Abstract—It is common knowledge that tractor-trailer vehicles are affected by jackknifing, a phenomenon that consists in the divergence of the trailer hitch angle and ultimately causes the vehicle to fold up. For the case of backwards motion, in which jackknifing can also occur at low speeds, we present a control method that drives the vehicle along a reference Cartesian trajectory while avoiding the divergence of the hitch angle. In particular, a feedback control law is obtained by combining two actions: a tracking term, computed using input-output linearization, and a corrective term, generated via IS-MPC, an intrinsically stable MPC scheme which is effective for stable inversion of nonminimum-phase systems. The proposed method has been verified in simulation and experimentally validated on a purposely built prototype.

I. INTRODUCTION

Many vehicles are equipped with trailers to increase their payload capacity as well as their maneuverability. Obviously, driving and controlling tractor-trailer systems is more complicated than single-body vehicles. One issue that arises during the motion of tractor-trailer vehicles is the so-called jackknifing phenomenon. This is a situation in which the hitch angle between the tractor and trailer grows so that the vehicle folds on itself, becoming uncontrollable and possibly leading to collisions.

As any caravan owner finds out by experience, jackknifing is a serious issue during backup maneuvers. However, truck drivers know that it may also arise in forward motion, when braking or turning abruptly. The nature of the phenomenon is however essentially different in the two cases: in backward motion, jackknifing is a kinematic issue which may occur even at very low speed, whereas in forward motion it is a dynamic (indeed, inertial) effect that is related to slippage and only takes place at high speed.

In the literature, there are a number of works that have proposed control schemes for tractor-trailer systems. For a vehicle with a zero-hooked trailer, the controller of [1] allows to track rectilinear/circular Cartesian paths using input/state linearization and time scale transformation. The same kind of reference paths were considered in other papers dealing with the general (i.e., zero or nonzero-hooked) 1-trailer system: for example, linear design techniques are used in [2]; the authors of [3] rely on exact linearization and the definition of a ghost vehicle; a controller is proposed in [4] that switches between different linear controllers for forward and backward motions; and finally [5] employs a Lyapunov-based approach.

Coming to trajectory tracking, the feedback control scheme of [6] can drive a general 1-trailer system along a generic backward trajectory by transforming the control inputs of a virtual vehicle that moves forward along the same trajectory. The approach of [7] is to design a low-level controller for the hitch angle, so as to obtain a simplification of the vehicle model and therefore of the associated control problem. Finally, in [8], a two-level trajectory tracking controller is proposed for a zero-hooked 1-trailer system.

In this paper, we focus on the kinematic jackknifing that occurs along backward motions. Our starting observation is that such phenomenon is a manifestation of the instability of the residual internal dynamics associated to output trajectory tracking; or, in other words, of the nonminimum-phase nature of the system in backward motion.

We therefore propose to build a feedback control law as the combination of two actions: a tracking term, computed using input-output linearization, and a corrective term, aimed at avoiding the internal state divergence and generated via Model Predictive Control (MPC). For the latter, we apply IS-MPC, an intrinsically stable framework that we have developed and successfully used for humanoid gait generation [9], [10], another context that requires stable inversion of a nonminimum-phase system.

In particular, since IS-MPC applies to linear systems, we compute the linear approximation of the tractor-trailer system around a suitable state trajectory and use it as a prediction model for IS-MPC block. The latter includes an explicit stability constraint whose role is to counteract the divergence of internal dynamics. The resulting method has been verified in simulation and experimentally validated on a purposely built prototype.

With respect to the above mentioned literature, the proposed control method does not pose any geometric limitation on the reference trajectory, applies to both zero- and nonzero-hooked trailer systems, and — thanks to the use of MPC — allows the inclusion of state and input constraints in the control problem. The latter feature is practically relevant, e.g., for taking into account mechanical limitations (joint limits) of the vehicle, for avoiding workspace obstacles during the motion, or to comply with the presence of actuator saturations.

