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On the Feedback Linearization of Robots with Variable Joint Stiffness

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***Abstract*— Physical human-robot interaction requires the development of safe and dependable robots. This involves the mechanical design of lightweight and compliant manipulators and the definition of motion control laws that allow to combine compliant behavior in reaction to possible collisions, while preserving accuracy and performance of rigid robots in free space. In this framework, great attention has been given to robots manipulators with relevant elasticity at the joints/transmissions. While the modeling and control of robots with elastic joints of finite but constant stiffness is a well-established topic, few results are available for the case of robot structures with variable joint stiffness –mostly limited to the 1-dof case. We present here a basic control study for a general class of multi-dof manipulators with variable joint stiffness, taking into account different possible modalities for changing the joint stiffness on the fly by an additional set of commands. It is shown that nonlinear control laws, based either on static or dynamic state feedback, are able to exactly linearize the closed-loop equations and allow to simultaneously impose a desired behavior to the robot motion and to the joint stiffness in an decoupled way. Illustrative simulations results are presented.**

***Index Terms*— Robot Manipulators, Elastic Joints, Variable Stiffness, Feedback Linearization, Nonlinear Systems.**

I. Introduction

One of the current challenges in robotics is the introduction of robots in the human environment and, as a consequence, the human-robot interaction/cooperation. The motivations of the growing interest on these topics can be found on the possibility of employing robots for human everyday activities, for operations in dangerous environments or for the assistance to elder or handicapped people. The main requirements for the introduction of robots in the human environment are safety and dependability of the robotic system [1], [2]. In [3] it has been shown how these requirements exclude the use of standard industrial robots for the interaction with humans, because of the intrinsic limitations on the safety of these devices due to the inertia of the links and to the magnitude of the torque that the actuators can apply.

A new generation of lightweight robots has been developed to overcome many of the limitations of standard industrial manipulators in terms of performance, portability and dexterity [4], [5]. In [6] it has been shown how these devices can ensure a safe interaction with an unknown environment and with humans. Anyway, these goals have been achieved with a significant increment of the cost of the overall robotic device, due to the use of composite materials and high-performance sensors and actuators.

As an alternative way to make robots intrinsically safe, it has been shown in [2] how the safety of robotic arms can be improved, besides maintaining a low level of inertia, introducing also an high compliance at the mechanical level both in the joints of the robot and in the interface between the robot and the environment. In order to obtain also an adequate level of both static and dynamic performances, the use of variable stiffness devices allows to satisfy all the requirements for a safe and accurate interaction with humans and unknown environments. With the aim of verifying the effectiveness of this approach, even if limited to the single joint case, several variable stiffness devices, and in particular antagonistic actuated joints, have been developed [3], [7], [8].

In this paper, the feedback linearization problem of robotic manipulators with variable joint stiffness has been analyzed. The feedback linearization of robot with elastic joints is a well-known problem, extensively treated by several researchers in its original formulation [9]–[11] or in the revised version in which the dynamic coupling between the joints and the links are considered [12], [13]. In [14], [15] the interaction of elastic joint robots with the external environment are analyzed. In all these works, the joint stiffness has been considered as constant parameters. More recently, some modifications to the *classic* problem have been proposed, such as in [16] where the effects of joint damping on the solution of the feedback linearization problem are discussed, or in [17] where the feedback linearization of antagonistic actuated robotic arms is carried out. This paper aims to show how the full state linearization and simultaneous control of both the position and the stiffness of the joints can be achieved via static or dynamic feedback for the general dynamic model of a robotic manipulator with variable joint stiffness. We suppose that the mechanical stiffness of the joint can be modulated by means of external control inputs.

II. Dynamic Model of Robots with Variable Joint Stiffness

Our starting point is the general dynamic model of robot manipulators with n elastic joints of finite, but constant stiffness. The model is composed by the dynamics of $2n$ rigid bodies (n links and n actuators), coupled through the elastic joints. Let $q \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^n$ be, respectively, the generalized coordinates of the driven links and of the driving actuators. Under the simplifying modeling assumption used in [9] (namely, in the contribution to the robot kinetic energy, the angular velocity of the motors is due only to their own

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spinning), the dynamic model can be written as [9]:

$$M(q)\,\ddot{q} + N(q,\dot{q}) + K\,(q-\theta) \;=\; 0 \qquad (1)$$

$$B\,\ddot{\theta} + K\,(\theta - q) \;=\; \tau, \qquad (2)$$

where $M(q)$ is the inertia matrix of the robot links, vector $N(q,\dot{q})$ contains the centrifugal, Coriolis, and gravity forces, $K = \text{diag}\{k_1, \ldots, k_n\} > 0$ is the joint stiffness matrix, $B = \text{diag}\{b_1, \ldots, b_n\}$ is the inertia matrix of the actuators, and $\tau \in \mathbb{R}^n$ are the motor torques. Damping at the joints can be also included —see, e.g., [16]. In the following, we will also use the equivalent notation

$$K\,(q-\theta) = \Phi k, \qquad (3)$$

with matrix

$$\Phi = \text{diag}\{(q_1-\theta_1),\,(q_2-\theta_2),\,\ldots,(q_n-\theta_n)\} \qquad (4)$$

and vector $k = \begin{bmatrix} k_1 & \ldots & k_n \end{bmatrix}^T \in \mathbb{R}^n$.

