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Stabilization of a PR Planar Underactuated Robot*

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Abstract

We consider the stabilization problem for an underactuated prismatic-rotational (PR) robot with the second joint passive and moving on the horizontal plane. After a controllability analysis, a nilpotent approximation of the system is derived and used for designing an open-loop polynomial command that reduces the state error in finite time. Under suitable hypotheses, the iterative application of this command, computed as a function of the state at the end of each iteration, leads to exponential convergence to the desired equilibrium configuration. Simulation results are reported, also in the presence of unmodeled viscous friction.

1 Introduction

Underactuated robots are mechanical systems with a number m of control inputs lower than the number n of generalized coordinates. A relevant case is represented by manipulators with passive joints, whose study is mainly motivated by the desire of controlling motion in the presence of actuation failures.

Planning and control of underactuated robots with passive joints are very challenging problems [1], due to the following specific features:

- The dynamics of passive joints is a second-order differential constraint that limits the set of feasible motion trajectories.
- Motion planning between two admissible configurations is an open problem for underactuated robots with only $m = 1$ control input.
- Stabilization by smooth static feedback is not possible, due to the violation of the necessary condition of Brockett [2]. In particular, the linearization at an equilibrium is not controllable in the absence of gravity (e.g., on a horizontal plane).

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Existing solutions to planning and control problems exploit specific control properties (see, e.g., [3]) such as accessibility (strictly related to the integrability of the second-order differential constraints [4]), small-time local controllability (the lack of STLC suggests the need of maneuvers even for small reconfigurations), and dynamic feedback linearizability (which allow a viable solution to motion planning and trajectory tracking problems). As a matter of fact, there are no general results for underactuated robots and research advances on a case-by-case basis with ad-hoc solutions.

We review the literature in the no gravity case — the most difficult. The stabilization to a desired rest configuration of a 2R planar robot with passive second joint ($n = 2$, $m = 1$) has been studied in [5], using a periodic input and Poincaré map analysis, and in [6], using the repeated application of open-loop commands (iterative state steering). A 3R planar manipulator with the last rotational joint passive ($n = 3$, $m = 2$) has been considered in [7]. This system is shown to be STLC and a rest-to-rest motion planner is designed using a sequence of elementary translations and rotations around the center of percussion (CP) of the third link. In [8], the position of the CP for the same robot (actually for an XXR arm, with any combination of prismatic or rotational actuated joints) is shown to be a linearizing output: using dynamic feedback linearization, smooth state-to-state trajectories can be planned and an exponentially stabilizing tracking controller is easily designed on the linear side of the problem. These works can be extended to the case of n planar bodies with the last $n-2$ joints rotational and passive, if the CP of the i-th link is centered on the $(i+1)$-th joint, for $i = 3, \ldots, n-1$ (see [9]).

In this paper, we consider the problem of stabilizing to a given equilibrium state a PR robot in the horizontal plane with only the first joint actuated ($m = 1$). This structure is the counterpart without gravity of an inverse pendulum on a cart. After performing partial feedback linearization of the underactuated dynamic model in Sect. 2, we detail the controllability analysis in Sect. 3. Following the same approach applied in [6] to the underactuated 2R robot, a *nilpotent approximation* of the robot dynamics is derived and used in the design of the stabilization strategy, based on the *iterative state steering* paradigm. In particular, Sect. 4 presents the specific conditions for state error contraction using a continuous (polynomial) open-loop command at each iteration and the transition maneuvers that may be needed in this case. Simulation results are reported in Sect. 5, where the whole control strategy is evaluated also in the presence of unmodeled viscous friction at both joints.

2 Dynamic Model of PR Robot

Consider the 2-dof robot with a prismatic and a rotational joint in Fig. 1, moving on the horizontal plane with a single actuator at the first joint. Let q_1 be the position of the center of mass of the first link, q_2 the second joint angle, m_1 and m_2 the masses of the two links, I_2 the baricentral inertia of the second link, and $d_2 \neq 0$ the distance of its center of mass from the second joint axis. The dynamic model is[1]

$$(m_1+m_2)\ddot{q}_1 - m_2 d_2 s_2 \ddot{q}_2 - m_2 d_2 c_2 \dot{q}_2^2 = \tau \quad (1)$$

$$-m_2 d_2 s_2 \ddot{q}_1 + (I_2 + m_2 d_2^2)\ddot{q}_2 = 0 \quad (2)$$

where $c_2 = \cos q_2$ and $s_2 = \sin q_2$.