The paper is organized as follows. In Section II we introduce the considered vehicle control problem and offer a related interpretation of the jackknife phenomenon. In Section III, we describe in general terms the proposed control approach, and then in detail the generation of the auxiliary trajectory and the IS-MPC algorithm. Simulations and experi-
ments are presented in Sect. IV and V, respectively. Finally, some future work is mentioned in Section VI.

II. THE CONTROL PROBLEM

In this section we introduce the considered vehicle, state the control problem and provide an interpretation of the jackknife phenomenon in this context.

A. Kinematic modeling

Consider the vehicle shown in Fig. 1 consisting of a car-like tractor towing a single\(^1\) trailer. Denote by \(x, y\) the coordinates of the tractor rear axle midpoint, and by \(\theta, \phi\) and \(\psi\) respectively the tractor heading, the steering angle and the hitch angle (i.e., the relative orientation of the trailer with respect to the tractor). Also, let \(\ell_1\) and \(\ell_2\) be the length of the tractor and the trailer, and \(\ell_h\) the distance between the tractor rear axle midpoint and the hitch joint axis. Throughout the paper we assume \(\ell_h > 0\) (nonzero hooking).

If no wheel slip occurs (an hypothesis that is consistent with the low speed typically associated to backup maneuvers), the kinematic model [11] of the vehicle is derived as

\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \frac{v \tan \phi}{\ell_1} \\
\dot{\psi} &= -\frac{v \tan \phi}{\ell_1} \left(1 + \frac{\ell_h}{\ell_2} \cos \psi\right) - \frac{v \sin \psi}{\ell_2} \\
\dot{\phi} &= \omega,
\end{align*}
\]

where \(v\) and \(\omega\) are respectively the driving and steering velocities, taken as control inputs.

In the following, we will denote by \(q = (x, y, \theta, \psi, \phi)\) the configuration vector of the vehicle.

B. Internal instability under tracking control

Assume that a Cartesian reference trajectory \((x_r(t), y_r(t))\) is assigned to be tracked by the vehicle. From a control viewpoint, this is simply an output trajectory — an associated state trajectory is not given and indeed must be determined by the controller.

The most direct way to design a tracking controller is to use input-output linearization via feedback. Ideally, one would like to track the reference trajectory with the vehicle representative point \((x, y)\). However, this cannot be achieved by static feedback because the decoupling matrix turns out to be singular. A possible workaround is to choose as output a different point \(P\) with coordinates \((x_P, y_P)\), as shown in Fig. 1. One easily finds

\[
\begin{pmatrix}
\dot{x}_P \\
\dot{y}_P
\end{pmatrix} = T(\theta, \phi) \begin{pmatrix} v \\ \omega \end{pmatrix}
\]

with

\[
T(\theta, \phi) = \begin{pmatrix} c\theta - \frac{t\phi}{t} (\ell_1 s\theta - d s(\theta + \phi)) & d s(\theta + \phi) \\ s\theta - \frac{t\phi}{t} (\ell_1 c\theta - d c(\theta + \phi)) & d c(\theta + \phi) \end{pmatrix}
\]

having set for compactness \(s = \sin, c = \cos, t = \tan\) in this expression. Since \(\det T = d/\cos \phi\), matrix \(T\) is invertible if \(d\) is nonzero. Under this assumption, one can achieve input-output linearization by using the feedback transformation

\[
\begin{pmatrix} v \\ \omega \end{pmatrix} = T^{-1}(\theta, \phi) u
\]

where \(u\) is the new control vector. If now we set \(u = u_{\text{track}}\), with

\[
u_{\text{track}} = \begin{pmatrix} k_x (x_r - x_P) \\ k_y (y_r - y_P) \end{pmatrix}
\]

and \(k_x, k_y > 0\), the tracking error will converge exponentially to zero for any initial condition.

However, the evolution of the remaining variables \(\theta, \psi\) and \(\phi\) (the so-called zero dynamics) is not controlled in this scheme. In particular, one may verify that if the vehicle moves backwards along the reference trajectory, variables \(\theta\) and \(\psi\) diverge\(^2\). This may be verified analytically by computing the linear approximation of the system dynamics (1) around the reference trajectory and observing that it is unstable due to the presence of two eigenvalues with positive real part; or in simulation, as will be shown in Sect. IV.

