Steady state and internal model principle for hybrid systems

L. Marconi

CASY-DEIS, University of Bologna

Workshop *"Current problems in Control Theory"*

in honor of Alberto Isidori

Roma, 24 September 2012

Due righe …… risolutive !

(in italian, sorry ☺)

Date: Wed, 24 Mar 1999 19:56:21 +0100
From: Isidori Alberto <isidori@giannutri.caspur.it>
To: andrea@zach.wustl.edu, lmarconi@deis.unibo.it

Hi Guys
vi sto scrivendo dalla Selva Nera, purtroppo non dal mio laptop.

Ho letto la nota di Serrani, e ho cercato di stabilire un ponte con l'idea proposta da Marconi nel corso del suo ultimo viaggio a Roma. Mi sembra che i due approcci possano essere collegati da una trasformazione di coordinate. I dettagli seguono.

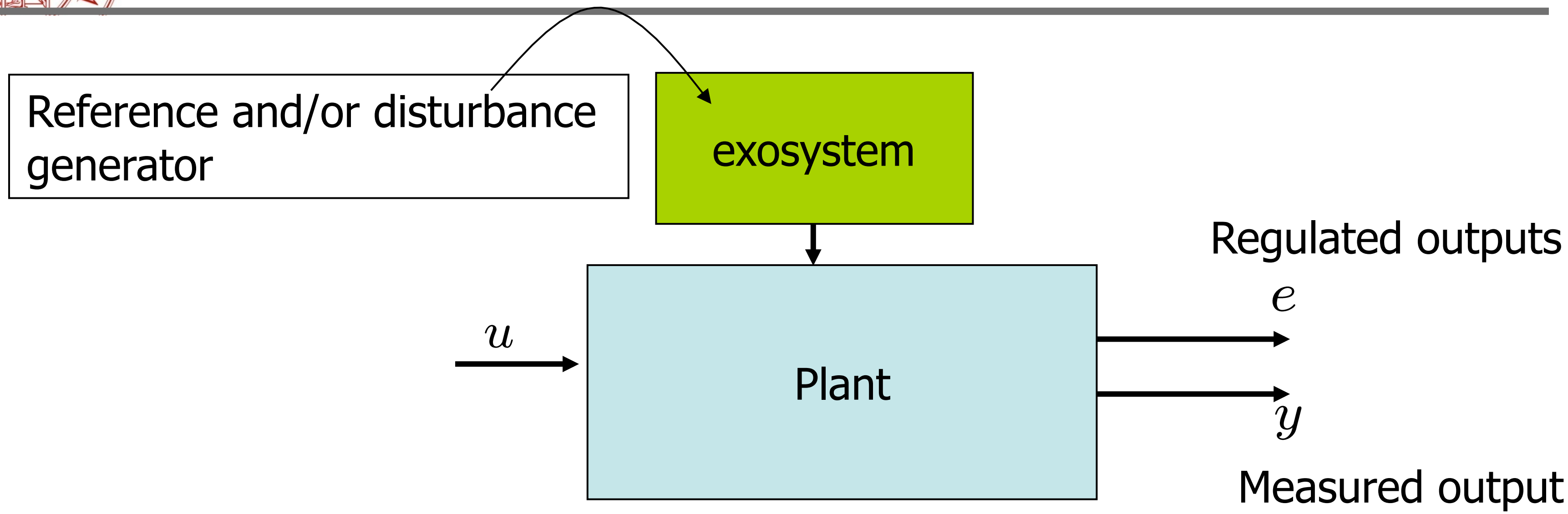
Da diverso tempo ho tra le mia cartelline una cartellina verde con su scritto "Choice of N". Vedrete da quanto segue che (se non mi sbaglio), posso finalmente - grazie alle vostre idee - archiviare la cartellina.

……

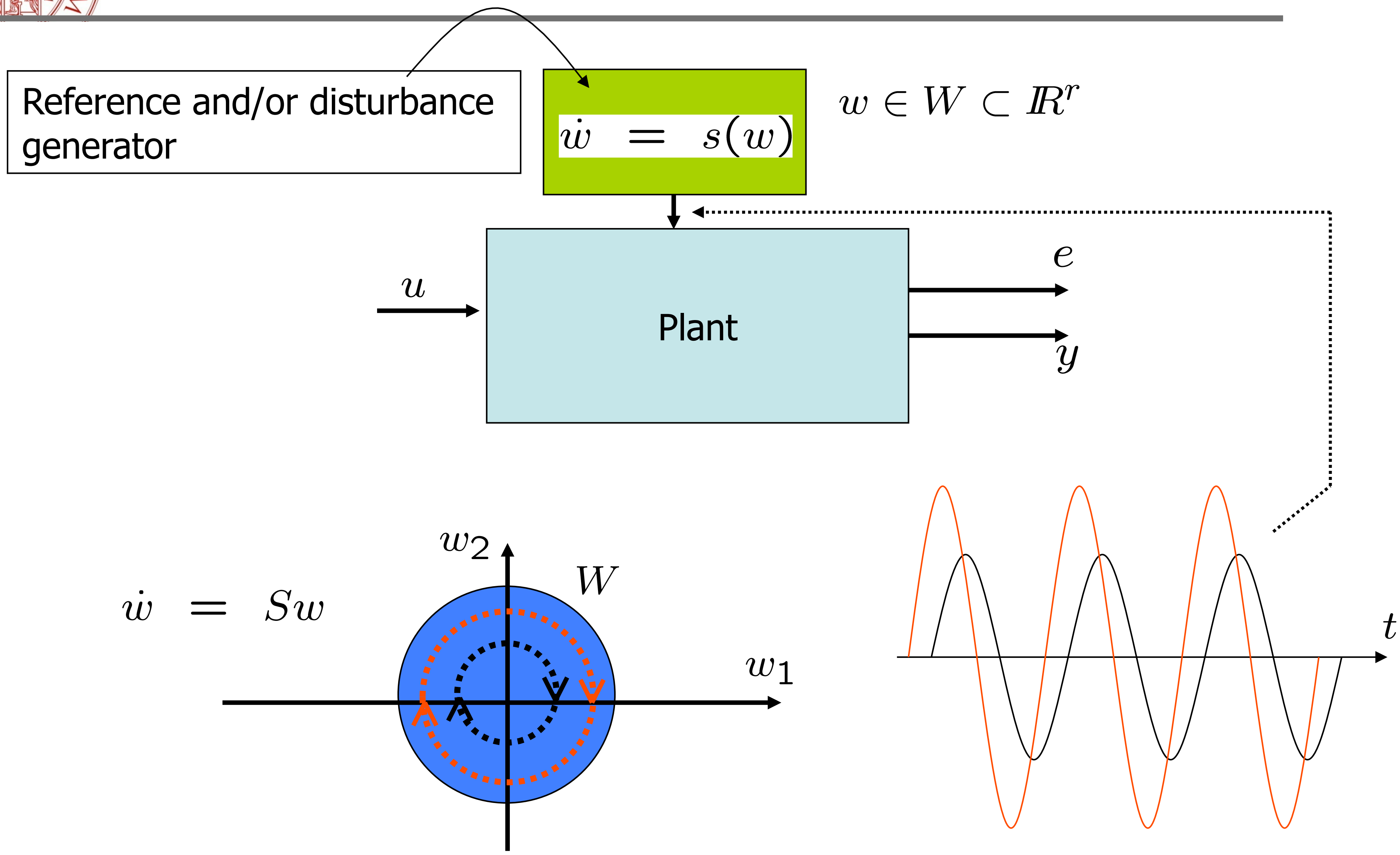
……

Fatemi sapere che ne pensate. Scusate il testo scarno: non avete idea cosa vuol dire usare una tastiera tedesca!
Tornero' a Roma Domenica. A presto
A.I.

