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Reconfiguration of redundant robots under kinematic inversion

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Abstract—We consider the problem of reconfiguring the joints of a redundant robot driven only through end-effector velocity commands. Depending on the chosen kinematic inversion scheme, this system behavior is subject to differential constraints that may be integrable or not. An analogy is established with non-holonomic systems, allowing the use of existing techniques to design cartesian inputs that bring the robot to the desired configuration. Two different control approaches are presented for a robot with a single degree of redundancy, based on the holonomy angle concept and on sinusoidal steering of chained-form systems. Simulation results are reported and generalizations are briefly discussed.

1. INTRODUCTION

Consider a kinematically redundant robot with n joints performing an m-dimensional task ($m < n$) and assume that the joint motion $q(t)$ is generated using a kinematic inversion scheme at the velocity level

$$\dot{q} = G(q)u = \sum_{i=1}^{m} g_i(q)u_i. \tag{1}$$

Here, G is any *generalized inverse* of the analytic Jacobian $J(q) = \partial f/\partial q$, with $f(q)$ the direct kinematic map and $u \in \mathbb{R}^m$ is a task velocity command. A generalized inverse satisfies $JGJ = J$, implying that any feasible vector u is exactly realized [1]. When a desired motion $p_d(t)$ of the robot end-effector is specified, a typical choice is to set $u = \dot{p}_d$ [2].

In this paper, the following problem is considered: find an input $u(t)$ in (1) such that the robot is driven from an initial configuration q_0 to a desired configuration q_d. Note that this amounts to reconfigure the n joints of the arm using only m velocity commands. The equations of motion (1) are those of a non-linear system without drift. From a mechanical point of view, the system is underactuated, because there are less inputs u than generalized variables q.

In the above formulation, we have assumed that the arm posture may be changed only by steering u in (1). This assumption restricts our interest to those robotic applications where commands are necessarily or more naturally defined at the task level.

A first situation of this kind occurs when multiple redundant arms manipulate an object, like in dextrous grasping with a multifingered robotic hand. As a matter of fact, the fundamental grasp constraint for these systems can be expressed in a form similar to (1) (e.g. see [3, pp. 237 and ff.]). If we wish to reconfigure the robots (say, for achieving maximal manipulability) without losing the grasp, we may simply plan a suitable *object trajectory* by choosing the velocity command $u(t)$ in the kinematic control scheme (1) so as to achieve the desired reconfiguration for the system.

As a second example, assume that a pick-and-place operation must be performed with a redundant robot controlled through the kinematic scheme (1). One may ask whether it is possible to design forward and backward *cartesian motions* that drive the robot from the 'pick' configuration to the 'place' configuration and vice versa with a cyclic joint trajectory, so that it can be repeated over time. With our approach, this problem is simply split into two separate and symmetric joint reconfiguration tasks.

Finally, it is expected that the definition of commands directly at the end-effector level will be particularly convenient for commercial redundant robots equipped with a *built-in kinematic inversion scheme G*. Indeed, the availability of joint-level commands will still be required for the execution of other kinds of tasks. For example, when performing a *self-motion* [4] the end-effector should be kept fixed, so that it is necessary to specify commands directly at the joint level—differently from the case considered here.

The existence of a task input u driving the arm to an arbitrary q_d is guaranteed if system (1) satisfies the so-called *accessibility condition* over the whole state space. With this hypothesis, a solution to the reconfiguration problem exists and can be found by exploiting its relationships with the motion planning problem for nonholonomic systems.

Denote by $\mathcal{A}$ the *accessibility distribution*, defined as the involutive closure of the distribution $\mathcal{A}_0 = \text{span}\{g_1(q), \ldots, g_m(q)\}$ under the repeated Lie bracket operation. Recall that the Lie bracket $[v_1, v_2]$ of two vector fields v_1 and v_2 is defined as

$$[v_1, v_2](q) = \frac{\partial v_2}{\partial q} v_1(q) - \frac{\partial v_1}{\partial q} v_2(q).$$

For the reconfiguration problem to be solvable, we need to check whether the accessibility condition

$$\dim \mathcal{A}(q) = n \tag{2}$$

holds globally [5].

An interesting connection can be recognized between the accessibility condition and the recently studied issue of cyclicity in redundant robots [6–8]. A given G in (1) is said to provide a *cyclic* inversion scheme if all closed task trajectories are mapped to closed motions in the joint space. Shamir and Yomdin [6] proved that this property is guaranteed under the following involutivity condition:

$$[g_i, g_j](q) \in \text{span}\{g_1(q), \ldots, g_m(q)\}, \quad \forall i, j = 1, \ldots, m, \ \forall q. \tag{3}$$

As a matter of fact, the fulfillment of this condition is rather exceptional [9]. In [10], it has been shown how to use condition (3) in order to modify a given (non-cyclic) generalized inverse so as to obtain a cyclic kinematic inversion scheme.