In practice, the divergence of \(\theta\) and, in particular, of \(\psi\) corresponds to the occurrence of the jackknife phenomenon for the vehicle. Its control interpretation is therefore straightforward: jackknifing is a manifestation of the instability of the zero dynamics associated to the tracking controller (2–3).

\(^1\)The control design to be presented does not exploit in any way the fact that a single trailer is present. Therefore, our method is in principle applicable to vehicles with more than one trailer, including the so-called general \(n\)-trailer system, which is non-flat.

\(^2\)As for the steering angle \(\phi\), it remains bounded if \(d > 0\), i.e., if point \(P\) is located behind the front axle of the tractor, as in Fig. 1. The situation is reversed along forward trajectories: \(\phi\) diverges if \(d > 0\) and remains bounded if \(d < 0\). Since in this paper we focus on backward trajectories, we will assume \(d > 0\).
III. THE PROPOSED APPROACH

We now describe the proposed method for dealing with the instability problem discussed above. First, we provide an overview of our approach; then we give a detailed discussion of its main ingredients.

A. Overview

The basic idea of our approach is to keep working on the input-output linearized system, adding to the pure tracking control action (3) a corrective action aimed at avoiding the onset of the instability

\[ u = u_{\text{track}} + u_{\text{corr}}, \]

with \( u_{\text{corr}} \) generated by intrinsically stable MPC (IS-MPC, see [10]). Since the latter applies to linear systems, we will derive the linear approximation of the vehicle dynamics (1) around an auxiliary state trajectory, whose construction is explained in Sect. III-B. Both the auxiliary trajectory and the linear approximation (which is obviously time-varying) are recomputed in real time. The latter is then fed to the IS-MPC block.

Figure 2 shows a block scheme of the proposed control approach. Due to the presence of an MPC module, our control algorithm works in discrete-time, producing control inputs \( u \) that are piecewise-constant over sampling intervals of duration \( \delta \).

B. Linearization around an auxiliary trajectory

The auxiliary state trajectory\(^3\) \( q_{\text{aux}}(t) \) should provide a basis for the approximate linearization procedure. Note that the assigned reference trajectory for the output does not directly entail a state trajectory. Our idea is then to generate a stable state trajectory (i.e., a trajectory along which \( \theta, \psi \) and \( \phi \) do not diverge) compatible with the reference output trajectory by reversing the evolution generated by pure tracking in forward motion, with an appropriate retrograde initialization.

In particular, call \( t_k \) the current time instant at which the computation is performed. Given a \( T > 0 \), let \( \tau_0 = t_k - T \) be the (past) initial time instant, and initialize\(^4\) the auxiliary trajectory at

\[ q_{\text{aux}}(\tau_0) = (x_P(\tau_0) \ y_P(\tau_0) \ \text{ATAN2}(y_P(\tau_0), x_P(\tau_0)) \ 0 \ 0). \]

The auxiliary state trajectory \( q_{\text{aux}} \) with the associated control input \( u_{\text{aux}} = u_{\text{track}}(q_{\text{aux}}) \) are generated by integrating model (1) from \( q_{\text{aux}}(\tau_0) \) up to \( \tau = t_k \) under the control law (2–3), and then reversing time \((t = 2t_k - \tau)\).

At this point, it is possible to compute the linear approximation of model (1), subject to the control law (2–4), around the auxiliary trajectory \( q_{\text{aux}} \). This leads to a linear model of the form

\[ \dot{e} = A(q_{\text{aux}}(t), u_{\text{aux}}(t))e + B(q_{\text{aux}}(t))u_{\text{corr}} \]

with \( e = q - q_{\text{aux}} \). This model is obviously time-varying due to the dependence on the auxiliary trajectory. For the subsequent developments, we approximate (5) with the following piecewise-time-invariant system

\[ \dot{e} = A_k e + B_k u_{\text{corr}}, \quad t \in [t_k, t_{k+1}] \]