The Framework of Output Regulation



The Framework of Output Regulation



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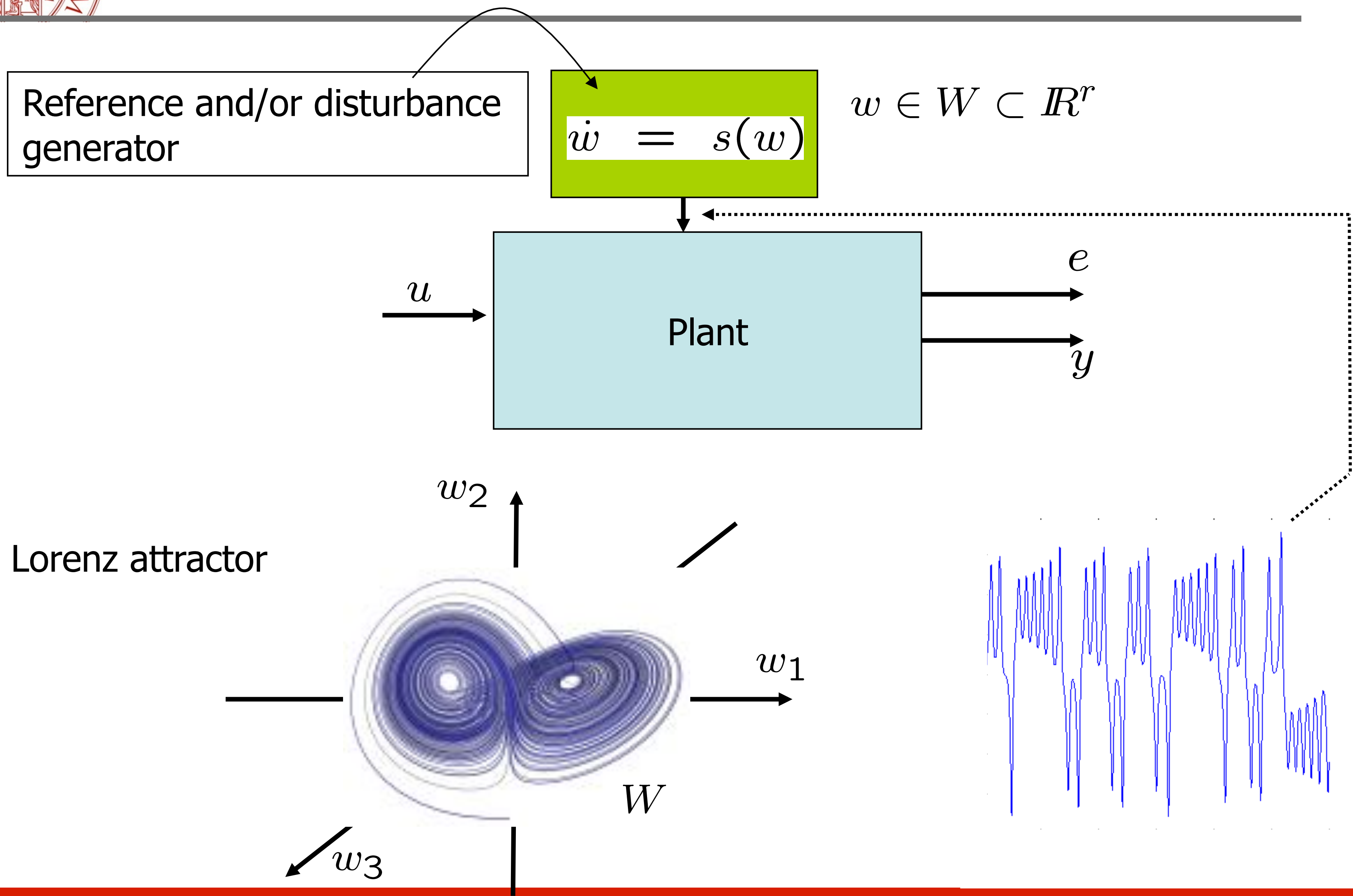
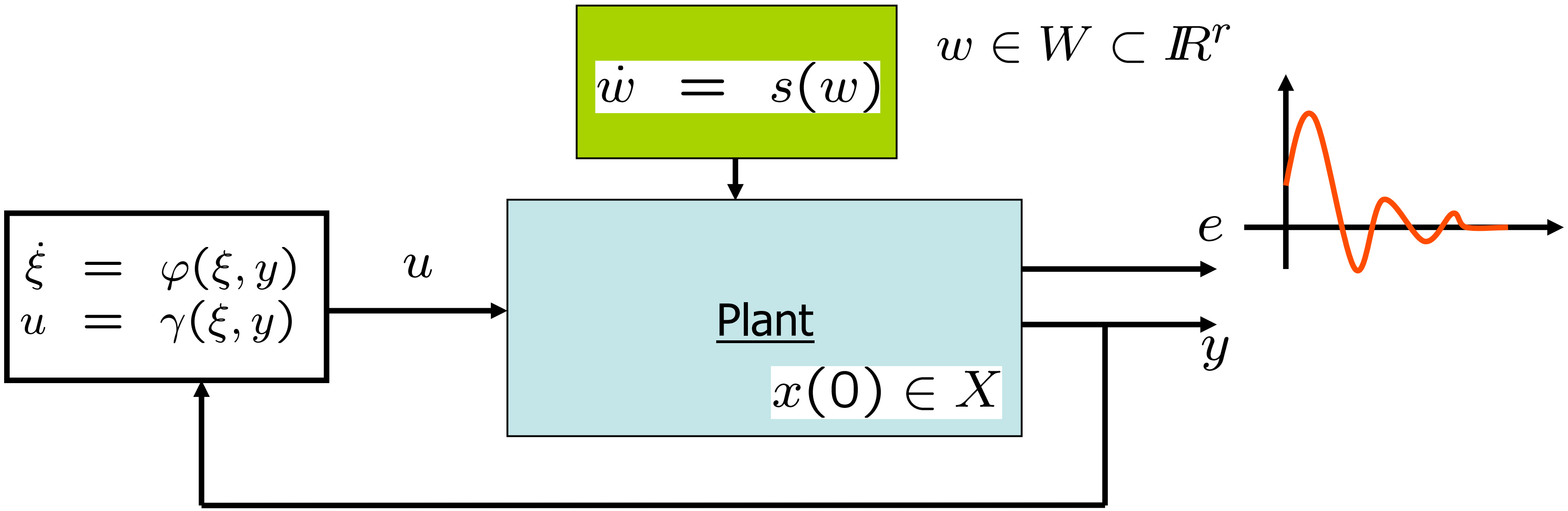


The Framework of Output Regulation



The problem of semiglobal output regulation

- Boundedness of closed-loop trajectories
- (Uniform) converge of the error to zero

for any initial condition in known compact sets $W \times X$ (semi-global)

Literature…… Pervasively AI !

B.A. Francis, W. M. Wonham, The internal model principle of control theory, Automatica, 1976

B.A. Francis, The linear multivariable regulator problem, SIAM JCO, 1977

A. Isidori, C.I. Byrnes, Output regulation of nonlinear systems, IEEE TAC 1990

J. Huang, C.F. Lin, On a robust nonlinear multivariable servomechanism problem, IEEE TAC, 1994

A. Serrani, A. Isidori, L. Marconi, Semiglobal nonlinear output regulation with adaptive internal model, IEEE TAC, 2001

R. Marino , P. Tomei, Output regulation for linear systems via adaptive internal model, IEEE TAC , 2003

C.I. Byrnes, A. Isidori, Limit sets, zero dynamics and internal models in the problem of nonlinear output regulation, IEEE TAC 2003

C.I. Byrnes, A. Isidori, Nonlinear internal models for output regulation, IEEE TAC, 2004

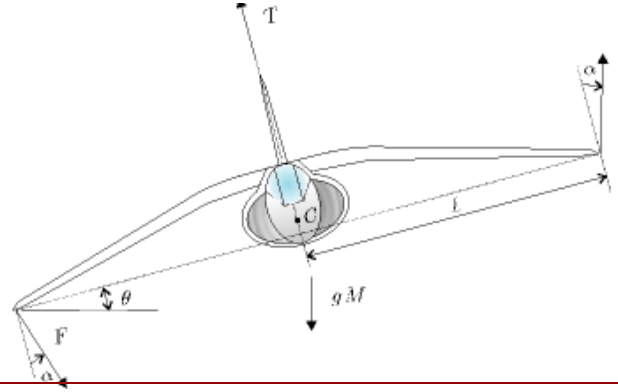
L. Marconi, L. Praly, A. Isidori, ``Output Stabilization via Nonlinear Luenberger Observers'', SIAM JCO,, 2007

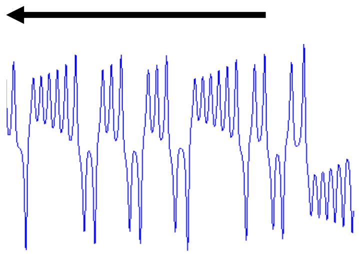
L. Marconi, L. Praly, ``Uniform Practical Output Regulation'', IEEE TAC , 2008.