The property of cyclicity is lost when the distribution associated to the vector fields g_i's is *not* involutive (however, depending on the initial condition q_0, it may still be possible to find special cyclic task paths leading to the repetition of the initial joint setting [8]). In the particular case of a single degree of redundancy, $n - m = 1$, the involutivity condition (3) is violated if and only if the accessibility condition (2) is satisfied. In the general case, $n - m > 1$, the violation of (3) becomes a necessary but not sufficient condition for accessibility. In any case, while the lack of cyclicity of a generalized inverse G is seen as a drawback when the end-effector task requires to trace closed paths, it represents a prerequisite for the solvability of the reconfiguration problem.

If the chosen kinematic inversion scheme is not cyclic, suggestions for the synthesis of reconfiguration methods are obtained by establishing the non-holonomic nature of the corresponding system. In fact, the joint velocities generated under (1) automatically satisfy $n - m$ differential constraints of the form

$$C(q)\dot{q} = 0, \tag{4}$$

where matrix C is implicitly defined by

$$\mathcal{N}(C(q)) = \mathcal{R}(G(q)) \quad \text{or equivalently} \quad \mathcal{R}(C^{\mathrm{T}}(q)) = \mathcal{N}(G^{\mathrm{T}}(q)), \tag{5}$$

where $\mathcal{N}(\cdot)$ and $\mathcal{R}(\cdot)$ denote the null space and the range space of a matrix, respectively. For system (1) to be accessible, it is necessary that constraints (4), although restricting the admissible generalized velocities, do not limit the attainable system configurations. This situation occurs when the differential constraints (4) are not integrable, i.e. *non-holonomic* [11]. In particular, the accessibility property, that guarantees the existence of a solution to our reconfiguration problem, is equivalent to the *maximal* non-holonomy of system (1), i.e. to the complete non-integrability of the set of constraints (4). We note also that these non-holonomic constraints should not be confused with the m *rheonomic* constraints

$$\psi(q, t) = f(q) - \int_0^t u(\tau)\,\mathrm{d}\tau + f(q_0) = 0$$

imposed by the direct kinematics mapping.

In view of the above relationship between accessibility and non-holonomy, for the reconfiguration of system (1) we can borrow existing techniques from the literature on motion planning for non-holonomic robotic systems, such as wheeled mobile robots and free-flying manipulators. For such systems, no continuously differentiable stabilizing feedback exists [12], so that one must resort to either open-loop control, or non-smooth and/or time-varying feedback. We will use both the idea of holonomy angle, similarly to [13], and of sinusoidal steering of chained-form systems, proposed in [14]. All our developments will be illustrated on a planar PPR arm, a robot with a single degree of redundancy with respect to the end-effector positioning task. The applicability of the results to more general cases will be briefly discussed in the concluding section.

2. INVERSION SCHEMES FOR A PPR ROBOT

Consider the PPR planar robot shown in Fig. 1, having one revolute and two prismatic joints. This robot is redundant for the task of positioning the tip of the end-effector in the plane with unspecified orientation ($n = 3$, $m = 2$). Denoting by ℓ the length of the third link, the direct kinematic equations are

$$p_x = q_1 + \ell c_3,$$
$$p_y = q_2 + \ell s_3,$$

where $s_3 = \sin q_3$ and $c_3 = \cos q_3$. The analytic Jacobian matrix

$$J(q) = \begin{bmatrix} 1 & 0 & -\ell s_3 \\ 0 & 1 & \ell c_3 \end{bmatrix}$$

has always full row rank.

2.1. A holonomic inversion scheme

We begin our analysis by identifying all cyclic generalized inverses $G = [g_1\ g_2]$ of J, i.e. those kinematic inversion schemes under which arbitrary reconfiguration is *not*

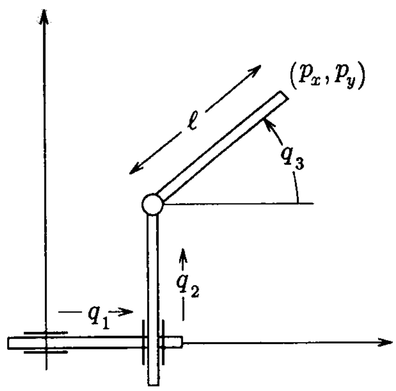

Figure 1. A PPR planar robot.

