Robotics II

June 11, 2025

Exercise 1

Consider the planar 3R robot with unitary link lengths in Fig. 1 controlled in velocity. The robot should execute a desired end-effector velocity $\boldsymbol{v}_e$ and move away from the obstacle the closest robot point with a velocity $\boldsymbol{v}_c$, with norm proportional to the inverse of the Cartesian distance to the obstacle (Cartesian clearance). In the shown configuration $\boldsymbol{q} = (30°, -30°, 30°)$, let $\boldsymbol{v}_e = (3, 0)$ [m/s]. Provide the numerical value of the joint velocity command $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ in the two cases: *i)* when the end-effector task has higher priority than the clearance task; and *ii)* vice versa. Verify and discuss the execution of the two tasks with your solutions, taking also into account the possible ill-conditioning of matrices involved in the task priority method (*Hint: if needed, introduce a tolerance on too small singular values in the pseudoinversion).*

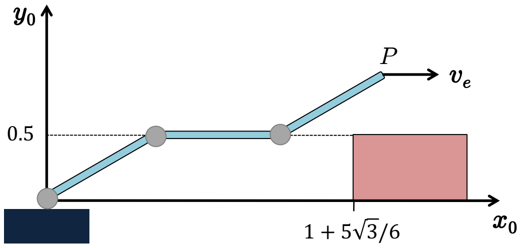

Figure 1: A planar 3R robot in the presence of an obstacle.

Exercise 2

The two-mass flexible system in a vertical plane illustrated in Fig. 2 is driven by a control force F applied to the first mass. The coordinates q_1 and q_2 have their zero in a position where the elastic spring is undeformed.

a) Derive the dynamic model assuming that friction can be neglected.

b) Determine all forced equilibrium configurations of the system.

c) Design the feedforward command $F_d(t)$ that reproduces a desired trajectory $q_{2,d}(t)$, for $t \geq 0$. What degree of differentiability should the trajectory $q_{2,d}(t)$ have? Which should be the initial state of the system so that exact reproduction is achieved from time $t = 0$ on?

d) Compute the feedforward $F_d(t)$ and the initial matched state $\boldsymbol{x}_d(0) = (q_{1d}(0), q_{2d}(0), \dot{q}_{1d}(0), \dot{q}_{2d}(0))$ for the desired output trajectory $q_{2,d}(t) = A \cos \omega t$, $t \geq 0$.

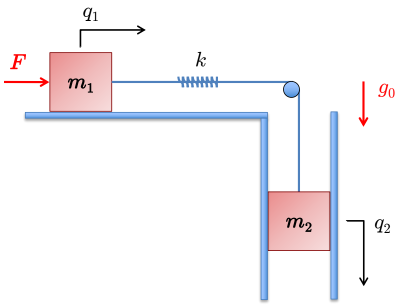

Figure 2: A system with two masses m_1 and m_2, coupled by an elastic transmission of stiffness k.

Exercise 3

A robot manipulator with n revolute joints moves under gravity in the absence of dissipative terms. The total energy of the robot is $E = T(\boldsymbol{q}, \dot{\boldsymbol{q}}) + U(\boldsymbol{q}) \geq 0$. Design a feedback control law for the joint torque $\boldsymbol{\tau}$ such that the closed-loop dynamics satisfies the desired behavior

$$\dot{E} = \begin{cases} -\gamma E - \beta \|\dot{\boldsymbol{q}}\|^2 & \text{for } \|\dot{\boldsymbol{q}}\| > \epsilon \\ -\gamma T & \text{for } \|\dot{\boldsymbol{q}}\| \leq \epsilon, \end{cases} \tag{1}$$

with $\beta > 0$, $\gamma > 0$, and for some (small) $\epsilon > 0$.

