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An Acceleration-based State Observer for Robot Manipulators with Elastic Joints

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***Abstract*— Robots that use cycloidal gears, belts, or long shafts for transmitting motion from the motors to the driven rigid links display visco-elastic phenomena that can be assumed to be concentrated at the joints. For the design of advanced, possibly nonlinear, trajectory tracking control laws that are able to fully counteract the vibrations due to joint elasticity, full state feedback is needed. However, no robot with elastic joints has sensors available for its whole state, i.e., for measuring positions and velocities of both motors and links. Several nonlinear observers have been proposed in the past, assuming different reduced sets of measurements. We introduce here a new observer which uses only motor position sensing, together with accelerometers suitably mounted on the links of the robot arm. Its main advantage is that the error dynamics on the estimated state is independent from the dynamic parameters of the robot links, and can be tuned with standard decentralized linear techniques (locally to each joint). We present an experimental validation of this observer for the three base joints of a KUKA KR15/2 industrial robot and illustrate the control use of the obtained results.**

I. INTRODUCTION

Flexibility of the motion transmission and reduction elements in a robot manipulator induce a vibratory behavior that degrades its dynamic accuracy. For industrial robots, this happens when adopting belts or long shafts to drive the links with remotely located actuators, or when harmonic drives or cycloidal gears are used so as to obtain large reduction ratios with compact, in-line, and power efficient devices. Time-varying dynamic displacements will be present on the robot axes, between the position of the motors and that of the driven links. This situation is typically modeled by the introduction of elasticity or visco-elasticity at the joints [1], [2]. The number of independent variables needed to describe the robot dynamics is then doubled with respect to the case of rigid joints. In order to recover performance during fast motion or in quasi-static contact tasks, suitable feedback control laws have to be designed that deal also with joint elasticity, beside facing the nonlinear and highly-coupled dynamics of the robot arm.

Dynamic models of different accuracy have been proposed for robots with elastic joints. In the case of electrical actuation, the most common model [3] assumes that the angular kinetic energy of the rotors of the motors is due only to their relative spinning around the driving axes –the so-called reduced model. A more complete dynamic model [4] includes also the inertial couplings existing between the motors and the links. In certain cases, depending on the kinematic architecture of the arm and on the localization of the motors, these couplings may turn to be configuration-independent [5]. These models possess different structural properties from the point of view of control. In particular, the reduced model of robots with elastic joints can be fully decoupled and exactly linearized by means of a nonlinear static state feedback [3], similarly to the well-known computed torque method for fully rigid robots. On the other hand, when considering the more complete dynamics, the same result can be achieved only by resorting to a more complex dynamic state feedback [5]. These control results, which are particularly relevant for trajectory tracking tasks, assume the availability of full state measurement of the elastic joint robot, namely of motor as well as link position and velocity.

However, a sensory set to directly measure this full state (or an equivalent set of variables) is hardly available. Any robot is equipped with position sensors and these, in the presence of joint elasticity, will typically measure the motor positions. Sometimes, but always more seldom, a tachometer may be present for the motor velocity. Recently, more sensory capabilities have been introduced in prototype arms, e.g., a magnetometer and a joint torque sensor in the DLR lightweight arm [6], [7]. Both allow to measure (directly or indirectly) the link position, but the former is rather noisy while the latter needs the knowledge of the joint stiffness to compute the position from the measured torque. No sensor measuring directly the link velocity is currently in use. This picture motivated the design of state observers to replace missing sensors.

There is a variety of existing state observers of elastic joint robots in the literature, assuming different combinations of sensed variables: motor position and elastic joint displacement, or link position and motor velocity [8], or link position and velocity and assuming that the latter is bounded by virtue of the control law [9], [10]. Unfortunately, the most successful ones (at least, those with theoretically proved convergence) use unrealistic combinations of measured outputs from the robot and typically assume perfect knowledge of the whole robot dynamics.

Indeed, the main use of state estimation is for control. Note, however, that regulation tasks can be successfully

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performed with $PD+$ control laws where only the motor position and an estimate of its velocity are needed [4], [7], [11]. For the more difficult task of trajectory tracking, there are two classes of results: those assuming directly that the state is given for control purposes [12], [13] and those designing an integrated observer-controller [14], [15].

In this paper, we propose a new state observer for elastic joint robots based on the use of accelerometers mounted on the links together with motor position measurements. The main advantages of this observer are its independence from the dynamic parameters of the arm (mass and its position, and inertia of the links), the easy tuning of the observer gain, and its predictable (linear) behavior. Our final motivation is the use of the reconstructed state for control purposes.

In the past, accelerometers have been already used for control [16]. However, direct acceleration feedback is critical from a theoretical point of view (there is no strict causality between applied torque and sensed accelerations) and may lead to instability if the control scheme is not carefully implemented. Instead, when acceleration measurements are filtered by the observer dynamics and feedback is performed using the recovered full state, degradation of the expected performance is only related to the missing "separation" property in nonlinear systems.