possible. To this end, note that all generalized inverses of a full rank Jacobian J can be obtained as

$$G(q) = G_1(q) + n_J(q) r^{\mathrm{T}}(q), \tag{6}$$

where G_1 is any generalized inverse of J, n_J is a vector spanning $\mathcal{N}(J)$ and $r(q)$ is an arbitrary vector in $\mathbb{R}^m$. For the PPR robot, it is convenient to choose G_1 in (6) as follows. Given a positive-definite matrix

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & w^2 \end{bmatrix},$$

where w is a constant with the same units of ℓ, the *W-weighted* pseudoinverse of J is the particular generalized inverse defined as

$$\begin{aligned} J_W^{\#}(q) &= W^{-1} J^{\mathrm{T}}(q) \big(J(q) W^{-1} J^{\mathrm{T}}(q) \big)^{-1} \\ &= \frac{1}{1 + (\ell/w)^2} \begin{bmatrix} 1 + (\ell/w)^2 c_3^2 & (\ell/w)^2 s_3 c_3 \\ (\ell/w)^2 s_3 c_3 & 1 + (\ell/w)^2 s_3^2 \\ -(\ell/w^2) s_3 & (\ell/w^2) c_3 \end{bmatrix}. \end{aligned} \tag{7}$$

Note that the weighting factor w^2 is introduced for achieving dimensional homogeneity.

Therefore, all the generalized inverses of the Jacobian for the PPR robot can be expressed as in (6), where $G_1 = J_W^{\#}$ and

$$n_J(q) = \frac{1}{1 + (\ell/w)^2} [\, \ell s_3 \quad -\ell c_3 \quad 1 \,]^{\mathrm{T}}.$$

Imposing the involutivity condition (3), we obtain

$$\det\big[\, g_1(r) \quad g_2(r) \quad [g_1(r), g_2(r)] \,\big] = 0,$$

with the two-dimensional vector function $r(q)$ as a parameter. Assuming $r = r(q_3)$, cyclic inverses are characterized by r_1 and r_2 in (6) satisfying

$$\ell^2 + \ell w^2 \left[c_3 \left(r^2 - \frac{\partial r_1}{\partial q_3} \right) - s_3 \left(r_1 + \frac{\partial r_2}{\partial q_3} \right) \right] + w^4 \left[r_1 \frac{\partial r_2}{\partial q_3} - r_2 \frac{\partial r_1}{\partial q_3} \right] = 0.$$

For example, a feasible choice is

$$G(q) = \begin{bmatrix} 1 + (\ell/w) s_3 & (\ell/w) s_3 \\ -(\ell/w) c_3 & 1 - (\ell/w) c_3 \\ 1/w & 1/w \end{bmatrix}, \tag{8}$$

corresponding to

$$r_1 = \frac{w^2+\ell^2}{w^3} + \frac{\ell}{w^2} s_3, \qquad r_2 = \frac{w^2+\ell^2}{w^3} - \frac{\ell}{w^2} c_3.$$

In fact, by using (5), the differential constraint (4) associated with the generalized inverse (8) is computed as

$$[\,1 \quad 1 \quad -w + \ell(c_3 - s_3)\,]\dot{q} = 0,$$

that can be integrated as

$$h(q) = q_1 + q_2 - wq_3 + \ell(s_3 + c_3) = c,$$

where c is a constant depending on the initial conditions.

The motion of the PPR robot is constrained to the level surface $\mathcal{H}_0$ of $h(q)$ corresponding to $q(0) = q_0$ (a two-dimensional manifold). The kinematic inversion scheme corresponding to the generalized inverse (8) is then holonomic. This implies that we cannot reconfigure the arm from q_0 to an arbitrary q_d, unless $q_d \in \mathcal{H}_0$. On the other hand, whenever the cartesian command $u(t)$ is cyclic with period T, we have necessarily $q(T) = q_0$. Thus, the generalized inverse (8) gives a cyclic inversion scheme.

