \beta < (\gamma - 1 - \|\bar{\phi}\|)/2\gamma$

Proof. At the end of iteration i equations (17a) and (18) imply that $u_i = g_\theta(q_i)$ and so

$$\|u_i - u_{i-1}\| = \|g_\theta(q_i) - g_\theta(q_{i-1})\| \leq \alpha \|q_i - q_{i-1}\| \leq \alpha (\|\theta_i - \theta_{i-1}\| + \|\delta_i - \delta_{i-1}\|) \quad (29)$$

where the second inequality in (3) has been used. From (17b), using the third inequality in (3) and hypothesis (a), we get

$$\|\delta_i - \delta_{i-1}\| \leq \|K^{-1}\| \|g_\delta(q_i) - g_\delta(q_{i-1})\| < \frac{\alpha}{\alpha\gamma} \|q_i - q_{i-1}\| \leq \frac{1}{\gamma} (\|\theta_i - \theta_{i-1}\| + \|\delta_i - \delta_{i-1}\|) \quad (30)$$

from which

$$\|\delta_i - \delta_{i-1}\| < \frac{1}{\gamma - 1} \|\theta_i - \theta_{i-1}\| \quad (31)$$

Combining (29) and (31), simple manipulations yield

$$\begin{aligned} \|u_i - u_{i-1}\| &< \alpha \frac{\gamma}{\gamma - 1} \|\theta_i - \theta_{i-1}\| = \alpha \frac{\gamma}{\gamma - 1} (\| -e_i + e_{i-1} + \theta_{d,i-1} - \theta_{d,i-2} \|) \\ &\leq \alpha \frac{\gamma}{\gamma - 1} (\|e_i\| + \|e_{i-1}\| + \|\Delta\theta_{d,i-1}\|) \end{aligned} \quad (32)$$

Further, from (18)

$$\|u_i - u_{i-1}\| = \frac{1}{\beta} \|K_p e_i\| \quad (33)$$

Using (32) with hypothesis (b), it follows that

$$\frac{1}{\beta} \alpha \|e_i\| < \frac{1}{\beta} \lambda_{\min}(K_p) \|e_i\| \leq \frac{1}{\beta} \|K_p e_i\| < \alpha \frac{\gamma}{\gamma - 1} (\|e_i\| + \|e_{i-1}\| + \|\Delta\theta_{d,i-1}\|) \quad (34)$$

Since hypotheses (c) and (d) guarantee that $\gamma - 1 - \gamma\beta > 0$ and $\beta(\gamma - 1) > 0$, reorganizing terms, we have

$$\|e_i\| < \frac{\gamma\beta}{\gamma - 1 - \gamma\beta} (\|e_{i-1}\| + \|\Delta\theta_{d,i-1}\|) \quad (35)$$

On the other hand, from (20), using inequality (31), it follows that

$$\begin{aligned} \|\theta_{d,i} - \theta_{d,i-1}\| &= \|\Delta\theta_{d,i}\| \leq \|\bar{\phi}\| \|\delta_i - \delta_{i-1}\| < \frac{\|\bar{\phi}\|}{\gamma - 1} \|\theta_i - \theta_{i-1}\| \\ &< \frac{\|\bar{\phi}\|}{\gamma - 1} (\|e_i\| + \|e_{i-1}\| + \|\Delta\theta_{d,i-1}\|) \end{aligned} \quad (36)$$

Substituting (35) in (36), we have finally

$$\|\Delta\theta_{d,i}\| < \frac{\|\bar{\phi}\|}{\gamma - 1 - \gamma\beta} (\|e_{i-1}\| + \|\Delta\theta_{d,i-1}\|) \quad (37)$$

For the convergence of both (35) and (37) the following map must have all eigenvalues inside the unitary disc:

$$\begin{bmatrix} \|e_i\| \\ \|\Delta\theta_{d,i}\| \end{bmatrix} = \begin{bmatrix} \frac{\gamma\beta}{\gamma-1-\gamma\beta} & \frac{\gamma\beta}{\gamma-1-\gamma\beta} \\ \frac{\|\bar{\phi}\|}{\gamma-1-\gamma\beta} & \frac{\|\bar{\phi}\|}{\gamma-1-\gamma\beta} \end{bmatrix} \begin{bmatrix} \|e_{i-1}\| \\ \|\Delta\theta_{d,i-1}\| \end{bmatrix} \quad (38)$$