Exercise 4

Consider the actuated pendulum in Fig. 3. This mechanical system is actuated by a direct-drive motor with inertia $I_m > 0$ and can be modeled as a massless link of length ℓ, with a payload of mass $m_p > 0$ and inertia $I_p > 0$ placed at the tip. The joint motion is subject to viscous friction with a coefficient $f_v > 0$. Assuming that only the link length ℓ, the payload mass m_p, and the acceleration of gravity g_0 are known, provide the explicit expression of an adaptive control law of *minimal* dimension that allows to asymptotically track any desired trajectory $q_d(t)$ having continuous velocity. For the proposed control law, prove global asymptotic stability of the zero trajectory error state ($e = q_d - q = 0$ and $\dot{e} = \dot{q}_d - \dot{q} = 0$).

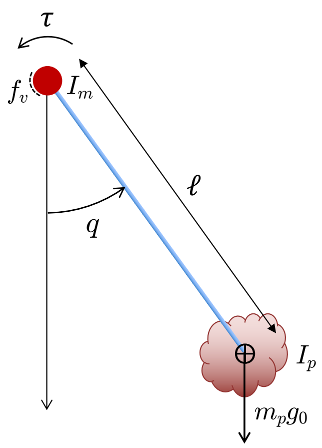

Figure 3: An actuated pendulum with payload.

Exercise 5

With reference to the interaction task in Fig. 4, assuming ideal conditions (rigid bodies, no friction), choose a suitable task frame and define natural and artificial constraints for the following two situations: *i)* the wedge slides in contact with the bottom of the rectangular groove without changing orientation; *ii)* the wedge oscillates around its contact edge without displacing it. When these tasks are performed with a 6R spatial robot using a hybrid force-motion control law, unnecessary stress between wedge and groove has to be avoided while enforcing contact maintenance. How many independent velocity and force control loops have to be designed in each situation?

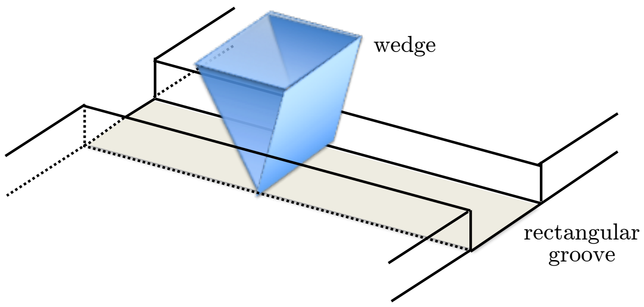

Figure 4: A wedge inside a rectangular groove.

[4 hours, open books]

Solution

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Exercise 1

Compute first the Cartesian clearance of the robot in the configuration $\boldsymbol{q} = (\pi/6, -\pi/6, \pi/6)$ [rad] shown in Fig. 1. With the help of Fig. 5, it is easy to see that the robot point of minimum distance from the obstacle is at the midpoint C of the last link. From the position vectors

$$B = \begin{pmatrix} 1 + \dfrac{5\sqrt{3}}{6} \\ 0.5 \end{pmatrix} \quad C = \begin{pmatrix} \cos\pi/6 + 1 + 0.5\cos\pi/6 \\ \sin\pi/6 + 0.5\sin\pi/6 \end{pmatrix} = \begin{pmatrix} 1 + \dfrac{3\sqrt{3}}{4} \\ 0.75 \end{pmatrix} \quad \Rightarrow \quad C - B = \begin{pmatrix} -\dfrac{\sqrt{3}}{12} \\ 0.25 \end{pmatrix},$$

we have

$$\text{clearance} = \|C - B\| = 0.2887\,\text{m}.$$

Accordingly, the desired velocity of point C for the clearance task is

$$\boldsymbol{v}_c = \frac{1}{\|C-B\|} \cdot \frac{C-B}{\|C-B\|} = \frac{C-B}{\|C-B\|^2} = \begin{pmatrix} -1.7321 \\ 3 \end{pmatrix} \qquad \Rightarrow \qquad \|\boldsymbol{v}_c\| = 3.4641 \text{ m/s},$$

whereas for the end-effector task the desired velocity is

$$\boldsymbol{v}_e = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \qquad \Rightarrow \qquad \|\boldsymbol{v}_e\| = 3 \text{ m/s}.$$

Note that the two desired task velocities have similar norms.

