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Regulation with on-line gravity compensation for robots with elastic joints

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Abstract—**In this paper a PD control with on-line gravity compensation is proposed for robot manipulators with elastic joints. The work extends the existing PD control with constant gravity compensation, where only the gravity torque needed at the desired configuration is used throughout motion. The control law requires measuring only position and velocity on the motor side of the elastic joints, and the on-line compensation scheme estimates the actual gravity torque using a biased measure of the motor position. It is proved via a Lyapunov argument that the control law globally stabilizes the desired robot configuration.**

Experimental results on an 8-d.o.f. robot manipulator with elastic joints show that this control scheme improves the transient behavior with respect to a PD controller with constant gravity compensation. In addition, it can be usefully applied in combination with a point-to-point interpolating trajectory leading to a reduction of final steady-state errors due to static friction and/or uncertainty in the gravity compensation.

I. INTRODUCTION

Control algorithms conceived for completely rigid robots may guarantee a stable behavior even if a certain degree of elasticity in the actuation system or in the link structure is present [1], [2]. The price to pay, however, is a typical degradation of robot performance. In fact, elasticity of mechanical transmissions may generate lightly damped vibrational modes, which reduce robot accuracy in tracking tasks [1]. Yet, it may become a source of instability in case of interaction between the robot and the environment [3].

When negative effects of mechanical elasticity are non-negligible, the control design has to be revisited in order to account for the elastic phenomena. In this paper, the case of elasticity at the joints is taken into account. This means that flexibility is assumed to be concentrated at the n robot joints and the number of Lagrangian configuration variables in the robot dynamics is doubled with respect to the rigid case, leading to a set of n motor and n link second-order nonlinear equations.

For robot manipulators with elastic joints, different control solutions are available for trajectory tracking as well as for regulation tasks [4]. For trajectory tracking tasks, one can resort to high-performing but complex control strategies, such as the linearizing and decoupling nonlinear feedback [5], [6] or an integral manifold approach based on a singular perturbation model of the robot dynamics [7], [8]. For regulation tasks, instead, it has been proved in [9] that a simple PD controller suffices to globally stabilize a robot with elastic joints about any desired configuration. The control law includes a constant gravity compensation term, which is evaluated at the desired reference position, and needs to feed back only position and velocity of the motors.

In the case of rigid robots, it is well known that global regulation to a desired configuration q_d can be achieved by a PD control law, either with a constant gravity compensation term $g(q_d)$ (and sufficiently large positional gains) [10] or with a nonlinear gravity compensation term $g(q)$ evaluated at the current configuration (or, on-line) [11]. In the presence of joint elasticity, putting to work an on-line gravity compensation is more complex than in the rigid case. On one hand, the gravity torque depends on the robot link positions whereas quite often only the motor positions are measurable. On the other hand, a PD control with on-line gravity compensation based on the motor positions θ (i.e., with $g(\theta)$) does not lead to the desired final equilibrium configuration. In addition, the stability analysis is complicated by the non-collocation between the available control torque (on the motor side) and the gravity torque to be compensated (acting on the link side of joint elasticity).

The contribution of this paper is a PD control law with on-line gravity compensation for robot manipulators with elastic joints, which requires only motor measurements and has guaranteed global stabilization properties. The main idea is to use a new variable, named 'gravity-biased' motor position, for evaluating (an estimate of) the gravity torque at each configuration. The typical feature of this controller is to improve the transient behavior of the original control law in [9]. In addition, using lower positional gains, and applying the scheme in combination with a point-to-point interpolating trajectory allows preventing motor saturation (typically occurring during the first instants of motion) and reduces the steady-state error due to unmodeled static friction and/or uncertainty in the gravity compensation term.

After recalling the dynamic modelling of robot manipulators with elastic joints in Section II, the PD control law with on-line gravity compensation is introduced in Section III. The analysis of the closed-loop equilibria and the proof of asymptotic stability via a Lyapunov argument are presented in Section IV and Section V, respectively. Finally, some experimental results obtained on the Dexter robot, an 8-dof cable-driven articulated arm, are reported in the Section V, where a comparison with the approach in [9] is carried out.

