13th Triennial World Congress, San Francisco, USA

CONTROLLABILITY THROUGH ACTIVE FORCES IN COOPERATING ROBOTS WITH GENERAL PAYLOADS [1]

Alessandro De Luca **Raffaella Mattone**

Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza"
Via Eudossiana 18, 00184 Roma, ITALY.
adeluca@giannutri.caspur.it mattone@labrob.ing.uniroma1.it

Abstract. A control-oriented formalism has been introduced by the authors for the modeling of multiple cooperating robots with different types of contacts and handling general payloads, including also payloads with extra degrees of freedom. The extra degrees of freedom of a cooperating robotic system are the generalized coordinates needed to characterize the payload configuration when the end-effector pose of each robot is kept fixed. While the modeling steps give an explicit characterization of this extra dynamics, from the control point of view we face the impossibility of assigning a desired dynamic behavior to the whole system. In particular, when designing active forces in the cooperation task for tracking some output motion trajectories, one should consider also the problem of internal stability of the closed-loop system. With the help of two simple case studies (a linear and a nonlinear one), we examine some possible control approaches that solve the problem for the given class of mechanical systems. The theory of output regulation and that of nonlinear noninteracting control with stability, turn out to be useful in this framework.

Keywords. Robotics; Cooperating Robots; Force Control; Controllability.

1. INTRODUCTION

Modeling and control of robotic systems in strict cooperation tasks is a main area of theoretical investigation and applied research. A typical cooperative system is constituted by two or more robots rigidly grasping and manipulating a single object in the free space. Dynamic models of cooperating robots have been developed by many authors [1]–[5]. In particular, in [6], we have introduced a new formalism for describing multiple cooperating robots, where the carried payload is seen from each robot as a dynamic environment. The introduced formalism allows to deal with general situations in which the commonly held object satisfies kinematic constraints or has dynamic interactions with the rest of the environment. A limiting hypothesis of most literature on modeling and control of cooperating robots, concerns the number of dynamic variables that model the payload, which should equal the number of degrees of freedom canceled by the type of robot grasp. On the other hand, there are situations in which two or more manipulators are required to carry payloads with extra degrees of freedom (dofs). In general, one can define the extra degrees of freedom of a cooperating robotic system as the additional generalized coordinates needed to characterize the payload configuration when the end-effector pose of each robot is kept fixed. Cooperating systems involving payloads with extra dofs can be found in many practical situations of interest, e.g. when large, and thus flexible [7]–[8], or articulated payloads have to be carried, or whenever the number of cooperating robots is not sufficient to impose the motion to all the dofs of the

[1] This work is supported by MURST 40% grant.

common payload. A kinematic analysis of the mobility of these systems has been carried out in [9]. As for the dynamic modeling, we modified in [10] the modeling formalism of [6], so as to cover these situations. While the modeling steps give an explicit characterization of the dynamics of the extra degrees of freedom of the payload, from the control point of view it is clear that it is not anymore possible to assign any desired dynamic behavior to the whole system. In fact, these mechanical systems are *underactuated*, namely they possess a number of independent control inputs which is strictly less than the number of generalized parameters that model the cooperation task. A basic question is whether the extra dofs are *controllable* or not, a property that should be checked in general in a nonlinear setting. This and related issues have been considered for example in [11], using however a linearized model of the cooperating system. Once this condition is verified, one should design control laws that fully exploit controllability in order to prevent an unstable behavior in the closed-loop system. We note that, with the system in motion, the evolution of the extra degrees of freedom act as a disturbance on the controlled variables of a standard hybrid cooperation task, i.e. on the payload dynamic variables and internal forces or, respectively, on the active and internal forces. In principle, these disturbances can be compensated using a model-based input-output decoupling controllers as in [10], [1]–[5]. However, in the presence of extra dofs, the question of internal stability arises. It was shown in [10] that, in the case of two planar robots pushing along a circular trajectory a two-body payload with one extra dof (the orientation angle of the attached swinging body), the relative angular position of the two bodies grows unbounded under nonlinear decoupling control.