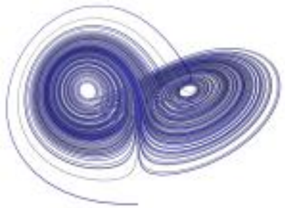
R. Marino, P. Tomei, Adaptive nonlinear regulation for uncertain minimum phase systems with unknown exosystem, CDC 2008

A Few Applicative Contexts

Lateral wind compensation

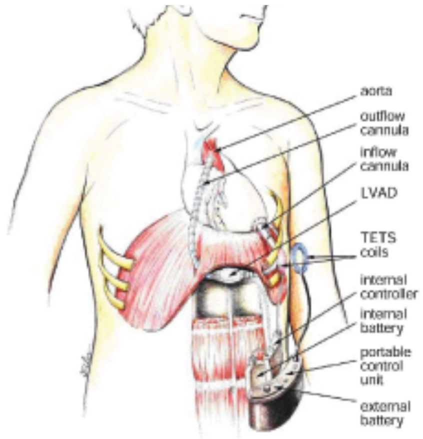


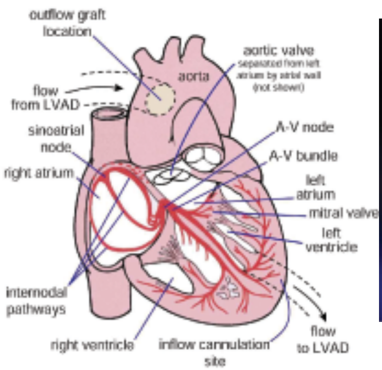


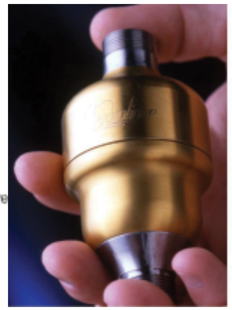


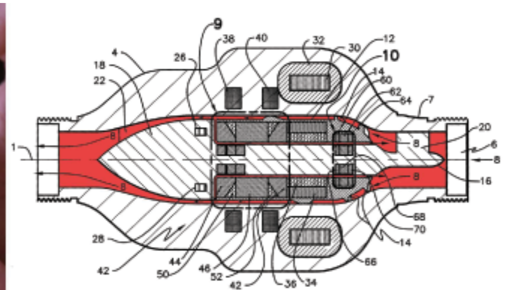
Dryden Wind
Turbulence Model

Disturbance compensation in left ventricular assist device









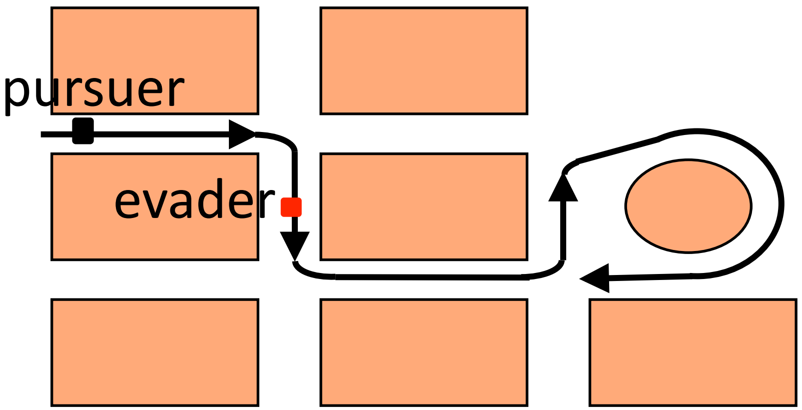
A Few Applicative Contexts

Asymmetries compensation in control of rotating machines

C. Bonivento, A. Isidori, L. Marconi, A. Paoli, ``Implicit Fault Tolerant Control: Application to Induction Motors'', Automatica 2004.

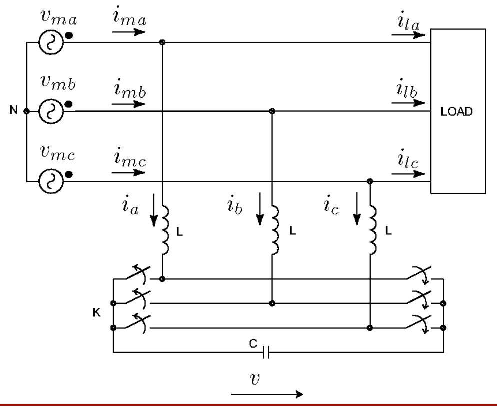
Recipient of the ``Outstanding Paper Prize Award''

