DETECTION AND ISOLATION OF SENSOR FAULTS IN NONLINEAR SYSTEMS: A CASE STUDY

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Abstract: In control systems, the most natural way to model failures of hardware components is introducing additional inputs (*fault inputs*) that equal the difference between actual and assumed values of the 'faulty' quantities. However, when applied to faults affecting the measure of the state of a nonlinear system, this natural modeling procedure results, in general, in non-standard models that are nonlinear in the fault inputs, whereas an affine dependence is desired to exploit the available fault detection and isolation techniques. Through the analysis of a case study (the IFATIS Heating System Benchmark), we illustrate how to alternatively model the failure of a sensor, in particular, by a set of (always concurrent) fault inputs, so that the standard, affine in the inputs, form of nonlinear systems is kept. With the model in this form, the recently developed idea of *Fault Set Detection and Isolation* (FSDI, (Mattone and De Luca, 2003*b*)) can be applied to detect sensor failures under the hypothesis of their non-concurrency.

Keywords: Fault detection and isolation, nonlinear observers, nonlinear control.

1. INTRODUCTION

In the control of complex dynamic plants, the problem of automatically detecting the occurrence of a faulty behaviour in one or more of the hardware components is crucial for a reliable continuous operation. When physical redundancy of components is not present in the plant, fault detection and isolation (FDI) is a prerequisite for a fault tolerant control architecture. Detection of a fault consists in the generation of a diagnostic signal triggered by a deviation of the plant from the expected behaviour, based on the processing of the available signals and/or the use of a dynamic model. Isolation of a fault consists in the discrimination of the occurrence of a specific fault out of a set of potential failures.

An FDI system is typically a dynamic system where each output signal (residual) is excited in response to the occurrence of a different fault, as a result of the processing of nominal commanded inputs and measured outputs of the monitored plant. Whenever a model of the nominal (faultless) plant is available, it is natural to model the presence of faults as additional inputs affecting the system dynamics. Several approaches to the model-based design of FDI systems have been proposed in the literature. For plants with linear dynamics, Kalman filters, Luenberger observers, parity space or parameter estimation techniques have been used since the 70's (see the survey in (Frank, 1990)). More recently, FDI techniques developed for linear systems have been extended in several ways to nonlinear systems that are affine in the (control and fault) inputs (De Persis and

Isidori, 2001; Hammouri *et al.*, 1999). In particular, in (De Persis and Isidori, 2001) a geometric condition is provided, that is necessary for the solution of the FDI problem in the presence of disturbances and possibly concurrent faults.

For the general class of nonlinear systems affine in the control and fault inputs, we analyzed in (Mattone and De Luca, 2003*b*) the problem of *Fault Set Detection and Isolation* (FSDI), namely that of designing a residual generator whose output is only sensible to the occurrence of faults in a given set. This problem is of interest when the necessary conditions in (De Persis and Isidori, 2001) for the detection and isolation of single faults are violated. For the case of non-concurrent faults, weaker necessary and sufficient conditions are given, and a procedure is proposed, that allows to detect and isolate even single faults by processing residuals that solve suitable FSDI problems.

The above results cannot be directly applied to the problem of detecting and isolating faults of the sensors measuring the state of a nonlinear system. In this case, in fact, the most natural procedure for modeling these failures, i.e., introducing additional inputs that equal the difference between actual and measured values of each state variable, would lead to a non- standard model, in particular, nonlinear in the fault inputs.

Through the use of a case study (the IFATIS Heating System Benchmark), we illustrate in this paper how to alternatively model the failure of sensors measuring the state (in this case, the temperatures and fluid levels in the tanks) by a set of (always concurrent) fault inputs, so that the standard, affine in all inputs, form of nonlinear systems is kept. With the model in this form, the methods based on the idea of Fault Set Detection and Isolation (Mattone and De Luca, 2003*b*) is applied to detect sensor failures under the hypothesis of their non-concurrency and verified by simulation. Other approaches to this application problem are based on approximations of the system dynamics, using a linear (Léger *et al.*, 2003) or bilinear (El Bahir and Kinnaert, 1998) model.