The paper is organized as follows. After reviewing the modeling of elastic joint robots in Sec. II, the basic observer design is presented assuming first that the link acceleration (i.e., at the joint level, beyond elasticity) is measured (Sec. III). Section IV presents an extension of the scheme that is implementable with standard accelerometers distributed along the arm. The approach has been validated through extensive experimentation on the KUKA KR15/2 industrial robot. In Sec. V, representative observation results are reported, together with their use within a tracking controller [17] based on inverse dynamics feedforward plus linear feedback from the reconstructed state. More details can be found in [18].

II. DYNAMIC MODELING

We consider robot manipulators as open kinematic chains of bodies, having N moving rigid links driven by electrical motors through N (rotational) *elastic* joints/transmissions. For the dynamic modeling of elastic joint robots, a doubling of generalized coordinates is needed in a Lagrangian formulation. Let $\boldsymbol{q} \in \mathbb{R}^N$ be the generalized coordinates associated to the link positions and $\boldsymbol{\theta} \in \mathbb{R}^N$ be the motor positions (as reflected through the gearboxes). Elasticity of the motion transmission elements is modeled by linear springs introduced at each joint. We consider also viscous damping of the springs at the joints as in [19].

With the standard assumptions in [3], the dynamic model is given by

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + & \\ \boldsymbol{D}(\dot{\boldsymbol{q}} - \dot{\boldsymbol{\theta}}) + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{\theta}) + \boldsymbol{d}(\boldsymbol{q},\dot{\boldsymbol{q}}) &= \boldsymbol{0} \\ \boldsymbol{B}\ddot{\boldsymbol{\theta}} + \boldsymbol{D}(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{q}}) + \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q}) &= \boldsymbol{\tau}, \end{aligned} \tag{1}$$

where $\boldsymbol{M}(\boldsymbol{q})$ is the symmetric, positive definite link inertia matrix, the Coriolis and centrifugal terms are factorized using the matrix $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ of Christoffel symbols, $\boldsymbol{g}(\boldsymbol{q})$ is the gravity vector, and $\boldsymbol{d}(\boldsymbol{q},\dot{\boldsymbol{q}})$ contains friction effects on the link side. The motor inertia matrix $\boldsymbol{B} = \text{diag}\{B_i\}$ and the joint stiffness matrix $\boldsymbol{K} = \text{diag}\{K_i\}$ are both positive definite, while $\boldsymbol{D} = \text{diag}\{D_i\} \geq \boldsymbol{0}$ is the viscosity matrix of the springs at the joints (all these matrices are diagonal). In the right-hand side of the second equation in (1), $\boldsymbol{\tau}$ contains the motor torques performing work on $\boldsymbol{\theta}$. We suppose that friction on the motor side has been already compensated by means of a nonlinear feedback of the motor velocity in order to achieve the given formulation (see [18] for further details). The two N-dimensional vector equations in (1) are referred to as the *link* and *motor* dynamics, respectively.

In the dynamic model (1), it is assumed that the angular components of the kinetic energy of the motors are due only to their own spinning. This results in no inertial cross-coupling among the link and motor dynamics. However, the analysis presented in this paper applies, with minor modifications, also to the more complete model of robots with elastic joints considered, e.g., in [5]:

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{S}\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + & \\ \boldsymbol{D}(\dot{\boldsymbol{q}} - \dot{\boldsymbol{\theta}}) + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{\theta}) + \boldsymbol{d}(\boldsymbol{q},\dot{\boldsymbol{q}}) &= \boldsymbol{0} \\ \boldsymbol{S}^T\ddot{\boldsymbol{q}} + \boldsymbol{B}\ddot{\boldsymbol{\theta}} + \boldsymbol{D}(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{q}}) + \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q}) &= \boldsymbol{\tau}, \end{aligned} \tag{2}$$

where the $(N \times N)$ $\boldsymbol{S}$ block in the overall robot inertia matrix is strictly upper triangular [4]. Depending on the arrangement of the motors along the serial kinematic chain, this matrix may be constant as in (2). This is the case of all planar manipulators with motors mounted on the joint axes and also for the KUKA KR15/2 robot considered in our experiments. For more general kinematic structures, this matrix becomes configuration dependent, i.e., $\boldsymbol{S}(\boldsymbol{q})$, leading to the appearance of further quadratic velocity terms in both the link and motor dynamics. This situation is not considered in this paper.

III. OBSERVER DESIGN

We present first an ideal design assuming that $\boldsymbol{\theta}$ and $\ddot{\boldsymbol{q}}$ are directly measured. The actual case of linear accelerometers mounted on the robot links is treated in Sect. IV. We shall see that it is possble to follow a classical Luenberger observer design.