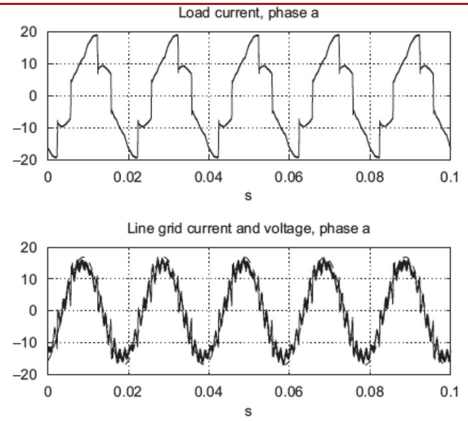
Pursuit evasion in urban environment



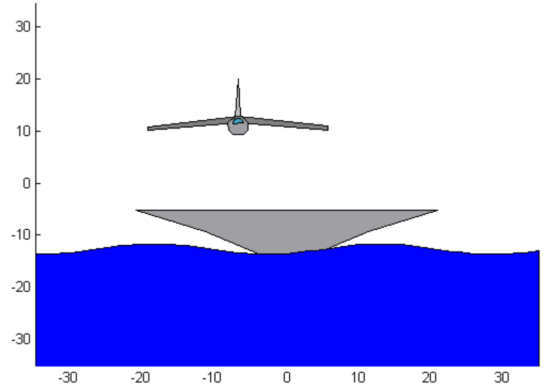
A Few Applicative Contexts

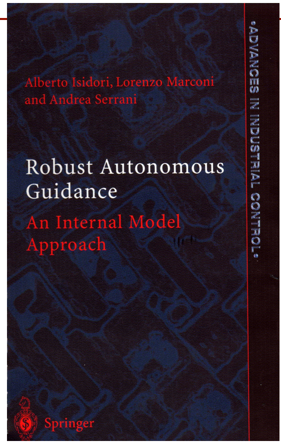
Harmonic pollution in electric grids



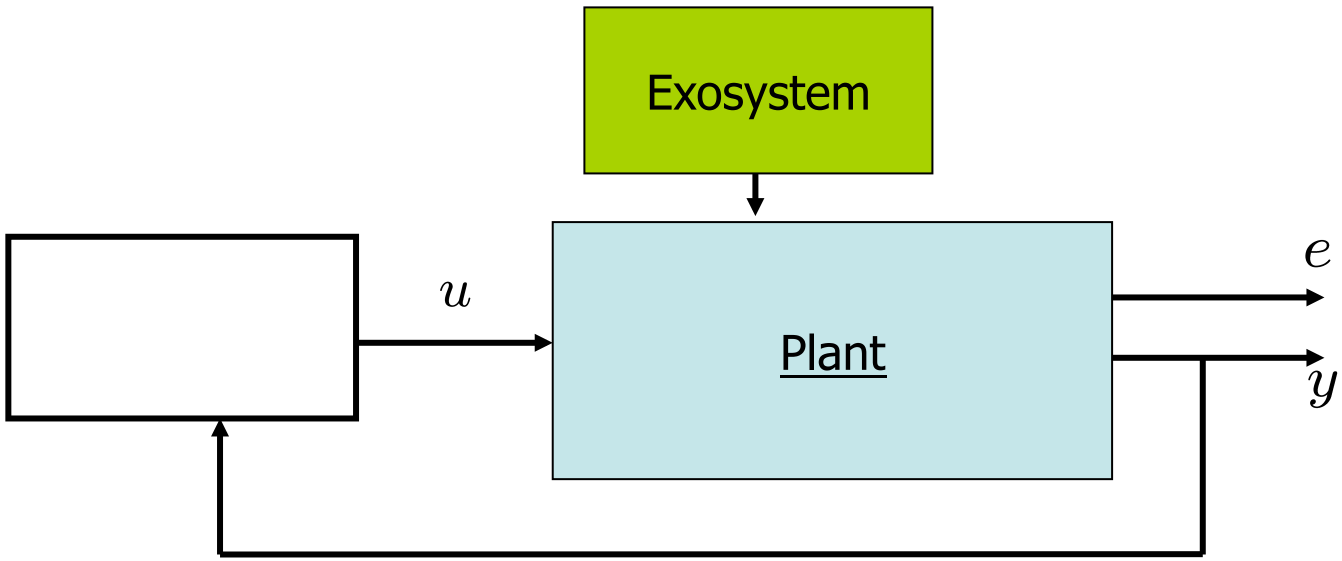


Landing on oscillating platform

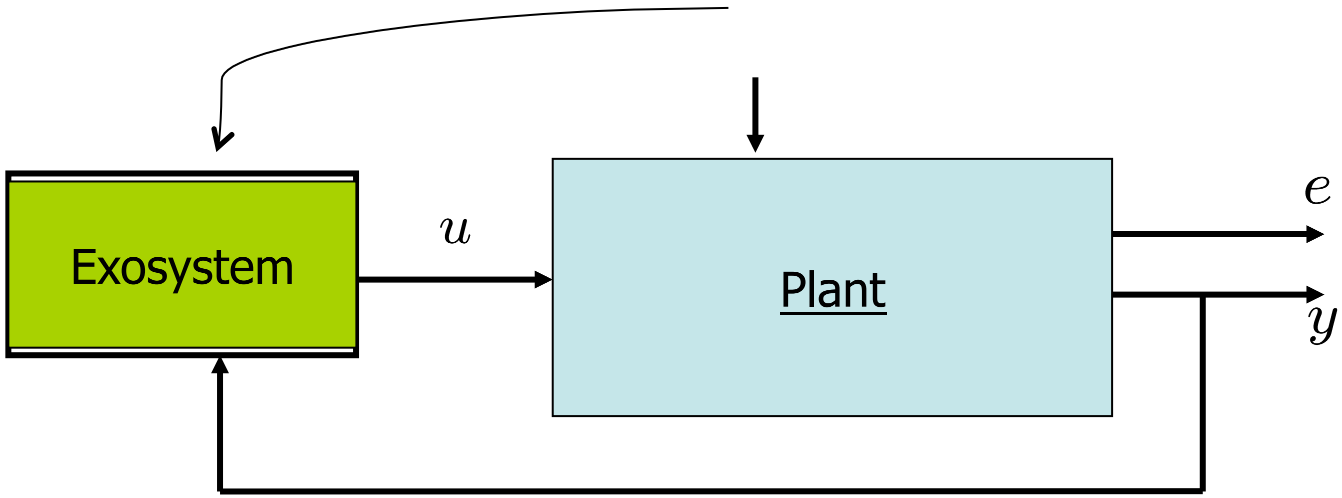




Internal Model Principle - 1



Internal Model Principle - 2



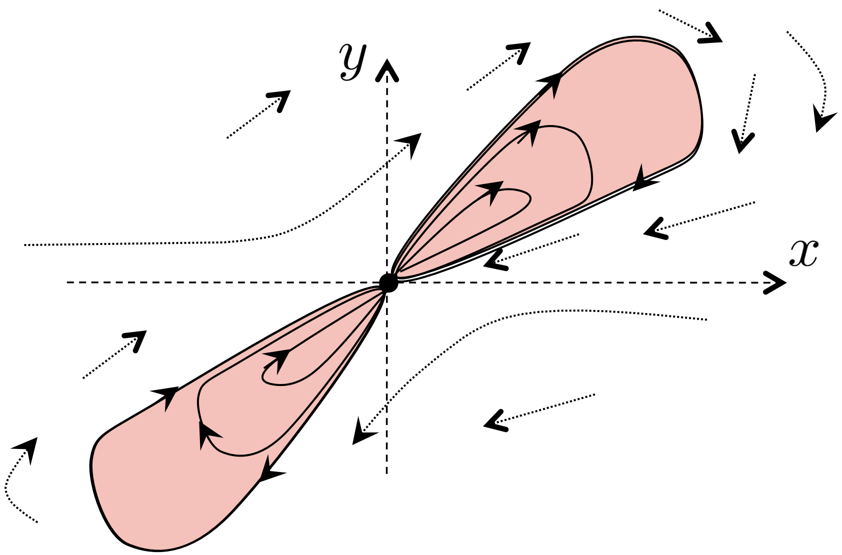
- In words: Every controller solving the problem must necessarily embed a "copy" of the exosystem

- A "cognitive perspective" (not by me!): think of waking up night-time and getting out the room in the dark.

Steady State Behavior

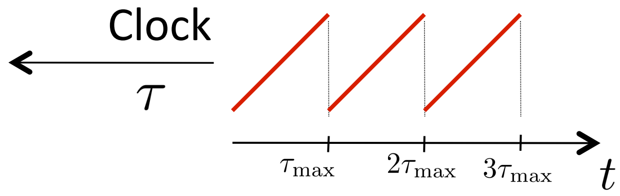
- Limiting behavior uniformly achieved by a (nonlinear) system
- Starting point in any (serious) regulator design:
  - ➡ Enforce a steady state
  - ➡ Shape the steady state so that the regulator error vanishes on it
- C.I. Byrnes and A. Isidori, *Limit sets, zero dynamics and internal models in the problem of nonlinear output regulation*, IEEE TAC 2003

$$
\begin{aligned}
\dot{x} &= \frac{x^2(y-x)+y^5}{(x^2+y^2)(1+(x^2+y^2)^2)}\\
\dot{y} &= \frac{y^2(y-2x)+y^5}{(x^2+y^2)(1+(x^2+y^2)^2)}
\end{aligned}
$$

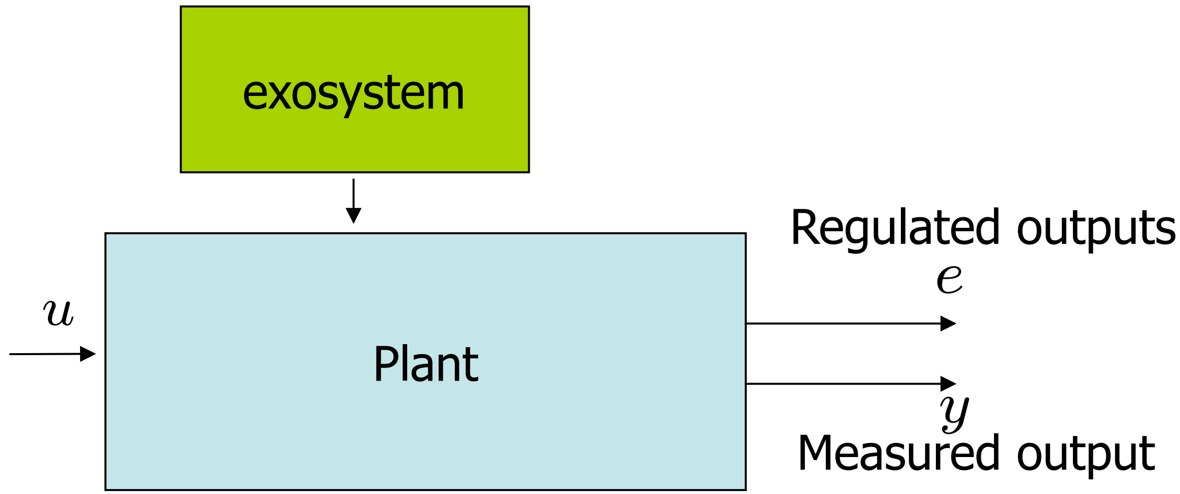


The framework in presence of clock-driven jumps

Joint work with A. Teel, UCSB



$$\left\{\begin{array}{rclc} \dot{\tau} & = & 1 & \tau \leq \tau_{\max} \\ \tau^{+} & = & 0 & \tau = \tau_{\max} \end{array}\right.$$