II. Dynamic model of robots with elastic joints

The following two assumptions are made in describing the dynamics of robots with elastic joints:

A1. The robot manipulator is an open kinematic chain of rigid bodies, driven by electrical actuators through elastic joints.

A2. Rotors of motors are uniform bodies balanced around their rotation axes.

The robot dynamic model can be written as follows [9]:

$$B(q)\ddot{q}_c + C(q_c, \dot{q}_c)\dot{q}_c + e(q) + K_e q_c = m \tag{1}$$

where $q_c = \left[q^T \; \theta^T\right]^T$ is the $(2n \times 1)$ vector of configuration variables, being q and θ the $(n \times 1)$ vectors of link positions and motor positions (reflected through the gears), respectively. In view of Assumptions **A1** and **A2**, the $(2n \times 2n)$ robot inertia matrix $B(q)$ and the $(2n \times 1)$ gravitational torque vector $e(q)$ are independent of θ. Moreover,

$$C(q_c, \dot{q}_c)\dot{q}_c = \dot{B}(q)\dot{q}_c - \frac{1}{2}\left(\frac{\partial}{\partial q_c}(\dot{q}_c^T B(q)\dot{q}_c)\right)^T$$

is the $(2n \times 1)$ vector of centrifugal and Coriolis torques, $K_e q_c$ represents the $(2n \times 1)$ vector of elastic torques and, on the right-hand side of (1), m is the $(2n \times 1)$ vector of external torques producing work on q_c.

Equation (1) can be rearranged into two equations, one for the link side and the other for the motor side, if the contributions to the robot dynamics are decomposed as follows. The $(2n \times 2n)$ robot inertia matrix $B(q)$ can be partitioned in four $(n \times n)$ block matrices

$$B(q) = \begin{bmatrix} B_1(q) & B_2(q) \\ B_2^T(q) & B_3 \end{bmatrix} \tag{2}$$

of which B_1 takes into account the inertial properties of rigid links, B_2 considers the coupling between each spinning actuator and the previous links, and B_3 is a constant diagonal matrix including the motor inertia (scaled through the squared gear ratios).

The $(2n \times 2n)$ matrix $C(q_c, \dot{q}_c)$, by resorting to the so-called decomposition in Christoffel symbols, can be expressed as

$$C(q_c, \dot{q}_c) = C_A(q, \dot{\theta}) + C_B(q, \dot{q}) \tag{3}$$

where

$$C_A(q, \dot{\theta}) = \begin{bmatrix} C_{A1}(q, \dot{\theta}) & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_B(q, \dot{q}) = \begin{bmatrix} C_{B1}(q, \dot{q}) & C_{B2}(q, \dot{q}) \\ C_{B3}(q, \dot{q}) & 0 \end{bmatrix}$$

being C_{A1}, C_{B1}, C_{B2}, C_{B3} suitable $(n \times n)$ matrices.

The gravitational torque takes on the form

$$e(q) = \begin{bmatrix} g(q) \\ 0 \end{bmatrix}$$

where $g(q) = \left(\frac{\partial U_g(q)}{\partial q}\right)^T$, being $U_g(q)$ the potential energy due to gravity.

The $(2n \times 2n)$ matrix K_e in the elastic torque can be written in terms of the $(n \times n)$ diagonal and positive definite matrix K of joint stiffness coefficients as follows:

$$K_e = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}$$

and, finally, the vector of generalized forces acting on q_c can be expressed as

$$m = \begin{bmatrix} 0 \\ u \end{bmatrix}$$

where u is the torque vector produced by the n motors.

Note finally that, under the assumptions in [5], the dynamic model in (1) simplifies to:

$$\begin{aligned} B_1(q)\ddot{q} + C_{B1}(q, \dot{q})\dot{q} + g(q) + K(q - \theta) &= 0 \\ B_3\ddot{\theta} + K(\theta - q) &= u, \end{aligned} \tag{4}$$

that is the so-called *reduced* dynamic model for robots with elastic joints.

It can be seen that, for the model (1), the following four properties hold [4]:

P1. The inertia matrix $B(q)$ is symmetric and positive definite for all q_c.

P2. The matrix $B_2(q)$ is strictly upper triangular.

P3. If a representation in Christoffel symbols is chosen for the elements of $C(q_c, \dot{q}_c)$, the matrix $\dot{B} - 2C$ is skew-symmetric.