2. THE IFATIS HEATING SYSTEM BENCHMARK

The process used as case study in this paper is composed of three cylindrical tanks: according to the scheme of Fig. 1, Tanks 1 and 2 are used for pre-heating the fluids supplied by Pump 1 and Pump 2. The fluid temperature in these tanks can be adjusted by means of two electrical resistors. The third tank allows mixing the fluids coming from the two pre-heating tanks.

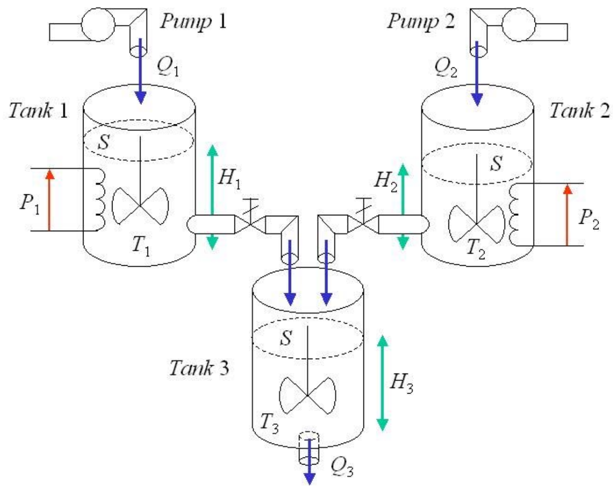

Fig. 1. Schematic diagram of the benchmark

As detailed in (Sauter *et al.*, 2003), the nonlinear model of the considered benchmark system can be written in the standard nonlinear, affine in the inputs, form

$$\begin{aligned} \dot{x} &= g_0(x) + \sum_{i=1}^{4} g_i(x)\, u_i \\ y &= x. \end{aligned} \tag{1}$$

In eq. (1), $x = (H_1, H_2, H_3, T_1, T_2, T_3)$ is the completely measurable state variable vector, with H_i and T_i, $i = 1, \ldots, 3$, the fluid level and, respectively, temperature in the i-th tank, $u = (Q_1, Q_2, P_1, P_2)$ is the input vector, with Q_i and P_i, $i = 1, 2$, the flow-rate and, respectively, heating power delivered to the i-th tank, and the expression of vector fields $g_i(x)$, $i = 0, \ldots, 4$, is given by

$$g_0 = \frac{1}{S}\begin{bmatrix} -\alpha_1\sqrt{x_1} \\ -\alpha_2\sqrt{x_2} \\ \alpha_1\sqrt{x_1} + \alpha_2\sqrt{x_2} - \alpha_3\sqrt{x_3} \\ 0 \\ 0 \\ -\dfrac{\alpha_1}{x_3}\sqrt{x_1}(x_6 - x_4) - \dfrac{\alpha_2}{x_3}\sqrt{x_2}(x_6 - x_5) \end{bmatrix},$$

$$g_1 = \frac{1}{S}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -\dfrac{(x_4 - T_{1i})}{x_1} \\ 0 \\ 0 \end{bmatrix}, \; g_2 = \frac{1}{S}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -\dfrac{(x_5 - T_{2i})}{x_2} \\ 0 \end{bmatrix},$$

$$g_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{S\mu c\, x_1} \\ 0 \\ 0 \end{bmatrix}, \quad g_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{1}{S\mu c\, x_2} \\ 0 \end{bmatrix}.$$

In the above expressions, S is the section of the tanks, c the specific heat of the fluid, μ its density, T_{1i}, T_{2i}, the fluid temperature at the input of tanks 1, 2, and α_i, $i = 1, \ldots, 3$, the (constant) ratio between the flow-rate at the output of i-th tank and the square root of the fluid height in the tank itself. Note that the output vector y coincides with the state, that the system vector fields $g_i(x)$, $i = 0, \ldots, 4$, are smooth in the state-space region of interest ($x_i > 0$, $i = 1, \ldots, 6$), and that $g_0(0) = 0$.