2.2. *A non-holonomic inversion scheme*

Consider now the kinematic control scheme directly corresponding to the W-weighted pseudoinverse (7), obtained from (6) for $G_1 = J_W^\#$ and $r = 0$. The joint velocities are then obtained as

$$\dot{q} = \frac{1}{1+(\ell/w)^2}\begin{bmatrix} 1+(\ell/w)^2 c_3^2 \\ (\ell/w)^2 s_3 c_3 \\ -(\ell/w^2)s_3 \end{bmatrix} u_1 + \frac{1}{1+(\ell/w)^2}\begin{bmatrix} (\ell/w)^2 s_3 c_3 \\ 1+(\ell/w)^2 s_3^2 \\ (\ell/w^2)c_3 \end{bmatrix} u_2. \tag{9}$$

Denoting by j_{W_1} and j_{W_2} the two input vector fields in (9), a simple computation shows that

$$[j_{W_1}, j_{W_2}] = \rho \begin{bmatrix} \ell s_3 \\ -\ell c_3 \\ 1/w \end{bmatrix}, \qquad \rho = \frac{1}{w}\frac{(\ell/w)^2}{(1+(\ell/w)^2)^2}.$$

It can be readily verified that the above Lie bracket does not belong to span $\{j_{W_1}, j_{W_2}\}$. As a consequence, $\dim \mathcal{A} = n = 3$ and system (9) is non-holonomic. In fact, the associated kinematic constraint computed from (5)

$$[\,\ell s_3 \quad -\ell c_3 \quad w^2\,]\dot{q} = 0$$

is not integrable.

Since accessibility holds for the PPR arm under weighted pseudoinversion, the reconfiguration problem is solvable by a proper choice of u_1 and u_2 in (9). In the next sections we shall present two different solution approaches.

3. RECONFIGURATION VIA HOLONOMY ANGLE METHOD

A peculiar feature of non-holonomic systems is that if m coordinates are forced to perform a cyclic motion over a time interval $[0, T]$, the remaining $n - m$ coordinates will display a *drift* at time $t = T$ with respect to their initial values. This drift is often called *holonomy angle* [15]. In this section, the basic idea is to plan a specific motion for a set of m coordinates so that the holonomy angle matches the value associated to the desired reconfiguration.

In order to apply this method to the PPR robot under the weighted pseudoinverse scheme (9), it is convenient to rewrite the system equations in terms of the new set of coordinates (p_x, p_y, q_3) as

$$\begin{aligned} \dot{p}_x &= u_1, \\ \dot{p}_y &= u_2, \\ \dot{q}_3 &= \alpha(-s_3 u_1 + c_3 u_2), \end{aligned} \tag{10}$$

with $\alpha = \ell/(w^2 + \ell^2)$. Since this change of coordinates is always invertible, there is a unique configuration $(p_{\rm d}, q_{3\rm d}) = (p_{x\rm d}, p_{y\rm d}, q_{3\rm d})$ corresponding to the desired $q_{\rm d}$.

Based on (10), it is straightforward to choose u_1 and u_2 in feedback so as to bring the end-effector position to $p_{\rm d}$. In particular, by using non-smooth controls, this first phase can be completed in a finite time t_1. Next, it is necessary to transfer the robot from $q(t_1)$, the configuration at the end of the first phase, to the final $q_{\rm d}$. Since $p(t_1) = p_{\rm d}$, this objective can be achieved by moving the end-effector on a closed path Γ. The resulting drift of the third joint is the holonomy angle

$$\gamma = \oint_\Gamma {\rm d}q_3(u) = \alpha \int_{t_1}^{t_1+T} (-s_3 u_1 + c_3 u_2)\,{\rm d}t, \tag{11}$$

where T is the (arbitrary) duration of the cycle Γ. In order to have a cyclic Cartesian motion, the integral of the command $u(t)$ should be zero over the time interval $[t_1, t_1 + T]$. For ease of analysis, a *parameterized* class of inputs is selected. In the second phase, we need to solve the following problem: given the desired reconfiguration $\gamma = q_{3\rm d} - q_3(t_1)$, determine the parameter values in the chosen class of inputs so as to satisfy (11). Accordingly, q_1 and q_2 will move to the desired angles $q_{1\rm d}$ and $q_{2\rm d}$. We note that, since the right-hand side of (10) depends on q_3, the system is not in the so-called *Čaplygin* form [13] and the value of γ in (11) will depend on the initial value $q_3(t_1)$.