P4. A positive constant α exists such that

$$\left\|\frac{\partial g(q)}{\partial q}\right\| = \left\|\frac{\partial^2 U_g(q)}{\partial q^2}\right\| \leq \alpha \tag{5}$$

where the matrix norm of a symmetric matrix $A(q)$ is given by $\lambda_{\max}(A(q))$, i.e., its largest (real) eigenvalue at q.[1] Inequality (5) holds for all q and implies

$$\| g(q_1) - g(q_2)\| \leq \alpha \|q_1 - q_2\|, \tag{6}$$

for any q_1, q_2. It should be explicitly remarked that this inequality holds whatever argument is used for evaluating the gravity vector.

These properties obviously hold also for the reduced dynamic model.

It should also be noted that, with the chosen generalized coordinates, the direct kinematics of the robot, i.e. the relation between the robot configuration variables and the Cartesian end-effector pose, depends only on the link position variables q.

[1] This is the matrix norm naturally induced by the Euclidean norm on vectors, e.g., $\|q\| = \sqrt{\sum_{i=1}^{n} q_i^2}$.

III. PD Control with On-Line Gravity Compensation

In this section, a control law is proposed which is aimed at regulating the robot link positions to a desired constant configuration q_d. It is a proportional-derivative action in the space of motor variables, as in [9]. The assumption is made that only the motor variables θ and $\dot{\theta}$ are measurable or, at least, θ is measurable and $\dot{\theta}$ is obtained by accurate numerical differentiation. An on-line gravity compensation in lieu of a constant gravity compensation is proposed as an improvement of the control law in [9].

The PD control with constant gravity compensation in [9] is expressed as:

$$u = K_P(\theta_d - \theta) - K_D\dot{\theta} + g(q_d), \tag{7}$$

with $K_P > 0$, $K_D > 0$ (both symmetric), and

$$\theta_d = q_d + K^{-1}g(q_d). \tag{8}$$

Via Lyapunov argument and La Salle's theorem, global asymptotic stability of the (unique) closed-loop equilibrium state $(q, \theta, \dot{q}, \dot{\theta}) = (q_d, \theta_d, 0, 0)$ was proved, under the assumption that the stiffness matrix K and the proportional gain matrix K_P comply with the following condition:

$$\lambda_{\min}(\bar{K}) := \lambda_{\min}\left(\begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix}\right) > \alpha. \tag{9}$$

The PD control law with on-line gravity compensation is addressed to improve transient behavior by performing some kind of gravity compensation at any configuration during motion. A gravity estimate based on the link variables cannot be considered, since the link variables q are assumed not to be measurable. In addition, it is easy to show that using $g(\theta)$, with the measured motor positions in place of the link positions, leads to an incorrect closed-loop equilibrium.

Thus, a new variable $\tilde{\theta}$ is introduced, i.e.

$$\tilde{\theta} = \theta - K^{-1}g(q_d), \tag{10}$$

that is a *gravity-biased* modification of the measured motor position θ. The PD control with on-line gravity compensation is subsequently defined as

$$u = K_P(\theta_d - \theta) - K_D\dot{\theta} + g(\tilde{\theta}), \tag{11}$$

where $K_P > 0$ and $K_D > 0$ are both symmetric (and typically diagonal) matrices. The variable $\tilde{\theta}$ shall provide the correct gravity compensation at steady state, even without a direct measure of q. As a matter of fact, the control law (11) can be implemented using only motor variables.

IV. Closed-Loop Equilibria

The equilibrium configurations of the closed-loop system (1), (11) are computed by setting $\dot{q} = \dot{\theta} = 0$ and $\ddot{q} = \ddot{\theta} = 0$. This yields

$$g(q) + K(q - \theta) = 0 \tag{12}$$

$$K(\theta - q) = K_P(\theta_d - \theta) + g(\tilde{\theta}). \tag{13}$$

From (12) it follows that, at any equilibrium, $\theta = q + K^{-1}g(q)$. Taking this into account and adding (12) to (13) leads to

$$K_P(\theta_d - \theta) + g(\tilde{\theta}) - g(q) = 0.$$

Indeed $(q, \theta) = (q_d, \theta_d)$ is a closed-loop equilibrium configuration, since $\tilde{\theta}_d := \theta_d - K^{-1}g(q_d) = q_d$ from (8) and (10) so that $g(\tilde{\theta}_d) = g(q_d)$.