In this paper, the joint stiffness matrix K in eqs. (1–2) will not be considered constant but, in general, a function of time:

$$K = K(t). \qquad (5)$$

The range of values that joint stiffness can assume depends largely on the technological implementation of variable stiffness. In [7], joint stiffness can be varied in the range $5.73 \cdot 10^{-4} \div 4.01 \cdot 10^{-2}$; in [8], it can be varied from 13.2 up to more than 400; finally, in [3] the feasible range is $0.2 \cdot 10^3 \div 2.2 \cdot 10^3$ (all units are $[Nm\ rad^{-1}]$).

Without loss of generality, we assume that all joints will have variable stiffness. Moreover, it is $K(t) > 0$ for all t, since it has no physical meaning to consider negative stiffness while if the stiffness drops to zero the joint/transmission would lead to an unactuated system.

Different causes may account for such stiffness variability. Joint stiffness can vary in response to extra command inputs to the robot system, as a function of the system configuration, or from a combination of the two. Therefore, different stiffness functions and behaviors can be defined depending on the system implementation.

The simplest situation is when the joint stiffness k_i can be directly changed by means of a (suitably scaled) additional command τ_{k_i}, for $i = 1, \ldots, n$. In vector form,

$$k = \tau_k. \qquad (6)$$

This situation is considered, e.g., in the paradigmatic 1-dof case of two masses connected through a spring of variable stiffness in [1], [2]. Therefore, the overall available input u and the robot state x are:

$$u = \begin{bmatrix} \tau \\ \tau_k \end{bmatrix} \in \mathbb{R}^{2n}, \quad x = \begin{bmatrix} q \\ \dot{q} \\ \theta \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^{4n}.$$

Indeed, the dynamics of change of the stiffness parameters of the joints (or, more in general, of the transmissions) may not be neglected. In this case, also in view of the mechanical nature of the system, we can model the variation of joint stiffness as a second-order dynamic system of the general form

$$\ddot{k} = \phi(x, k, \dot{k}, \tau_k), \qquad (7)$$

in which the dependence include also the stiffness and their time derivatives. In this case, equations (1–2) should be complemented by (7) in order to represent the complete dynamic model of a robot with variable joint stiffness. This model covers a situation where the additional actuation τ_k modifies the motion transmission configuration so to change its stiffness, e.g., by pre-compressing nonlinear springs or by moving some mechanical parts. A slight generalization of this model can be used also to describe the case of antagonistic variable stiffness devices, such as those considered in [3], [7], [8], [17] –see the Appendix.

As a result, the state vector of the robot is extended and becomes:

$$x_e = \begin{bmatrix} q^T & \dot{q}^T & \theta^T & \dot{\theta}^T & k^T & \dot{k}^T \end{bmatrix}^T \in \mathbb{R}^{6n}, \qquad (8)$$

so that eq. (7) can be rewritten as:

$$\ddot{k} = \phi(x_e, \tau_k). \qquad (9)$$

We note that for $n = 1$, the number of state variable will be equal to that needed to describe the dynamics of the variable stiffness actuating device in [3].

In all cases, the objective will be to simultaneously control the following set of outputs

$$y = \begin{bmatrix} q \\ k \end{bmatrix} \in \mathbb{R}^{2n},$$

namely the link positions (and thus, through the robot direct kinematics, the end-effector pose) and the joint stiffness. The implications on feedback linearization and decoupling control of the different models of joint stiffness variation will be investigated in Sec. IV.

III. Inverse Dynamics of Variable Stiffness Robots

For robot manipulators having elastic joints with variable stiffness, we consider here the problem of determining the expression of the actuation commands τ_d and $\tau_{k,d}$ needed to perform an assigned motion task, with a predefined and simultaneous variation of the stiffness at the joints. These commands can be then used for defining the feedforward action in a control scheme.

We assume that the motion is specified in terms of a desired smooth trajectory $q = q_d(t)$ for the link variables (possibly coming from the kinematic inversion of a Cartesian trajectory) and that a desired time evolution of the joint stiffness matrix $K = K_d(t)$ (or, equivalently, of the vector $k = k_d(t)$) is also given. These 'reference output trajectories' may be the outcome of some optimization process, like the safe brachistochrone solution in [2].

For illustration, the procedure is detailed using the simple model (6) for joint stiffness actuation. Therefore, we have simply $\tau_{k,d} = k_d(t)$ and only the computation of the nominal motor torque τ_d is of actual interest.