For a matrix of the form

$$\begin{bmatrix} x & x \\ y & y \end{bmatrix} \quad (39)$$

the eigenvalues are

$$\lambda_1 = 0, \quad \lambda_2 = x + y \quad (40)$$

which in our case become

$$\lambda_1 = 0, \quad \lambda_2 = \frac{\gamma\beta + \|\bar{\phi}\|}{\gamma-1-\gamma\beta} \quad (41)$$

The convergence condition is then satisfied thanks to hypothesis (d). \square

We note explicitly that in the case of robots with elastic joints the convergence hypotheses (c) and (d) become

$$(c') \quad \gamma > 2$$

$$(d') \quad 0 < \beta < (\gamma - 2)/2\gamma < \frac{1}{2}.$$

Incidentally, for rigid robots the joint stiffness K goes to ∞ and so γ can be chosen arbitrarily large without violating (a). Condition (c') is always satisfied, while (d') becomes $\beta < \frac{1}{2}$ as in Reference 5.

6. EXPERIMENTS WITH A TWO-LINK FLEXIBLE ROBOT

The robot used in the experiments is a direct drive planar arm with two revolute joints and two links, the second of which—the forearm—is very flexible.^{7,10} A sketch of the arm is shown in Figure 2. To introduce gravity effects, the arm has been tilted by approximately 8° .

The first link (the rigid one) is 300 mm long. The second one has been designed to be very flexible in a plane orthogonal to the two motors axes (the motion plane) and stiff in the other directions and torsionally. It is composed, as depicted in Figure 3, of two parallel sheets of harmonic steel coupled by nine equispaced aluminium frames which prevent torsional deformations. Its length is 700 mm.

The total mass of the flexible link is 1.8 kg. The inertias of the first and second links are $J_1 = 0.447 \text{ kg m}^2$ and $J_2 = 0.303 \text{ kg m}^2$ respectively. The first two natural frequencies of vibration are $f_1 = 4.7 \text{ Hz}$ and $f_2 = 14.4 \text{ Hz}$, leading to stiffness coefficients $k_1 = 872 \text{ N}$ and $k_2 = 8186 \text{ N}$ respectively.

The motors driving the two links can provide maximum torques of 3.5 and 1.8 Nm respectively and are supplied by two 20 kHz PWM amplifiers. Each motor is equipped with a 20 000 pulses/turn encoder and a DC tachometer.

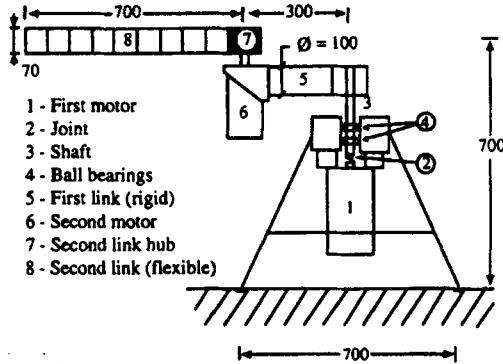


Figure 2. The two-link flexible arm

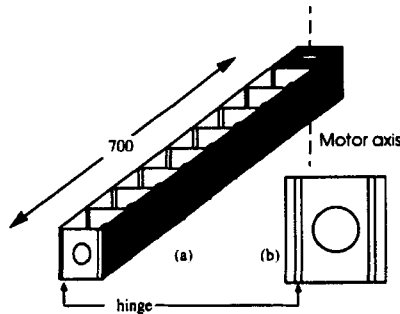


Figure 3. The flexible link

The second-link deformation is measured by an optical sensor with a precision of about 0.1° .⁷ The light of a lamp positioned at the end of the link is reflected by a rotating mirror on a detector. When this is illuminated, the position of the mirror is measured by an encoder. In this way it is possible to know the lamp displacement during the experiments and to reconstruct the link deflection to good approximation.

