Robotics 2

February 7, 2025

Exercise 1

The 2P2R planar robot in Fig. 1 should move its end-effector along a desired trajectory $\boldsymbol{p}_d(t) \in \mathbb{R}^2$ driven by the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^4$. The motion of each joint is restricted to $[q_{min,i}, q_{max,i}]$, with ranges $[-1, 5]$ [m], for $i = 1, 2$, and $[-\pi/2, \pi/3]$ [rad], for $i = 3, 4$. Design the commands $\dot{\boldsymbol{q}}$ that realize the task at best and try to locally increase the available joint range, by suitably exploiting:

- the Projected Gradient (PG) method;
- the Reduced Gradient (RG) method;
- the Task Priority (TP) method.

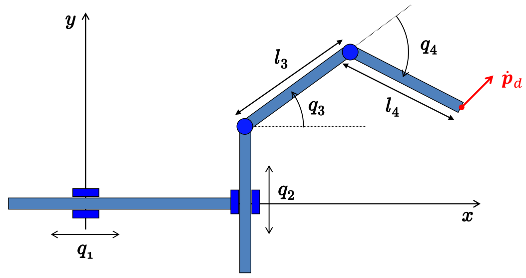

Figure 1: A 2P2R planar robot.

Provide then the numerical values of the PG, RG and TP commands at time $t = 0$ in the situation:

$$l_3 = l_4 = 1 \text{ [m]} \qquad \boldsymbol{q}(0) = \boldsymbol{0} \qquad \boldsymbol{p}_d(t) = \begin{pmatrix} 2 - t \\ \sin 3t \end{pmatrix} \text{ [m]}.$$

Discuss the obtained results. How would you modify these velocity commands so as to compensate for possible end-effector position errors that may arise during task execution?

Exercise 2

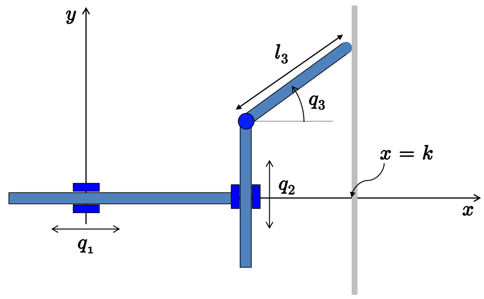

Figure 2: A PPR planar robot with its end effector in constrained motion on a linear surface.

Consider the PPR planar robot in Fig. 2, moving on a horizontal plane and having its end effector constrained to an ideal (rigid, frictionless) linear surface. For this robot in constrained motion:

- provide the explicit symbolic expressions of its *reduced dynamics*, i.e., the minimum number of second-order differential equations relating the pseudoacceleration $\dot{\boldsymbol{v}}$ to the pseudovelocity $\boldsymbol{v}$, the configuration $\boldsymbol{q}$, and the generalized input torque at the joints $\boldsymbol{\tau}$;

- design a suitable control law for $\boldsymbol{\tau}$ so as to realize a motion of the end effector on the linear surface with constant velocity V, while inducing a constant reaction force f at the contact;
- evaluate whether the robot is redundant for this hybrid force-velocity task; if so, pursue a control design that minimizes the norm of the torque $\boldsymbol{\tau}$ needed to achieve the above task.

Exercise 3

The robotic actuation device shown in Fig. 3 is a servomechanism with a DC motor coupled to a rotating balanced load through a flexible transmission.

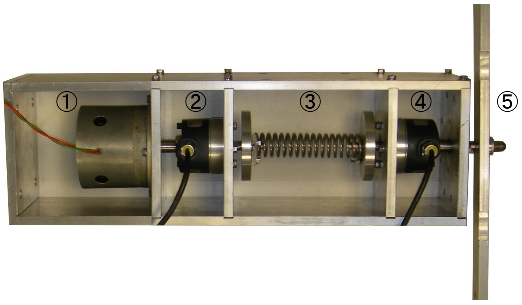

Figure 3: A 1-dof actuation device with a flexible transmission

- Explain the role to each component numbered (from 1 to 5) in the figure.
- Using an energy (Lagrangian) approach, derive a dynamic model of the system that is appropriate for trajectory planning, motion control, and simulation. The flexible transmission can be modeled as an elastic element, i.e., it behaves as a torsional spring of constant stiffness. In addition, viscous friction is present on the motor and load sides of the transmission. The DC motor is commanded by the armature current. Write down the complete dynamic equations and draw an associated block diagram for simulation purposes (e.g., in Simulink style).
- Plan a smooth rest-to-rest trajectory that rotates the load from an initial angle q_0 to a final q_f in a given time T, without residual vibrations. Determine the motor current in input that commands the motor so that the planned trajectory is exactly realized when the system is initially undeformed and at rest.

[210 minutes (3.5 hours); open books]