Note first that we can differentiate eq. (1) twice with respect to time, without introducing derivatives of the input torque τ. We have thus

$$M(q)\, q^{[3]} + \dot{M}(q)\, \ddot{q} + \dot{N}(q,\dot{q}) + \dot{K}\,(q-\theta) + K\,(\dot{q}-\dot{\theta}) = 0, \tag{10}$$

and

$$\begin{aligned} M(q)\, q^{[4]} + 2\,\dot{M}(q)\, q^{[3]} + \ddot{M}(q)\, \ddot{q} + \ddot{N}(q,\dot{q}) + \\ + K\,(\ddot{q}-\ddot{\theta}) + 2\,\dot{K}\,(\dot{q}-\dot{\theta}) + \ddot{K}\,(q-\theta) = 0. \end{aligned} \tag{11}$$

From eq. (1), evaluated along the desired output trajectory, we obtain the reference motion for the motor position:

$$\theta_d = q_d + K_d^{-1}\left(M(q_d)\ddot{q}_d + N(q_d,\dot{q}_d)\right). \tag{12}$$

Evaluating now (10) along the desired trajectory, and using eq. (12) for eliminating the presence of θ_d, yields for the reference motor velocity

$$\begin{aligned} \dot{\theta}_d \;&=\; \dot{q}_d + K_d^{-1}\Big(M(q_d)q_d^{[3]} + \dot{M}(q_d)\ddot{q}_d + \dot{N}(q_d,\dot{q}_d) \\ &\quad - \dot{K}_d K_d^{-1}(M(q_d)\ddot{q}_d + N(q_d,\dot{q}_d))\Big). \end{aligned} \tag{13}$$

In the final step, by solving eq. (2) with respect to $\ddot{\theta}$

$$\ddot{\theta} = B^{-1}\left[\tau - K(\theta - q)\right], \tag{14}$$

and substituting it into eq. (11), we obtain an expression involving the motor torque τ, which appears pre-multiplied by matrix KB^{-1} that is always non-singular. Therefore, we can evaluate this expression along the desired trajectory and solve for the reference motor torque as:

$$\tau_d = M(q_d)\ddot{q}_d + N(q_d,\dot{q}_d) + BK_d^{-1}\alpha_d\left(q_d,\dot{q}_d,\ddot{q}_d,q_d^{[3]},q_d^{[4]}\right) \tag{15}$$

with

$$\begin{aligned} \alpha_d = {} & M(q_d)\, q_d^{[4]} + 2\,\dot{M}(q_d)\, q_d^{[3]} \\ & + (\ddot{M}(q_d) + K_d)\ddot{q}_d + \ddot{N}(q_d,\dot{q}_d) \\ & - 2\dot{K}_d K_d^{-1}(M(q_d)q_d^{[3]} + \dot{M}(q_d)\ddot{q}_d + \dot{N}(q_d,\dot{q}_d)) \\ & - \left(\ddot{K}_d K_d^{-1} + 2\dot{K}_d K_d^{-1}\dot{K}_d\right)(M(q_d)\ddot{q}_d + N(q_d,\dot{q}_d)) \end{aligned} \tag{16}$$

where again eqs. (12) and (13) have been used to eliminate the explicit dependence on θ_d and $\dot{\theta}_d$. In the above expressions, the contribution due to the rigid robot dynamics, to the joint elasticity, and to the time-varying nature of the latter can be clearly recognized.

As a result of this analysis, some minimal smoothness requirements are imposed on the desired link motion $q_d(t)$ and on the desired stiffness profile $K_d(t)$ in order to achieve their exact reproducibility on a time interval $[0,T]$ of interest by the application of (15). For this, it is in fact necessary that

$$q_d(t) \in \mathbb{C}^4 \quad \text{and} \quad k_d(t) \in \mathbb{C}^2.$$

The considered model does not take into account dissipative effects or hard nonlinearities. If one wishes to include these in the inverse dynamics computation, if should be noted that the link dynamics (1) needs to be differentiated twice whereas the motor dynamics (2) is never differentiated. Therefore, discontinuous models of friction or actuator dead-zones on the motor side can be considered without problems. On the other hand, any such discontinuous phenomena acting on the link side should be approximated by a smooth model. Furthermore, in the presence of actuator saturations, it is possible to keep the command torques τ_d within the saturation limits by a suitable time scaling of the manipulator trajectory [18].

IV. Feedback Linearization of Robots with Variable Joint Stiffness

The analysis on the feedback linearization control [19] of robots with joints of constant elasticity carried out in previous works [9], [12], [16] is considered as a starting point to develop a general approach to the solution of the feedback linearization problem for robots with variable joint stiffness.