For the dynamic model (1), define as input $\boldsymbol{u} \in \mathbb{R}^N$, state $\boldsymbol{x} \in \mathbb{R}^{4N}$, and measured output $\boldsymbol{y} \in \mathbb{R}^{2N}$ the following vector quantities:

$$\boldsymbol{u} = \boldsymbol{\tau}, \quad \boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \\ \boldsymbol{x}_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{q} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{q}} \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta} \\ \ddot{\boldsymbol{q}} \end{bmatrix}. \tag{3}$$

The associated nonlinear state equations are

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x}_3 \\ \boldsymbol{x}_4 \\ \boldsymbol{B}^{-1}\left[\boldsymbol{K}(\boldsymbol{x}_2 - \boldsymbol{x}_1) + \boldsymbol{D}(\boldsymbol{x}_4 - \boldsymbol{x}_3)\right] \\ \boldsymbol{f}_4(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{B}^{-1} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{u} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}\boldsymbol{u}, \tag{4}$$

with the output given by

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{f}_4(\boldsymbol{x}) \end{bmatrix} = \boldsymbol{h}(\boldsymbol{x}). \tag{5}$$

Note that all nonlinear dynamic components are collected in the term

$$\begin{aligned} \boldsymbol{f}_4(\boldsymbol{x}) &= \boldsymbol{M}^{-1}(\boldsymbol{x}_2)\left[\boldsymbol{K}(\boldsymbol{x}_1 - \boldsymbol{x}_2) + \boldsymbol{D}(\boldsymbol{x}_3 - \boldsymbol{x}_4)\right. \\ &\quad \left. - \boldsymbol{C}(\boldsymbol{x}_2, \boldsymbol{x}_4)\boldsymbol{x}_4 - \boldsymbol{g}(\boldsymbol{x}_2)\right]. \end{aligned}$$

Therefore, the drift vector field $\boldsymbol{f}(\boldsymbol{x})$ and the output vector function $\boldsymbol{h}(\boldsymbol{x})$ in eqs. (4–5) have the internal structures

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{A}\boldsymbol{x} \\ \boldsymbol{f}_4(\boldsymbol{x}) \end{bmatrix}, \quad \boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{C}\boldsymbol{x} \\ \boldsymbol{f}_4(\boldsymbol{x}) \end{bmatrix},$$

where the linear terms are characterized by the $(3N \times 4N)$ matrix $\boldsymbol{A}$ and the $(N \times 4N)$ matrix $\boldsymbol{C}$ given by

$$\begin{aligned} \boldsymbol{A} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{B}^{-1}\boldsymbol{K} & \boldsymbol{B}^{-1}\boldsymbol{K} & -\boldsymbol{B}^{-1}\boldsymbol{D} & \boldsymbol{B}^{-1}\boldsymbol{D} \end{bmatrix} \\ \boldsymbol{C} &= \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}. \end{aligned}$$

The following result holds.

Theorem 1: For the robot model (1), define the linear dynamic observer

$$\dot{\boldsymbol{\xi}} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\xi} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y}_1 - \boldsymbol{C}\boldsymbol{\xi}) + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{y}_2, \tag{6}$$

where $\boldsymbol{\xi} \in \mathbb{R}^{4N}$ *is the observer state and the* $(4N \times N)$ *gain matrix* $\boldsymbol{L} = \begin{bmatrix} \boldsymbol{L}_1 & \boldsymbol{L}_2 & \boldsymbol{L}_3 & \boldsymbol{L}_4 \end{bmatrix}^T$ *has blocks* $\boldsymbol{L}_j = diag\{L_{j1}, \ldots, L_{jN}\}$ $(j = 1, \ldots, 4)$*. Then, the state estimation error* $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{\xi} \in \mathbb{R}^{4N}$ *can be made globally exponentially stable with an arbitrary decay rate.*

Proof: The proof is straightforward. The error dynamics of the state observation process can be written as

$$\begin{aligned} \dot{\boldsymbol{e}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}\boldsymbol{u} - \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\xi} - \boldsymbol{G}\boldsymbol{u} - \boldsymbol{L}\boldsymbol{C}(\boldsymbol{x} - \boldsymbol{\xi}) - \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{f}_4(\boldsymbol{x}) \end{bmatrix} \\ &= \left(\begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} - \boldsymbol{L}\boldsymbol{C} \right) \boldsymbol{e} = \boldsymbol{A}_{\rm obs}\boldsymbol{e}, \end{aligned}$$

with the pair

$$\left\{ \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix}, \boldsymbol{C} \right\}$$

being always observable. Therefore, any set of desired eigenvalues for $\boldsymbol{A}_{\rm obs}$ can be imposed by a suitable choice of the observer gain matrix $\boldsymbol{L}$. Noting that all blocks in the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ are diagonal, the associated matrix $\boldsymbol{L}$ can be taken with the given block-diagonal structure. ◁

Some remarks are in order.