We will consider as input class a sequence of constant velocity commands in which only one input is active at each instant:

$$u(t) = \begin{cases} u_1(t) = 4\Delta/T,\ u_2(t) = 0, & t \in [0, T/4], \\ u_1(t) = 0,\ u_2(t) = 4\Delta/T, & t \in [T/4, T/2], \\ u_1(t) = -4\Delta/T,\ u_2(t) = 0, & t \in [T/2, 3T/4], \\ u_1(t) = 0,\ u_2(t) = -4\Delta/T, & t \in [3T/4, T]. \end{cases}$$

For a given T, this class is parameterized by Δ. The path Γ traced by the robot end-effector will be an xy-square of side Δ in the counterclockwise direction, located right and above p_{d} for $\Delta > 0$, or left and below p_{d} for $\Delta < 0$. For the special case $\ell = w = 1 (\alpha = 1/2)$, the forward integration of the third equation in (10) gives

$$\tan\left(\frac{q_3(t_1+T)}{2} + \frac{\pi}{4}\right) = \frac{(\mathrm{e}^{\Delta/2} + \mathrm{e}^{-\Delta/2}) + (1 + 2\mathrm{e}^{-\Delta/2} - \mathrm{e}^{\Delta}) \tan(q_3(t_1)/2)}{(1 + 2\mathrm{e}^{\Delta/2} - \mathrm{e}^{\Delta}) - (\mathrm{e}^{\Delta/2} + \mathrm{e}^{-\Delta/2}) \tan(q_3(t_1)/2)}. \quad (12)$$

Imposing the desired reconfiguration, i.e. $q_3(t_1 + T) = q_{3\mathrm{d}}$, and using the transformation $x = \mathrm{e}^{\Delta/2}$, leads to the following fourth-order polynomial equation to be solved:

$$c_1 x^4 + (1 - 2c_2 + c_1 c_2)x^3 + (c_1 - c_2)x^2 + (1 + 2c_1 + c_1 c_2)x - c_1 = 0, \quad (13)$$

where

$$c_1 = \tan\frac{q_3(t_1)}{2} \qquad c_2 = \tan\left(\frac{q_{3\mathrm{d}}}{2} + \frac{\pi}{4}\right).$$

Only the real positive roots of (13) are relevant: for a root $x > 1$, we have $\Delta > 0$ and an upper right square; for a root $0 < x < 1$, $\Delta < 0$ and a lower left square. Besides, $x = 1$ ($\Delta = 0$) is a root only if $\gamma = 0$ (no reconfiguration needed).

This method has been simulated for $q_0 = (1, 1, \pi/3)$, $q_{\mathrm{d}} = (2, 1, 2\pi/3)$ and $T = 4$ s. In this case, the initial end-effector position p_0 coincides with the final end-effector position p_{d}, and the first phase is not necessary. The desired reconfiguration $\gamma = \pi/3$ is obtained along a square path of side $\Delta = 1.578$, corresponding to the solution of (13) with the smallest magnitude. Figures 2 and 3 show the motion of the arm along the square path and the evolution of the joint variables, respectively. The peaks correspond to the sudden changes of the end-effector velocity at the corners of the square path Γ.

We finally note that:

- In most cases, the smallest value of Δ is of interest, so as to obtain the reconfiguration with the smallest end-effector motion.
- For some special values of $q_3(t_1)$ and $q_{3\mathrm{d}}$, no real root of (13) exists. In these degenerate cases, two or more cyclic motions of the end-effector are needed to build up the desired joint reconfiguration. It can be proven that this strategy is always successful.
- Equation (13) can also be used for exploring the existence of a solution $\Delta \neq 0$ such that the holonomy angle $\gamma = 0$, i.e. $q(t_1 + T) = q(t_1)$. In other words, even if the kinematic inversion scheme (9) is non-holonomic, there may exist *holonomic paths*, i.e. particular end-effector cycles along which no drift occurs in the joint coordinates. The occurrence of such paths is not frequent. For example, in the case $\ell = 1$, it can be shown that no joint configuration is repeatable along squares of side $\Delta < 3.52$.

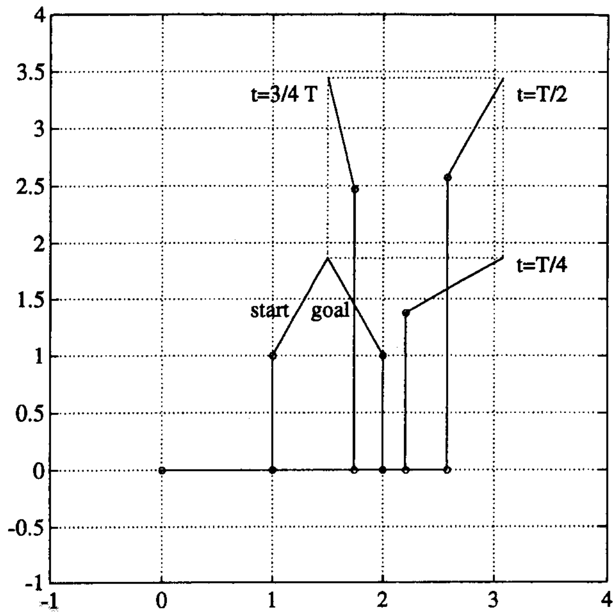

Figure 2. Reconfiguration of the PPR robot with the holonomy angle method.