It is necessary to assume that the joint stiffness are measurable quantities. This last assumption is not restrictive because, if practical implementations of variable stiffness devices are considered [3], [7], [8], the joint stiffness is directly related to the system state variables. Then, the knowledge of the system state allows to compute the joint stiffness.

Preliminarly, we note that feedback linearization is certainly not the simplest control strategy for the considered system. However, it allows to prescribe linear tracking performance with arbitrary dynamics and to obtain exact trajectory reproduction in the nominal case.

A. Static Feedback Linearization

In this section, we assume that the joint stiffness depend on the (relative) positions of both the joints and the actuators [17]. We suppose also that there is no coupling between the stiffness of the joints or, in other words, that the stiffness of the i-th joint is influenced only by the position of the joint itself, by the position of the i-th actuator and by the input τ_{k_i} that is used to modulate the stiffness. It is then possible to write k as generic nonlinear functions of the system state variables q and θ:

$$\ddot{k}_i = \beta_i(q_i,\theta_i) + \gamma_i(q_i,\theta_i)\,\tau_{k_i},\; i = 1,\,\ldots,\,n$$

or, in a more compact form:

$$\ddot{k} = \beta(q,\theta) + \gamma(q,\theta)\,\tau_k \tag{17}$$

where k is the vector of the joint stiffness. Note that this last equation is the form of eq. (9), then all the consideration made in the previous section for this stiffness model are valid.

Eq. (17), together with eq. (1) and (2), and recalling the definition of $K = \mathrm{diag}\{k_1,\,\ldots,\,k_n\}$, define the complete model of the robotic manipulator with configuration dependent joint stiffness.

The stiffness k_i has been written as a second order differential equation (see eq. (17)) to highlight the fact that this model represent the dynamics of a mechanism, actuated by the input τ_{k_i}, that modifies the configuration of the joint to changes its stiffness, i.e. by pre-compressing nonlinear springs or by moving some mechanical parts.

From eq. (17) it is possible to see that the vector relative degree of the output k_i is two, while for the output q, eq. (10) remains unchanged and, by considering also eq. (17), eq. (11) can be rewritten as:

$$\begin{aligned} &M\, q^{[4]} + 2\, \dot{M}\, q^{[3]} + \ddot{M}\, \ddot{q} + \ddot{N} \\ &\quad + K\, \left(\ddot{q} - B^{-1}\left[\tau - K(\theta - q)\right]\right) \\ &\quad + 2\, \dot{K}\, (\dot{q} - \dot{\theta}) + \Phi\, (\beta + \gamma\, \tau_k) = 0 \end{aligned} \tag{18}$$

where both the inputs τ and τ_k appears, so we can conclude that the vector relative degrees of q is four. Then, by recalling the state definition (8), we can state that the condition for the solution of the feedback linearization problem on the sum of the relative degrees of the output information is satisfied, since the vector state dimension is 6 while the vector relative degrees of q and k are 4 and 2 respectively.

Then, the overall system can be written in a more compact form:

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \alpha(x_e) \\ \beta(q, \theta) \end{bmatrix} + Q(x_e) \begin{bmatrix} \tau \\ \tau_k \end{bmatrix} \tag{19}$$

where

$$\begin{aligned} \alpha(x_e) = -M^{-1} \Big[&2\, \dot{M}\, q^{[3]} + (\ddot{M} + K)\, \ddot{q} + \ddot{N} \\ &+ K B^{-1} K\, (\theta - q) + 2\, \dot{K}\, (\dot{q} - \dot{\theta}) + \Phi\, \beta \Big] \end{aligned} \tag{20}$$

and $Q(x_e)$ is the so called decoupling matrix:

$$Q(x_e) = \begin{bmatrix} M^{-1} K B^{-1} & M^{-1} \Phi\, \gamma(q, \theta) \\ 0_{n \times n} & \gamma(q, \theta) \end{bmatrix} \tag{21}$$

To achieve non interacting control of both the positions and the stiffness of the joints of the robot, the decoupling matrix $Q(x_e)$ must be non singular. From eq. (21) it is possible to see that this last condition is satisfied if both the diagonal matrices K and $\gamma(q, \theta)$ are non singular, or in other words:

$$\left. \begin{array}{l} k_i > 0 \\ \gamma_i(q_i, \theta_i) \neq 0 \end{array} \right\} \; \forall\, i = 1, \ldots, n \tag{22}$$

If these conditions are satisfied, by defining the static control law:

$$\begin{bmatrix} \tau \\ \tau_k \end{bmatrix} = Q^{-1}(x_e) \left(- \begin{bmatrix} \alpha(x_e) \\ \beta(q, \theta) \end{bmatrix} + \begin{bmatrix} v_q \\ v_k \end{bmatrix} \right) \tag{23}$$

we obtain the full linearized form of the overall system:

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} v_q \\ v_k \end{bmatrix}$$

where v_q and v_k are the new inputs of the linearized system used to control, respectively, the positions and the stiffness of the joints of the manipulator.