In order to solve this control problem, other approaches should be attempted. The key idea is to define the active forces in the cooperating task not only with the goal of imposing a desired behavior to the main motion of the payload (the one described by the dynamic variables of the payload), but also so as to control indirectly the extra dofs dynamics. In this paper, some convenient techniques are proposed and analyzed, that achieve asymptotic tracking of the output trajectories while preserving internal stability in the closed loop. The paper is organized as follows. Starting from a brief description of the modeling approach (Sect. 2), we carry out in Sect. 3 the analysis of two simple case studies, with linear and, respectively, nonlinear dynamics of the extra dofs. In these examples we illustrate two strategies: *output regulation* [12] and *noninteracting control with stability* [13]. On the basis of the performed theoretical analysis, we draw in Sect. 4 some conclusions of practical interest, that can be easily extended to other systems in the class we are considering (mechanical systems which are input-output decoupled and linearized, having a residual, unobservable dynamics).

2. ROBOTS HANDLING PAYLOADS WITH EXTRA DOFS

In this section we briefly recall the modeling procedure for cooperating robots introduced in [6,10], which covers the presence of extra degrees of freedom in the payload. The first step is the characterization of the payload dynamics. In particular, the *extra variables*, s_{OI}, are the generalized coordinates needed to characterize the payload configuration when the end-effector pose of each robot is kept fixed. The *dynamic variables*, s_{OD}, complete the dynamic description of the payload. These variables are used also to individuate the position of the contact points between payload and robots, altough a further set of *kinematic variables*, s_K, is in general necessary to complete the characterization of the end-effector pose of the robots *from the environment side*. At the task level, the kinematic description of the robots by means of s_{OD}, s_K, and their derivatives, is equivalent to the usual one, obtained through the joint variables and velocities, q, $\dot{q}$, and the standard Jacobian of the robots, $J(q)$. Equating these two kinematic descriptions gives the contact constraints in the differential form

$$J\dot{q} = T_K \dot{s}_K + T_D \dot{s}_{OD}, \tag{1}$$

with proper Jacobians T_K and T_D, and where matrix dependence has been dropped.

The second step is the parametrization of the contact forces F by means of the two sets of variables λ_A and λ_R. They parametrize, respectively, the net *active forces* producing motion of the payload, and the *internal forces*, i.e. those forces that do not perform work on any degree of freedom of the system. In particular, one has

$$F = Y_A \lambda_A + Y_R \lambda_R, \tag{2}$$

where Y_A, Y_R can be chosen on the basis of energy arguments and have intrinsic orthogonality properties with the columns of T_D, T_K.

In order to obtain the overall dynamic model, the joint accelerations $\ddot{q}$ are explicited from the robots dynamic model, and substituted in the differentiated expression of the contact constraints. Then, two alternatives are available, corresponding to eliminating from the model the accelerations $\ddot{s}_{OD}$ or the active forces λ_A, expliciting them from the dynamic model of the payload. Due to the definition of s_{OD} and λ_A, both choices are always feasible, resulting in the following alternative forms for system description:

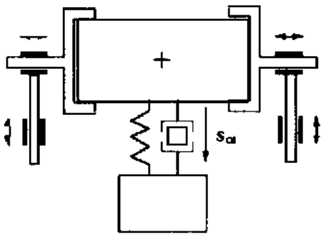

Fig. 1. A cooperating system with linear extra dynamics.

$$Q_A \begin{bmatrix} \lambda_A \\ \lambda_R \\ \ddot{s}_K \end{bmatrix} = m_A + G\tau, \quad \ddot{s}_{OI} = H_{IA}\lambda_A + n_{IA}, \quad (3)$$

$$Q_D \begin{bmatrix} \ddot{s}_{OD} \\ \lambda_R \\ \ddot{s}_K \end{bmatrix} = m_D + G\tau, \quad \ddot{s}_{OI} = H_{ID}\ddot{s}_{OD} + n_{ID}, (4)$$

where all the involved matrices and vectors depend on the dynamic models of the robots and the payload, and on the chosen parametrization of the system motion. The robots input torques at the joints are collected in vector τ.