The uniqueness of such an equilibrium has to be demonstrated. Thus, adding $K(\theta_d - q_d) - g(q_d) = 0$ to both (12) and (13) yields

$$\begin{aligned} K(q - q_d) - K(\theta - \theta_d) &= g(q_d) - g(q) \\ -K(q - q_d) + (K + K_P)(\theta - \theta_d) &= g(\tilde{\theta}) - g(q_d), \end{aligned}$$

or

$$\bar{K}\begin{bmatrix} q - q_d \\ \theta - \theta_d \end{bmatrix} = \begin{bmatrix} g(q_d) - g(q) \\ g(\tilde{\theta}) - g(q_d) \end{bmatrix} \tag{14}$$

if the matrix $\bar{K}$ defined in (9) is used.

Assuming that condition (9) holds true implies

$$\begin{aligned} \left\| \bar{K}\begin{bmatrix} q - q_d \\ \theta - \theta_d \end{bmatrix} \right\|^2 &\geq \lambda^2_{\min}(\bar{K}) \left\| \begin{bmatrix} q - q_d \\ \theta - \theta_d \end{bmatrix} \right\|^2 \\ &= \lambda^2_{\min}(\bar{K}) \left(\|q - q_d\|^2 + \|\theta - \theta_d\|^2 \right), \end{aligned} \tag{15}$$

while, using inequality (6) and the identity $\tilde{\theta} - q_d = \theta - \theta_d$ yields

$$\begin{aligned} \left\| \begin{matrix} g(q_d) - g(q) \\ g(\tilde{\theta}) - g(q_d) \end{matrix} \right\|^2 &= \|g(q_d) - g(q)\|^2 + \|g(\tilde{\theta}) - g(q_d)\|^2 \\ &\leq \alpha^2 \left(\|q - q_d\|^2 + \|\theta - \theta_d\|^2 \right). \end{aligned} \tag{16}$$

By comparing (15) with (16) it follows that, when $\lambda_{\min}(\bar{K}) > \alpha$, the equality (14) holds only for $(q, \theta) = (q_d, \theta_d)$, which is thus the unique equilibrium configuration of the closed-loop system (1), (11).

V. Proof of Asymptotic Stability

To demonstrate asymptotic stability of the closed-loop system, a candidate Lyapunov function is defined in terms of an auxiliary configuration-dependent function $P(q, \theta)$. This is expressed as:

$$\begin{aligned} P(q, \theta) &= \frac{1}{2}(q - \theta)^T K (q - \theta) + \frac{1}{2}(\theta_d - \theta)^T K_P (\theta_d - \theta) \\ &+ U_g(q) - U_g(\tilde{\theta}). \end{aligned} \tag{17}$$

Under the assumption (9), this function has a unique minimum in (q_d, θ_d). In fact, the necessary condition for a minimum of $P(q, \theta)$ is

$$\begin{aligned} \nabla P(q, \theta) &= \begin{bmatrix} \nabla_q P \\ \nabla_\theta P \end{bmatrix} = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{bmatrix} q \\ \theta \end{bmatrix} \\ &+ \begin{bmatrix} g(q) \\ K_P(\theta - \theta_d) - g(\tilde{\theta}) \end{bmatrix} = 0. \end{aligned} \tag{18}$$

Equation (18) is exactly in the form (12), (13), which in turn is equivalent to (14). As in Section IV, it can be demonstrated that $\nabla P(q,\theta) = 0$ only at (q_d, θ_d). Moreover, the sufficient condition for a minimum

$$\nabla^2 P(q_d,\theta_d)=\begin{bmatrix} K & -K \\ -K & K+K_P \end{bmatrix}+\begin{bmatrix} \frac{\partial g(q)}{\partial q} & 0 \\ 0 & -\frac{\partial g(\tilde{\theta})}{\partial \theta} \end{bmatrix}\Bigg|_{q=q_d,\theta=\theta_d} > 0$$

is satisfied, using again assumption (9).