Note that the linearization law (23) is an algebraic function of the state x_e and of the auxiliary inputs v_q and v_k. To highlight this fact, the expression $\alpha(x_e)$ in eq. (20) can be rewritten by substituting $\ddot{q}$ and $q^{[3]}$ from eq. (1) and (10). Another important remark is that discontinuous phenomena in the motor dynamics (2), e.g., due to dry friction, as well as general nonlinearities in the joint stiffness dynamics (17) can be handled within this control design, since no differentiation of these relations is required.

B. Dynamic Feedback Linearization

By taking into account the following very simple stiffness variation model:

$$k_i = \tau_{k_i} \tag{24}$$

the full state linearization problem cannot be solved by means of a static state feedback, because the vector relative degree of the stiffness output becomes zero, and then the condition on the sum of the vector relative degrees of the output information is not longer satisfied.

By rewriting eq. (1) and (24), recalling the definition (4), in the following form:

$$\begin{bmatrix} \ddot{q} \\ k \end{bmatrix} = \begin{bmatrix} -M^{-1} N \\ 0_{n \times n} \end{bmatrix} + \begin{bmatrix} 0_{n \times n} & -M^{-1} \Phi \\ 0_{n \times n} & I_{n \times n} \end{bmatrix} \begin{bmatrix} \tau \\ \tau_k \end{bmatrix} \tag{25}$$

it is possible to see that the decoupling matrix of the system is singular, so it is not possible to achieve neither the non-interacting control and the full state linearization of this system via static state feedback.

From the structure of the decoupling matrix, one can see that a dynamic extension on the input τ_k is needed to satisfy the condition on the vector relative degree of the outputs. Then, we define the auxiliary control input u_k by adding a chain of two integrators on the input τ_k (see also Fig. 1):

$$\ddot{\tau}_k = u_k$$

In this way the vector relative degree of k is 2, while, for what concerns q, by differentiating eq. (1) twice with respect to time we can write:

$$\begin{aligned} &M\, q^{[4]} + 2\, \dot{M}\, q^{[3]} + \ddot{M}\, \ddot{q} + \ddot{N} \\ &\quad + K\, \left(\ddot{q} - B^{-1}\left[\tau - K(\theta - q)\right]\right) \\ &\quad + 2\, \dot{\Phi}\, \dot{\tau}_k + \Phi\, u_k = 0 \end{aligned} \tag{26}$$

in which both the input τ and the input u_k appear. This allows to state that the vector relative degree of q is 4. By substituting τ_k and $\dot{\tau}_k$ with k and $\dot{k}$ respectively, the system can be then rewritten as:

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} \alpha(x_e) \\ 0_{n \times n} \end{bmatrix} + Q(x_e) \begin{bmatrix} \tau \\ u_k \end{bmatrix} \tag{27}$$

where

$$Q(x_e) = \begin{bmatrix} M^{-1} K B^{-1} & -M^{-1} \Phi \\ 0_{n \times n} & I_{n \times n} \end{bmatrix} \tag{28}$$

$$\begin{aligned} \alpha(x_e) = -M^{-1} \Big[&2\, \dot{M}\, q^{[3]} + (\ddot{M} + K)\, \ddot{q} \\ &+ \ddot{N} + 2\, \dot{\Phi}\, \dot{\tau}_k + K B^{-1} K\, (\theta - q) \Big], \end{aligned} \tag{29}$$

from which it follows that the decoupling matrix is non-singular if K is non-singular, or, in other words, if the joint stiffness are strictly positive. This condition has been already considered before to maintain the physical meaning of the dynamic model of the manipulator.

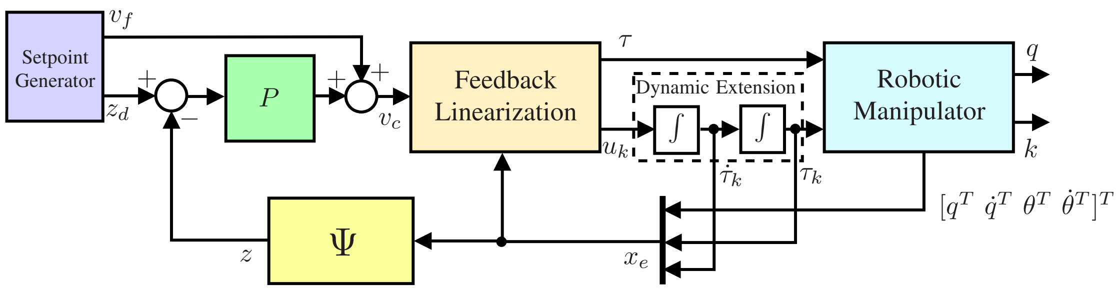

Fig. 1. Scheme of the feedback linearization controller with dynamic extension.

