

Robust stabilization of the plate-ball manipulation system

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Abstract

We consider the plate-ball system as a typical example of manipulation by rolling contacts. While there exist techniques for planning motions of this nonholonomic mechanism in nominal conditions, our objective in this paper is the robust execution of maneuvers in the presence of model perturbations. To this end, we adopt an iterative steering paradigm based on the use of a nilpotent approximation of the system. Simulation results are reported to confirm the robustness achieved with the proposed feedback controller.

1 Introduction

Rolling manipulation has recently attracted the interest of robotic researchers as a convenient way to achieve dexterity with a relatively simple mechanical design (see [1–3] and the references therein). In fact, the nonholonomic nature of rolling contacts between rigid bodies can guarantee the controllability of the manipulation system (hand+manipulated object) with a reduced number of actuators. More in general, this is another example of the minimalistic trend in the field of robotics, aimed at designing devices of reduced complexity for performing complex tasks.

The archetypal example of rolling manipulation is the plate-ball system [4–7]: the ball (the manipulated object) can be brought to any contact configuration by maneuvering the upper plate (the first finger), while the lower plate (the second finger) is fixed. Despite its mechanical simplicity, the planning and control problems for this device already raise challenging theoretical issues. In fact, in addition to the well-known limitations coming from its nonholonomic nature (e.g., the lack of smooth stabilizability), the plate-ball system is neither flat nor nilpotentizable; therefore the classical techniques (e.g., see [8]) for planning and stabilization of nonholonomic systems cannot be applied.

To this date, only the planning problem has been

attacked with some success; e.g., see the symbolic algorithm of [5] (which contains an error but admits a suitable modification) and the numerical algorithm of [3]. Like for any planner based on open-loop control, however, the successful execution of maneuvers is not preserved in the presence of perturbations — some sort of feedback is necessary to induce a degree of robustness.

In this paper, we prove that robust stabilization of the plate-ball mechanism can be simply achieved through iterative application of an appropriate open-loop control law designed for the nilpotent approximation of the system. This paradigm, based on the theoretical results in [9], has already been effectively used for the stabilization of *general* (i.e., non-flat) nonholonomic systems, such as off-hooked trailer vehicles [10] or underactuated robots in the absence of gravity [11].

The paper is organized as follows. In Sect. 2, the model of the plate-ball system is given together with its nilpotent approximation. Section 3 describes our stabilization strategy, which makes use of a contracting open-loop control (Sect. 3.1) within an iterative scheme (Sect. 3.2). The robust performance of the method is confirmed by simulation in Sect. 4.

2 The plate-ball system

Consider the system shown in Fig. 1, consisting of a spheric ball of radius ρ rolling between two horizontal plates. The lower plate is fixed, while the upper is actuated and can translate horizontally.

2.1 Kinematic model

Denote by u and v the coordinates (latitude and longitude, respectively) of the contact point on the sphere, by x , y the cartesian coordinates of the contact point on the lower plane, and by ψ the angle between the x axis and the plane of the meridian through the contact point (see Fig. 1). We assume $-\pi/2 < u < \pi/2$ and $-\pi < v < \pi$, so that the contact point belongs always

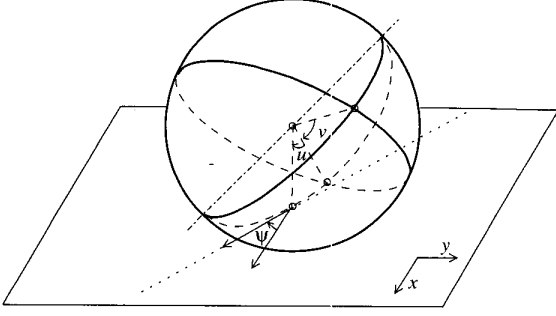


Figure 1: The plate-ball system. The upper plate is not shown in the figure for the sake of clarity.

to the same coordinate patch for the sphere.

The manipulation system is completely described by the kinematics of contact between the sphere and the lower plate [4]:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{\psi} \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos \psi / \rho \\ -\sin \psi / \rho \cos u \\ \tan u \sin \psi / \rho \\ 1 \\ 0 \end{pmatrix} w_x + \begin{pmatrix} -\sin \psi / \rho \\ -\cos \psi / \rho \cos u \\ \tan u \cos \psi / \rho \\ 0 \\ 1 \end{pmatrix} w_y, \quad (1)$$

where w_x and w_y are the cartesian components of the translational velocity of the sphere, which we assume to be directly controlled¹.