By setting $P_d := P(q_d, \theta_d) = g^T(q_d)K^{-1}g(q_d)$, the candidate Lyapunov function can be written as

$$V(q,\theta,\dot{q},\dot{\theta}) = \frac{1}{2}\dot{q}_c^T B(q)\dot{q}_c + P(q,\theta) - P_d \geq 0. \quad (19)$$

Indeed, V is zero only at the desired equilibrium state $q = q_d$, $\theta = \theta_d$, $\dot{q} = \dot{\theta} = 0$.

Along the trajectories of the closed-loop system (1), (11), the time derivative of V becomes

$$\begin{aligned}
\dot{V} &= \dot{q}_c^T B(q)\ddot{q}_c + \tfrac{1}{2}\dot{q}_c^T \dot{B}(q)\dot{q}_c + \left(\dot{q} - \dot{\theta}\right)^T K(q-\theta) \\
&\quad - \dot{\theta}^T K_P(\theta_d - \theta) + \dot{q}^T\left(\tfrac{\partial U_g(q)}{\partial q}\right)^T - \dot{\theta}^T\left(\tfrac{\partial U_g(\tilde{\theta})}{\partial \theta}\right)^T \\
&= \dot{q}_c^T\left(-C(q_c,\dot{q}_c)\dot{q}_c - e(q) - K_e q_c + m + \tfrac{1}{2}\dot{B}(q)\dot{q}_c\right) \\
&\quad + \dot{q}^T\left(K(q-\theta) + g(q)\right) - \dot{\theta}^T K(q-\theta) \\
&\quad - \dot{\theta}^T\left(K_P(\theta_d - \theta) + g(\tilde{\theta})\right) \\
&= \dot{q}^T\left(-K(q-\theta) - g(q) + K(q-\theta) + g(q)\right) \\
&\quad + \dot{\theta}^T\left(K(q-\theta) - K(q-\theta) + K_P(\theta_d - \theta)\right) \\
&\quad + \dot{\theta}^T\left(-K_D\dot{\theta} + g(\tilde{\theta}) - K_P(\theta_d - \theta) - g(\tilde{\theta})\right) \\
&= -\dot{\theta}^T K_D \dot{\theta} \leq 0,
\end{aligned} \quad (20)$$

where the identity $\dot{\tilde{\theta}} = \dot{\theta}$ and the skew-symmetry of matrix $\dot{B} - 2C$ have been used.

Since $\dot{V} = 0$ if and only if $\dot{\theta} = 0$, substituting $\dot{\theta}(t) \equiv 0$ into the closed-loop equations yields

$$B_1(q)\ddot{q} + C_{B1}(q,\dot{q})\dot{q} + g(q) + Kq = K\theta = \text{const} \quad (21)$$

$$\begin{aligned} B_2^T(q)\ddot{q} + C_{B3}(q,\dot{q})\dot{q} - Kq \\ = -K\theta + K_P(\theta_d - \theta) + g(\tilde{\theta}) = \text{const}. \end{aligned} \quad (22)$$

By virtue of Property **P2** and the expression of $C_{B3}(q,\dot{q})$, from (22) it follows that $\dot{q}(t) \equiv 0$. This in turn simplifies (21) to

$$g(q) + K(q - \theta) = 0. \quad (23)$$

It has already been shown that the system (22), (23) has the unique solution $(q,\theta) = (q_d,\theta_d)$, provided that condition (9) holds true. Therefore, $q = q_d$, $\theta = \theta_d$, $\dot{q} = \dot{\theta} = 0$ is the largest invariant subset contained in the set of states such that $\dot{V} = 0$. By La Salle's Theorem, global asymptotic stability of the desired set point can be concluded.

The PD control law with on-line gravity compensation can be applied also to the reduced dynamic model (4). In the demonstration of global asymptotic stability in correspondence of the unique equilibrium point $(q,\theta,\dot{q},\dot{\theta}) = (q_d,\theta_d,0,0)$, the sole difference with respect to the case of dynamic model (1) is that the analysis through La Salle's Theorem is simplified. Invoking Property **P2** is not required.