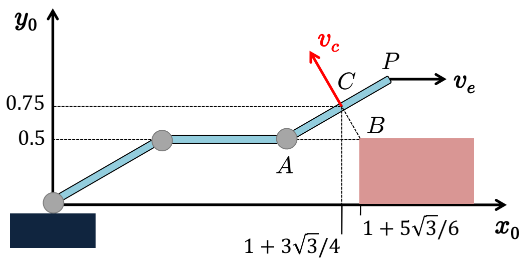

Figure 5: The minimum distance between the robot and the obstacle is $\|C - B\| = 0.2887$ m.

The Jacobians of the two tasks are

$$\boldsymbol{J}_e(\boldsymbol{q}) = \begin{pmatrix} -(s_1 + s_{12} + s_{123}) & -(s_{12} + s_{123}) & -s_{123} \\ c_1 + c_{12} + c_{123} & c_{12} + c_{123} & c_{123} \end{pmatrix}$$

$$\boldsymbol{J}_c(\boldsymbol{q}) = \begin{pmatrix} -(s_1 + s_{12} + 0.5\,s_{123}) & -(s_{12} + 0.5\,s_{123}) & -0.5\,s_{123} \\ c_1 + c_{12} + 0.5\,c_{123} & c_{12} + 0.5\,c_{123} & 0.5\,c_{123} \end{pmatrix}.$$

When evaluated at the given configuration, they provide

$$\boldsymbol{J}_e = \begin{pmatrix} -1 & -0.5 & -0.5 \\ 2.7321 & 1.8660 & 0.8660 \end{pmatrix} \qquad \boldsymbol{J}_c = \begin{pmatrix} -0.75 & -0.25 & -0.25 \\ 2.2990 & 1.4330 & 0.4330 \end{pmatrix},$$

which are both full (row) rank matrices. The corresponding null-space projection matrices (having rank 1) are

$$\boldsymbol{P}_e = \boldsymbol{I} - \boldsymbol{J}_e^{\#}\boldsymbol{J}_e = \frac{1}{3}\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \qquad \boldsymbol{P}_c = \boldsymbol{I} - \boldsymbol{J}_c^{\#}\boldsymbol{J}_c = \frac{1}{3}\begin{pmatrix} 0.5 & -0.5 & -1 \\ -0.5 & 0.5 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$

When the end-effector velocity task has higher priority ($e \prec c$) than the clearance velocity task, the control solution is

$$\dot{\boldsymbol{q}}_{e \prec c} = \boldsymbol{J}_e^{\#}\boldsymbol{v}_e + (\boldsymbol{J}_c\boldsymbol{P}_e)_{\varepsilon}^{\#}\left(\boldsymbol{v}_c - \boldsymbol{J}_c\boldsymbol{J}_e^{\#}\boldsymbol{v}_e\right) = \begin{pmatrix} 3.9282 \\ 1.2679 \\ -15.1244 \end{pmatrix} \qquad \|\dot{\boldsymbol{q}}_{e \prec c}\| = 15.6775\,[\text{rad/s}]. \qquad (2)$$

The notation $\boldsymbol{A}_\varepsilon^\#$ denotes a robust pseudoinversion of matrix $\boldsymbol{A}$: non-zero singular values of matrix $\boldsymbol{A}$ that are below a small tolerance $\varepsilon > 0$ are treated then as zeros.[1] In fact, we have that the matrix

$$\boldsymbol{J}_c\boldsymbol{P}_e = \begin{pmatrix} -0.0833 & 0.0833 & 0.0833 \\ 0.1443 & -0.1443 & -0.1443 \end{pmatrix} \qquad \Rightarrow \qquad \sigma_{\min}\left(\boldsymbol{J}_c\boldsymbol{P}_e\right) = 1 \cdot 10^{-15} \neq 0$$

is ill-conditioned, so that a standard pseudoinverse (with the default value $\varepsilon = 0$) would produce