2.1 Partial feedback linearization

For analysis and control design, it is convenient to perform a partial feedback linearization [1] of the system dynamics. Using the (globally defined) state feedback

$$\tau = \left[m_1 + m_2 - \frac{m_2^2 d_2^2 s_2^2}{I_2 + m_2 d_2^2} \right] a - m_2 d_2 c_2 \dot{q}_2^2,$$

with a as new control variable, system (1-2) becomes

$$\ddot{q}_1 = a \quad (3)$$

$$\ddot{q}_2 = \frac{m_2 d_2}{I_2 + m_2 d_2^2} s_2\, a = K s_2\, a, \quad (4)$$

[1]If the actuation is reversed (first joint passive), the second-order differential equation associated to the first joint is twice integrable, leading to the holonomic constraint

$$h(q,t) = (m_1+m_2)q_1 + m_2 d_2 c_2 + k_2 t + k_1 = 0.$$

The system is thus not controllable. Similarly, eq. (2) becomes integrable when $d_2 = 0$.

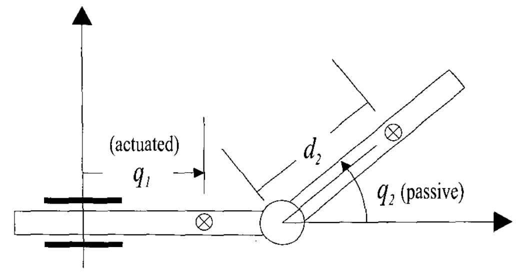

Figure 1: PR planar robot

or, defining the state $x = (q_1, q_2, \dot{q}_1, \dot{q}_2) \in I\!R^4$,

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ K s_2 \end{bmatrix} a = f(x) + g(x) a. \quad (5)$$

3 System Analysis

It is easy to see that the linear approximation of eq. (5) around an equilibrium point is not controllable. In a nonlinear setting, we study first the accessibility of system (5). Define a *regular* point $x_0 = (x_{10}, x_{20}, 0, x_{40})$, i.e., a state having zero velocity for the first joint. For $x_{40} = 0$, this is an equilibrium state x_e.

In order to test accessibility, we use the Lie algebra rank condition (LARC) [3] at a regular point x_0. By computing the Lie brackets of vector fields f and g, the accessibility matrix $\mathcal{A}_0 = \mathcal{A}(x_0)$ has the form

$$\mathcal{A}_0 = \Big[f \quad g \quad [f,g] \quad [g,[f,g]] \Big]_{x=x_0} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ x_{40} & 0 & -K s_{20} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & K s_{20} & K c_{20} x_{40} & 2K^2 s_{20} c_{20} \end{bmatrix}, \quad (6)$$

with $\det \mathcal{A}_0 = -2K^2 c_{20} s_{20} x_{40}$. Matrix $\mathcal{A}_0$ is singular iff *(i)* $x_{20} = \pm k\frac{\pi}{2}$ or *(ii)* $x_{40} = 0$. When the system is not at an equilibrium, it can be shown that the configuration singularity *(i)* can be overcome by considering further brackets (in particular, $[f,[g,[f,g]]]$). Therefore, the LARC condition holds at any regular point.

The accessibility from an equilibrium state x_e is more restricted. Since $x_{40} = 0$, the drift vector field in eq. (5) vanishes ($f(x_e) = 0$) and the accessibility matrix should be built differently. We have

$$\mathcal{A}_e = \Big[g \quad [f,g] \quad [g,[f,g]] \quad [f,[g,[f,g]]] \Big]_{x=x_e},$$

which is singular iff $x_{2e} = \pm k\frac{\pi}{2}$. When $q_2 = \{0, \pi\}$ (i.e., for even k), there is an intrinsic loss of 'controllability': no actuation will move q_2 when the second link is at rest and stretched or folded along the prismatic joint axis. On the other hand, by considering further brackets, accessibility holds for $x_{2e} = \pm\frac{\pi}{2}$ (odd k).

Due the presence of a drift vector field f in eq. (5), accessibility does not imply controllability. The only analytic way to prove controllability is to apply the sufficient conditions for small-time local controllability (STLC) given in [10] and refined in [11]. Based on these results, specialized STLC tests for systems in the form (3)–(4) have been given in [12]. Unfortunately, these conditions fail for system (5) as the 'bad' Lie bracket $[g,[f,g]]$, which gives rank to the accessibility matrix (6), cannot be expressed as a linear combination of 'good' brackets of lower length [11]. Nevertheless, controllability will be shown in a constructive way, by explicitly designing a stabilizing control law.