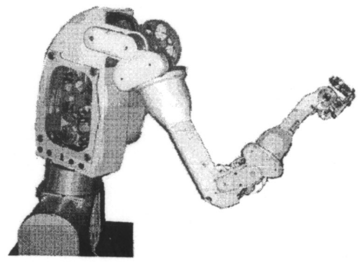

Fig. 1. The Dexter arm

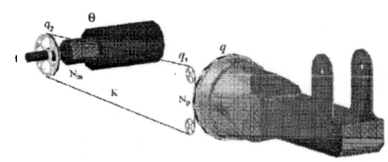

Fig. 2. A cable-driven joint/link pair

VI. EXPERIMENTAL RESULTS

As a first attempt to verify global asymptotic stability of the PD control with on-line gravity compensation, the same simulation tests reported in [9] were repeated, in order to make a comparison with the PD control with constant gravity compensation. The results have shown that the PD gains used in [9] wipe out the differences between the two control schemes by overcoming the action of gravity estimate. In addition, the torque values generated at the motion starting point are so high (due to the initial error) that they could be one cause of motor saturation. Indeed, the stability analysis does not take into account this situation.

Thus, both control laws have been implemented on a robot manipulator with elastic joints and the experimental results have been compared.

The robot used for the experiments is an 8-d.o.f. cable-driven robot manipulator, named Dexter (Fig. 1). It has a mechanical transmission system realized by pulleys and steel cables. As an example, Fig. 2 shows one of the Dexter joints.

The cable actuation permits a decreasing distribution of the link masses from the robot base up to the end effector, by lightening the robot mechanical structure. The values of the Dexter link masses and centers of gravity are reported in Table I.

	r_x [mm]	r_y [mm]	r_z [mm]	m [Kg]
Link 0	0	6.92	27.72	9.43
Link 1	−139.35	174.49	46.08	12.05
Link 2	0	−6.11	34.59	1.63
Link 3	90.72	133.77	−0.24	2.49
Link 4	0.01	−3.72	20.30	0.82
Link 5	−24.01	141.05	0.11	0.54
Link 6	−0.05	2.36	6.78	0.27
Link 7	20.35	1.81	33.26	0.09

TABLE I

COORDINATES OF THE LINK CENTERS OF GRAVITY IN THE COORDINATE SYSTEM FIXED ON EACH LINK AND VALUES OF THE MASSES (COURTESY OF SCIENZIA MACHINALE S.R.L.)

The robot dynamics model is expressed in terms of 16 position variables, of which eight variables define the motor positions, and the remaining ones define the link positions. Eight incremental encoders allow measuring motor positions during motion.

The cable stiffness coefficients for the Dexter arm are reported in Tables II and III. As one can observe, joints 1 and 2 have higher stiffness values with respect to the other joints. This indicates that joints 1 and 2 have a low level of elasticity which can be neglected, in general.

The effect of the elasticity is not negligible in joints 3–8.

	Joint 1	Joint 2	Joint 3	Joint 4
Stiffness coefficient	10^5	10^5	$6.34 \cdot 10^3$	$3.60 \cdot 10^3$

TABLE II
STIFFNESS COEFFICIENTS FOR THE JOINTS 1–4 OF THE DEXTER ARM, EXPRESSED IN Nm/rad

	Joint 5	Joint 6	Joint 7	Joint 8
Stiffness coefficient	$2.69 \cdot 10^3$	$1.69 \cdot 10^3$	$1.23 \cdot 10^2$	$2.06 \cdot 10^2$

TABLE III
STIFFNESS COEFFICIENTS FOR JOINTS 5–8 OF THE DEXTER ARM, EXPRESSED IN Nm/rad

The PD control law is written in C^{++} programming language and runs on a PC Pentium II under DOS Operating System. The motor commands are sent to the actuation system each 10 ms, by means of two MEI 104/DSP-400 board controllers.

The issue of motor saturation becomes evident in the experiments on the Dexter arm. The regulation tasks to a constant desired configuration cannot be accomplished if a constant gravity compensation is used. The initial error is so high that the motor actuators saturate.

In the case of on-line gravity compensation the task can be performed, but only for short distances (nearby 3-4 cm in the Cartesian space) between the initial configuration and the constant desired configuration.

Thus, in order to overcome the critical point of motor saturation, a point-to-point quintic polynomial trajectory (with zero velocity and acceleration boundary conditions) has been planned that guides the robot manipulator from an initial joint configuration q_i to the desired reference configuration q_d in a given time interval.