$$\left(\boldsymbol{J}_c\boldsymbol{P}_e\right)^\# = 10^{15} \cdot \begin{pmatrix} 1.3020 & 0.7517 \\ 0.7470 & 0.4313 \\ 0.5551 & 0.3205 \end{pmatrix},$$

with a clear loss of accuracy. If this pseudoinverse were used in (2), also the first priority task would not be executed correctly. However, using a small tolerance ε such that $\sigma_{\min}\left(\boldsymbol{J}_c\boldsymbol{P}_e\right) < \varepsilon$, this singular value is treated as 0. Robust pseudoinversion of $\boldsymbol{J}_c\boldsymbol{P}_e$ with $\varepsilon = 10^{-10}$ gives

$$\left(\boldsymbol{J}_c\boldsymbol{P}_e\right)_\varepsilon^\# = \begin{pmatrix} -1 & 1.7321 \\ 1 & -1.7321 \\ 1 & -1.7321 \end{pmatrix},$$

which is the matrix that has been used in (2). With this numerical solution, the two resulting task velocities are

$$\boldsymbol{v}_{e,\text{actual}} = \boldsymbol{J}_e\,\dot{\boldsymbol{q}}_{e \prec c} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \boldsymbol{v}_e \qquad \boldsymbol{v}_{c,\text{actual}} = \boldsymbol{J}_c\,\dot{\boldsymbol{q}}_{e \prec c} = \begin{pmatrix} 0.5179 \\ 4.2990 \end{pmatrix} \neq \boldsymbol{v}_c,$$

showing that the highest priority task is executed correctly, while $\|\boldsymbol{v}_c - \boldsymbol{v}_{c,\text{actual}}\| = 2.5981$ m/s.

On the other hand, when the clearance task has higher priority ($c \prec e$) than the end-effector task, the control solution is

$$\dot{\boldsymbol{q}}_{c \prec e} = \boldsymbol{J}_c^\#\boldsymbol{v}_c + \left(\boldsymbol{J}_e\boldsymbol{P}_c\right)^\#\left(\boldsymbol{v}_e - \boldsymbol{J}_e\boldsymbol{J}_c^\#\boldsymbol{v}_c\right) = \begin{pmatrix} 8.4282 \\ -8.4282 \\ -9.9282 \end{pmatrix} \qquad \|\dot{\boldsymbol{q}}_{c \prec e}\| = 15.5125\,[\text{rad/s}]. \tag{3}$$

In this case, the matrix

$$\boldsymbol{J}_e\boldsymbol{P}_c = \begin{pmatrix} 0.0833 & -0.0833 & -0.1667 \\ -0.1443 & 0.1443 & 0.2887 \end{pmatrix} \qquad \Rightarrow \qquad \sigma_{\min}\left(\boldsymbol{J}_e\boldsymbol{P}_c\right) = 0$$

is not ill-conditioned: the smallest singular value is in fact zero (up to machine precision) and is treated as such in the pseudoinversion. Thus, no robust pseudoinversion is needed

$$\left(\boldsymbol{J}_e\boldsymbol{P}_c\right)^\# = \left(\boldsymbol{J}_e\boldsymbol{P}_c\right)_\varepsilon^\# = \begin{pmatrix} 0.5 & -0.8660 \\ -0.5 & 0.8660 \\ -1 & 1.7321 \end{pmatrix}.$$

With solution (3), the two resulting task velocities are

$$\boldsymbol{v}_{c,\text{actual}} = \boldsymbol{J}_c\,\dot{\boldsymbol{q}}_{c \prec e} = \begin{pmatrix} -1.7321 \\ 3 \end{pmatrix} = \boldsymbol{v}_c \qquad \boldsymbol{v}_{e,\text{actual}} = \boldsymbol{J}_e\,\dot{\boldsymbol{q}}_{c \prec e} = \begin{pmatrix} 0.75 \\ -1.2990 \end{pmatrix} \neq \boldsymbol{v}_e,$$

showing again that the clearance task, which is now the highest priority task, has been executed correctly, while $\|\boldsymbol{v}_e - \boldsymbol{v}_{e,\text{actual}}\| = 2.5981$ m/s. Somewhat surprisingly, this error norm on the secondary end-effector task is the same as before, when the secondary task was the clearance velocity.