Remark 1: The most relevant feature of the observer (6) is that no knowledge of the link dynamics is required. Only the dynamic parameters relative to motor inertia, transmission stiffness and damping at each joint are needed.

Remark 2: The proposed observer has a decentralized (and linear) structure. This implies that only quantities local to joint i are needed for observing the state of this joint. In particular, only the measures of θ_i and $\ddot{q}_i$ are used.

Remark 3: The eigenvalue assignment problem can be solved in closed form, as the parallel of N independent fourth-order assignment problems. In particular, for the generic visco-elastic joint i, assume that a set of four eigenvalues $\lambda_{ji} \in \mathbb{C}^-$, $j = 1, \ldots, 4$, is specified (if complex, in conjugate pairs). The associated characteristic polynomial is

$$p_i^*(\lambda) = \lambda^4 + a_{3i}\lambda^3 + a_{2i}\lambda^2 + a_{1i}\lambda + a_{0i},$$

with the coefficients a_{ji}'s uniquely determined by the desired λ_{ji}'s. Simple calculations lead to the following expressions of the gains that impose the desired observation error dynamics:

$$L_{1i} = a_{3i} - \frac{D_i}{B_i}, \quad L_{2i} = \frac{B_i}{K_i} a_{1i} - \frac{B_i D_i}{K_i^2} a_{0i},$$

$$L_{3i} = a_{2i} - \frac{D_i}{B_i} a_{3i} + \frac{D_i^2 - K_i B_i}{B_i^2}, \quad L_{4i} = \frac{B_i}{K_i} a_{0i}.$$

These explicit expressions are helpful for the fine tuning of the observer gains.

Consider next the case of the dynamic model (2). One can repeat all the above developments, by taking now into account the presence of the inertial coupling matrix $\boldsymbol{S}$. One notable difference is that, in this case, the instantaneous acceleration $\ddot{\boldsymbol{q}}$ of the links becomes also a function of the applied torque $\boldsymbol{\tau}$. Due to lack of space, we report here only the final result.

Theorem 2: For the robot model (2), define the linear dynamic observer

$$\dot{\boldsymbol{\xi}} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\xi} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y}_1 - \boldsymbol{C}\boldsymbol{\xi}) + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ -\boldsymbol{B}^{-1}\boldsymbol{S}^T \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{y}_2, \tag{7}$$

with the $(4N \times N)$ *gain matrix* $\boldsymbol{L}$ *defined as in Theorem 1. Then, the state estimation error* $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{\xi} \in \mathbb{R}^{4N}$ *can be made globally exponentially stable with an arbitrary decay rate.*

Remark 4: The observer (7) is still linear, but it is not anymore decentralized because of the presence of the motor-link inertial couplings. However, matrix $\boldsymbol{S}$ contains just data about the motor inertial components (possibly, including those along directions different from the joint axes of the robot) and this is the only additional information needed with respect to (6). Otherwise, the properties of the state observation error remain the same as before.

IV. IMPLEMENTABLE OBSERVER

The direct measure of the link acceleration $\ddot{\boldsymbol{q}}$ (at the elastic joints) is technically difficult and one must then rely on the use of (multi-axis) accelerometers mounted on the robot structure, which measure linear Cartesian accelerations (including the constant gravitational acceleration $\boldsymbol{g}_0$) of selected points of the manipulator.

The 3D-position on the robot arm of one such accelerometer is a function of the link coordinates $\boldsymbol{q}$ only and is expressed in world coordinates by the kinematic map

$$ {}^W\boldsymbol{p}_A = \boldsymbol{f}_A(\boldsymbol{q}). \tag{8} $$

Differentiating twice eq. (8) yields

$$ {}^W\ddot{\boldsymbol{p}}_A = \boldsymbol{J}_{\boldsymbol{f}_A}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_{\boldsymbol{f}_A}(\boldsymbol{q})\dot{\boldsymbol{q}}, $$

where $\boldsymbol{J}_{\boldsymbol{f}_A} = \partial \boldsymbol{f}_A / \partial \boldsymbol{q}$. Then, the acceleration measured along the direction of a unitary vector ${}^S\boldsymbol{v}_A$ attached to the accelerometer is expressed, in sensor coordinates, as

$$ {}^S\ddot{p}_{A,\boldsymbol{v}} = {}^S\boldsymbol{v}_A^T\, {}^S\boldsymbol{R}_W(\boldsymbol{q}) \left({}^W\ddot{\boldsymbol{p}}_A + {}^W\boldsymbol{g}_0 \right), \tag{9} $$

where ${}^S\boldsymbol{R}_W$ is a rotation matrix. By stacking equations of the form (9) for $M \geq N$ different directions/accelerometers, we obtain an expression for the measured acceleration output in terms of the link variables $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, and $\ddot{\boldsymbol{q}}$. Thus, the actual robot measurement available for the observer design will be

$$ \boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2' \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta} \\ {}^S\boldsymbol{T}_A(\boldsymbol{q})\ddot{\boldsymbol{q}} + {}^S\boldsymbol{W}_A(\boldsymbol{q}, \dot{\boldsymbol{q}}) \end{bmatrix}. \tag{10} $$