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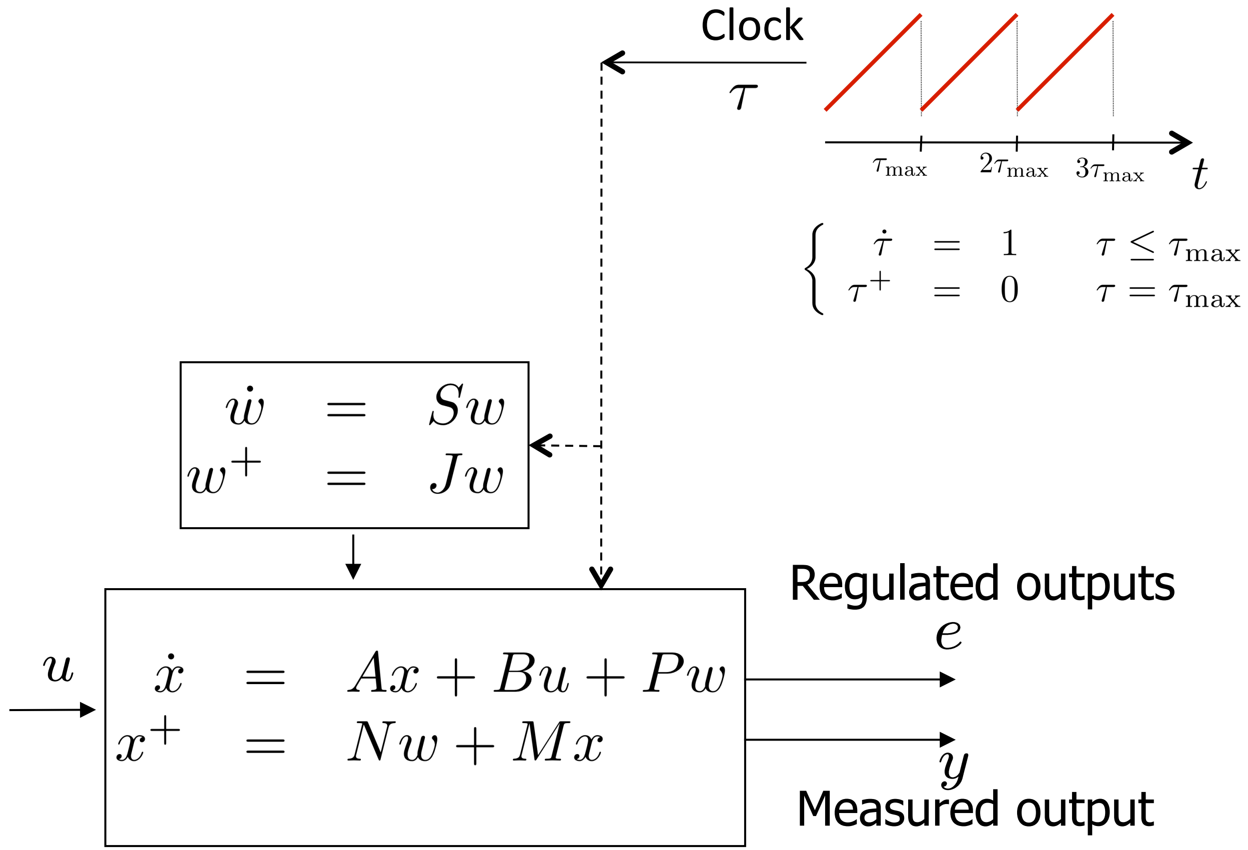

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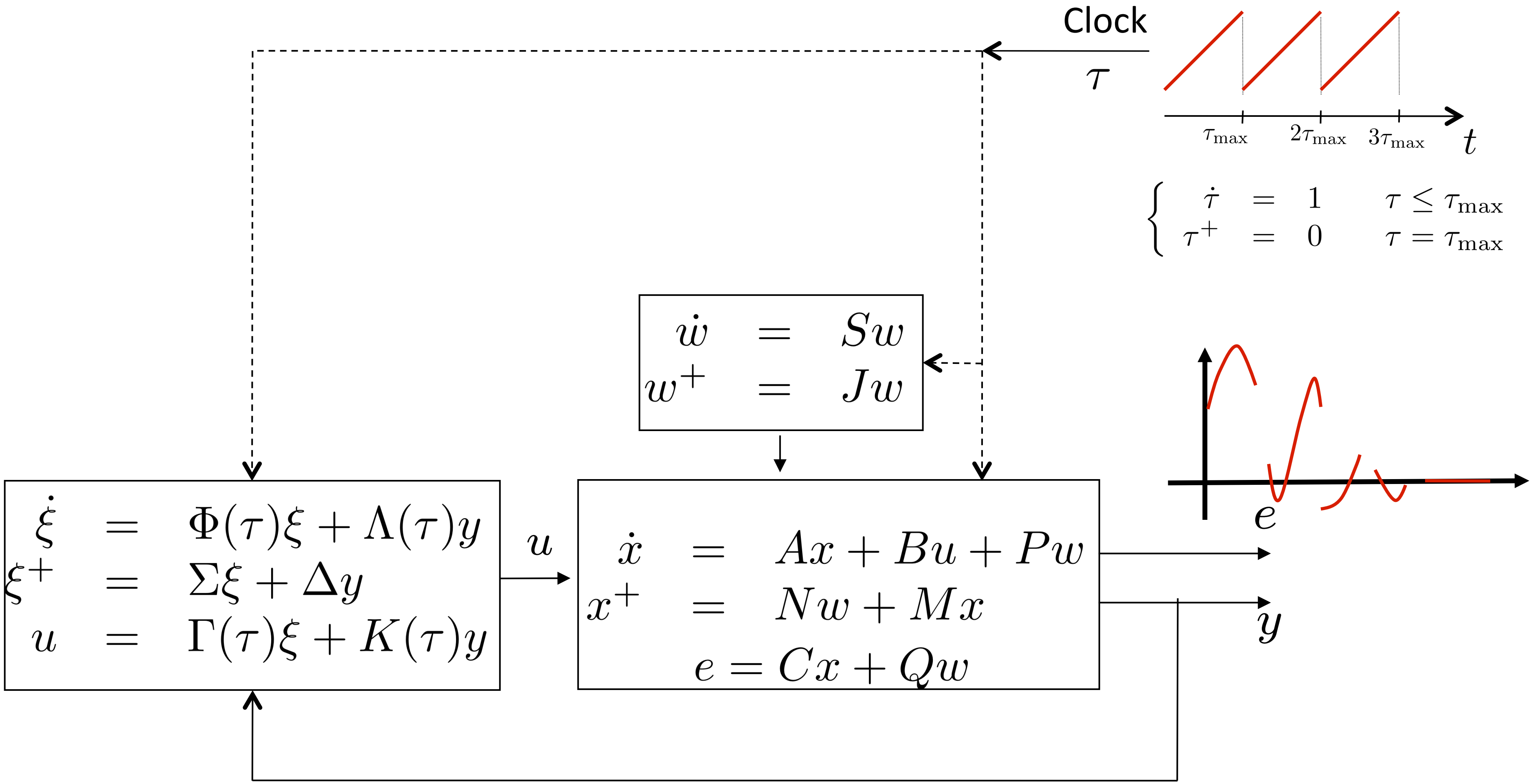
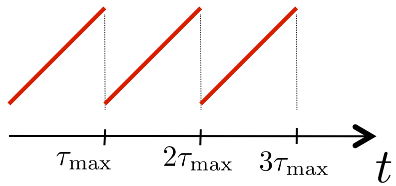


Steady state for (the class of) hybrid systems

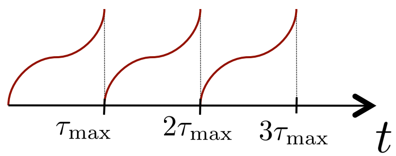
Clock

$$\begin{aligned} \dot{\tau} &= 1 & \tau &\le \tau_{\max} \\ \tau^+ &= 0 & \tau &= \tau_{\max} \end{aligned}$$



$$\begin{aligned} \dot{w} &= Sw \\ w^+ &= Jw \end{aligned}$$

$$\begin{aligned} \dot{z} &= F(\tau)z + G(\tau)w \\ z^+ &= Nw + Mz \end{aligned}$$



$$\lim_{(t+j)\to\infty} z(t,j) = ??$$

Steady state for hybrid systems - 1

$$\begin{array}{rcl} \dot{w} & = & Sw \\ w^+ & = & Jw \end{array} \longrightarrow \begin{array}{rcl} \dot{z} & = & F(\tau)z + G(\tau)w \\ z^+ & = & Nw + Mz \end{array}$$

Let $\phi(\tau)$ be such that

$d\phi(\tau)/d\tau = F(\tau)\phi(\tau) \quad \phi(0) = I_n$

A general result: if $\mathrm{eig}(M\phi(\tau_{\max})) \cap \mathrm{eig}(J\exp(S\tau_{\max})) = \{0\}$

there exists a $\mathcal{C}^1$ $\Pi : [0, \tau_{\max}] \to \mathbb{R}^{n \times s}$ solution of

$$\begin{array}{rcl} \dfrac{d\Pi(\tau)}{d\tau} & = & F(\tau)\Pi(\tau) - \Pi(\tau)S + G(\tau) \\ 0 & = & M\Pi(\tau_{\max}) - \Pi(0)J + N \end{array}$$

As a consequence $\mathcal{M} = \{((\tau, w), z) \in \mathcal{W} \times \mathbb{R}^n \ : \ z = \Pi(\tau)w\}$

is invariant.