In view of the nilpotent approximation procedure, it is convenient to perform the input transformation

$$\begin{pmatrix} w_x \\ w_y \end{pmatrix} = \begin{pmatrix} -\sin \psi \cos u & \cos \psi \\ -\cos \psi \cos u & \sin \psi \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \quad (2)$$

obtaining the triangular system

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{\psi} \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 \\ 1/\rho \\ -\sin u / \rho \\ -\sin \psi \cos u \\ -\cos \psi \cos u \end{pmatrix} w_1 + \begin{pmatrix} 1/\rho \\ 0 \\ 0 \\ \cos \psi \\ -\sin \psi \end{pmatrix} w_2. \quad (3)$$

Note that the input transformation (2) is always defined, except for $u = \pm\pi/2$ which is however outside our coordinate patch.

2.2 Nilpotent approximation

Nilpotent approximations [12, 13] of nonlinear systems are high-order local approximations that are useful

¹Recall that the translational velocity of the sphere is half the translational velocity of the upper plane.

when tangent linearization does not retain controllability, as in nonholonomic systems. In particular, the computation of (approximate) steering controls can be performed symbolically, thanks to the closed-form integrability of the nilpotent system, which is polynomial and triangular by construction.

Thanks to the particular structure of our iterative steering strategy (see Sect. 3), it is sufficient to compute the nilpotent approximation at configurations of the form $\bar{q} = (0, 0, 0, \bar{x}, \bar{y})$. Applying the procedure in [13] to system (3), one obtains the so-called *privileged* coordinates by the following change of variables

$$\begin{aligned} z_1 &= \rho v \\ z_2 &= \rho u \\ z_3 &= \rho^2 \psi \\ z_4 &= -\rho^3 u + \rho^2(x - \bar{x}) \\ z_5 &= \rho^3 v + \rho^2(y - \bar{y}). \end{aligned} \quad (4)$$

This transformation is globally valid due to the fact that the degree of nonholonomy is 3 everywhere.

The approximate system is then computed by differentiating eqs. (4) and expanding the input vector fields in Taylor series up to a suitably defined order:

$$\begin{aligned} \dot{\hat{z}}_1 &= w_1 \\ \dot{\hat{z}}_2 &= w_2 \\ \dot{\hat{z}}_3 &= -\hat{z}_2 w_1 \\ \dot{\hat{z}}_4 &= -\hat{z}_3 w_1 \\ \dot{\hat{z}}_5 &= \frac{1}{2} \hat{z}_2^2 w_1 - \hat{z}_3 w_2. \end{aligned} \quad (5)$$

The approximation is polynomial and triangular; in particular, the dynamics of z_1 and z_2 is exact.

Another nilpotent approximation for the plate-ball system is given in [14].

3 The stabilization strategy

Assume that we wish to transfer the plate-ball system from q^0 to q^d , respectively the initial and desired contact configuration. Without loss of generality, we assume that $q^d = (0, 0, 0, 0, 0)$; this can always be achieved by properly defining the reference frames on the sphere and the lower plane.

Our objective is to devise a stabilization strategy which is robust w.r.t. the presence of model perturbations (e.g., on the sphere radius ρ). To this end, it is necessary to embed some form of feedback in the scheme. A natural way to realize this is represented by the iterative steering (IS) paradigm [9].

The essential tool of this method is a contracting open-loop control law, which can steer the system

closer to the desired state q^d in a finite time. If such control is Hölder-continuous w.r.t. the desired reconfiguration, its iterated application (i.e., from the state reached at the end of the previous iteration), guarantees exponential convergence of the state to q^d . The resulting control is a time-varying law which depends on a sampled feedback action. A certain degree of robustness is also achieved: a class of non-persistent perturbations is rejected, and the error is ultimately bounded in the presence of persistent perturbations.

3.1 A contracting open-loop control

To comply with the IS paradigm outlined above, we must design an open-loop control which steers system (1) (or system (3)) from q^0 to a point closer in norm to $q^d = (0, 0, 0, 0, 0)$. Since the plate-ball manipulation system is controllable [5], such an open-loop control certainly exists. However, the necessary and sufficient condition for flatness [15] are not satisfied; equivalently, the system cannot be put in chained form, as already noticed in [3]. Therefore, we cannot use conventional techniques for generating the required open-loop control.

A possibility is to use the planning method of [3]; however, such numerical method is computationally intensive and therefore unsuitable for the real-time iteration of the open-loop control. Moreover, a symbolic expression of the control would be needed for guaranteeing the continuity properties required by the IS approach. We therefore settle for an approximate (but symbolic) solution; this is on the other hand consistent with the IS framework, which only requires the error to contract at each iteration.