To validate the theory of the previous section, some experiments have been carried out to regulate the arm tip in different positions. We show here four of them in which, starting every time from the lower extended position $(\theta_1, \theta_2, y) = (0, 0, 0)$, we try to reach various configurations with and without deflection of the second link. We remark that the tip and second-joint positions are expressed with respect to the first-link position. The gains that have been used for the four experiments are

$$K_p = \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \quad K_d = \begin{bmatrix} 2 \\ 0.8 \end{bmatrix} \tag{42}$$

In the first experiment (Figures 4(a) and 5–7) we desire to put the first link to 90° and align the tip with it. Every 4 s we update the control and it is possible to see how, even if this update is done while the robot is still moving, within three steps we are able to reach with zero error the position $(90, \hat{\theta}_2, 0)$. The hat indicates that the position of the second joint is unknown or, better, is not interesting owing to the fact that we are regulating the tip. We can deduce it from the measure of the link deflection reported in Figure 7. In this configuration, fully extended, we have the maximum of control effort for both motors and the maximum of deflection for the second link.

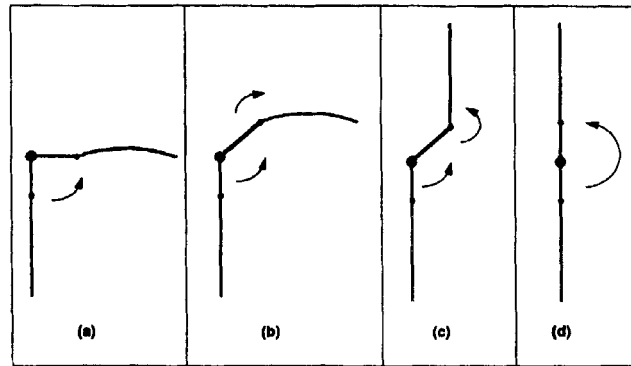


Figure 4. The four experiments

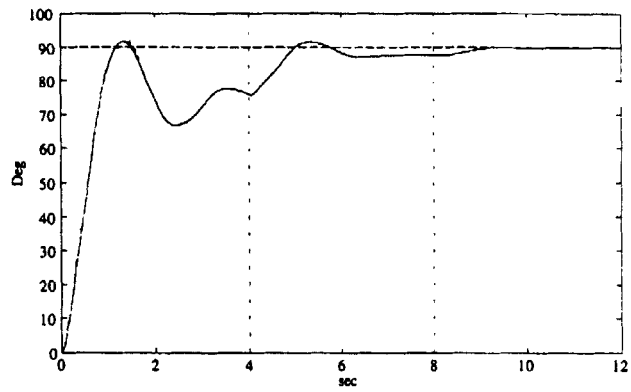


Figure 5. Experiment 1: position of first link

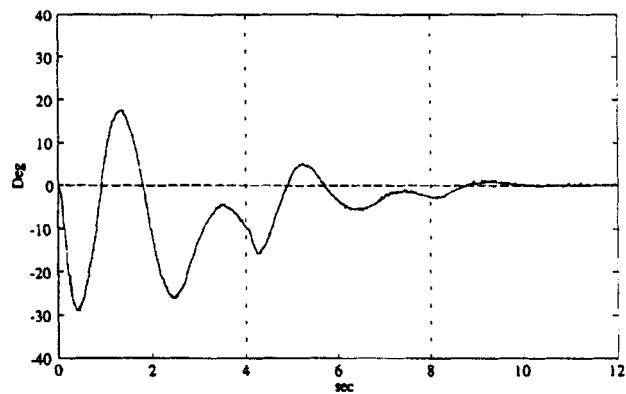


Figure 6. Experiment 1: tip position w.r.t. first link

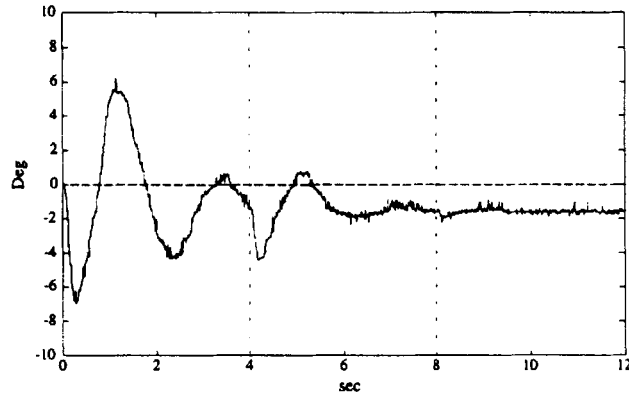


Figure 7. Experiment 1: deflection of second link

In the second experiment (Figures 4(b) and 8–10) the desired position is $(135, \hat{\theta}_2, -45)$. Also here, three steps of 4 s are sufficient to achieve the regulation.

In the third experiment (Figures 4(c) and 11–13) the goal is $(135, \hat{\theta}_2, 45)$, but this time we need six iterations to converge. The learned control effort oscillates around the desired one for both motors and the final deflection is zero owing to the vertical final position of the second link. Also, the final control torque applied at the second joint is obviously zero, which shows the ability of the algorithm to converge to zero control effort even when intermediate values are different from zero.

Finally, in the fourth experiment (Figures 4(d) and 14–16) we try to position the robot in the vertical extended pose $(180, \hat{\theta}_2, 0)$. In this case, owing to the fact that the gravity compensation must be zero as well as the final deflection and that in the starting position we have the same conditions, theoretically we should reach this goal in one step. Owing to some dry friction, this is not true and a residual error of a few degrees is obtained. This error is then compensated by the algorithm in the next step starting at 10 s.