Furthermore, if $|\mathrm{eig}(M\phi(\tau_{\max}))| < 1$ then $\mathcal{M}$ is GES

Steady state for hybrid systems - 2

$$\begin{aligned} \dot{w} &= Sw \\ w^+ &= Jw \end{aligned} \longrightarrow \begin{aligned} \dot{z} &= F(\tau)z + G(\tau)w \\ z^+ &= Nw + Mz \end{aligned}$$

Let $\phi(\tau)$ be such that

$$d\phi(\tau)/d\tau = F(\tau)\phi(\tau) \qquad \phi(0) = I_n$$

A general result: if $\mathrm{eig}(M\phi(\tau_{\max})) \cap \mathrm{eig}(J\exp(S\tau_{\max})) = \{0\}$

"hybrid non resonance condition"

there exists a $\mathcal{C}^1$ $\Pi : [0, \tau_{\max}] \to \mathbb{R}^{n \times s}$ solution of

$$\begin{aligned} \frac{d\Pi(\tau)}{d\tau} &= F(\tau)\Pi(\tau) - \Pi(\tau)S + G(\tau) \\ 0 &= M\Pi(\tau_{\max}) - \Pi(0)J + N \end{aligned}$$

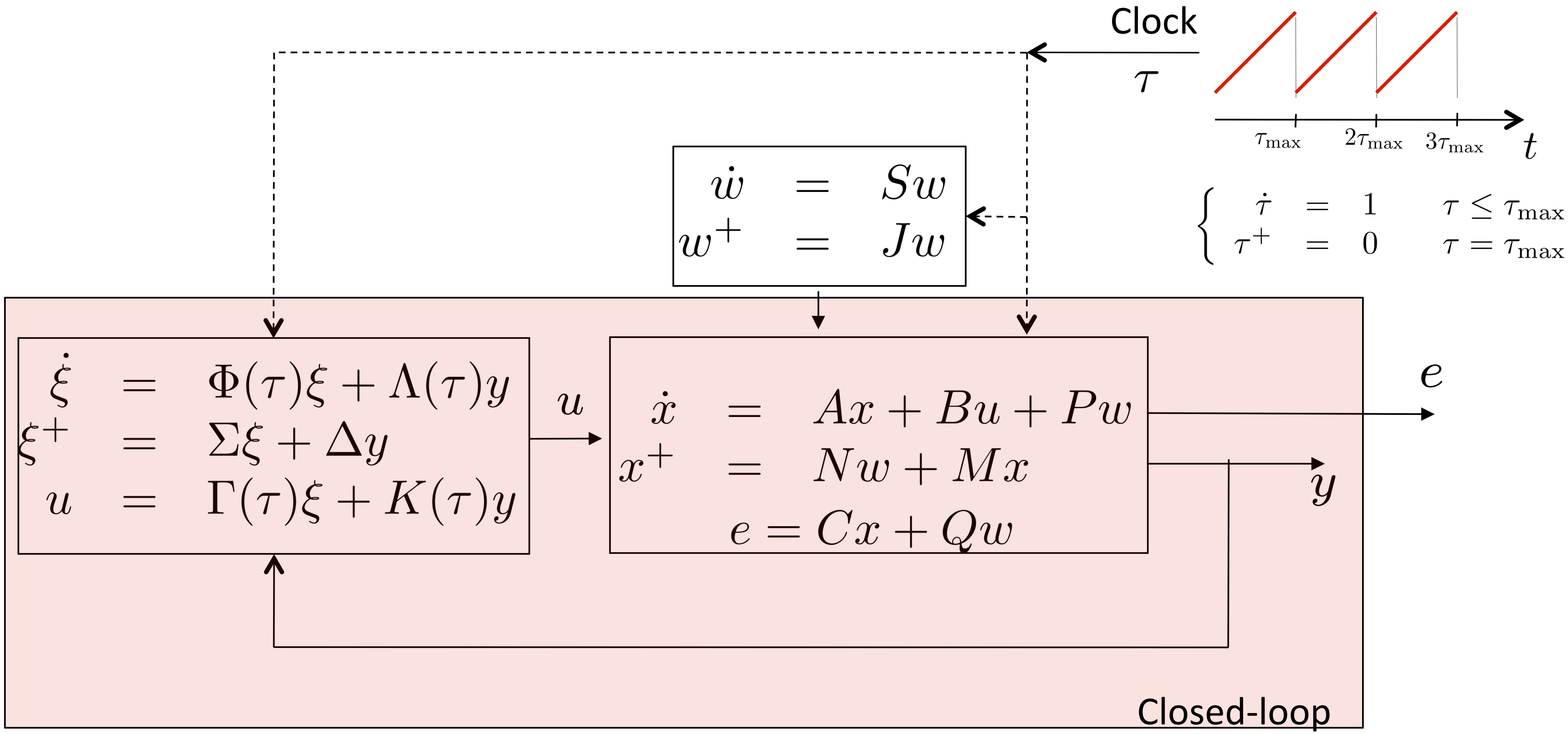
As a consequence $\mathcal{M} = \{((\tau, w), z) \in \mathcal{W} \times \mathbb{R}^n \ : \ z = \Pi(\tau)w\}$

is invariant.

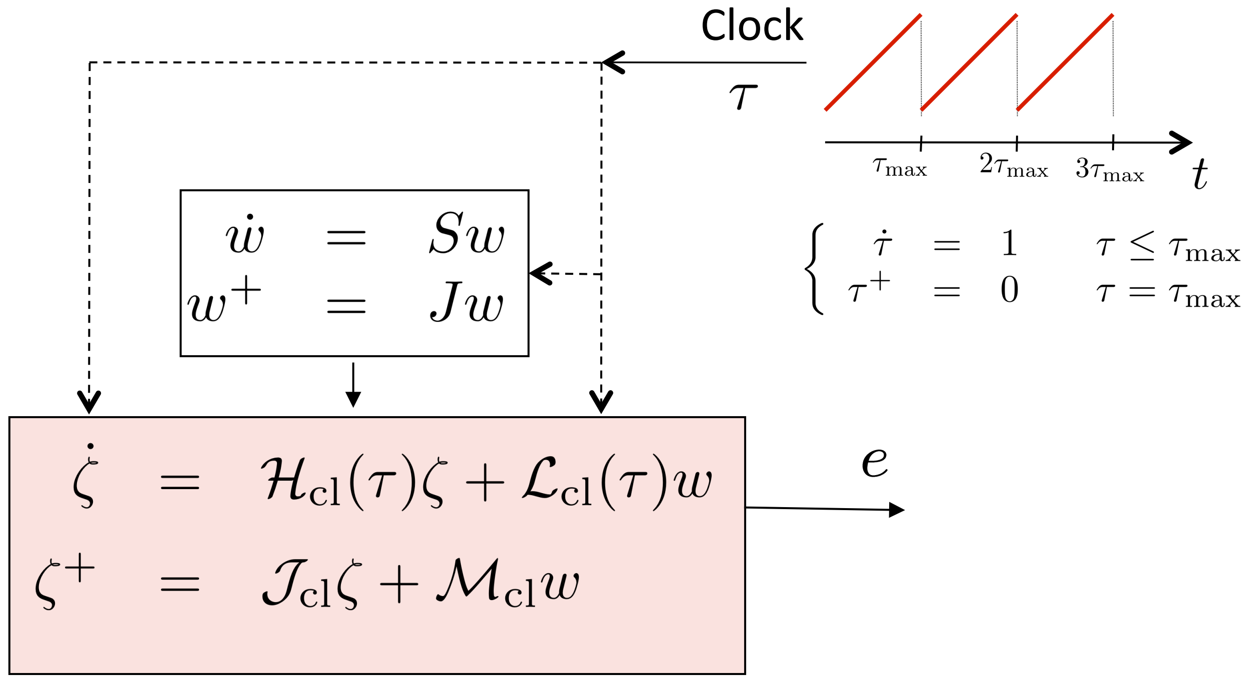
"Stability requirement"