Assuming that the accelerometers are correctly located on the robot arm, we note that the link acceleration $\ddot{\boldsymbol{q}}$ can be recovered in principle from eq. (10) by using the (pseudo-)inverse of the $(M \times N)$ full row-rank matrix ${}^S\boldsymbol{T}_A$ as

$$ \ddot{\boldsymbol{q}} = {}^S\boldsymbol{T}_A^{\dagger}(\boldsymbol{q}) \left(\boldsymbol{y}_2' - {}^S\boldsymbol{W}_A(\boldsymbol{q}, \dot{\boldsymbol{q}}) \right). \tag{11} $$

Consider again, for simplicity, the robot dynamic model (1), together with its state-space description (3) and the new output (10), and define the robot state observer as

$$ \begin{aligned} \dot{\boldsymbol{\xi}} \;&=\; \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\xi} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y}_1 - \boldsymbol{C}\boldsymbol{\xi}) \\ &\quad + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ {}^S\boldsymbol{T}_A^{\dagger}(\boldsymbol{\xi}_2) \end{bmatrix} \left(\boldsymbol{y}_2' - {}^S\boldsymbol{W}_A(\boldsymbol{\xi}_2, \boldsymbol{\xi}_4) \right). \end{aligned} \tag{12} $$

It can be shown that the dynamics of the state estimation error becomes

$$ \dot{\boldsymbol{e}} = \boldsymbol{A}_{\text{obs}}\boldsymbol{e} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{f}_{\text{res}}(\boldsymbol{x}, \boldsymbol{\xi}) \end{bmatrix}, $$

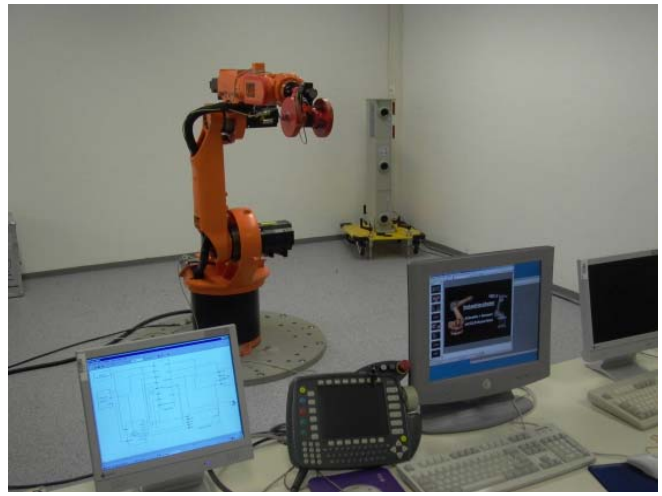


Fig. 1. Test bed for experimental evaluation

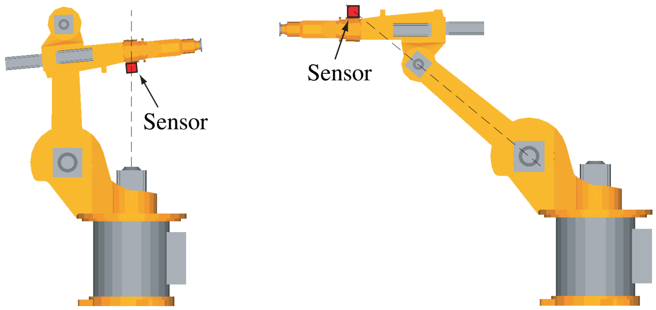
being $\boldsymbol{A}_{\text{obs}}$ defined as in the proof of Theorem 1 and with the N-dimensional residual vector

$$ \begin{aligned} \boldsymbol{f}_{\text{res}}(\boldsymbol{x}, \boldsymbol{\xi}) &= \left[\boldsymbol{I} - {}^S\boldsymbol{T}_A^{\dagger}(\boldsymbol{\xi}_2)\, {}^S\boldsymbol{T}_A(\boldsymbol{x}_2) \right] \boldsymbol{f}_4(\boldsymbol{x}) \\ &\quad + {}^S\boldsymbol{T}_A^{\dagger}(\boldsymbol{\xi}_2) \left[{}^S\boldsymbol{W}_A(\boldsymbol{\xi}_2, \boldsymbol{\xi}_4) - {}^S\boldsymbol{W}_A(\boldsymbol{x}_2, \boldsymbol{x}_4) \right]. \end{aligned} $$

The convergence of the observation error to a small ultimately bounded region around zero can be shown using 'high-gain' considerations and a Lyapunov analysis, as detailed in [18]. Essentially, the proof relies on bounding the quantities in $\boldsymbol{f}_{\text{res}}(\boldsymbol{x}, \boldsymbol{\xi})$ (which may be enforced by the presence of a feedback controller generating the input torque $\boldsymbol{\tau}$) and choosing the observer gain matrix $\boldsymbol{L}$ large enough. Note also that, when the robot starts from rest, the initial error $\boldsymbol{e}(0)$ can be made arbitrarily small (possibly, zero) using the motor position measurement and only the additional knowledge of the gravity vector $\boldsymbol{g}$ in eq. (1). In particular, using the iterative algorithm proposed in [7], also the initial estimate of the link position can be determined so that $\boldsymbol{\xi}_2(0) = \boldsymbol{q}_2(0)$.