3.1 Coordinate transformation

Following the algorithm in [13], we use the Lie algebraic structure of system (5) to derive a suitable change of coordinates around a regular point x_0. Let

$$y = \mathcal{A}_0^{-1}(x - x_0), \qquad y \in I\!R^4. \tag{7}$$

We check whether y are *privileged* coordinates. For, we need to compare the lengths of the vector fields in $\mathcal{A}_0$ with the orders ω_i of the new coordinates y_i, $i = 1,\dots,4$. The length of a vector field is the number of Lie bracket operations that define it plus one: f and g have length 1, $[f,g]$ has length 2, and $[g,[f,g]]$ has length 3. The order of y_i is the minimum number of directional derivatives of y_i along f or g to be performed such that the result is nonzero at x_0. Privileged coordinates have orders equal to the lengths of the vector fields in $\mathcal{A}_0$, i.e., $\{1,1,2,3\}$. Easy computations show that $\omega_1 = 1$, $\omega_2 = 1$, $\omega_3 = 2$, but $\omega_4 = 2 \neq 3$ (in fact, $L_g L_f y_4(x_0) = L_f L_g y_4(x_0) = \frac{x_{40}}{2Ks_{20}} \neq 0$) and thus y_4 is not a privileged coordinate. However, by simple inspection, a set of privileged coordinates is:

$$z_i = y_i, \quad i = 1,2,3, \qquad z_4 = y_4 - \frac{x_{40}}{2Ks_{20}} y_1 y_2.$$

The invertible coordinate transformation $z = \Phi(x)$ is

$$\begin{aligned}
z_1 &= -K\frac{s_{20}}{x_{40}}(x_1 - x_{10}) + \frac{1}{x_{40}}(x_2 - x_{20}) \\
z_2 &= x_3 - x_{30} \\
z_3 &= -(x_1 - x_{10}) \\
z_4 &= \frac{x_{40}(x_1 - x_{10})}{2Ks_{20}} - \frac{x_3 - x_{30}}{2Kc_{20}} + \frac{x_4 - x_{40}}{2K^2 c_{20} s_{20}} \\
&\quad + \frac{1}{2}\left[(x_1 - x_{10}) - \frac{1}{Ks_{20}}(x_2 - x_{20})\right](x_3 - x_{30}).
\end{aligned} \tag{8}$$

The system equations in the z coordinates are ready for an high-order approximation that, by construction, preserves the same accessibility of the original system.

3.2 Nilpotent approximation

The nilpotent approximation of $\dot{z} = f_z(z) + g_z(z)a$ at $z_0 = \Phi(x_0) = 0$ involves Taylor series expansion of each coordinate z_i up to the order $\omega_i - 1$. Having the coordinates z_1 and z_2 order $\omega_1 = \omega_2 = 1$, the first and second components of f_z and g_z are approximated by constant terms; their third component is approximated by linear terms, since z_3 has $\omega_3 = 2$; as for the last component, having z_4 order $\omega_4 = 3$, we will truncate its expansion at the second order in z_1 and z_2 and at the first order in z_3. The approximated system is

$$\begin{aligned}
\dot{z} &= \begin{bmatrix} 1 \\ 0 \\ -z_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{x_{40}^2}{4Kc_{20}} z_1^2 - \frac{1}{2} z_3 \end{bmatrix} a \\
&= \hat{f}_z(z) + \hat{g}_z(z)\, a.
\end{aligned} \tag{9}$$

The nilpotent approximation is polynomial and has a triangular structure. Therefore, when applying a parametrized input a, it can be easily integrated in closed form. In addition, the linear part of the dynamics (5) is fully preserved in eq. (9). The evolution of the robot second joint and of its nilpotent approximation (9) are shown in Fig. 2, when applying a cyclic polynomial acceleration (see eq. (11)) to the first joint for $T = 2$ s. The approximation describes well the behavior of the system, but its accuracy decreases with amplitude and duration of the command. Deriving a bound on the error is still an open research issue.