Since the state, and then the dimension of state space, is not changed with respect to the previous analysis, it follows that the conditions for the solution of the full state linearization and non-interacting control problem on both the relative degrees of the outputs and the non-singularity of the decoupling matrix are now satisfied.

By defining the control law:

$$\begin{bmatrix} \tau \\ u_k \end{bmatrix} = Q^{-1}(x_e)\left(-\begin{bmatrix} \alpha(x_e) \\ 0_{n\times n} \end{bmatrix} + \begin{bmatrix} v_q \\ v_k \end{bmatrix}\right) \tag{30}$$

we obtain the linearized form of the (27):

$$\begin{bmatrix} q^{[4]} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} v_q \\ v_k \end{bmatrix}$$

Note that, also in this case, the linearization law (30) is an algebraic function of the state x_e and of the auxiliary inputs v_q and v_k. To highlight this fact, the expression $\alpha(x_e)$ in eq. (29) can be rewritten by substituting $\ddot{q}$ and $q^{[3]}$ from eq. (1) and (10). With respect to the previous case, the state linearization has been now achieved by means of dynamic feedback because of the introduction of the double integrator (dynamic extension) on the stiffness control input τ_k.

C. Control Strategy

The state feedback linearization defined in the previous sections allows to control both the positions and the stiffness of the joint of the robot by means of two totally independent linear controllers, composed by a static state feedback plus feedforward action:

$$v_q = q_d^{[4]} + \sum_{i=0}^{3} P_{q_i}(q_d^{[i]} - q^{[i]}) \tag{31}$$

$$v_k = \ddot{k}_d + P_{k_1}(\dot{k}_d - \dot{k}) + P_{k_0}(k_d - k) \tag{32}$$

with diagonal gain matrices P_{k_1}, P_{k_0}, P_{q_i}, $i = 0, \ldots, 3$ such that:

$$\lambda^4 + \lambda^3 p_{q_{3j}} + \lambda^2 p_{q_{2j}} + \lambda p_{q_{1j}} + p_{q_{0j}} = 0 \tag{33}$$

$$\lambda^2 + \lambda p_{k_{1j}} + p_{k_{0j}} = 0 \tag{34}$$

with $j = 1, \ldots, n$, are Hurwitz polynomials, where $p_{q_{ij}}$ and $p_{k_{ij}}$ are the j-th term of the main diagonal of the gain matrix P_{q_i} and P_{k_i} respectively, while $q_d^{[i]}$, $i = 0, \ldots, 4$ are the vector of the desired joint position and their time derivative up to the 4-th order and k_d, $\dot{k}_d$ and $\ddot{k}_d$ are the vector of the desired stiffness trajectories and their time derivative up to the 2-th order. By means of these linear controllers, it is possible to achieve the asymptotic tracking of the position and stiffness trajectories if $q_d(t) \in \mathbb{C}^4$ and $k_d(t) \in \mathbb{C}^2$.

This control approach can be viewed as a static state feedback in the state space of the linearized system. Eq. (31) and (32) can be rewritten in a more compact form by grouping the control signals and the desired position and stiffness trajectories in this convenient way:

$$v_c = \begin{bmatrix} v_q \\ v_k \end{bmatrix}, \quad v_f = \begin{bmatrix} q_d^{[4]} \\ \ddot{k}_d \end{bmatrix}$$

$$z_d = \begin{bmatrix} q_d^T & \dot{q}_d^T & \ddot{q}_d^T & q_d^{[3]^T} & k_d^T & \dot{k}_d^T \end{bmatrix}^T$$

To this end, it is useful also to define the state vector z of the linearized system and the nonlinear coordinate transformation between the state of the original system and the state of the linearized system:

$$z = \begin{bmatrix} q^T & \dot{q}^T & \ddot{q}^T & q^{[3]^T} & k^T & \dot{k}^T \end{bmatrix}^T = \Psi(x_e) =$$

$$\begin{bmatrix} q \\ \dot{q} \\ -M^{-1}\left[N + \Phi\, k\right] \\ -M^{-1}\left[-\dot{M}\, M^{-1}\left[N + \Phi\, k\right] + \dot{N} + \Phi\, \dot{k} + \dot{\Phi}\, k\right] \\ k \\ \dot{k} \end{bmatrix}$$

It is important to note that, also in this case, both the state linearization and the outer linear control loop depends only on the state information x_e and any time derivative of the outputs must be computed.

The controller (31), (32) can be then rewritten as:

$$v_c = v_f + P[z_d - z] = v_f + P[z_d - \Psi(x_e)] \tag{35}$$

where

$$P = \begin{bmatrix} P_{q_0} & P_{q_1} & P_{q_2} & P_{q_3} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & P_{k_0} & P_{k_1} \end{bmatrix}$$

A scheme of the proposed controller is depicted in Fig. 1.