Now, both controllers can perform the motion and ensure asymptotic stability of the closed-loop system.

The proportional gains are different for the two cases: in the case of PD control with constant gravity compensation $K_P = \text{diag}\{80, 80, 30, 20, 16, 8, 2, 2\}$, while $K_P = \text{diag}\{110, 110, 50, 35, 26, 15, 4, 4\}$ in the PD control with on-line gravity compensation. The rationale for the different values of proportional gains is that, if the same K_P matrix is used for the two controllers, the initial error produced by the constant gravity estimate at q_d results in a higher torque value with respect to the case of on-line gravity estimate. Thus, for the saturation issue, a reduction of K_P is needed.

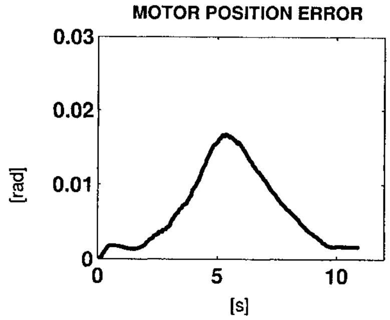

Fig. 3. Motor error norms with on-line gravity compensation (desired time-varying joint trajectory)

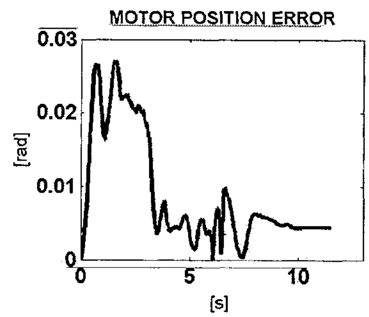

Fig. 4. Motor error norms with constant gravity compensation (desired time-varying joint trajectory)

The derivative gains are equal and set to $K_D = \text{diag}\{10, 10, 9, 3, 2.5, 2, 0.1, 0.1\}$.

Figures 3 and 4 report the norm of the motor position error for both controllers. They are relative to a point-to-point motion from the initial configuration $q_i = [1.57\ 0.00\ 10.91\ 2.82\ -3.90\ 2.55\ 2.80\ 2.92]^T$ rad to the desired reference configuration $q_d = [1.57\ 0.30\ 12\ 2.82\ -3.4\ 2.60\ 3.42\ 3.39]^T$ rad (that is not an equilibrium configuration) in a time interval of 10 s plus 2 s for the adjustment.

Figures 5 and 6 show motor positions over time, as recorded by the encoders on the motor shaft during motion. Only motor variables 6, 7, 8 are shown because they are more involved than the others in the motion performed and, thus, they are meaningful in delineating the differences between the two controllers.

Three main elements emerge from the experimental trials as basic differences between the two control schemes.

The first one is the difference in the time course of the error as well as the motor variables, that is smoother in the PD control with on-line gravity compensation with respect to the PD control with constant gravity compensation.

The second element is the error magnitude during transients: the error in the constant gravity case turns out to be larger than in the on-line case.

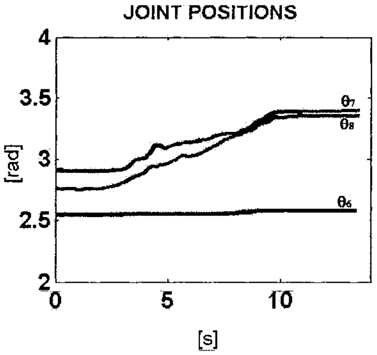

Fig. 5. Motor positions for PD control with on-line gravity compensation (desired time-varying joint trajectory)

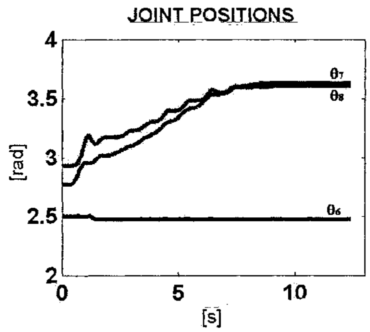

Fig. 6. Motor positions for PD control with constant gravity compensation (desired time-varying joint trajectory)

Finally, the third element is the error magnitude at the steady state. Operating with a real system, like a robot manipulator, includes the possibility that real effects, such as static friction or else inaccurate estimate of gravity torque, can affect robot performance in regulation tasks, and determine a steady-state error that is different from zero. In particular, in the Dexter arm it has been observed that when the gravity is compensated only at the desired final configuration q_d, the error maintains greater also at the steady state with respect to the case of on-line gravity compensation. An increase of the K_P matrix aimed at reducing the error at the steady state cannot be performed, in view of the closeness to the motor saturation, as explained above.