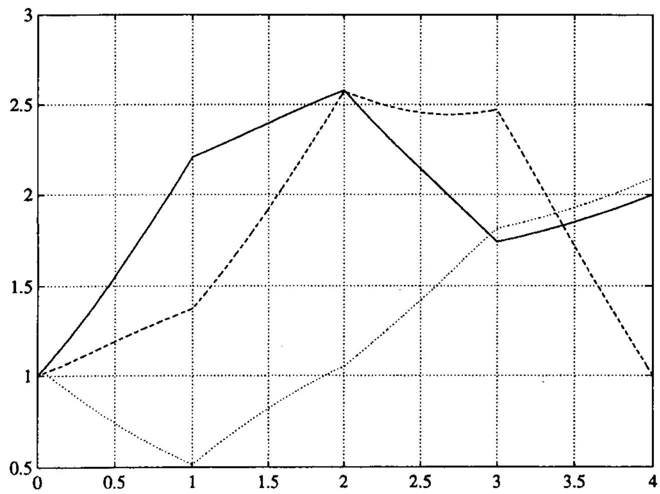

Figure 3. Joint evolutions of the PPR robot with the holonomy angle method: q_1 (—), q_2 (– –), q_3 (· · ·).

4. RECONFIGURATION VIA SINUSOIDAL STEERING

A powerful approach to non-holonomic motion planning relies on the existence of canonical forms for which the problem can be efficiently solved. The most common is the so-called *chained form* [14]. In particular, a two-input system

$$\dot{x} = G(x)v = g_1(x)v_1 + g_2(x)v_2 \tag{14}$$

admits a chained-form representation if there exists an invertible state-dependent input transformation $v = \beta(x)w$ and a change of coordinates $\xi = \phi(x)$ (a feedback transformation) such that

$$\begin{aligned}\dot{\xi}_1 &= w_1,\\ \dot{\xi}_2 &= w_2,\\ \dot{\xi}_i &= \xi_{i-1}w_1, \quad i = 3,\dots,n.\end{aligned} \tag{15}$$

In view of the application to the PPR robot, we restrict our attention to the case $n = 3$.

Define the distributions

$$\begin{aligned}\Delta_0 &= \operatorname{span}\{g_1, g_2, [g_1, g_2]\},\\ \Delta_1 &= \operatorname{span}\{g_2, [g_1, g_2]\}.\end{aligned}$$

If, for some open set $U \subset \mathbb{R}^3$, we have $\dim \Delta_0 = 3$, Δ_1 is involutive on U and there exists a smooth function $h_1\colon\ U \mapsto \mathbb{R}^3$ such that

$$\mathrm{d}h_1 \cdot \Delta_1 = 0 \quad \text{and} \quad \mathrm{d}h_1 \cdot g_1 = 1, \tag{16}$$

then there exists a feedback transformation (local on U) that puts the system into chained form.

In fact, in the above hypotheses, one can find a smooth function $h_2\colon\ U \mapsto \mathbb{R}^3$ satisfying

$$\mathrm{d}h_2 \cdot g_2 = 0 \quad \text{and} \quad \mathrm{d}h_2 \cdot [g_1, g_2] \neq 0, \tag{17}$$

so that the change of coordinates is

$$\phi(x) = \begin{bmatrix} h_1(x) \\ L_{g_1}h_2(x) \\ h_2(x) \end{bmatrix},$$

and the input transformation is

$$\beta(x) = \begin{bmatrix} 1 & 0 \\ L_{g_2}^2 h_2(x) & L_{g_1}L_{g_2}h_2(x) \end{bmatrix}^{-1},$$

where $L_g h = (\partial h/\partial x) \cdot g$ denotes the Lie derivative of h with respect to g.