3. FAULT MODELING

We focus here on the modeling of failures of the sensors $S_{x_1}, \ldots, S_{x_6}$ providing measures of the state variables $x_1, \ldots, x_6$. For simplicity, no input failures or disturbances are considered in the model. Furthermore, we assume that different sensor failures never affect the system at the same time (non-concurrency): as we will show, this assumption is fundamental to be able to detect and isolate any single sensor failure. The most natural way to model a sensor failure would be defining a fault input that equals the difference between actual and measured value of the considered quantity, i.e., such that

$$x_k = x_{km} + f_k, \tag{2}$$

being x_k the k-th state variable, x_{km} the measure provided by sensor S_{x_k}, and f_k the corresponding fault input. On the other hand, model (1) is nonlinear in the state x, so that substituting (2) into (1) would lead to a non-standard model, in particular, nonlinear in the sensor fault inputs. Furthermore, the product between control and fault inputs would appear in the model. Thus, a different modeling procedure is required for this class of failures, in order to preserve the useful property that the system model is affine in all control and fault inputs.

To get this result, we apply the following procedure to the generic failure F_k of sensor S_{x_k}:

(1) For any different (possibly nonlinear) expression $\varphi_i(x, u)$ involving x_k and such that the model is affine in $\varphi_i(x, u)$, define the fault input $f_{k,i} = \varphi_i(x, u) - \varphi_i(x, u)\,|_{x_k = x_{km}}$ and compute the corresponding fault vector field $l_{k,i}(x)$. Note that $f_{k,i}$ is only affected by a failure of sensor S_{x_k} (which is consistent with the assumption of non-concurrency [1]), and is zero whenever $x_k = x_{km}$. As a result of this modeling step, any occurrence of the expression $\varphi_i(x, u)$ in the system model can be replaced by $\varphi_i(x, u)\,|_{x_k = x_{km}} + f_{k,i}$, which guarantees that the model is affine in the fault input $f_{k,i}$.

(2) Define the further fault input $f_{k,\nu_k} = \dot{x}_k - \dot{x}_{km}$. The introduction of this additional fault input allows writing the system equations in terms of the new (measured) variable x_{km}, whose relevance will be clear in the following.

(3) If, for some indices i, j and some real function $\alpha_{kij}(x)$, we can write $l_{k,i} = \alpha_{kij}(x) l_{k,j}$, then set $f_{k,j} = \alpha_{kij}(x) f_{k,i} + f_{k,j}$ and eliminate [2] $f_{k,i}$.

Remark: Note that, although non-concurrency of physical failures has been assumed, the faults $f_{k,1}, \ldots, f_{k,\nu_k}$ introduced to model the single physical failure F_k of sensor S_{x_k} will always be concurrent [3] (but never concurrent with any other $f_{i,j}$, $i \neq k$).

As a result of the general modeling procedure described above, we have introduced in our case the following fault inputs.

- For modeling a failure of sensor S_{x_1}:

$$\begin{aligned}
f_{1,1} &= \frac{\alpha_1}{S}\left(\sqrt{x_1} - \sqrt{x_{1m}}\right), \\
f_{1,2} &= \left(-\frac{x_4 - T_{1i}}{S} u_1 + \frac{u_3}{S\mu c}\right)\left(\frac{1}{x_1} - \frac{1}{x_{1m}}\right), \\
f_{1,3} &= \dot{x}_1 - \dot{x}_{1m}.
\end{aligned}$$

- For modeling a failure of sensor S_{x_2}:

$$\begin{aligned}
f_{2,1} &= \frac{\alpha_2}{S}\left(\sqrt{x_2} - \sqrt{x_{2m}}\right), \\
f_{2,2} &= \left(-\frac{x_5 - T_{2i}}{S} u_2 + \frac{u_4}{S\mu c}\right)\left(\frac{1}{x_2} - \frac{1}{x_{2m}}\right), \\
f_{2,3} &= \dot{x}_2 - \dot{x}_{2m}.
\end{aligned}$$

- For modeling a failure of sensor S_{x_3}:

$$\begin{aligned}
f_{3,1} &= -\frac{\alpha_3}{S}\left(\sqrt{x_3} - \sqrt{x_{3m}}\right) - (\dot{x}_3 - \dot{x}_{3m}), \\
f_{3,2} &= -\frac{1}{S}\left(\alpha_1\sqrt{x_1}(x_6 - x_4) + \alpha_2\sqrt{x_2}(x_6 - x_5)\right) \\
&\quad \cdot \left(\frac{1}{x_3} - \frac{1}{x_{3m}}\right).
\end{aligned}$$