V. EXPERIMENTAL RESULTS

The proposed observer has been evaluated on a KUKA KR15/2 industrial robot. This robot has an articulated kinematics with $N = 6$ joints and can handle payloads up to 15 kg within a work envelope volume of roughly 15 m^3. Kinematic data are given in [20], while a multi-body identification of dynamic parameters can be found, e.g., in [21]. The robot test bed used for experimental evaluation is shown in Fig. 1. Because of the stiff aluminum links and the cycloidal gearboxes known for their torsional elasticity, this robot is suitably modeled as a manipulator with rigid links but visco-elastic joints. Like most industrial robots, the KUKA KR15/2 is equipped with motor position sensors. For the implementation of the observer in Sec. IV, additional acceleration sensors were needed. However, due to hardware limitations, only three additional signals could be read into the CPU of the robot controller. Based on eq. (11), this allows the calculation of at most three link accelerations



(a) Sensor above joint axis 1

(b) Sensor in line with axes 2 and 3

Fig. 2. Robot configurations with singular transformation matrix ${}^{S}\boldsymbol{T}_{A}$

$\ddot{q}_i$. The application of the observer was therefore limited to the motion of three robot joints, chosen to be the main ones (proximal to the base). A 3-axial MEMS accelerometer has been mounted on the fourth link, as shown in Figs. 1–2. In particular, Figure 2 depicts the robot configurations where the transformation matrix ${}^{S}\boldsymbol{T}_{A}$ in eq. (10) is singular. This happens when the sensor is either above joint axis 1 or is aligned with the normal to joint axes 2 and 3. In order to avoid these singularities, during the experiments the workspace was limited to appropriate regions.

One interesting feature of the KUKA KR15/2 robot is that the motors for the last three (hand) joints are all mounted on link 4. Accordingly, this implies that rows 4 to 6 of matrix $\boldsymbol{S}$ are identically zero. Exploitation of additional kinematic properties, like orthogonality of joint axes, further simplifies the structure of $\boldsymbol{S}$ leading to a constant matrix with upper left (3×3) block given by

$$\boldsymbol{S}_{1\ldots3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & {}^{R_3}I_{R_{3_{zz}}} \\ 0 & 0 & 0 \end{bmatrix}.$$

Finally, in order to reduce the computation time of the implemented algorithm, it has been found useful to divide the observer calculations into two parts: a transformation of the sensed Cartesian accelerations into link accelerations and an observer in the joint space. In fact, the observer of Theorem 2 is very simple since it basically consists of a decoupled, linear design with only one constant coupling between joint axes 2 and 3. In our setup, the observer was implemented using C language.

For evaluating the performance of the proposed observer, the actual values of the robot state had to be independently measured during motion. To this purpose, an optical co-ordinate measurement system for the link position $\boldsymbol{q}$ and precision gyroscopes for the link velocity $\dot{\boldsymbol{q}}$ were used as additional sensors.

A. Observer

Due to the decoupled characteristics, the observer dynamics could be easily tuned working individually on each axis. This was done by matching measured and observed states for several motions and operating points via multi-objective optimization of the observer eigenvalues and of the system parameters (kept within reasonable bounds from the nominal values). Well damped eigenvalues with angular frequencies around $\omega = 250$ rad/s turned out to be a good choice. For illustration, typical matching of observed and ground-truth evolution of the states are given for joint axis 1 in Figs. 3 and 4, respectively during a short and a longer robot motion. The distance traveled by the first joint axis is 5 deg for the short motion and 30 deg for the longer motion.

From these, it can be seen that for the motor states $\boldsymbol{\theta}$ and $\dot{\boldsymbol{\theta}}$ the observed evolution is very close to the measured (real) one, which is not so surprising since a direct measurement of the motor position is available as input to the observer. Indeed, also for the link states $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ the matching between observed and measured values is pretty good. Since the errors in link position estimation were barely visible when the link angle was displayed, the joint torsion $\boldsymbol{\theta}-\boldsymbol{q}$ has been reported instead. Some estimation errors can still be found, but their peaks are fairly small.