Our open-loop controller requires two phases:

- I. Drive the first three variables u , v and ψ to zero. This amounts to steering the ball to the desired contact configuration regardless of the variables x and y , i.e., of the cartesian position of the contact point. Denote by $q^I = (0, 0, 0, x^I, y^I)$ the contact configuration at the end of this phase.
- II. Bring x and y closer to x^d and y^d (in norm), while guaranteeing that u , v and ψ return to their desired zero value.

Since the first three equations of (3) can be easily transformed in chained form (see Appendix), phase I can be performed in a finite time T_1 by choosing one of many available steering controls (see [8]). However, the latter should comply with the Hölder-continuity requirement w.r.t. the desired reconfiguration; relevant examples are given in [9].

For the second phase, a possible choice is to perform a cyclic motion of period T_2 on u , v and ψ , giving final values $x(T_1 + T_2) = x^H$, $y(T_1 + T_2) = y^H$ closer to zero than $x(T_1) = x^I$, $y(T_1) = y^I$. To design a control law that produces such a motion, we shall exploit the nilpotent approximation of the plate-ball system.

Consider the nilpotent approximation (5) at q^I . The synthesis of a control law that transfers in a finite time T_2 the state \hat{z} from $z^I = 0$ to z^H (respectively, the images of q^I and $q^H = (0, 0, 0, x^H, y^H)$), computed through eqs. (4) can be done as follows. Choose the open-loop control inputs as

$$w_1 = a_1 \cos \omega t + a_2 \cos 4\omega t \quad (6)$$

$$w_2 = a_3 \cos 2\omega t, \quad (7)$$

with $a_1, a_2, a_3 \in \mathbb{R}$ and $\omega = 2\pi/T_2$. The integration of eqs. (5) gives

$$\begin{aligned} z_1(T_2) &= z_2(T_2) = z_3(T_2) = 0 \\ z_4(T_2) &= k_1 a_1^2 a_3 \\ z_5(T_2) &= k_2 a_2 a_3^2, \end{aligned} \quad (8)$$

having set $k_1 = -T_2^3/32\pi^2$ and $k_2 = T_2^3/128\pi^2$.

In order to obtain $z_4(T_2) = z_4^H$ and $z_5(T_2) = z_5^H$, coefficients a_1 and a_2 in (6-7) must be chosen as

$$a_1 = \sqrt{\frac{z_4^H}{k_1 a_3}} \quad a_2 = \frac{z_5^H}{k_2 a_3^2}. \quad (9)$$

Substitution of eq. (9) in eq. (8) proves that the value of a_3 is immaterial as long as (i) $a_3 \neq 0$ when $z_4^H \neq 0$ or $z_5^H \neq 0$, and (ii) $\text{sign}(a_3) = -\text{sign}(z_4^H)$. Therefore, denoting by $\|\cdot\|$ denotes the euclidean norm, we let

$$a_3 = -\text{sign}(z_4^H) \cdot \left\| \begin{pmatrix} z_4^H \\ z_5^H \end{pmatrix} \right\|^{1/2r} \quad r > 1, \quad (10)$$

This choice, guarantees for a_1 , a_2 and a_3 the Hölder-continuity property required by the IS paradigm.

The other condition to be met by our two-phase open-loop control is contraction from q^0 to q^H . It is easy to show that, with a suitable definition of norm, such condition is satisfied. This is true in spite of fact that the use of the nilpotent dynamics (5) for computing $z_4(T_2)$ and $z_5(T_2)$ induces an approximation error² on x and y , which increases with the required reconfiguration. In fact, the contraction property can be preserved by requiring a sufficiently small contraction.

²Note that u , v and ψ return to zero, as verified by integration of the first three equations of the original system (3). Thus, the open-loop controls (6-7) are exactly cyclic in u , v and ψ .

3.2 Iterative steering

We now clarify the use of the proposed open-loop controller within the iterative steering framework.

Starting from the initial contact configuration, apply the open-loop control of phase I for the required time T_1 . Using the values x^I, y^I at the end of this phase, the desired z_4^I and z_5^I are generated as

$$z_4^I = \beta_1 z_4^d \quad z_5^I = \beta_2 z_5^d, \quad (11)$$

where $\beta_1 < 1, \beta_2 < 1$ are the chosen contraction rates and z_4^d, z_5^d are the images of $x^d = 0, y^d = 0$ computed inverting eqs. (4), in which $\bar{x} = x^I, \bar{y} = y^I$.

At this point, eqs. (9–10) are used to compute coefficients a_i , and the phase II open-loop controls (6–7) are applied to system (3). After $T_1 + T_2$ seconds from the initial time, the system state is sampled and the two-phase control procedure is repeated.