where \( A_k = A(q_{\text{aux}}(t_k), u_{\text{aux}}(t_k)) \) and \( B_k = B(q_{\text{aux}}(t_k)) \). This 5-dimensional system can be partitioned in two subsystems, one stable and one unstable. In formulas, there exists a change of coordinates such that in the new coordinates \( \hat{e} \) the system takes the form

\[ \begin{pmatrix} \dot{\hat{e}}_s \\ \dot{\hat{e}}_u \end{pmatrix} = \begin{pmatrix} \Lambda_{s,k} & 0 \\ 0 & \Lambda_{u,k} \end{pmatrix} \begin{pmatrix} \hat{e}_s \\ \hat{e}_u \end{pmatrix} + \begin{pmatrix} G_{s,k} \\ G_{u,k} \end{pmatrix} u_{\text{corr}}, \]

with the two subsystems respectively 3- and 2-dimensional. This is consistent with the previous discussion on the internal instability, and in particular with the fact that the dynamics of both \( \theta \) and \( \psi \) are unstable along backward trajectories.

\(^3\)An alternative to linearizing (1) around a trajectory would be to compute the linearization around the current system state; since this state is not an equilibrium for the closed-loop dynamics, an affine system would be obtained, to which IS-MPC can still be applied. We will not pursue this possibility in the present paper.

\(^4\)The specific initialization of \( \theta, \psi \) and \( \phi \) is not important as long as it produces an auxiliary state trajectory that tracks the reference trajectory in forward motion. The above choice (with point \( P \) on the reference trajectory, the tractor oriented as the forward tangent to the reference trajectory, trailer aligned with the tractor, and zero steering angle) guarantees the desired behavior. As for \( T \), it should be sufficiently large for the transient to be practically over in \( t_k \). Moreover, it should be larger than the control horizon of the MPC trajectory (see Sect. III-C).
C. IS-MPC

In the proposed control scheme, the role of IS-MPC is to compute the control correction term $u_{\text{corr}}$ so as to guarantee internal stability.

Denote by $T_c = N \cdot \delta$ the control horizon over which control corrections are generated. As prediction model, we use the block-partitioned model (6).

To guarantee internal stability, we take inspiration from [10] and introduce a stability constraint that imposes a condition on the future control inputs so as to guarantee that the internal dynamics does not diverge (in particular, it is the condition under which the free evolution exactly cancels the divergent component of the forced evolution). In the present case, the stability constraint takes the following form

$$\sum_{i=k}^{\infty} \Lambda_{u,i}^{-1}e^{-\Lambda_{u,i}(t_i-t_k)} \left(e^{-\Lambda_{u,i}\delta} - I\right) G_{u,i} u_{\text{corr},i} = \tilde{e}_u(t_k).$$

(7)

The left hand side contains the variables of the MPC problem, i.e., $u_{\text{corr},i}$ for $i = k, \ldots, k+N$, but also the corrective actions $u_{\text{corr},i}$ for $i > k + N$ after the control horizon. The latter, collectively referred to as the tail, are obviously unknown, and they must be conjectured in order to obtain a causal expression that can be computed at $t_k$. Possible tails in IS-MPC [10] include the truncated tail, which corresponds to setting $u_{\text{corr},i} = 0$ for $i > k + N$; and the periodic tail, obtained by replication of the corrective actions within the control horizon. This replication may be infinite or finite; in the second case, the remaining part of the tail is truncated.

In addition to the stability constraint, the MPC framework also allows us to introduce practically relevant kinematic constraints on the hitch angle $\psi$ and the steering angle $\phi$:

$$|\psi| \leq \psi_{\text{max}} \quad |\phi| \leq \phi_{\text{max}}$$

(8)

where $\psi_{\text{max}}$ and $\phi_{\text{max}}$ are the mechanical limits on the corresponding joints. These constraints are still linear when expressed in the transformed state coordinates $\tilde{\varepsilon}$.

The IS-MPC algorithm solves at each iteration the following QP problem:

$$\min \sum_{i=1}^{N} \|u_{\text{corr},i}\|^2$$

subject to:

- stability constraint (7)
- kinematic constraints (8)

The actual expression of the stability constraint (7) will depend on the chosen tail (truncated, infinite-periodic, finite-periodic). As customary with MPC, only the first corrective action $u_{\text{corr},k}$ is actually used as real-time control, and a new QP problem is set up and solved at the next sampling instant.