B. Observer-based controller

The reconstructed robot states have been used also for implementing motion controllers. Tests have been performed using a proportional state controller, with diagonal matrix gain $\boldsymbol{K}_P > 0$, supplemented by a feedforward torque $\boldsymbol{\tau}_d$:

$$\boldsymbol{\tau} = \boldsymbol{K}_P(\boldsymbol{x}_d - \boldsymbol{\xi}) + \boldsymbol{\tau}_d. \tag{13}$$

In (13), the estimated value $\boldsymbol{\xi}$ of the robot state $\boldsymbol{x}$ is the on-line output of the implementable observer (12). The desired state evolution $\boldsymbol{x}_d(t)$ and the nominal torque $\boldsymbol{\tau}_d(t)$ were obtained by an efficient inverse dynamics algorithm

$$(\boldsymbol{x}_d, \boldsymbol{\tau}_d) = \mathtt{INVDYN}(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d, \boldsymbol{q}_d^{(3)}, \ldots),$$

where $\boldsymbol{q}_d(t)$ is the desired smooth rest-to-rest link trajectory. The controller was designed to place the slowest frequencies of the closed-loop system around $\omega = 30$ s^{-1}, which is close to the robot natural frequency. Details on the design of this controller can be found in [18]. When compared with a motion control law using only motor states, the controller using full state information shows significant improvements in damping and overshoot. Representative examples of performance are given in Figs. 5 and 6.

VI. CONCLUSIONS

A new state observer for robots with visco-elastic joints has been proposed, using the motor position and the acceleration at selected points along the robot arm as measured quantities. The observer design has been presented in a step-wise fashion, assuming first the availability of acceleration of the joint variables associated to the links (i.e., the link accelerations beyond joint elasticity), and then considering an implementation with standard accelerometers mounted on the links. Experimental validation of the observer itself and of a trajectory tracking controller based on the reconstructed state has been conducted on a KUKA KR15/2 industrial robot with satisfactory performance.