For the PPR robot, it is convenient to use the system representation (10), and to set $x = (p_y, p_x, q_3)$ and $v = (v_1, v_2) = (u_2, u_1)$. Correspondingly, we have

$$g_1 = \begin{bmatrix} 1 \\ 0 \\ \alpha c_3 \end{bmatrix}, \qquad g_2 = \begin{bmatrix} 0 \\ 1 \\ -\alpha s_3 \end{bmatrix}.$$

Being

$$[g_1, g_2] = [0 \quad 0 \quad -\alpha^2\,]^{\mathrm{T}},$$

it is easy to see that Δ_1 is involutive. Applying the constructive part of the proof of Frobenius Theorem (see [5, p. 26]) to solve the partial differential equations (16) and (17), a feasible state transformation is obtained as

$$\begin{aligned}\xi_1 &= p_y,\\ \xi_2 &= \frac{\alpha e^{\alpha p_x}\cos q_3}{\eta(p_x, q_3)},\\ \xi_3 &= 2\arctan\left(e^{\alpha p_x}\tan\frac{q_3}{2}\right),\end{aligned} \tag{18}$$

where $\eta(p_x, q_3) = \cos^2(q_3/2) + e^{2\alpha p_x}\sin^2(q_3/2) \neq 0$. The corresponding transformation for the original input u in (10) is given by

$$\begin{aligned}u_1 &= \frac{1}{2}\sin q_3\cos q_3\,\frac{1+e^{2\alpha p_x}}{\eta(p_x, q_3)}\,w_1 + \frac{\eta(p_x, q_3)}{\alpha^2 e^{\alpha p_x}}\,w_2,\\ u_2 &= w_1,\end{aligned} \tag{19}$$

with w_1 and w_2 external inputs to be designed. Indeed, the feedback transformation (18)–(19) is just one possible choice, with the nice feature of being *globally* defined.

The initial configuration q_0 and the desired configuration q_d are mapped through (18) into ξ_0 and ξ_d, respectively. With the robot kinematic control system in chained form, the choice of sinusoidal steering is particularly advantageous, as shown in [14]. The reconfiguration task may be executed in two phases:

(1) Steer the *base* variables ξ_1 and ξ_2 to their desired values ξ_{1d} and ξ_{2d} in a finite time t_1, using non-smooth feedback laws for w_1 and w_2. The remaining variable ξ_3 will move to $\xi_3(t_1)$.

(2) Use sinusoidal open-loop commands

$$\begin{aligned}w_1 &= A_1\sin\big(2\pi(t-t_1)/T\big),\\ w_2 &= A_2\cos\big(2\pi(t-t_1)/T\big),\end{aligned}$$

where $t \in [t_1, t_1 + T]$ and $T > 0$ is arbitrary. In this way, ξ_1 and ξ_2 will cycle over T returning to their desired values. In order to bring ξ_3 from $\xi_3(t_1)$ to its desired value ξ_{3d} at $t = t_1 + T$, the amplitudes are chosen as

$$\begin{aligned}A_1 &= \frac{2}{T}\sqrt{\pi\left|\xi_{3d} - \xi_3(t_1)\right|},\\ A_2 &= A_1\,\mathrm{sign}\big(\xi_{3d} - \xi_3(t_1)\big).\end{aligned}$$

This method has been simulated for the same reconfiguration task of the previous section. The duration of the two phases is $t_1 = 0.6$ s and $T = 1$ s, respectively. In this simulation, we have used for simplicity a linear feedback within the first phase. The arm is practically still at t_1, corresponding to 30 times the time constant with

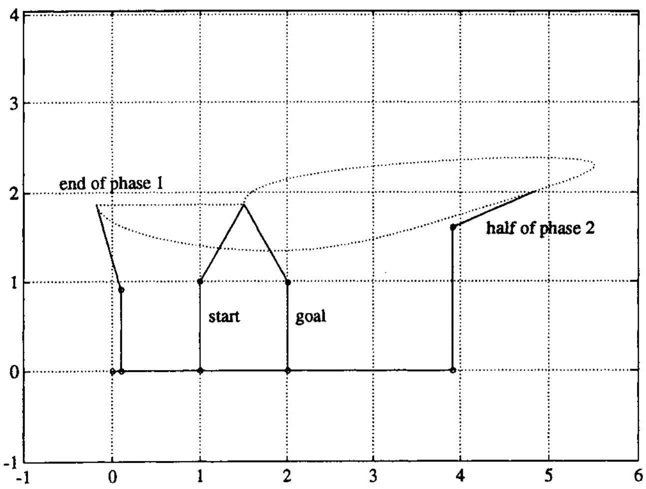

Figure 4. Reconfiguration of the PPR robot with sinusoidal steering.