IV. SIMULATIONS

The proposed method has been simulated in MATLAB for a tractor-trailer vehicle with the same kinematic characteristics of our prototype (to be described in the next section), i.e., $\ell_1 = 0.25$ m, $\ell_2 = 0.26$ m, $\ell_h = 0.07$ m, $\psi_{\text{max}} = 45^\circ$, $\phi_{\text{max}} = 15^\circ$. The parameters needed by our control scheme have been set to $d = 0.05$ m and $\ell_1 = \ell_2 = 1$. The sampling interval is $\delta = 0.1$ s, while the MPC control horizon is $T_c = 1$ s. A finite-periodic tail consisting of 4 replications was used in the stability constraint (7).

In the first simulation, the vehicle is assigned a rectilinear reference trajectory to track. As shown in Fig. 3, inclusion of the corrective action generated by IS-MPC in the control law $(u = u_{\text{track}} + u_{\text{corr}})$ successfully prevents the occurrence of the jackknife phenomenon, producing stable tracking of the reference trajectory. For comparison, Fig. 4 shows what happens if no correction is added $(u = u_{\text{track}})$: as expected, jackknife occurs as both $\theta$ and $\phi$ diverge.

An eight-shaped reference trajectory is assigned in the second simulation. The results, shown in Figs. 5–6, confirm the effectiveness of our method. The same kind of performance was achieved over a variety of reference trajectories.

The accompanying video contains video clips of the above two simulations.

V. EXPERIMENTS

To carry out an experimental validation of our method, we have built a prototype tractor-trailer system using a commercial radio-controlled model (Carson Unimog U300, a 1:12 replica of the Mercedes Unimog U300 truck) which has been modified and instrumented so as to allow the implementation of the proposed controller. In particular, we replaced the original electronics with an Arduino Uno microcontroller board; moreover, we added an H bridge for driving the two DC motors, encoders on wheels and a Bluetooth module for communication.

The control parameters (including the sampling interval, the MPC control horizon and the choice of the tail for the stability constraint) are identical to those used in the simulations.

Experimental results on rectilinear (Fig. 7–8) and circular (Fig. 9–10) reference trajectories fully corroborate the positive outcome of the simulations. See the accompanying video for video clips of these experiments.

VI. CONCLUSIONS

Tractor-trailer vehicles are affected by jackknifing, a phenomenon that consists in the divergence of the trailer hitch angle and ultimately causes the vehicle to fold up. For the case of backwards motion, in which jackknifing can also occur at low speeds, we have presented a control method that can drive the vehicle along a reference Cartesian trajectory while avoiding the divergence of the hitch angle. In particular, our feedback control law was designed as the combination of two actions: a tracking term, computed using input-output linearization, and a corrective term, generated via IS-MPC, an intrinsically stable MPC scheme which is
Fig. 3. Simulation 1: Stable tracking of a rectilinear trajectory by the proposed method (top, the tractor is in red); the associated velocity inputs $v$ and $\omega$ (bottom).

Fig. 4. Simulation 1: Jackknife occurs if no corrective action is added (zoom on the initial part of the motion).

Fig. 5. Simulation 2: Stable tracking of an eight-shaped trajectory by the proposed method (top); the associated velocity inputs $v$ and $\omega$ (bottom).

Fig. 6. Simulation 2: Jackknife occurs if no corrective action is added (zoom on the initial part of the motion).
effective for stable inversion of nonminimum-phase systems. The proposed method has been verified in simulation and experimentally validated on a purposely built prototype.

In the future, we intend to expand this approach by addressing several points, such as:

- devising a robust version of the proposed controller to handle external disturbances, following the approach in [12];
- taking into account the presence of saturations of the input velocities, which is possible with the proposed framework but was not addressed here;
- the application to the case of multiple trailers;
- the extension of the proposed control method for counteracting the dynamic jackknife phenomenon associated to wheel slippage in high-speed forward motion.
REFERENCES