- For modeling a failure of sensor S_{x_4}:

$$\begin{aligned}
f_{4,1} &= -\frac{u_1}{Sx_1}\left(x_4 - x_{4m}\right) - (\dot{x}_4 - \dot{x}_{4m}), \\
f_{4,2} &= -\frac{\alpha_1}{Sx_3}\sqrt{x_1}\left(x_4 - x_{4m}\right).
\end{aligned}$$

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[1] In absence of this assumption, we should apply the described procedure not only to the failure of any single hardware component of interest, but also to all possible combinations of faulty/faultless devices.

[2] As $\mathrm{span}\{l_{k,i}\} \subseteq \mathrm{span}\{l_{k,j}\}$, the two faults $f_{k,i}$, $f_{k,j}$ would be in any case indistinguishable (see [1,6]).

[3] The only exception is represented by the fault $f_{k,\nu_k} = \dot{x}_k - \dot{x}_{km}$, that is zero whenever the measure of x_k is affected by a constant bias.

- For modeling a failure of sensor S_{x_5}:

$$f_{5,1} = -\frac{u_2}{Sx_2}(x_5 - x_{5m}) - (\dot{x}_5 - \dot{x}_{5m}),$$
$$f_{5,2} = -\frac{\alpha_2}{Sx_3}\sqrt{x_2}\,(x_5 - x_{5m}).$$

- For modeling a failure of sensor S_{x_6}:

$$f_{6,1} = -\frac{1}{Sx_3}(\alpha_1\sqrt{x_1} + \alpha_2\sqrt{x_2})(x_6 - x_{6m}) \\ -(\dot{x}_6 - \dot{x}_{6m}).$$

Before writing the resulting overall model of the benchmark system, we take as new state variables the outputs of sensors $S_{x_1}, \ldots, S_{x_6}$, so that the new state is indeed measurable. This assumption is particularly convenient, as it guarantees to be able to build a residual generator that solves any assigned FDI problem, whenever the necessary conditions for its solvability are met (De Persis and Isidori, 2001; Mattone and De Luca, 2003*b*). The final model, including the effect of all, non-concurrent failures of the state sensors, is then

$$\begin{aligned} \dot{x}_m &= g_0(x_m) + \sum_{i=1}^{4} g_i(x_m)\,u_i \\ &\quad + \sum_{k=1}^{6}\sum_{j=1}^{\nu_k} l_{k,j}(x_m) f_{k,j} \\ y &= x_m, \end{aligned} \tag{3}$$

where the expression of fault vector fields $l_{k,j}$, $k = 1, \ldots, 6$, $j = 1, \ldots, \nu_k$, is given by

$$l_{1,1} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ -\dfrac{x_{6m} - x_{4m}}{x_{3m}} \end{bmatrix},\; l_{1,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},\; l_{1,3} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$l_{2,1} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ -\dfrac{x_{6m} - x_{5m}}{x_{3m}} \end{bmatrix},\; l_{2,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},\; l_{2,3} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$l_{3,1} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, l_{3,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix};\; l_{4,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, l_{4,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix};$$

$$l_{5,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},\; l_{5,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix};\quad l_{6,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

4. NECESSARY AND SUFFICIENT CONDITIONS FOR FDI

Having modeled the physical failure F_k of the generic sensor S_{x_k} by a set of always concurrent fault inputs, the problem of detecting and isolating F_k may be formulated as a typical problem of *Concurrent Fault Set Detection and Isolation* (CFSDI, see (Mattone and De Luca, 2003*b*)), in particular, that of finding a residual generator whose output r is affected by *at least one* of the faults $f_{k,1}, \ldots, f_{k,\nu_k}$, is *not* affected by any other fault input, and asymptotically converges to zero whenever all fault inputs $f_{k,1}, \ldots, f_{k,\nu_k}$ are zero. Under the assumption of possible concurrency of physical failures, the following condition would be necessary and (under full state availability) sufficient to solve this problem:

$$\exists\, j \in \{1, \ldots, \nu_k\}: \quad \text{span}\{l_{k,j}\} \not\subseteq \bar{\Phi}_k, \tag{4}$$

where $\Phi_k = \text{span}\{l_{i,j},\ i \neq k,\ j = 1, \ldots, \nu_i\}$ and $\bar{\Phi}_k$ denotes the involutive closure of Φ_k. The fulfilment of condition (4) means that, for any physical failure F_k, at least one of the associated fault inputs can be exactly isolated from all faults related to the other physical failures. In the case under consideration, it is readily verified that condition (4) does not hold for any F_k, so that the problem admits no solution under the assumption that multiple sensor failures may concur. This justifies the assumption of non-concurrency made in Sect. 3; in fact, in this case, condition (4) weakens to:

$$\begin{array}{ll} \forall i \neq k,\ \exists j \in \{1, \ldots, \nu_k\}: & \text{span}\{l_{k,j}\} \not\subseteq \bar{P}_i \\ \text{OR} \quad \exists h \in \{1, \ldots, \nu_i\}: & \text{span}\{l_{i,h}\} \not\subseteq \bar{P}_k, \end{array} \tag{5}$$

where $P_i = \text{span}\{l_{i,1}, \ldots, l_{i,\nu_i}\}$ and $\bar{P}_i$ denotes the involutive closure of P_i. Condition (5) implies that, for any couple of physical failures F_i and F_k, a residual signal exists, that is excited by at least one fault input associated to one of the two failures, but not excited by any of the faults corresponding to the other. As condition (5) holds in our case for any $k = 1, \ldots, 6$, it is possible (see again (Mattone and De Luca, 2003*b*)) to design a bank of residual generators such that any failure F_k affects a different combination of residuals, thus allowing fault isolation under the assumption of non-concurrency.

5. RESIDUAL GENERATOR DESIGN

The problem of designing a dynamical system whose output is only affected by some of the inputs clearly appears to be dual to the well-known problem of input-output decoupling (Isidori, 1995) and, in the case of full state availability, can be solved by means of similar techniques. Following the geometric approach, in particular, the basic ingredient is a suitable change of coordinates, inducing a decomposition of the system into subsystems that are affected/not affected by the desired subsets of inputs. Then, a residual generator may be designed as a nonlinear observer of a suitable subsystem dynamics (in particular, that affected by the faults to be detected and not affected by all other faults and disturbances), having as output residual the observation error. Being the design of this observer based on the faultless model of the system, its output will asymptotically (in particular, exponentially) converge to zero in the absence of faults, while the occurrence of the 'affecting' faults drives the residual to be generically nonzero. We do not detail here this design procedure, but just provide, for any residual generator RG_i, $i = 1, \ldots, 6$, designed to solve the problem under consideration, the expression of the observed variable $X_i = \psi_i(x_m)$, the dynamics of the corresponding observer (i.e., the residual generator), and the expression of the resulting residual dynamics.

RG_1 $(X_1 = x_{1m})$:

$$\begin{aligned} \dot{\xi}_1 &= -\frac{\alpha_1}{S}\sqrt{x_{1m}} + \frac{1}{S}u_1 + K\left(X_1 - \xi_1\right), \\ r_1 &= X_1 - \xi_1, \quad \Rightarrow \\ \dot{r}_1 &= -K\,r_1 - f_{1,1} - f_{1,3}. \end{aligned} \tag{6}$$

RG_2 $(X_2 = x_{2m})$:

$$\begin{aligned} \dot{\xi}_2 &= -\frac{\alpha_2}{S}\sqrt{x_{2m}} + \frac{1}{S}u_2 + K\left(X_2 - \xi_2\right), \\ r_2 &= X_2 - \xi_2, \quad \Rightarrow \\ \dot{r}_2 &= -K r_2 - f_{2,1} - f_{2,3}. \end{aligned} \tag{7}$$