The values of z_4^I and z_5^I are updated at each iteration using eq. (11) (with constant β_1, β_2). In fact, as transformation (4) depends on the approximation point, the same is true for z_4^d, z_5^d . Note also that:

- Since the conditions of the IS paradigm [9] have been satisfied, it is guaranteed that the manipulation system state q exponentially converges to the desired contact configuration q^d .
- In the absence of perturbations, there is no need to repeat phase I after the first iteration.
- In perturbed conditions, it is necessary to analyze the structure of the perturbation itself. If certain requisites (see [9, Th. 2]) are met, the perturbation will be rejected on the simple basis of the stable behavior of the nominal system.

4 Simulation results

Two simulations are now presented to show the effectiveness of the proposed stabilization strategy: in the first, perfect knowledge of the system is assumed (*nominal case*), while in the second we have included a perturbation on the ball radius ρ (*perturbed case*).

In the first simulation, the radius $\rho = 1$ is exactly known and phase I has already been executed. The initial and desired configurations are $q^0 = (0, 0, 0, 0.5, 0.5)$ and $q^d = (0, 0, 0, 0, 0)$, respectively. In each iteration, the open-loop control (6–7) is applied with $T_2 = 1$ sec, $r = 1.5$ in eq. (10), and contraction rates $\beta_1 = \beta_2 = 0.4$ in eq. (11).

Figures 2 and 3 illustrate the exponential convergence of the state variables along the iterations. The complete cartesian path of the contact point is shown in Fig. 4: note how the path of the single iterations

‘shrinks’ with time. The contraction of the positioning error is visible in Fig. 5, which reports the path of the contact point during iterations 1, 4, 7 and 10.

In the second simulation, q^0, q^d as well as the control parameters are the same of the previous simulation, but a 10% perturbation on the value of the ball radius has been introduced; only its nominal value $\rho = 1$ is known and can be used for computing the control law. The theoretical framework of the IS paradigm (see [9, Th. 2]) guarantees that this kind of perturbation will be rejected by the iterative steering scheme.

Figures 6 and 7 confirm that exponential convergence is preserved despite the perturbation — only at a slightly smaller rate. The cartesian path of the contact point is very similar to the nominal case, as shown in Fig. 8, although Fig. 9 reveals that the paths in the single iterations are deformed.

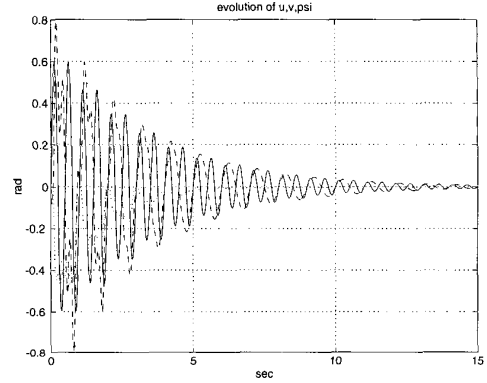


Figure 2: Nominal system: Evolution of u (solid), v (dashed) and ψ (dotted)

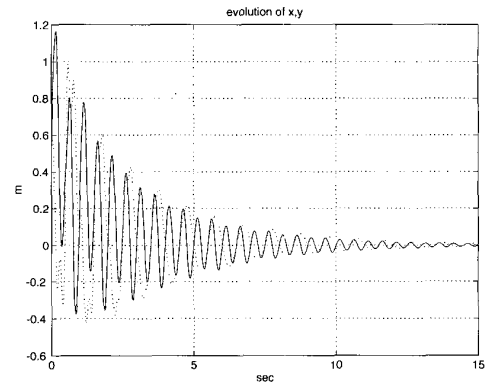


Figure 3: Nominal system: Evolution of x (solid) and y (dotted)

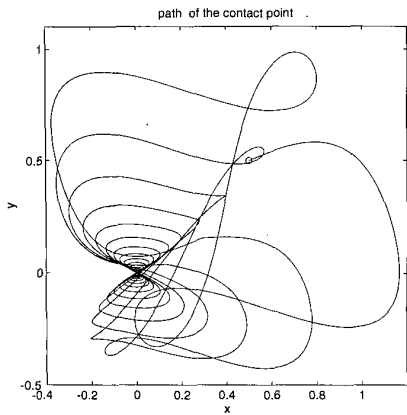


Figure 4: Nominal system: Cartesian path of the contact point (the small circle indicates q^0)