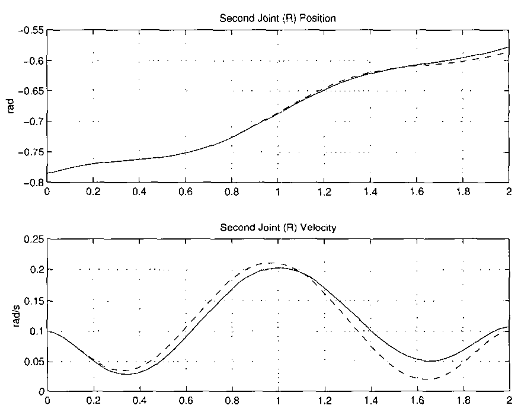

Figure 2: Real (–) and approximated (- -) second joint position (top) and velocity (bottom) under a cyclic motion of the first joint

4 Control Strategy

Our objective is to stabilize the underactuated robot to an equilibrium configuration $q_d = (q_{1d}, q_{2d})$ (with zero final velocity). In view of eq. (3), the first joint can be regulated to $(q_{1d}, 0)$ in a finite time T_1 (e.g., using a terminal feedback controller). This first phase, called *alignment* of the active joint, can also be implemented in practice with a PD controller

$$a = k_p(q_{1d} - q_1) - k_d\dot{q}_1, \qquad k_p, k_d > 0. \qquad (10)$$

At the end of the alignment phase, one has in general $q_2(T_1) \neq q_{2d}$ and $\dot{q}_2 \neq 0$. We design then a *cyclic* open-loop command a for a period T so as to bring the first joint back to $(q_1, \dot{q}_1) = (q_{1d}, 0)$ and simultaneously steer the unactuated joint closer to $(q_2, \dot{q}_2) = (q_{2d}, 0)$ at the end of the cycle. When this second phase, called *contraction*, is applied iteratively (namely, starting from the state reached at the end of the previous cycle) we obtain the iterative state steering control method [14]. If the contracting command is Hölder-continuous with respect to the initial conditions at the beginning of each cycle, asymptotic stabilization is achieved with exponential rate of convergence.

Sometimes it is not possible to switch directly from the alignment to the contraction phase, since the latter may be successfully executed only when starting from specific *contraction sets* in the robot state space. In that case, it is necessary to insert a *transition* phase in order to reach the initial conditions sufficient for contraction.

4.1 Contraction phase

The design of a cyclic *and* contracting command on system (5) is not easy. The nilpotent approximation (9) is used to this end, assuming that each cycle starts from a regular state. For simplicity, we shall reset time to zero at the end of the previous phase.

A 5-th order polynomial acceleration $a(t)$ is chosen as a parametric command for the first joint. Its coefficients are determined from the following boundary conditions: cyclicity, i.e., $q_1(0) = q_1(T)$ $(= q_{1d})$ and $\dot{q}_1(0) = \dot{q}_1(T)$ $(= 0)$; vanishing of acceleration $(\ddot{q}_1(0) = \ddot{q}_1(T) = 0)$, implying continuity of a across cycles; zero integral of $q_1(t)$ in $[0, T]$, to simplify subsequent integration. Letting $\lambda = t/T$, we obtain

$$a(t) = \frac{A}{T^2}\left(3\lambda - 30\lambda^2 + 90\lambda^3 - 105\lambda^4 + 42\lambda^5\right) \quad (11)$$

with A and T left as free parameters.

The nilpotent approximation (9) can be easily integrated under the action of the open-loop command (11). Similarly to [6], we have

$$z_1(T) = T, \quad z_2(T) = z_3(T) = 0, \quad z_4(T) = -\beta\frac{A^2}{T},$$

where $\beta = \frac{3}{80080}$. Using the inverse transformation $x = \Phi^{-1}(z)$ obtained from eqs. (8), it is

$$\Delta q_2 = q_2(T) - q_2(0) = \dot{q}_2(0)T \qquad (12)$$

$$\Delta \dot{q}_2 = \dot{q}_2(T) - \dot{q}_2(0) = -2K^2 c_{20} s_{20} \beta \frac{A^2}{T}. \qquad (13)$$

Separate contraction of the second joint position and velocity errors are guaranteed if

$$\Delta q_2 = (1 - \eta_1)(q_{2d} - q_2(0)) \qquad (14)$$

$$\Delta \dot{q}_2 = (\eta_2 - 1)\dot{q}_2(0), \qquad (15)$$

with contraction rates $\eta_1, \eta_2 \in [0, 1)$. Solving for T and A from eqs. (12–13) and (14–15) yields

$$T = (1 - \eta_1)\frac{q_{2d} - q_2(0)}{\dot{q}_2(0)} \qquad (16)$$

$$A = \sqrt{\frac{T(1 - \eta_2)\dot{q}_2(0)}{K^2\beta \sin 2q_2(0)}}. \qquad (17)$$