RG_3 $(X_3 = x_{3m})$:

$$\begin{aligned} \dot{\xi}_3 &= \frac{1}{S}\left(\alpha_1\sqrt{x_{1m}} + \alpha_2\sqrt{x_{2m}} - \alpha_3\sqrt{x_{3m}}\right) \\ &\quad + K\left(X_3 - \xi_3\right), \\ r_3 &= X_3 - \xi_3, \quad \Rightarrow \\ \dot{r}_3 &= -K r_3 + f_{1,1} + f_{2,1} + f_{3,1}. \end{aligned} \tag{8}$$

RG_4 $(X_4 = x_{1m}(x_{4m} - T_{1i}))$:

$$\begin{aligned} \dot{\xi}_4 &= -\frac{\alpha_1}{S}\left(x_{4m} - T_{1i}\right)\sqrt{x_{1m}} + \frac{1}{S\mu c}u_3 \\ &\quad + K\left(X_4 - \xi_4\right), \\ r_4 &= X_4 - \xi_4, \quad \Rightarrow \\ \dot{r}_4 &= -K r_4 - \left(x_{4m} - T_{1i}\right)\left(f_{1,1} + f_{1,3}\right) \\ &\quad + x_{1m}\left(f_{1,2} + f_{4,1}\right). \end{aligned} \tag{9}$$

RG_5 $(X_5 = x_{2m}(x_{5m} - T_{2i}))$:

$$\begin{aligned} \dot{\xi}_5 &= -\frac{\alpha_2}{S}\left(x_{5m} - T_{2i}\right)\sqrt{x_{2m}} + \frac{1}{S\mu c}u_4 \\ &\quad + K\left(X_5 - \xi_5\right), \\ r_5 &= X_5 - \xi_5, \quad \Rightarrow \\ \dot{r}_5 &= -K r_5 - \left(x_{5m} - T_{2i}\right)\left(f_{2,1} + f_{2,3}\right) \\ &\quad + x_{2m}\left(f_{2,2} + f_{5,1}\right). \end{aligned} \tag{10}$$

fault / residual	r_1	r_2	r_3	r_4	r_5	r_6
$f_{1,1}$	1	0	1	1	0	1
$f_{1,2}$	0	0	0	1	0	0
$f_{1,3}$	1	0	0	1	0	0
F_1	**1**	**0**	**1**	**1**	**0**	**1**
$f_{2,1}$	0	1	1	0	1	1
$f_{2,2}$	0	0	0	0	1	0
$f_{2,3}$	0	1	0	0	1	0
F_2	**0**	**1**	**1**	**0**	**1**	**1**
$f_{3,1}$	0	0	1	0	0	1
$f_{3,2}$	0	0	0	0	0	1
F_3	**0**	**0**	**1**	**0**	**0**	**1**
$f_{4,1}$	0	0	0	1	0	0
$f_{4,2}$	0	0	0	0	0	1
F_4	**0**	**0**	**0**	**1**	**0**	**1**
$f_{5,1}$	0	0	0	0	1	0
$f_{5,2}$	0	0	0	0	0	1
F_5	**0**	**0**	**0**	**0**	**1**	**1**
F_6	**0**	**0**	**0**	**0**	**0**	**1**

Table 1. Residual Matrix/Isolation Logics relating failures $F_1, \ldots, F_6$, and the associated fault inputs, to the residuals $r_1, \ldots, r_6$ of eqs. (6–11)

RG_6 $(X_6 = x_{3m}x_{6m})$:

$$\begin{aligned} \dot{\xi}_6 &= \frac{\alpha_1}{S}\sqrt{x_{1m}}x_{4m} + \frac{\alpha_2}{S}\sqrt{x_{2m}}x_{5m} + \\ &\quad \frac{\alpha_3}{S}\sqrt{x_{3m}}x_{6m} + K\left(X_6 - \xi_6\right), \\ r_6 &= X_6 - \xi_6, \quad \Rightarrow \\ \dot{r}_6 &= -K r_6 + x_{4m}f_{1,1} + x_{5m}f_{2,1} + x_{6m}f_{3,1} \\ &\quad + x_{3m}\left(f_{3,2} + f_{4,2} + f_{5,2} + f_{6,1}\right). \end{aligned} \tag{11}$$