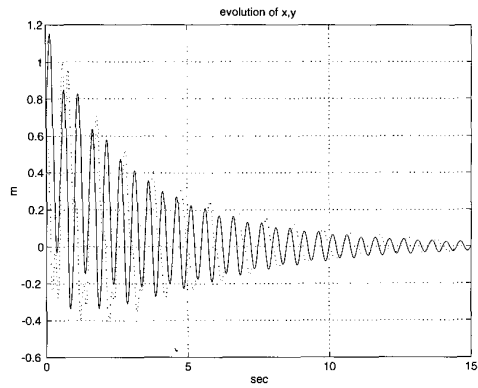


Figure 7: Perturbed system: Evolution of x (solid) and y (dotted)

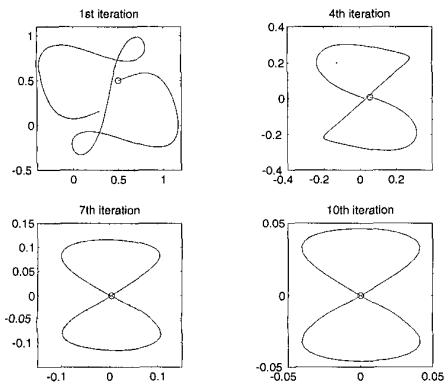


Figure 5: Nominal system: Cartesian paths of the contact point during the 1st, 4th, 7th and 10th iterations (the small circle indicates the starting configuration of each iteration). Notice the different scale in the plots.

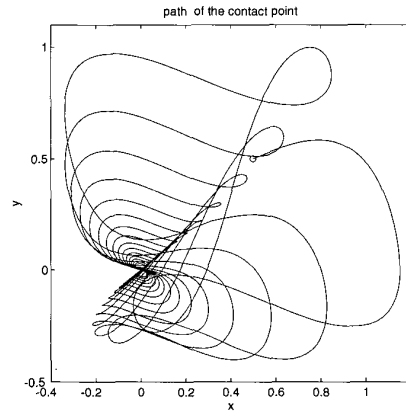


Figure 8: Perturbed system: Cartesian path of the contact point (the small circle indicates q^0)

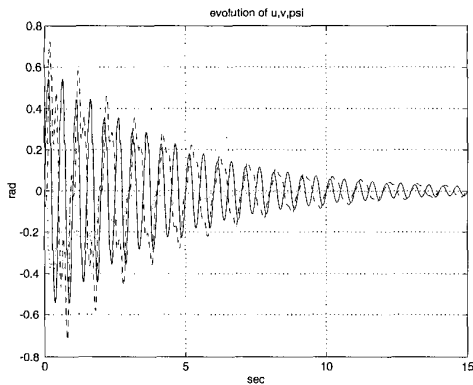


Figure 6: Perturbed system: Evolution of u (solid), v (dashed) and ψ (dotted)

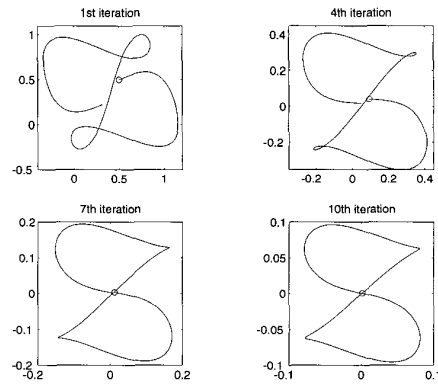


Figure 9: Perturbed system: Cartesian paths of the contact point during the 1st, 4th, 7th and 10th iterations (the small circle indicates the starting configuration of each iteration).

5 Conclusions

We have presented a feedback method for executing robust maneuvers with a plate-ball manipulation device in the presence of perturbations. Beside its practical interest, this problem is challenging from a theoretical viewpoint because the considered nonholonomic system is outside the class for which well-established planning and control techniques exist.

The proposed solution is based on an iterative steering scheme, which makes use of a nilpotent approximation of the system for designing the open-loop control law to be applied repeatedly. The performance of the algorithm, which can be established relying on the iterative steering theoretical framework, has been confirmed by simulations, both in the nominal case and in the presence of a perturbation on the ball radius.

Another advantage of the proposed technique, which could be useful for performing manipulation in the presence of obstacles, is the possibility of shaping the system trajectory during the generic iteration through the choice of the open-loop control. Finally, we point out that the same iterative approach may be successfully applied to other manipulation systems, such as the impulsive manipulator based on tapping described in [16].

Appendix

The first three equations of system (3) can be put in chained form by the following coordinate change

$$\begin{aligned}x_1 &= -v \\x_2 &= \sin u \\x_3 &= \psi\end{aligned}$$

and input transformation

$$\begin{aligned}v_1 &= -w_1/\rho \\v_2 &= \cos u w_2/\rho.\end{aligned}$$

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