Since it should be $0 < T < \infty$ and $0 < A < \infty$, expressions (16) and (17) are valid, and thus contraction is allowed, iff the state belongs to the contractions sets:

$$\begin{cases} \dot{q}_2(0) > 0 \\ q_{2d} > q_2(0) \\ q_2(0) \in \text{I or III} \end{cases} \quad \text{or} \quad \begin{cases} \dot{q}_2(0) < 0 \\ q_{2d} < q_2(0) \\ q_2(0) \in \text{II or IV}, \end{cases}$$

where roman numbers define the four (open) quadrants of the 2π angle. If at the end of the alignment phase the robot is not in a contraction set, a *transition* phase is required. However, once the proper contraction set is reached, it is never left during the iterated application of the contracting open-loop command.

4.2 Transition phase

There are different situations where the contraction phase cannot be started. Assume that q_{2d} is in quadrant I. In order to reach the proper contraction set, it is necessary to bring q_2 to quadrant I, with $q_2 < q_{2d}$ and $\dot{q}_2 > 0$. The following cases may happen:

T1. $\dot{q}_2(0) > 0$, $q_2(0)$ is in quadrant I, but $q_2(0) > q_{2d}$. Set $a = 0$ to keep the second joint velocity constant until its position reenters quadrant I passing through 0 (thus having at a certain time $q_2 < q_{2d}$).

T2. $\dot{q}_2(0) > 0$, but $q_2(0)$ is not in quadrant I. It is sufficient to apply $a = 0$ as in the previous case.

T3. $\dot{q}_2(0) < 0$. To invert the sign of $\dot{q}_2$, note from eq. (13) that the velocity variations $\Delta\dot{q}_2$ have opposite sign w.r.t. $\sin 2q_2(0)$. If $q_2(0)$ belongs to quadrant II or IV, use the command (11) with any value for time T and amplitude A so that $\Delta\dot{q}_2$ will be positive, driving $\dot{q}_2(T)$ toward a positive value. If $q_2(0)$ is instead in quadrant I or III, set $a = 0$ until q_2 enters quadrant II or IV. After a finite number of such cycles, $\dot{q}_2 > 0$ will be attained and we recover one of the two previous cases or directly enter the contraction phase.

Similar transition maneuvers can be carried out when the desired position q_{2d} of the second joint is in one of the other quadrants.

5 Simulation Results

The task is to stabilize the equilibrium configuration $q_d = (0, \pi/4)$ starting from $q_0 = (1, -\pi/4)$ with zero velocity. The nominal data for the PR robot are: $d_2 = 0.5$ m, $I_2 = 1$ kgm^2, $m_1 = m_2 = 1$ kg. The PD gains for the alignment phase are $k_p = 7$, $k_d = 5$. The contraction rates are chosen as $\eta_1 = 0.5$, $\eta_2 = 0.6$.

Figures 3 and 4 show the position and velocity of the joints. At $T_1 = 3.4$ s the alignment phase is virtually completed; the second joint is still in quadrant IV with a positive velocity $\dot{q}_2 = 0.06$ rad/s. A transition is then needed (case T2 in Sect. 4.2) and we set $a = 0$ until the second joint has safely reached quadrant I ($q_2 \simeq \pi/8$ rad at $t = 14.9$ s). In the contraction phase, the second joint exponentially approaches the desired position with zero final velocity. After 9 contraction cycles (for a total of 32.6 s), the desired equilibrium is reached within a tolerance $\varepsilon = |q_2 - q_{2d}| + |\dot{q}_2| \leq 0.005$. Figure 5 shows the resulting acceleration a and the force τ on the first joint.

In order to test for the robustness of the stabilizing strategy, we have applied the same previous control law in the presence of viscous friction at both joints: the terms $f_1\dot{q}_1$ and $f_2\dot{q}_2$ were added in the lhs of eq. (1) and (2), respectively, with $f_1 = f_2 = 0.02$. As a consequence, eqs. (3–4) will be perturbed by nonlinear terms and all control phases will be affected accordingly. In Figs. 6–7, the alignment phase is still achieved by the PD law (10) in about the same time, while the longer transition phase ends at $t = 18.3$ s. Sufficient error contraction is preserved but, after each contraction phase, a re-alignment of the first joint is needed. Convergence is obtained after 7 contraction+realignment cycles (for a total of 49.2 s).