The *residual matrix* relating the modeled faults and the designed residuals $r_1, \ldots, r_6$ of eqs. (6–11) is represented in Table 1. For each fault input $f_{k,j}$, the entries of the associated row are 1 (respectively, 0), if the corresponding residual is affected (respectively, not affected) by the fault input. Furthermore, for any physical failure F_k, $k = 1, \ldots, 6$, an additional row, obtained as the logical OR of the rows corresponding to the associated fault inputs $f_{k,j}$, $j = 1, \ldots, \nu_k$, has been introduced (and highlighted in boldface) to summarize the relationships between physical failures and designed diagnostic signals. Note that all boldface rows of the residual matrix are different, i.e., any failure F_k affects a different combination of residuals, confirming the fact that the designed diagnostic system solves the FDI problem for the considered benchmark system, under the assumption of non-concurrency. Furthermore, note that the matrix in Table 1 (and, in particular, the submatrix in boldface) can be used to set up a combinatorial *isolation logics* mapping the excited/not excited residuals r_i's to residuals r_{x_i} $(i = 1, \ldots, 6)$ one-to-one related to the physical failures.

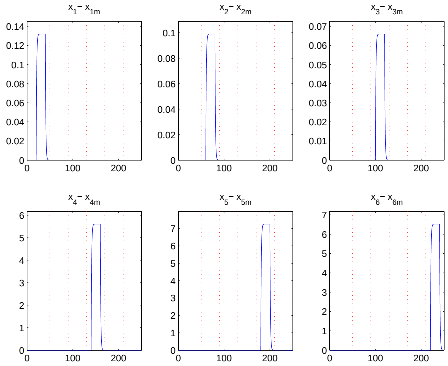

Fig. 2. Bias-type faults of state sensors

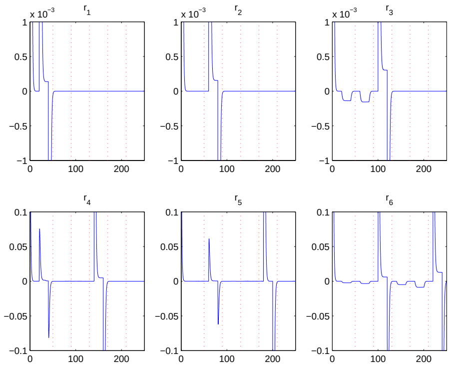

Fig. 3. Behaviour of residuals $r_1, \dots, r_6$ corresponding to the faults of Fig. 2

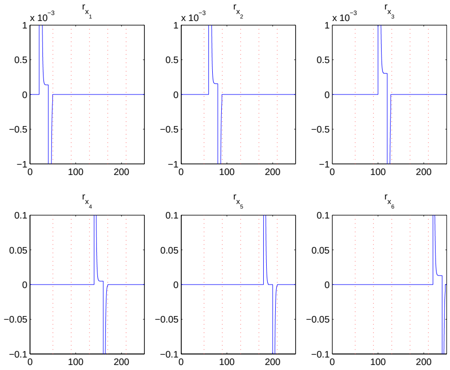

Fig. 4. FDI residuals after the isolation logics

6. SIMULATION RESULTS

Simulation results for the case of non-concurrent, bias-type faults of sensors $S_{x1}, \dots, S_{x6}$ are reported in Figs. 2–4. In Fig. 3, in particular, it can be observed how multiple residuals are excited by the occurrence of each of the faults of Fig. 2, in agreement with Table 1, while the implementation of the isolation logics gives back a perfect decoupling between each fault and the associated residual (see Fig. 4).

7. CONCLUSIONS

A method has been proposed for modeling failures of the sensors measuring the state of a nonlinear system, so as to preserve the affine form in all (control and fault) inputs. As a result, for any physical sensor failure, a set of (always concurrent) fault inputs is introduced in the model, so that the problem of detecting and isolating any single sensor failure can be formulated according to the recently developed concept of Fault Set Detection and Isolation (Mattone and De Luca, 2003*b*). The method has been applied to the IFATIS Heating System Benchmark. For this case study, we have provided a bank of residual generators that solve the sensor fault detection and isolation problem under the hypothesis of non-concurrence of the failures. Simulations confirm the validity of this global nonlinear approach.

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