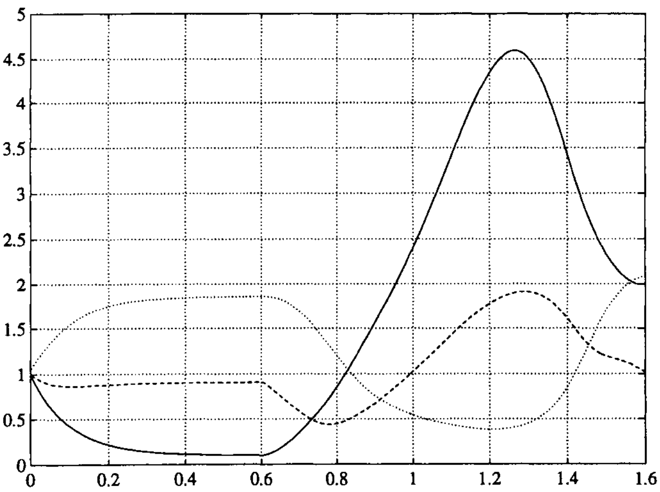

Figure 5. Joint evolutions of the PPR robot with sinusoidal steering: q_1 (—), q_2 (– –), q_3 (· · ·).

the chosen gains. The resulting motion is shown in Figs 4 and 5. Both joint and end-effector evolutions are smoother than those obtained with the first method.

Note that:

- As in the holonomy angle method, the first phase is performed in feedback, while the second specifies $w(t)$ in a feedforward mode. However, in both phases the state-dependent input transformation (19) must be applied in order to obtain the actual task command $u(t)$.
- Sometimes it is convenient to use multiple sinusoidal phases, each achieving part of the reconfiguration for ξ_3. In this way, it may be possible to limit the required end-effector displacement along the cycle.

- While the holonomy angle method prescribes the execution of a cycle over the task coordinates p_x and p_y, in the sinusoidal steering method the cycle takes place over the transformed base variables ξ_1 and ξ_2.

5. CONCLUSION

We have shown that the possibility of reconfiguring redundant robots driven only through end-effector commands is related to the non-holonomy of the differential constraints associated with the chosen kinematic inversion scheme.

For robots with a single degree of redundancy, accessibility is equivalent to non-cyclicity. In this case, two solution approaches have been presented for the reconfiguration problem, based on the established analogy with motion planning for non-holonomic systems. As useful tools, we used the holonomy angle concept and the transformation in chained form. The synthesis of the two control laws was carried out on a PPR robot under weighted pseudoinversion. Both schemes are nonsmooth and prescribe an open-loop phase, and thus are sensitive to numerical approximations or errors in the initial conditions.

In the presence of multiple degrees of redundancy, it will be necessary to check the accessibility condition even if cyclicity has already been excluded. In fact, a subset of the differential constraints associated with the chosen kinematic inversion scheme may be integrable. In these intermediate situations, reconfiguration is possible only within a lower-dimensional submanifold of the joint space.

While the holonomy angle concept is effective only for robots with a single degree of redundancy, the sinusoidal steering method may be generalized to an arbitrary number of joints and to higher degrees of redundancy by steering with sinusoids at integrally related frequencies, as in [14]. In particular, the results of [16] on the existence of chained forms guarantee that the associated reconfiguration strategy can be applied to any redundant robot with up to $n = 4$ rotary and/or prismatic joints and $m = 2$ task inputs, provided that the inverse kinematic scheme is non-holonomic. However, deriving the chained form representation can certainly become a difficult job.

Finally, we briefly mention a natural extension of the problem considered in this paper, i.e. reconfigure the arm by using a kinematic inversion scheme that includes a *null-space term*:

$$\dot{q} = G(q)u + \big(I - G(q)J(q)\big)v(q), \tag{20}$$

where $v(q)$ is typically chosen as the gradient of a configuration-dependent criterion, e.g. distance from singularities or workspace obstacles [17]. The adoption of such a scheme could be desirable, because reconfiguration is attempted while maintaining a certain degree of control of the joint motion.

It can be shown that, provided the chosen generalized inverse G is non-holonomic, it is still possible to drive the robot to a desired joint configuration q_d by a proper choice of u in (20). While a proof of this fact is not given here, we wish to emphasize the two following facts:

- The additional configuration-dependent term in (20) represents a *drift* for the corresponding non-linear control system (i.e. net motion with zero input). As a consequence, the accessibility condition (2) becomes only a *necessary* condition for controllability. By resorting to the concept of *small-time local controllability* [18], it is possible to derive a sufficient condition for the solvability of the reconfiguration problem, that is satisfied if G is non-holonomic.
- The presence of a drift term must be taken into account also for the synthesis of a reconfiguration method. While sinusoidal steering cannot be used, because the system cannot be put in chained form, the holonomy angle method can still be applied, provided that one compensates for the perturbation introduced by the drift.

A detailed discussion of these and related issues may be found in [19].

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