[1] In MATLAB, the command is the usual one, `pinv(A,tol)`, with an additional parameter `tol` $= \varepsilon$. To display more digits use the command `format long`. For the machine precision of MATLAB, see `https://it.mathworks.com/matlabcentral/answers/116515-which-is-the-machine-precision-of-matlab` .

Exercise 2

The kinetic and potential energy for this mechanical system are

$$T = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 \qquad U = U_g + U_e = -m_2 g_0 q_2 + \frac{1}{2} k \left(q_1 - q_2\right)^2 .$$

Therefore, applying the Euler-Lagrance equations, the dynamic model is

$$m_1 \ddot{q}_1 + k\left(q_1 - q_2\right) = F \tag{4}$$

$$m_2 \ddot{q}_2 + k\left(q_2 - q_1\right) - m_2 g_0 = 0. \tag{5}$$

Due to the presence of the constant gravity term $m_2 g_0 \neq 0$, there are no unforced (i.e., with $F_e = 0$) equilibria. For the forced equilibrium configurations, setting $\ddot{q}_1 = \ddot{q}_2 = 0$, we obtain from (4) and (5)

$$q_1 = q_{1e} \qquad q_2 = q_{2e} = q_{1e} + \frac{m_2 g_0}{k} \qquad F = F_e = k\left(q_{1e} - q_{2e}\right) = -m_2 g_0 < 0,$$

i.e., an infinite set of configurations parametrized here by the value q_{1e}, all corresponding to the same equilibrium command.

For the inverse dynamics problem, given a desired trajectory $q_{2,d}(t)$, from (5) we obtain

$$q_{1,d}(t) = q_{2,d}(t) + \frac{m_2}{k}\left(\ddot{q}_{2,d}(t) - g_0\right).$$

Differentiating once and twice this equation gives

$$\dot{q}_{1,d}(t) = \dot{q}_{2,d}(t) + \frac{m_2}{k}\,\dddot{q}_{2,d}(t) \qquad \ddot{q}_{1,d}(t) = \ddot{q}_{2,d}(t) + \frac{m_2}{k}\frac{d^4 q_{2,d}(t)}{dt^4}$$

which, substituted in (4), lead to the desired feedforward command

$$F_d(t) = m_1 \ddot{q}_{2,d}(t) + \frac{m_1 m_2}{k}\frac{d^4 q_{2,d}(t)}{dt^4} + m_2\left(\ddot{q}_{2,d}(t) - g_0\right),$$

which is expressed only in terms of the output trajectory $q_{2,d}(t)$ and its time derivatives. As a result, in order to be reproduced exactly, the desired trajectory $q_{2,d}(t)$ should be at least four times differentiable with respect to its time argument t.

Moreover, exact reproduction requires that the initial state is matched with the desired trajectory at time $t = 0$. From the previous relations

$$\begin{aligned} q_1(0) &= q_{2,d}(0) + \frac{m_2}{k}\left(\ddot{q}_{2,d}(0) - g_0\right) \\ q_2(0) &= q_{2,d}(0) \\ \dot{q}_1(0) &= \dot{q}_{2,d}(0) + \frac{m_2}{k}\,\dddot{q}_{2,d}(0) \\ \dot{q}_2(0) &= \dot{q}_{2,d}(0). \end{aligned}$$

Finally, for the smooth desired trajectory $q_{2,d}(t) = A\cos\omega t$, the above computations yield the desired force command as

$$F_d(t) = A\omega^2\left(\frac{m_1 m_2}{k}\,\omega^2 - (m_1 + m_2)\right)\cos\omega t - m_2 g_0,$$

with the initial condition on the system state

$$\boldsymbol{x}_d(0) = \begin{pmatrix} A - \dfrac{m_2}{k}\left(A\omega^2 + g_0\right) \\ A \\ 0 \\ 0 \end{pmatrix}.$$