Furthermore, if $|\mathrm{eig}(M\phi(\tau_{\max}))| < 1$ then $\mathcal{M}$ is GES

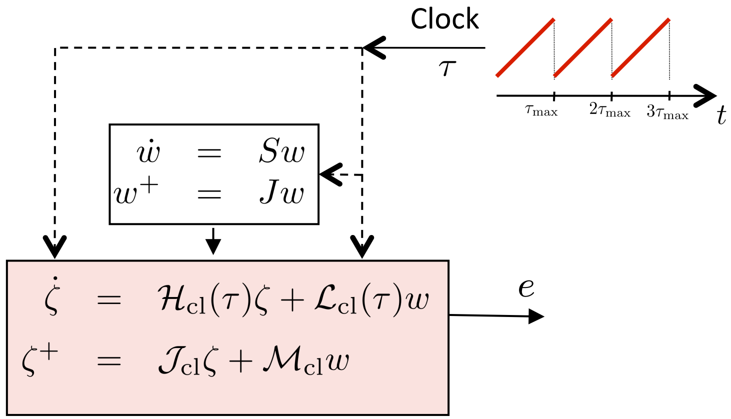
Steady state for hybrid systems



Steady state for hybrid systems



Steady state for hybrid systems



Result: If

Stability Requirement (SR):

$$|\mathrm{eig}(\mathcal{J}_{\rm cl}\phi_{\rm cl}(\tau_{\max}))| < 1$$

Non Resonance condition (NR):

$$\mathrm{eig}(\mathcal{J}_{\rm cl}\phi_{\rm cl}(\tau_{\max})) \cap \mathrm{eig}(J\exp(S\tau_{\max}) = \{\emptyset\}$$

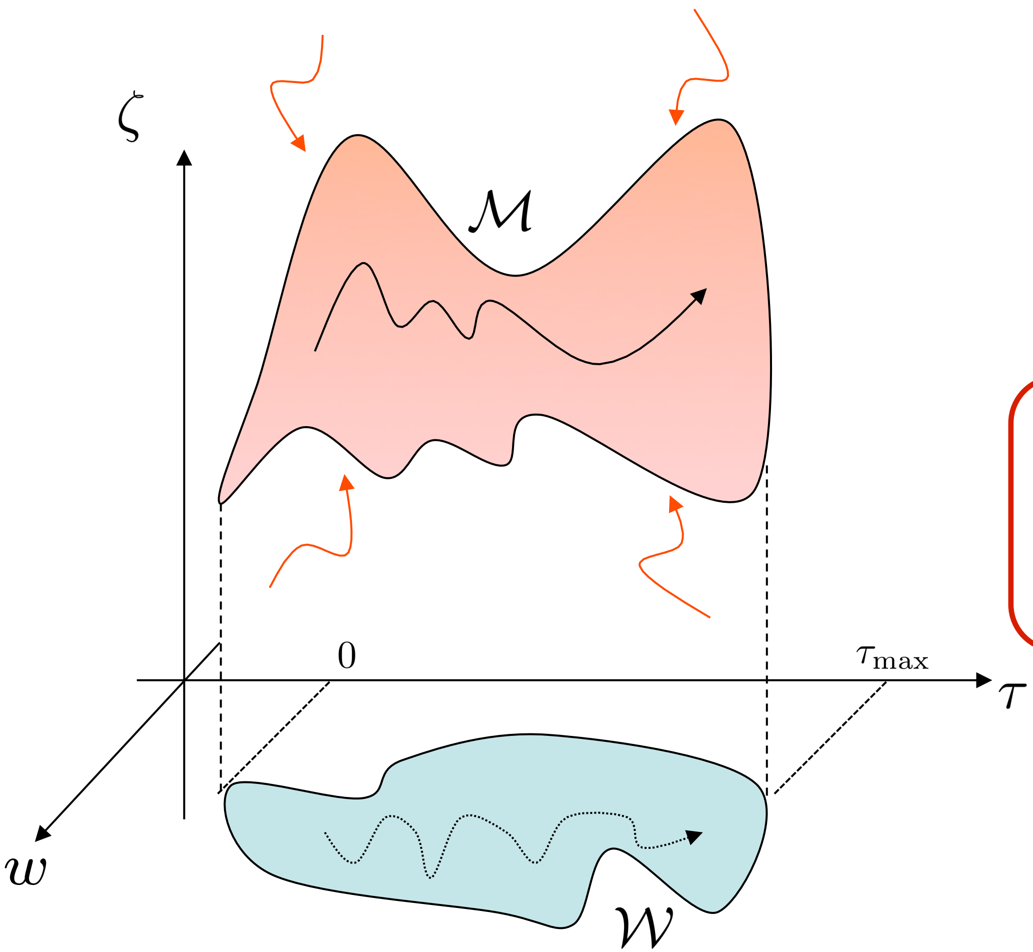
Then there exists a $\mathcal{C}^1\ \Pi : [0,\tau_{\max}] \to \mathbb{R}^{n\times s}$ solution of

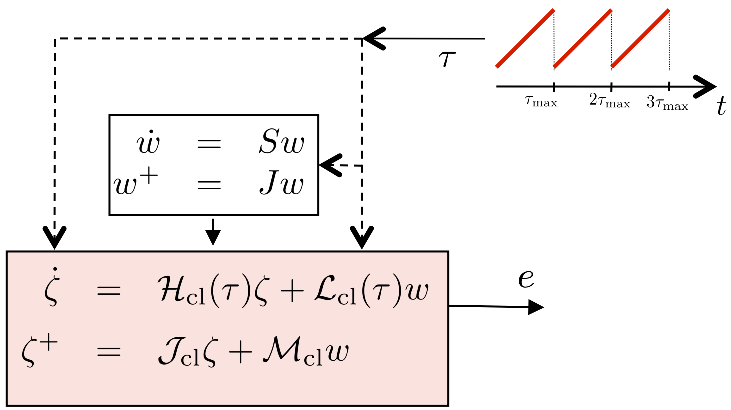
$$\begin{aligned} \frac{d\Pi(\tau)}{d\tau} &= \mathcal{H}_{\rm cl}(\tau)\Pi(\tau) - \Pi(\tau)S + \mathcal{L}_{\rm cl}(\tau) \\ 0 &= \mathcal{J}_{\rm cl}(\tau)\Pi(\tau_{\max}) - \Pi(0)J + \mathcal{M}_{\rm cl} \end{aligned}$$

with $\mathcal{M} = \{((\tau,w),\zeta) \in \mathcal{W}\times\mathbb{R}^{n+m} \ : \ \zeta = \Pi(\tau)w\}$ GES

Steady state for hybrid systems

$$\mathcal{M} = \{((\tau, w), \zeta) \in \mathcal{W} \times \mathbb{R}^{n+m} \ : \ \zeta = \Pi(\tau) w\}$$

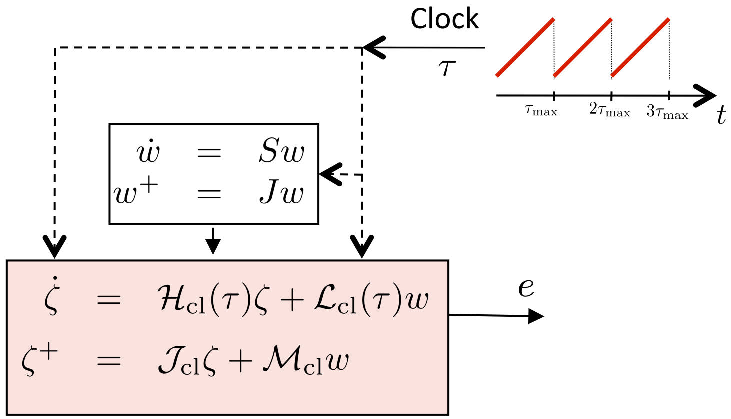




Regulation objective fulfilled if:

$$C\Pi_x(\tau) + Q \equiv 0 \qquad \forall\, \tau \in [0, \tau_{\max}]$$

Towards a hybrid internal model principle



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Then there exists a $\mathcal{C}^1$ $\Pi : [0, \tau_{\max}] \to \mathbb{R}^{n\times s}$ solution of