Under the assumptions of *nonredundancy* of the robots, and nonsingularity of the robot Jacobians, the above representations can be input-output decoupled and linearized by feedback. For example, model (4) becomes

$$\begin{bmatrix} \ddot{s}_{OD} \\ \lambda_R \\ \ddot{s}_K \end{bmatrix} = \begin{bmatrix} u_D \\ u_R \\ u_K \end{bmatrix}, \quad (5)$$

$$\ddot{s}_{OI} = H_{ID}u_D + n_{ID}, \quad (6)$$

where $u = (u_D, u_R, u_K)$ is the new system input. We refer to the dynamics of eq. (6) as *extra dynamics*, distinguishing it from the output dynamics (5) which is now linear and decoupled. Note that this choice of the system outputs is suggested not just by formal considerations. In fact, variables s_{OD}, s_K, and λ_R (as well as λ_A) are related by means of invertible functions to the joint angles and velocities, q, $\dot{q}$, and the contact forces F, i.e. the system variables that can be typically measured.

3. OUTPUT TRACKING WITH INTERNAL STABILITY

In the previous section we have shown how the system model can be put in a suitable form, namely input-output decoupled and linearized, so that the trajectory tracking problem has a trivial solution in the case of standard payloads, i.e., when the s_{OI} variables are not in the picture. On the other hand, the presence of extra degrees of freedom introduces an unobservable dynamics which may exhibit an unstable behavior, when the outputs are required to follow particular trajectories. Thus, the issue of internal stability must also be addressed in this case. In order to examine suitable approaches to the solution of this problem, we present two simple case studies, having an *extra dynamics* which is, respectively, linear and nonlinear. In both cases we concentrate on the payload dynamics, which involves the more important control problems, and leave out the kinematic degrees of freedom and the internal forces, which can be directly and independently controlled and do not influence the payload motion.

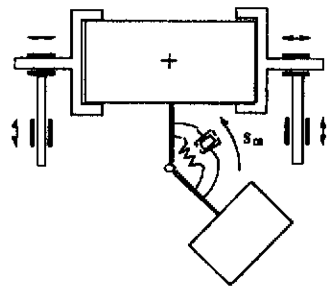

Fig. 2. A cooperating system with nonlinear extra dynamics.

3.1 *A linear example*

Consider two planar, cartesian robots as in Fig.(1), rigidly grasping one object of a payload constituted by two rigid objects connected by linear spring and damper. The s_{OD} variables are the X and Y coordinates of the grasped object, while the single extra dof is the relative displacement, s_{OI}, of the second body of the payload with respect to the grasped one, i.e. the deformation of the spring. This system is linear and, after performing the modeling steps and the input-output decoupling, its second order model is

$$\ddot{s}_{OD} = u,$$
$$\ddot{s}_{OI} = -c_1 k s_{OI} - c_1 d \dot{s}_{OI} + c_2 u_2,$$

where k and d are, respectively, the elastic and the damping coefficients, while the positive constants c_i depend on the masses of the two payload bodies. Choosing as state vector $x = (s_{OD}, s_{OI}, \dot{s}_{OD}, \dot{s}_{OI}) \in I\!R^6$, the matrices that characterize the state-space representation of the system are

$$A = \begin{bmatrix} 0_{3,3} & & I_3 & \\ 0_{2,3} & & 0_{2,3} & \\ 0_{1,2} & -c_1 k & 0_{1,2} & -c_1 d \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3,2} \\ 0_{2,2} \\ 0 \;\; c_2 \end{bmatrix}, (7)$$

$$C = \begin{bmatrix} I_2 & 0_{2,1} & 0_{2,2} & 0_{2,1} \end{bmatrix}, \quad (8)$$

where $0_{i,j}$ is the $i \times j$ matrix of zero entries, and I_l is the l-dimensional identity matrix.