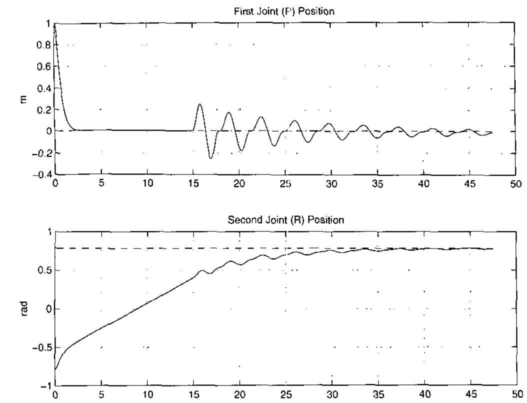

Figure 3: Joint positions (–) and their reference (- -)

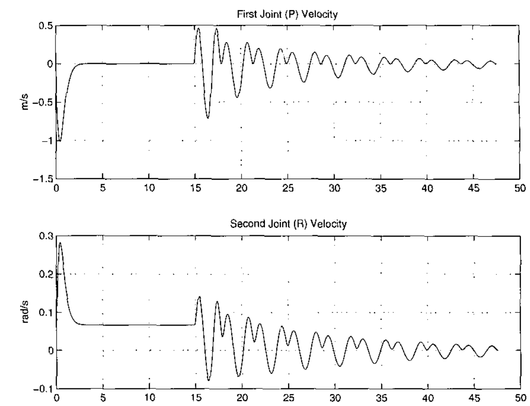

Figure 4: Joint velocities

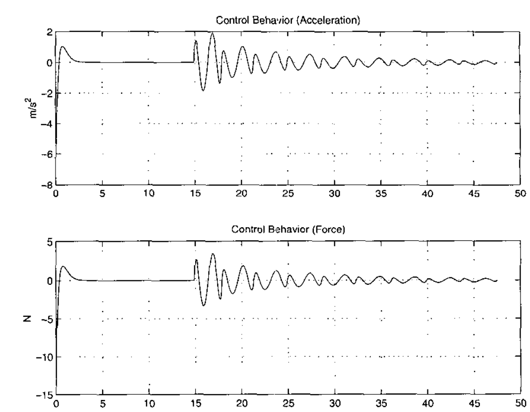

Figure 5: Acceleration a and force τ commands

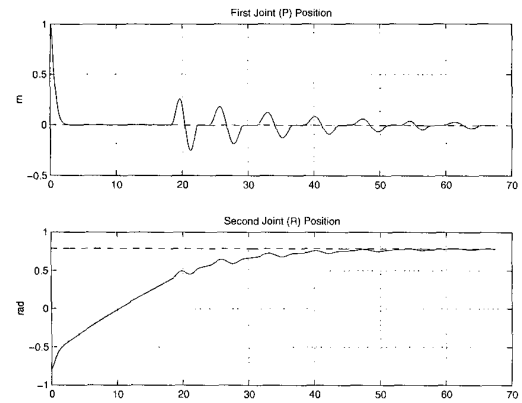

Figure 6: Joint positions (–) and their reference (- -) in the presence of viscous friction

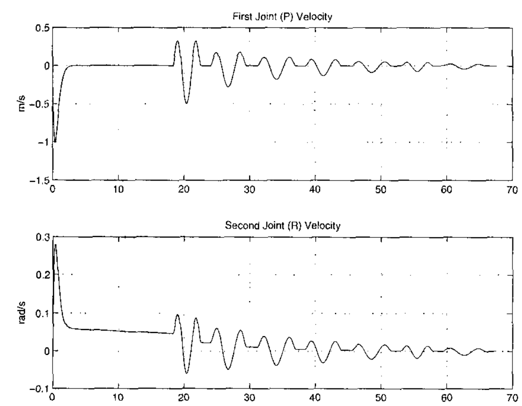

Figure 7: Joint velocities in the presence of viscous friction

6 Conclusions

The stabilization problem for a PR planar robot with passive second joint has been solved using the iterative state steering approach. The overall control strategy is able to asymptotically stabilize a given equilibrium configuration with an exponential rate of convergence. This result does not hold when starting at rest with the second link stretched or folded. Robustness of the control strategy has been evaluated in simulations w.r.t. unmodeled viscous friction. An issue left for further research is related to change of the accessibility structure of the system at $q_2 = \pm\pi/2$ and/or $\dot{q}_2 = 0$. The existence of a globally valid basis for the accessibility of the system would avoid control singularities.

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