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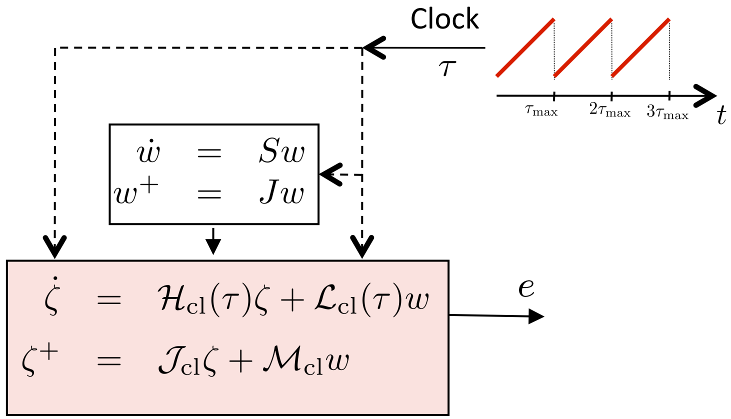
with $\mathcal{M} = \{((\tau, w), \zeta) \in \mathcal{W} \times \mathbb{R}^{n+m} \ : \ \zeta = \Pi(\tau)w\}$ GES

Towards a hybrid internal model principle



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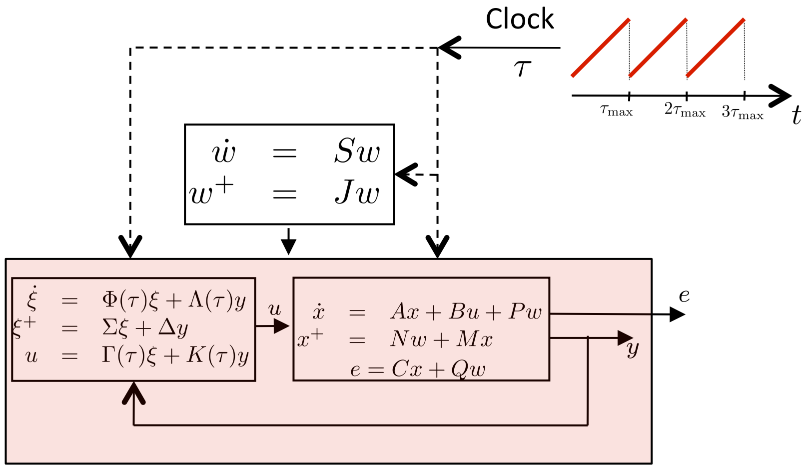
$$C\Pi_x(\tau) + Q \equiv 0$$

with $\mathcal{M} = \{((\tau, w), \zeta) \in \mathcal{W} \times \mathbb{R}^{n+m} \ : \ \zeta = \Pi(\tau)w\}$ GES

Towards a hybrid internal model principle



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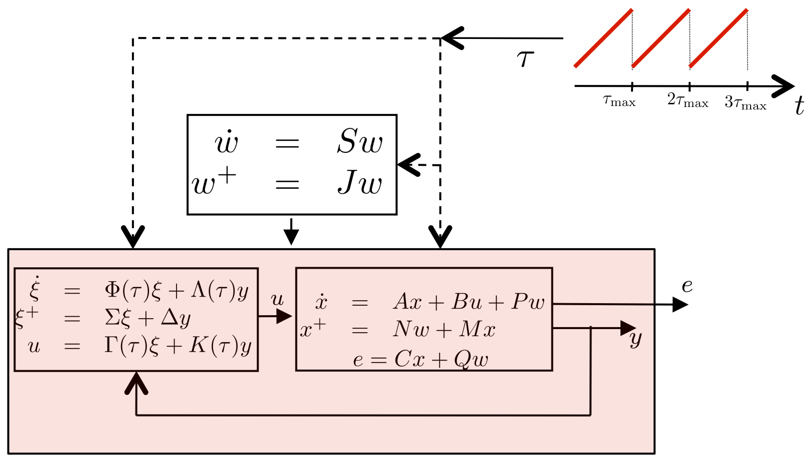
$$\Pi(\tau) = \begin{pmatrix} \Pi_x(\tau) \\ \Pi_\xi(\tau) \end{pmatrix}$$

Then there exists a $\mathcal{C}^1$ $\Pi : [0, \tau_{\max}] \to \mathbb{R}^{n \times s}$ solution of

$$\begin{aligned} \frac{d\Pi(\tau)}{d\tau} &= \mathcal{H}_{\mathrm{cl}}(\tau)\Pi(\tau) - \Pi(\tau)S + \mathcal{L}_{\mathrm{cl}}(\tau) \\ 0 &= \mathcal{J}_{\mathrm{cl}}(\tau)\Pi(\tau_{\max}) - \Pi(0)J + \mathcal{M}_{\mathrm{cl}} \\ C\Pi_x(\tau) &+ Q \equiv 0 \end{aligned}$$

with $\mathcal{M} = \{((\tau, w), \zeta) \in \mathcal{W} \times \mathbb{R}^{n+m} \ : \ \zeta = \Pi(\tau)w\}$ GES

Towards a hybrid internal model principle



$$
\begin{aligned}
\frac{d\Pi_x(\tau)}{d\tau} &= A\Pi_x(\tau) - \Pi_x(\tau)S + P + BR(\tau) \\
0 &= M\Pi_x(\tau_{\max}) - \Pi_x(0)J + N \\
0 &= C\Pi_x(\tau) + Q
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Pi_\xi(\tau)}{d\tau} &= \Phi(\tau)\Pi_\xi(\tau) - \Pi_\xi(\tau)S \\
0 &= \Sigma\Pi_\xi(\tau_{\max}) - \Pi_\xi(0)J \\
R(\tau) &= \Gamma(\tau)\Pi_\xi(\tau)
\end{aligned}
$$

Internal model principle for (the class of) hybrid systems

Assume that (SR), (NR) are fulfilled and $\mathcal{W}$ invariant.

If $\lim_{t+j\to\infty} e(t,j) = 0$ (uniformly) then necessarily there exist

$R : [0, \tau_{\max}] \to \mathbb{R}^{1\times s}$, $\Pi_x : [0, \tau_{\max}] \to \mathbb{R}^{n\times s}$, $\Pi_\xi : [0, \tau_{\max}] \to \mathbb{R}^{m\times s}$ s.t.

$$
\begin{aligned}
\frac{d\Pi_x(\tau)}{d\tau} &= A\Pi_x(\tau) - \Pi_x(\tau)S + P + BR(\tau) \\
0 &= M\Pi_x(\tau_{\max}) - \Pi_x(0)J + N \\
0 &= C\Pi_x(\tau) + Q
\end{aligned}
$$

Hybrid regulated Eq.

FBI–LIKE

If and only if

$$
\begin{aligned}
\frac{d\Pi_\xi(\tau)}{d\tau} &= \Phi(\tau)\Pi_\xi(\tau) - \Pi_\xi(\tau)S \\
0 &= \Sigma\Pi_\xi(\tau_{\max}) - \Pi_\xi(0)J \\
R(\tau) &= \Gamma(\tau)\Pi_\xi(\tau)
\end{aligned}
$$

Internal model property

Internal model principle for continuous-time systems

Assume that (SR), (NR) are fulfilled and $\mathcal{W}$ invariant.

If $\lim_{t+j\to\infty} e(t,j) = 0$ (uniformly) then necessarily there exist

R, Π_x, Π_ξ s.t.

$$
\begin{aligned}
0 &= A\Pi_x - \Pi_x S + P + BR \\
0 &= C\Pi_x + Q
\end{aligned}
$$

Hybrid regulated Eq.

Francis eq

If and only if

$$
\begin{aligned}
0 &= \Phi\,\Pi_\xi - \Pi_\xi S \\
R &= \Gamma\,\Pi_\xi
\end{aligned}
$$

Internal model property

Additional results achieved so far

- Systematic way of designing regulators (min, nonmin-phase)
- Design of robust regulators (TAC, accepted)
- “Kicking” internal model (discrete time stabilization)
- Unknown clock (CDC’11)
- Robust spline tracking (CDC’12)
- Output regulation with sampled error measurement: “double internal model” (TAC, accepted)

Clock

$$\begin{array}{rcl} \dot{\xi}_h & = & \Phi_h \xi_h \\ u & = & \Gamma_h \xi_h \end{array}$$

u

$$\begin{array}{rcl} \dot{w} & = & Sw \\ \dot{x} & = & Ax + Bu + Pw \end{array}$$

e

$$\begin{array}{rcl} \xi_h^+ & = & \Sigma_h \xi_d + \Delta_h e \\ \xi_d{}^+ & = & \Sigma_d \xi_d + \Delta_d e \end{array}$$