VII. Conclusion

In this paper elasticity at the robot joints in regulation tasks has been taken into account, and a proportional-derivative control action is proposed to compensate it. The work has resumed the PD control on motor variables with constant gravity compensation in [9] and has extended it to an on-line gravity compensation. The main purpose is to improve the transient behavior thanks to the adoption of a gravity-biased motor position variable in the estimate of gravity torque.

As in [9], the control law requires using only the position sensors on the motor shafts.

The control law has been demonstrated to stabilize robot manipulators with elastic joints. In particular, asymptotic stability has been proved through the direct Lyapunov method and La Salle's Theorem, and the control performance has been evaluated by means of experiments on an 8-d.o.f. robot manipulator with elastic joints.

The results have shown that the PD control in [9] can cause motor saturation in view of the large error generated by a constant gravity compensation. The use of an interpolating trajectory guiding the robot to the desired final position has been proposed, in order to reduce the maximum torque values at the motors.

A comparison has been carried out between the control law in [9] and the proposed PD control with on-line gravity compensation. The experimental results have shown a better transient behavior and also a reduction of the position error at steady state, caused by static friction and/or inaccurate estimate of gravity torque, when the on-line gravity compensation is used.

Finally, it is worth mentioning that the PD control law with on-line gravity compensation has been extended to the Cartesian space, in order to regulate robot compliance at the end effector [12].

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References

[1] E.I. Rivin, "Effective rigidity of robot structure: analysis and enhancement", *American Control Conference*, Boston, MA, 1985.

[2] L.M. Sweet, M.C. Good, "Re-definition of the robot motion control problem: Effects of plant dynamics, drive system constraints, and user requirements", *23rd IEEE Conference on Decision and Control*, Las Vegas, NV, 1984, pp. 724–731.

[3] M.W. Spong, "On the force control problem for flexible joint manipulators", *IEEE Transactions on Automatic Control*, 1989, vol. 34, pp. 107–111.

[4] C. Canudas de Wit, B. Siciliano, G. Bastin, (Eds.), *Theory of Robot Control*, Springer-Verlag, London, UK, 1996.

[5] M.W. Spong, "Modeling and control of flexible joint robots", *ASME Journal of Dynamic Systems, Measurement, and Control*, 1987, vol. 109, pp. 310–319.

[6] A. De Luca, P. Lucibello, "A general algorithm for dynamic feedback linearization of robots with elastic joints", *IEEE International Conference on Robotics and Automation*, Leuven, Belgium, 1998, pp. 504–510.

[7] M.W. Spong, K. Khorasani, P.V. Kokotovic, "An integral manifold approach to the feedback control of flexible joint robots", *IEEE Journal of Robotics and Automation*, 1987, vol. 3, pp. 291–300.

[8] K. Khorasani, P.V. Kokotovic, "Feedback linearization of a flexible manipulator near its rigid body manifold", *Systems and Control Letters*, 1985, vol. 6, pp. 187–192.

[9] P. Tomei, "A simple PD controller for robots with elastic joints", *IEEE Transactions on Automatic Control*, 1991, vol. 36, pp. 1208–1213.

[10] S. Arimoto, F. Miyazaki, "Stability and robustness of PID feedback control for robot manipulators of sensory capability", in M. Brady and R.P. Paul, (Eds.), *Robotics Research*, MIT Press, Cambridge, MA, 1984.

[11] H. Asada, J.-J.E. Slotine, *Robot Analysis and Control*, Wiley, New York, 1986.

[12] L. Zollo, B. Siciliano, A. De Luca, E. Guglielmelli, P. Dario, "Compliance control for a robot with elastic joints", *11th International Conference on Advanced Robotics*, Coimbra, Portugal, 2003, pp. 1411–1416.