In order to solve the problem of output trajectory tracking with internal stability, we can attempt,in this case, a linear regulation approach [12]. For the sake of simplicity, suppose that the whole state of the system is measurable, i.e. that *complete information* on the system is available. Suppose also that the reference trajectories of interest, $y_d(t)$, are *canonical*, i.e., can be generated by an autonomous, linear *exosystem* with dynamic matrix S. We want to solve the problem by means of a static state feedback law of the form

$$u = \alpha x + \beta y_d, \tag{9}$$

with α and β constant matrices of proper dimensions. In particular, consider the following hypotheses:

(H1) *The eigenvalues of S have nonnegative real part.*
(H2) *The matrix pair (A,B) is stabilizable.*

Note that (H1) implies that the problem cannot be trivially reduced to the stabilization of the system. If (H1) and (H2) are verified, a necessary and sufficient condition for the solution of the problem is the existence of two matrices Π and Γ that solve

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma, \\ 0 &= -C\Pi + I, \end{aligned} \tag{10}$$

where I is the identity matrix of proper dimensions. A sufficient condition for the solvability of the above equations is that the following matrix

$$\Sigma = \begin{pmatrix} A - \lambda I & B \\ C & 0 \end{pmatrix}, \tag{11}$$

has full row rank for each λ which is an eigenvalue of S.

Let us check the above conditions for system (7–8). It can be easily verified that the pair (A,B) is controllable, and thus stabilizable, if and only if the spring coefficient k is not zero (in general, a position-dependent term in the extra dynamics must be present for the controllability of the system). Moreover, matrix Σ is full rank for every λ that is not an eigenvalue of the extra dynamics (this is always true for second order linear systems that are input-output decoupled). Thus, we can design a feedback law of the form (9) that makes the s_{OD} variables asymptotically track any canonical trajectory (except the natural modes of the extra dynamics), while preserving the internal stability of the system. In particular, we can choose

$$u = Kx + (\Gamma - K\Pi) y_d, \tag{12}$$

with Π and Γ solutions of eqs.(10), and K any matrix that assigns to $A + BK$ eigenvalues with negative real part.

3.2 *A nonlinear example*

Consider the cooperating system in Fig.(2), quite similar to that of the previous case study. The s_{OI} variable is torsional and the system turns out to be nonlinear. After the modeling steps, and the input-output decoupling and linearizing feedback, its reduced dynamic model is

$$\begin{aligned} \ddot{s}_{OD} &= u, \\ \ddot{s}_{OI} &= -c_1 k s_{OI} - c_1 d \dot{s}_{OI} \\ &\quad -c_2 \cos(s_{OI}) u_1 - c_2 \sin(s_{OI}) u_2, \end{aligned} \tag{13}$$

or, equivalently, in first order form

$$\begin{aligned} \dot{x} &= \begin{pmatrix} \dot{s}_{OD} \\ \dot{s}_{OI} \\ 0 \\ 0 \\ f_I \end{pmatrix} + \begin{pmatrix} 0_{2,1} \\ 0_{1,1} \\ 1 \\ 0 \\ g_{I1} \end{pmatrix} u_1 + \begin{pmatrix} 0_{2,1} \\ 0_{1,1} \\ 0 \\ 1 \\ g_{I2} \end{pmatrix} u_2 \\ &= f(x) + g_1(x) u_1 + g_2(x) u_2, \\ y &= s_{OD} = h(x), \end{aligned} \tag{14}$$

where $x = (s_{OD}, s_{OI}, \dot{s}_{OD}, \dot{s}_{OI}) \in \mathbb{R}^6$, $u = (u_1, u_2)$, and c_1, c_2 are positive constants depending on the inertial parameters of the system.

Although we could apply a nonlinear version of the regulation approach [12], we pursue here a different strategy that achieves the asymptotic output tracking, while preserving the internal stability of the whole system. In particular, we look for a *noninteracting control with global stability* [13], i.e., a feedback law that makes the system input-output decoupled and globally asymptotically stable at a given equilibrium point $x^0 = (s^0_{OD}, 0, 0, 0)$. Then, by proving further the input-to-state (or bounded input-bounded state) stability of the extra dynamics, we will assure the desired behavior to the system. As in the linear case, some assumptions allow to obtain conditions for the existence of a control law that solves the problem. The first two hypotheses are:

(H1) *The system has uniform vector relative degree.*
(H2) *The system is globally strongly accessible.*

The first one guarantees the nonsingularity of the input-output decoupling matrix and is always verified in all cases we are interested in, as it is appearent from eq. (5). The second hypothesis corresponds to the stabilizability of (A,B) in the linear case, but it is much weaker in general. Property (H2) holds for model (14), as can be readily verified building the strong accessibility distribution of the system [14]. In order to state the third hypothesis, some definitions are necessary. Let $\mathcal{G}$ be the distribution spanned by vector fields g_i, $h_i(x)$ the ith output function, and $\mathcal{K}_i$ the distribution spanned by the vector fields which are *orthogonal* to dh_i, $i = 1, \ldots, m$ (in (14), $m = 2$). Let then $\mathcal{V}_i^*$ and $\mathcal{R}_i^*$ be, respectively, the *maximal locally controlled invariant* and the *maxi-*

mal local controllability distribution for system (14) contained in $\cap_{j\neq i}\mathcal{K}_j$, both supposed to exist. Denote by $\mathcal{R}^* = \cap_{i=1}^m \sum_{j\neq i} \mathcal{R}_j^*$. These distributions can be computed by means of the algorithms given in [13] and are said to be *regularly computable* (at x^0) if the distributions given at each step by these algorithms have constant dimension in a neighborhood of x^0. Then, the last hypothesis we need is

(H3) *The distributions $\mathcal{V}_i^*$, $\mathcal{R}_i^*$, $i = 1,\dots,m$, are regularly computable at any x. Moreover, the distributions $\sum_{j\in I}\mathcal{R}_j^*$, $\mathcal{R}^*$, $(\sum_{j\in I}\mathcal{R}_j^*)\cap\mathcal{G}$, $I \subset \{1,\dots,m\}$, have constant dimension at any x.*

For system (14), after performing some computation steps, we have that, at any x,

$$\mathcal{V}_1^* = \mathcal{R}_1^* = \mathcal{K}_2 = \text{span}\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_6}\right\},$$

$$\mathcal{V}_2^* = \mathcal{R}_2^* = \mathcal{K}_1 = \text{span}\left\{\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}\right\},$$

$$\mathcal{R}^* = \text{span}\left\{\frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_6}\right\},$$

so that also (H3) is verified.

At this point, for a generic nonlinear system, one should apply *any* invertible static feedback law that makes the system *noninteracting*. In our case, this provides already the closed-loop system in the form (14), which holds globally. In this situation, we have on one side chains of double input-output integrators that can be globally asymptotically stabilized to the origin through a (linear) feedback which depends only on the local state, i.e. $u_i = u_i(s_{OD,i}, \dot{s}_{OD,i})$, for $i = 1, 2$. On the other hand, there is the second-order extra dynamics of s_{OI}. More in general, this dynamics, which is *unobservable* from the output, takes the form

$$\dot{z} = \tilde{f}(\xi, z) + \tilde{G}(\xi, z)u,$$

where ξ are the states of the input-output dynamics. In our example, we have $z = (s_{OI}, \dot{s}_{OI})$ and, since we are looking for asymptotic stabilization of x^0, we can set $\xi = (s_{OD} - s_{OD}^0, \dot{s}_{OD})$. Then, the whole system can be made noninteractive and globally asymptotically stable in x^0 if and only if the dynamics

$$\dot{z} = \tilde{f}(0, z), \tag{15}$$

is globally asymptotically stable at the origin.

Thus, the only condition to verify is that the zero dynamics of the system, i.e. the extra dynamics with zero input, is globally asymptotically stable. For the case study (14) this is verified if and only if k and d are both strictly positive. If this is the case, the static feedback law that stabilizes the outputs to the origin, also globally stabilizes the whole system to x^0.

If the condition on the dynamics (15) were not verified, one may attempt to solve the problem by enlarging the class of admissible controllers to dynamic feedback laws of the form

$$u_i = \alpha_i(x, w) + \sum_{j=1}^m \beta_{ij}(x, w)v_j, \quad i = 1,\dots,m,$$

$$\dot{w} = \gamma(x, w) + \sum_{j=1}^m \delta_j(x, w)v_j.$$

However, in our case study it can be shown that the necessary condition for obtaining internal stability even when using dynamic state feedback (see [13]) is still the asymptotic stability of dynamics (15).