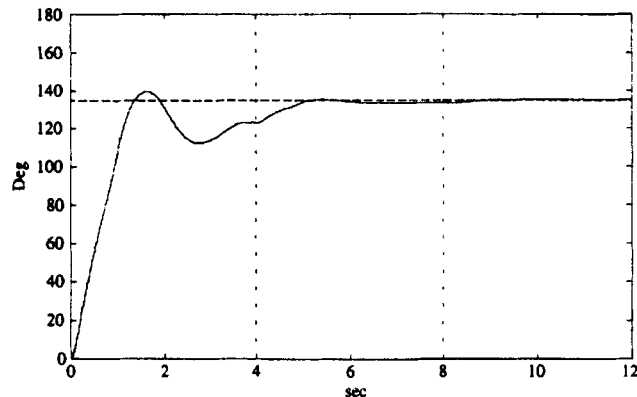


Figure 8. Experiment 2: position of first link

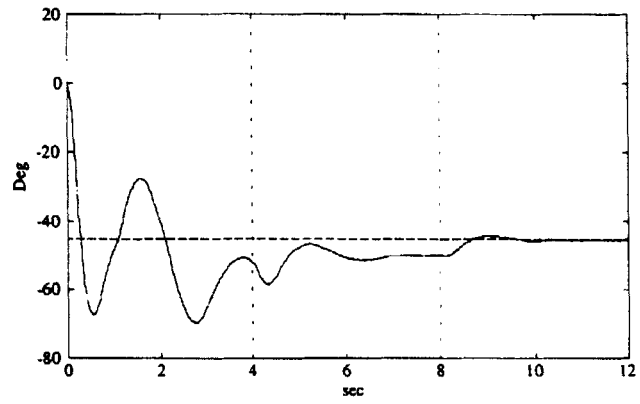


Figure 9. Experiment 2: tip position w.r.t. first link

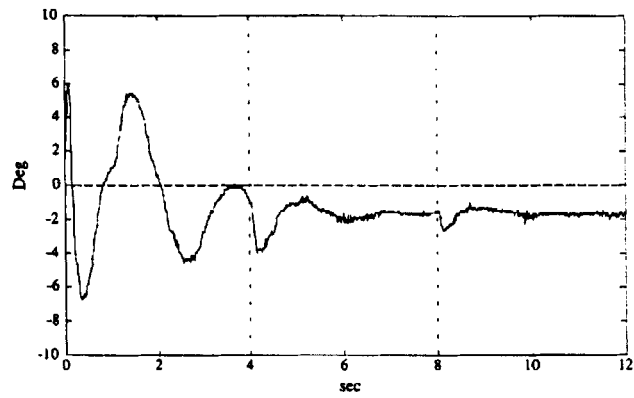


Figure 10. Experiment 2: deflection of second link

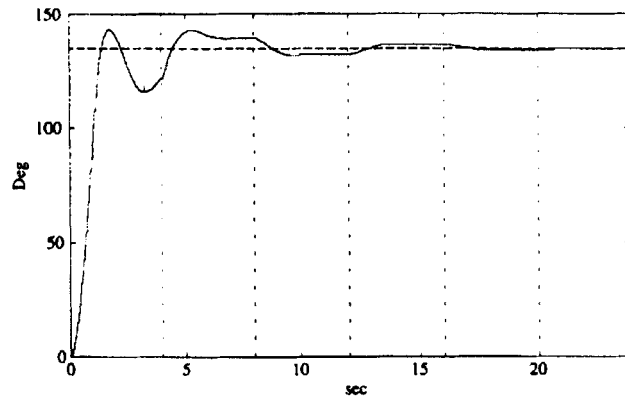


Figure 11. Experiment 3: position of first link

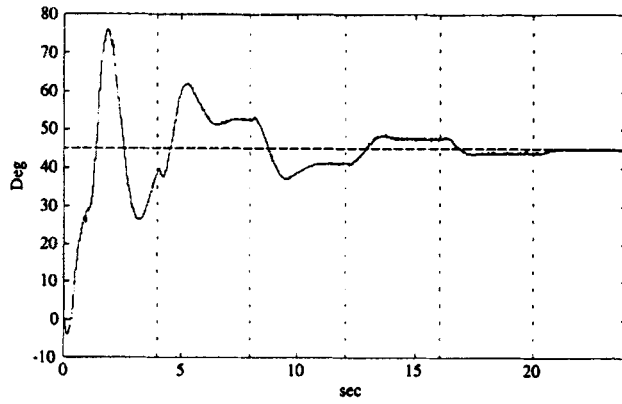


Figure 12. Experiment 3: tip position w.r.t. first link

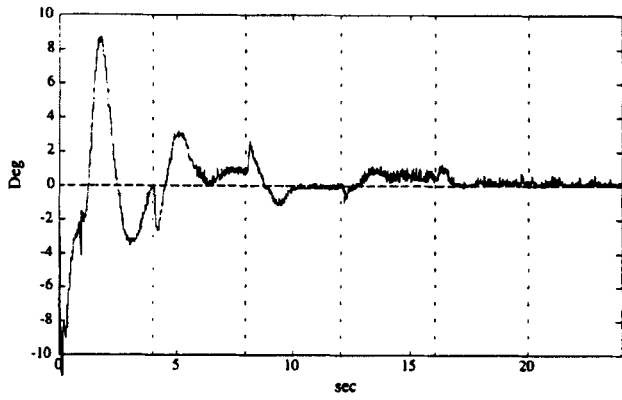


Figure 13. Experiment 3: deflection of second link

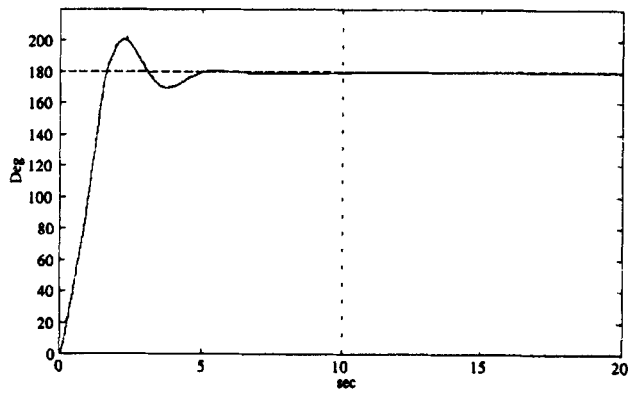


Figure 14. Experiment 4: position of first link

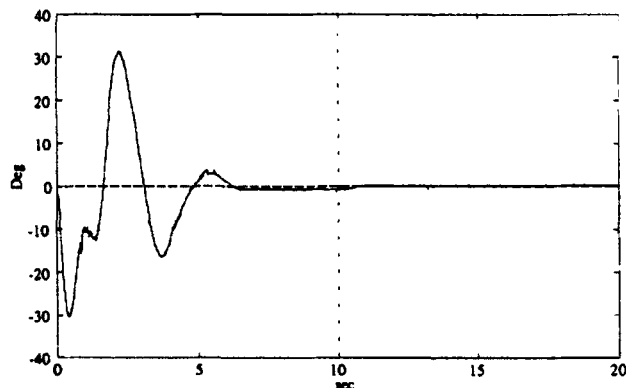


Figure 15. Experiment 4: tip position w.r.t. first link

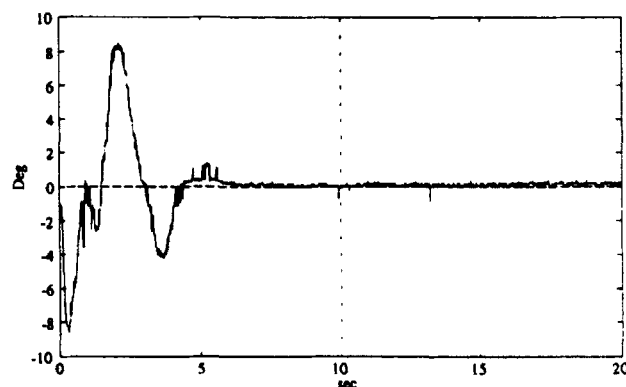


Figure 16. Experiment 4: deflection of second link

7. CONCLUSIONS

A new iterative one-stage scheme able to directly regulate under gravity the end-effector position in robots with joint elasticity or link flexibility has been presented and its convergence properties have been shown.