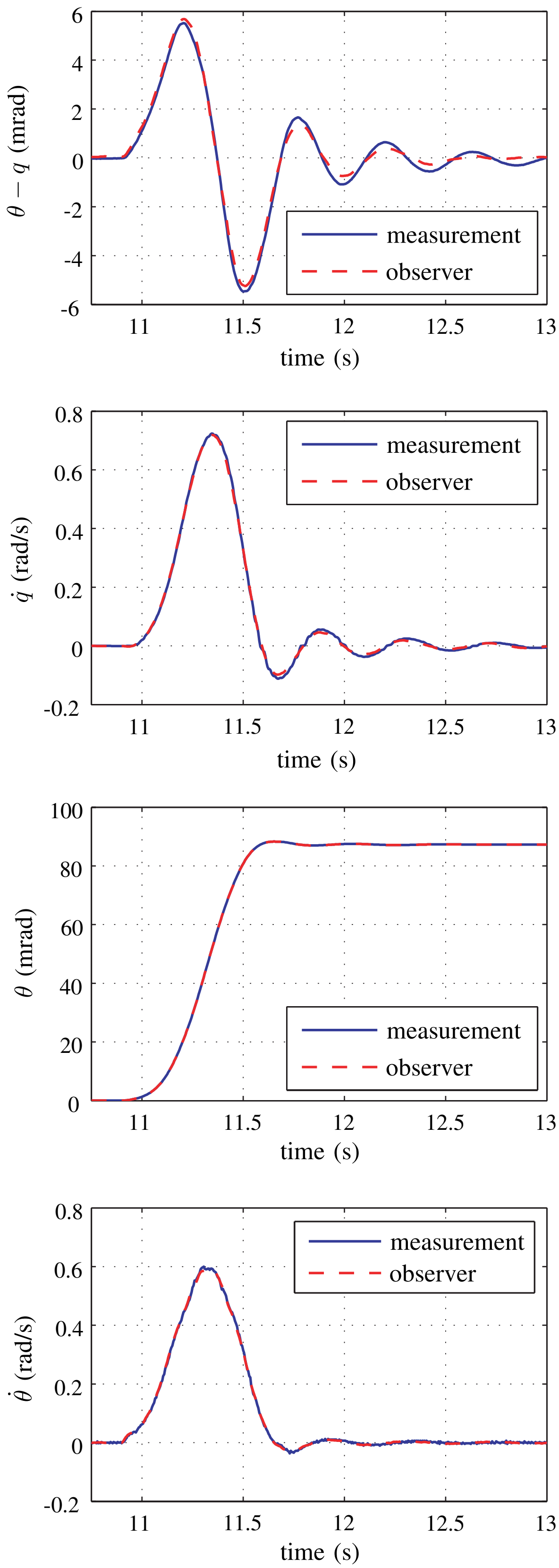


Fig. 3. Observer performance on a short robot motion

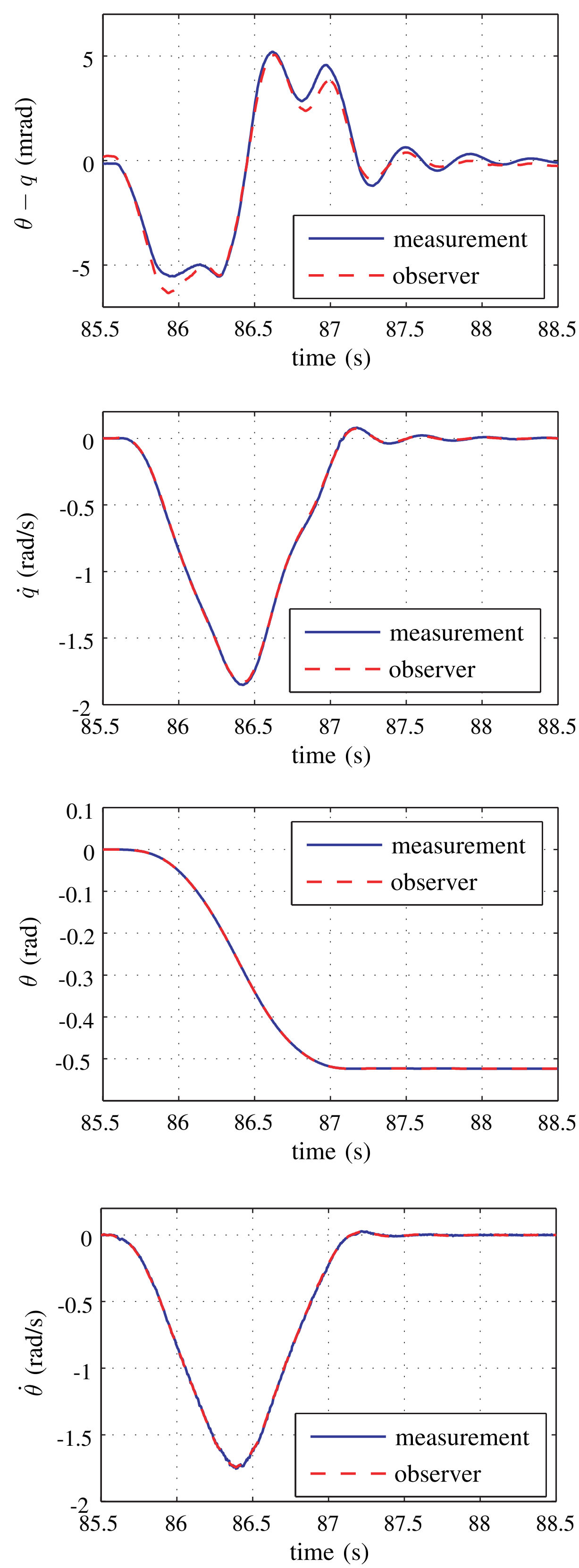


Fig. 4. Observer performance on a longer robot motion

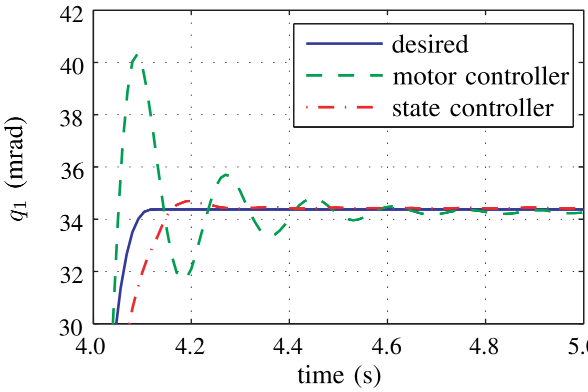


Fig. 5. Comparison of observer-based full state control and motor control on a short robot motion

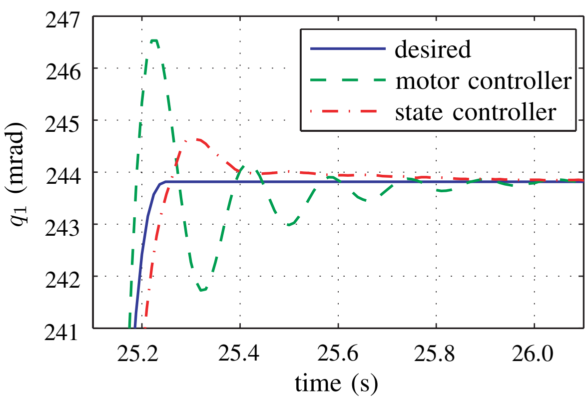


Fig. 6. Comparison of observer-based full state control and motor control on a longer robot motion

The main features of this acceleration-based observer are the following: *i)* knowledge of the robot link dynamics is not needed; *ii)* a linear and decoupled behavior of the state observation error is obtained when link acceleration is available, both when neglecting or including the presence of inertial couplings between the motors and the links; *iii)* tuning of the observer gains is made easy by its (fully or dominantly) decentralized structure; *iv)* the use of accelerometers is handled similarly, though with the caution to avoid kinematic singularities associated to the location of these sensors on the arm.

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