Exercise 3

For any robot, conservation of energy under the absence of dissipative terms implies $\dot{E} = \dot{\boldsymbol{q}}^T \boldsymbol{\tau}$. Being the total energy $E = \frac{1}{2}\,\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + U(\boldsymbol{q})$, the required behavior (1) is

$$\dot{E} = \dot{\boldsymbol{q}}^T \boldsymbol{\tau} = \begin{cases} -\dfrac{\gamma}{2}\,\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \gamma\, U(\boldsymbol{q}) - \beta\, \dot{\boldsymbol{q}}^T \dot{\boldsymbol{q}} & \text{for } \|\dot{\boldsymbol{q}}\| > \epsilon \\ -\dfrac{\gamma}{2}\,\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} & \text{for } \|\dot{\boldsymbol{q}}\| \leq \epsilon. \end{cases} \tag{6}$$

Thus, the torque $\boldsymbol{\tau} \in \mathbb{R}^n$ can be chosen as

$$\boldsymbol{\tau} = \begin{cases} -\dfrac{\gamma}{2}\,\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \gamma\, U(\boldsymbol{q})\,\dfrac{\dot{\boldsymbol{q}}}{\|\dot{\boldsymbol{q}}\|^2} - \beta\, \dot{\boldsymbol{q}} & \text{for } \|\dot{\boldsymbol{q}}\| > \epsilon \\ -\dfrac{\gamma}{2}\,\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} & \text{for } \|\dot{\boldsymbol{q}}\| \leq \epsilon. \end{cases} \tag{7}$$

Note that in this way $\boldsymbol{\tau}$ remains bounded in norm when $\dot{\boldsymbol{q}} \to \boldsymbol{0}$.

Exercise 4

Under the given assumptions, the dynamic model is

$$\left(I_m + I_p + m_p \ell^2\right)\ddot{q} + f_v \dot{q} + m_p \ell g_0 \sin q = \tau. \tag{8}$$

Defining $I = I_m + I_p + m_p\ell^2$, the unknown part of the model can be linearly parametrized as

$$\boldsymbol{Y}(\dot{q}, \ddot{q})\,\boldsymbol{a} = \begin{pmatrix} \ddot{q} & \dot{q} \end{pmatrix} \begin{pmatrix} I \\ f_v \end{pmatrix}.$$

Thus, an adaptive trajectory tracking controller of minimal dimension $p = 2$ can be defined as

$$\tau = \boldsymbol{Y}(\dot{q}_r, \ddot{q}_r)\,\hat{\boldsymbol{a}} + m_p \ell g_0 \sin q + k_D s = \hat{I}\,\ddot{q}_r + \hat{f}_v \dot{q}_r + m_p \ell g_0 \sin q + k_D s \tag{9}$$

$$\dot{\hat{\boldsymbol{a}}} = \begin{pmatrix} \dot{\hat{I}} \\ \dot{\hat{f}}_v \end{pmatrix} = \boldsymbol{\Gamma}\, \boldsymbol{Y}^T(\dot{q}_r, \ddot{q}_r)\, s = \begin{pmatrix} \gamma_1 \ddot{q}_r \\ \gamma_2 \dot{q}_r \end{pmatrix} s, \tag{10}$$

with $\dot{q}_r = \dot{q}_d + \lambda\,(q_d - q)$, $s = \dot{q}_r - \dot{q} = \dot{q}_d - \dot{q} + \lambda\,(q_d - q) = \dot{e} + \lambda e$, and being $k_D > 0$, $\lambda = k_P/k_D > 0$, $\gamma_1 > 0$, and $\gamma_2 > 0$.