The algorithm introduced is a one-stage learning process able to build up first the desired gravity compensation and second the required set-point correction due to elasticity for motor variable regulation.

The convergence conditions applied to the case of robots with elastic joints are less restrictive than those for flexible link robots.

In the performed experiments it was found that it is not necessary to wait for the reading of the intermediate steady state configurations for updating the control command. Moving to the next iteration while the variation in the arm configuration drops below a fixed tolerance avoids the drawback of theoretically infinite time needed for exact convergence.

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REFERENCES

1. Takegaki, M. and S. Arimoto, 'A new feedback method for dynamic control of manipulators', *Trans. ASME, J. Dyn. Syst., Meas., Control*, **102**, 119–125 (1981).
2. Tomei, P., 'Adaptive PD controller for robot manipulators', *IEEE Trans. Robotics Automat.*, **RA-7**, 565–570 (1991).
3. Tomei, P., 'A simple PD controller for robots with elastic joints', *IEEE Trans. Automatic Control*, **AC-36**, 1208–1213 (1991).
4. De Luca, A. and B. Siciliano, 'Regulation of flexible arms under gravity', *IEEE Trans. Robotics Automat.*, **RA-9**, 463–467 (1993).
5. De Luca, A. and S. Panzieri, 'Learning gravity compensation in robots: Rigid arms, joint elasticity, link flexibility', *Int. J. Adapt. Control Signal Process.*, **7**, 417–433 (1993).
6. De Luca, A. and S. Panzieri, 'An iterative scheme for learning gravity compensation in flexible robot arms', *Automatica*, **30**, 993–1002 (1994).
7. De Luca, A., L. Lanari, P. Lucibello, S. Panzieri and G. Ulivi, 'Control experiments on a two-link robot with a flexible forearm', *Proc. 29th IEEE Conf. on Decision and Control*, Honolulu, HI, 1990, IEEE, New York, 1990, pp. 520–527.
8. Book, W. J., 'Recursive Lagrangian dynamics of flexible manipulator arms', *Int. J. Robotics Res.*, **3**, 87–101 (1984).
9. De Luca, A. and B. Siciliano, 'Closed-form dynamic model of planar multi-link lightweight robots', *IEEE Trans. Syst., Man, Cyber.*, **SMC-21**, 826–839 (1991).
10. Lucibello, P. and G. Ulivi, 'Design and realization of a two-link direct drive robot with a very flexible forearm', *Int. J. Robotics Automat.*, **8**, 113–128 (1993).