V. Simulation of a Two-Link Planar Manipulator

The validity of the proposed approach is now reported by presenting the simulation results of a planar two-link robotic arm with variable joint stiffness. Due to space limitations, the well-known dynamic model of the arm and the solution of the previous equations for this system are omitted. Only the simulation results in the case of full state linearization via dynamic feedback described in Sec. IV-B are reported,

Description	Symbol	Value	
Joint inertia	J_j	1.15e-2	$kg \cdot m^2$
Joint viscous friction coeff.	d_j	0.001	$N \cdot s \cdot m^{-1}$
Joint mass	m_j	0.541	kg
Link center of mass	l_c	0.085	m
Link length	l	0.3	m
Motors inertia	b_m	6.6e-5	$kg \cdot m^2$
Motors viscous frict. coeff.	d_m	0.00462	$N \cdot s \cdot m^{-1}$
Position error weight	w_q	10^6	rad^{-1}
Stiffness error weight	w_k	10^4	$rad\ N^{-1}\ m^{-1}$
Actuator torque weight	w_τ	1	$N^{-1}\ m^{-1}$

TABLE I

PARAMETERS OF THE 2-LINK PLANAR MANIPULATOR.

because this case is more complicated from the implementation point of view with respect to the static one, and because there are no significant difference in the response of the full state linearization via static or dynamic feedback.

In the simulation scheme, the trajectories are generated through proper filters to compute also their derivatives up to the appropriate order. The control strategy has been chosen as in eq. (35) and the matrix P is obtained from the solution of the CARE[1] equation with a diagonal state weights matrix. The parameters of the 2-link planar manipulator and of the controller used in simulation are reported in Tab. I. The links of the manipulator are considered identical for simplicity. The elements of the resulting feedback matrix P are all diagonal:

$$P_{q0_{ii}} = 3162.3\,,\ P_{q1_{ii}} = 1101.9\,,\ P_{q2_{ii}} = 192.0$$
$$P_{q3_{ii}} = 19.6\,,\ P_{k0_{ii}} = 316.2\,,\ P_{k1_{ii}} = 25.1$$

In Fig.2 the positions and the stiffness of the joints of the two-link planar manipulator are reported, together with their trajectory tracking errors. Note that the tracking errors are practically zero, as expected. Both step and sinusoidal joint trajectories are used together with coordinated movements to show the stabilizing properties of the controller. It is important to note that the joint stiffness trajectories are not affected by the changes of the joint positions and vice versa.

VI. CONCLUSIONS

In this paper, the feedforward control action needed to perform a desired motion profile on a robotic manipulator with joint variable stiffness has been computed and the problem of feedback linearization of these devices has been analyzed. The key point in the analysis of this problem is the definition of the stiffness model, and in particular of the way the inputs of the system act to modulate the stiffness of the joints. Two cases has been considered, with different (vector) relative degree of both the position and the stiffness information. The results of this analysis are summarized in Tab. II. If also the damping of the transmission system is considered, only input-output linearization can be achieved [16].

The simultaneous non-interactive stiffness-position control can be implemented by means of an outer linear control loop, that can be seen as a static state feedback in the state space of the linearized system. The asymptotic trajectory tracking problem can then be solved with arbitrary dynamics if the

[1]Continuous Algebraic Riccati Equation

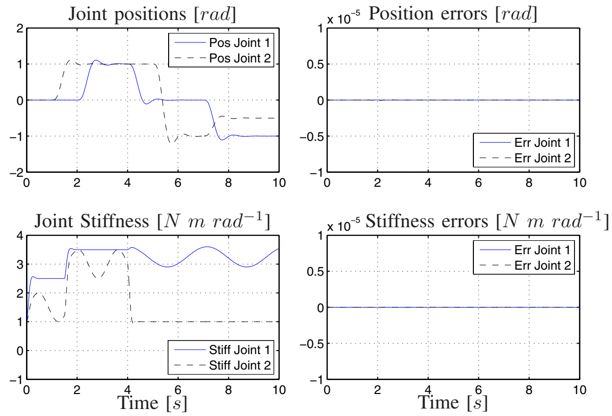

Fig. 2. Full state linearization via dynamic feedback: (a) Joint positions, (b) position errors, (c) joint stiffness and (d) stiffness errors.

position and the stiffness trajectories are continuous together with their time derivatives up to the 4th and 2nd order respectively. The experimental validation of the proposed approach will be considered in the next future.

The mixed case, in which both rigid and elastic joints are present, with constant or variable stiffness, can be easily solved on the base of the analysis reported in this paper. Also the case of joints with different stiffness variation models can be easily considered.

Type of nonlinear state feedback	Stiffness model	
	$\ddot{k}_i = \beta + \gamma \tau_{k_i}$	$k_i = \tau_{k_i}$
Static	full linearization	not enough
Dynamic	not needed	full linearization
Dimension	0	$2\,n$

TABLE II

SUMMARY OF THE RESULTS ON THE FEEDBACK LINEARIZATION .