Once the global asymptotic stability of the system has been assured, one has finally to check if the boundedness of the system state is preserved, when a bounded input is applied. This is required because we want to perform tracking of output trajectories and not just a stabilization to an equilibrium point. Equivalently, one has to test the *input-to-state stability* (ISS) of the extra dynamics of the system (see, e.g., [15] for a review of various equivalent definitions). Verifying the ISS property is not a trivial task, in general. Recall that all functions $\gamma : \mathbb{R}_{\geq} \to \mathbb{R}_{\geq}$, which are continuous, strictly increasing and satisfy $\gamma(0) = 0$ are called class $\mathcal{K}$ functions. A nonlinear system is input-to-state stable if and only if there exist a *storage* function (i.e. continuously differentiable, radially unbounded and definite positive), $V(x)$, and two class $\mathcal{K}$ functions α and χ so that the implication

$$|x| \geq \chi(|u|) \Longrightarrow \dot{V}(x, u) \leq -\alpha(|x|), \tag{16}$$

holds for each state x and control value u. For the extra dynamics of the case study (14) we have $\dot{x} = Ax + G(x)u$, with $x = (s_{OI}, \dot{s}_{OI})$, and

$$A = \begin{pmatrix} 0 & 1 \\ -c_1 k & -c_1 d \end{pmatrix}, G(x) = \begin{pmatrix} 0 & 0 \\ -c_2\cos x_1 & -c_2 \sin x_1 \end{pmatrix}.$$

If $\dot{x} = Ax$ is asymptotically stable, it is always possible to find a symmetric, positive definite matrix Q such that $QA + A^TQ = -P$, being P any symmetric, positive definite matrix. Then, choose $V(x) = x^TQx$. We have

$$\dot{V} = -x^TPx + 2x^TQG(x)u \leq$$
$$\leq -x^TPx + |x^T(2QG(x))u|.$$

Assuming $|x| \geq |u|$, it holds

$$\dot{V} \leq -x^TPx + |x^T(2QG(x))x|.$$

As matrix $G(x)$ is bounded (only trigonometric functions are present) implication (16) is certainly verified for some values of the parameters k, d, c_1, c_2 and the corresponding system is input-to-state stable.

4. CONCLUSIONS

For two simple, but representative case studies of cooperating robots handling payloads with extra dofs, we have analyzed the possibility of achieving asymptotic output tracking preserving the internal stability of the system. In both cases we started from the input-output decoupled model, obtained through the application of model-based hybrid force-motion controllers that are of standard use in the literature on cooperating robot control.

In the linear case we have shown that, if the system is controllable, output tracking with internal stability can be achieved for any canonical output trajectory not coincident with the natural modes of the extra dynamics. This is an outcome of output regulation theory.

In the nonlinear case study we have tried to apply the theory of noninteracting control with stability. The analysis was more complicated because, in order to have output tracking and internal stability, one has first to prove the global stabilizability of the system at an equilibrium point, and then the input-to-state stability of the extra dynamics. In our case study, the global asymptotic stability of the zero dynamics (the extra dynamics with zero inputs) is a necessary and sufficient condition for obtaining noninteraction with global stability via static state feedback. Furthermore, if this condition is satisfied, the same feedback law that stabilizes the output, globally stabilizes also the whole system.

Finally we have proved that, for suitable values of the parameters characterizing the considered system, this is also input-to-state stable, so that internal stability is assured when a bounded input is applied in order to stabilize the output to a desired trajectory.

When compairing the two proposed control approaches one recognizes that the design of a regulator is simpler even in a nonlinear setting. However we destroy in this case the perfect decoupling of output variables, with respect to the more complex design of a noninteracting controller with internal stability.

We remark that the whole analysis performed in the nonlinear case study, strongly relies only on the structure of the system and thus can be easily extended to all mechanical systems within this class.

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