Having canceled the known (non-adaptive) gravity term with the control law, the proof of global asymptotic stability of the tracking error is proven in a similar way, with the simplifications due to the scalar case. Let the Lyapunov candidate function be

$$V = \frac{1}{2}\,I s^2 + \frac{1}{2}\,R\, e^2 + \frac{1}{2}\tilde{\boldsymbol{a}}^T \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{a}} \geq 0,$$

with $R > 0$ and $\tilde{\boldsymbol{a}} = \boldsymbol{a} - \hat{\boldsymbol{a}}$. Its time derivative is

$$\dot{V} = I s\, \dot{s} + R\, e\, \dot{e} - \tilde{\boldsymbol{a}}^T \boldsymbol{\Gamma}^{-1} \dot{\hat{\boldsymbol{a}}}. \tag{11}$$

The closed-loop equation obtained from (8) with (9) is

$$I\ddot{q} + f_v \dot{q} + \cancel{m_p \ell g_0 \sin q} = \hat{I}\,\ddot{q}_r + \hat{f}_v \dot{q}_r + \cancel{m_p \ell g_0 \sin q} + k_D s.$$

Subtracting both sides from $I\ddot{q}_r + f_v \dot{q}_r$, we obtain

$$I\dot{s} + f_v s = \boldsymbol{Y}(\dot{q}_r, \ddot{q}_r)\tilde{\boldsymbol{a}} - k_D s,$$

which, substituted in (11) and using the update $\dot{\hat{\boldsymbol{a}}}$ in (10), yields

$$\begin{aligned} \dot{V} &= -\left(f_v + k_D\right) s^2 + R\, e\, \dot{e} + \cancel{s\, \boldsymbol{Y}(\dot{q}_r, \ddot{q}_r)\tilde{\boldsymbol{a}}} - \cancel{\tilde{\boldsymbol{a}}^T \boldsymbol{\Gamma}^{-1} \boldsymbol{\Gamma}\, \boldsymbol{Y}^T(\dot{q}_r, \ddot{q}_r)\, s} \\ &= -\left(f_v + k_D\right)\left(\dot{e} + \lambda e\right)^2 + R\, e\, \dot{e} = -\left(f_v + k_D\right)\dot{e}^2 - \left(f_v + k_D\right)\lambda^2 e^2 \leq 0, \end{aligned}$$

having set $R = 2\lambda\,(f_v + k_D) > 0$. The conclusion follows from Barbalat lemma + LaSalle theorem.

Exercise 5

With reference to Fig. 6 and the task frame defined therein, the interaction of the objects in contact is defined by the geometry and the tasks will be executed in the required way when imposing the following natural and artificial constraints:

natural constraints	artificial constraints for *i)* sliding	artificial constraints for *ii)* oscillating
${}^t f_x = 0$	${}^t v_x = v_{x,d} \neq 0$	${}^t v_x = 0$
${}^t v_y = 0$	${}^t f_y = 0$	${}^t f_y = 0$
${}^t v_z = 0$	${}^t f_z = f_{z,d} > 0$	${}^t f_z = f_{z,d} > 0$
${}^t \omega_x = 0$	${}^t \mu_x = 0$	${}^t \mu_x = 0$
${}^t \mu_y = 0$	${}^t \omega_y = 0$	${}^t \omega_y = \omega_{y,d} \neq 0$
${}^t \omega_z = 0$	${}^t \mu_z = 0$	${}^t \mu_z = 0.$

Accordingly, in both cases the hybrid force/motion control law will have $k = 2$ velocity/position and $6 - k = 4$ force loops.

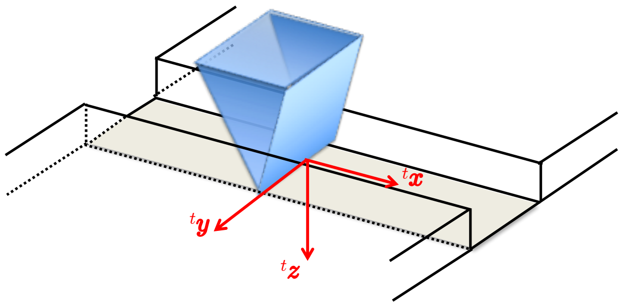

Figure 6: The task frame for the interaction task of Fig. 4.

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