Throughout the paper, we have assumed ideal conditions, a perfect knowledge of the robot dynamics models and the availability of full state measures. The presented results can be used as a starting point for the definition of adaptive and robust controllers for robots with variable joint stiffness, possibly based only on the feedback of the position information of both the links and the actuators, supposing the velocity information not available.

APPENDIX
ANTAGONISTIC VARIABLE STIFFNESS DEVICES

In antagonistic variable stiffness devices [3], [7], [8], [17], a couple of actuators for each joint is present, and both these actuators contribute to determine the position and the stiffness of the joint to which the actuators are connected. The dynamic model of the whole manipulator can be written by grouping the actuators in two sets, denominated here as α and β, described by two equations similar to (2), one for each actuators set:

$$M(q)\,\ddot{q} + N(q,\dot{q}) + \eta_\alpha - \eta_\beta = 0 \qquad (36)$$
$$B\,\ddot{\theta}_\alpha + \eta_\alpha = \tau_\alpha \qquad (37)$$
$$B\,\ddot{\theta}_\beta + \eta_\beta = \tau_\beta \qquad (38)$$

where $\eta_{\alpha_i} = \eta_{\alpha_i}(q_i, \theta_{\alpha_i}\,\theta_{\beta_i})$ and $\eta_{\beta_i} = \eta_{\beta_i}(q_i, \theta_{\alpha_i}\,\theta_{\beta_i})$ represent the coupling torques between the i-th joint of the robot and its actuators, while τ_α and τ_β are the torques applied by the actuator set α and β respectively.

By making the model of elastic joint robots more general, e.g., by means of a suitable change of coordinates, it is possible to write the dynamic model of antagonistic variable stiffness devices in a form similar to eq. (1), (2) and (9). This generalization is meaningful since it allows to apply the general concepts presented in this paper to different technological implementations of variable stiffness devices. By introducing the auxiliary variables $p = \frac{\theta_\alpha - \theta_\beta}{2}$ and $s = \theta_\alpha + \theta_\beta$ representing the positions of the generalized joint actuators and the state of the virtual stiffness actuators respectively, it is possible to write:

$$M(q)\,\ddot{q} + N(q,\dot{q}) + F(s)g(q-p) = 0 \qquad (39)$$

$$2B\ddot{p} + F(s)g(p-q) = \tau \qquad (40)$$

$$B\ddot{s} + h(q-p,s) = \tau_k \qquad (41)$$

where $\tau = \tau_\alpha - \tau_\beta$ and $\tau_k = \tau_\alpha + \tau_\beta$, $F(s)$ is a diagonal matrix whose elements are strictly positive functions representing the generalized joint stiffness, $g(q-p)$ is a vector whose elements are odd strictly monotonically increasing functions representing the generalized joint displacements, while $h(q-p,s)$ is a vector whose elements are functions such that $h_i(0,0) = 0$.

As case studies, for the antagonistic actuated robot described in [8], [17], in which transmission elements with exponential force/compression characteristic are used, the following relations hold:

$$\begin{aligned} f_i(s_i) &= e^{a\,s_i} \\ g_i(q_i - p_i) &= b\sinh\big(c\,(q_i - p_i)\big) \\ h_i(q_i - p_i, s_i) &= d\left[\cosh\big(c\,(q_i - p_i)\big)\,e^{a\,s_i} - 1\right] \end{aligned}$$

where a, b, c, and d are suitable constants and $f_i(s_i)$ are the elements of the diagonal of the matrix $F(s)$.

For the same variable stiffness device, if transmission elements with quadratic force/compression characteristic are considered [7], [17], the following relations hold:

$$\begin{aligned} f_i(s_i) &= a_1\,s_i + a_2 \\ g_i(q_i - p_i) &= q_i - p_i \\ h_i(q_i - p_i, s_i) &= b_1\,s_i^2 + b_2\,(q_i - p_i)^2 \end{aligned}$$

where a_1, a_2, b_1, and b_2 are suitable constants and $f_i(s_i)$ are the elements of the diagonal of the matrix $F(s)$.

For the variable stiffness actuation joint (VSA), described in [3], the third-order polynomial approximation of the transmission model reported in [20] can be used to transform the system in the desired form:

$$\begin{aligned} f_i(s_i) &= a_1\,s_i^2 + a_2\,s_i + a_3 \\ g_i(q_i - p_i) &= q_i - p_i \\ h_i(q_i - p_i, s_i) &= b_1\,s_i^3 + b_2\,(q_i - p_i)^2 s_i + b_3\,s_i \end{aligned}$$

where a_i, b_i, $i = 1, \ldots, 3$ are suitable constants and $f_i(s_i)$ are the elements of the diagonal of the matrix $F(s)$.

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