

Robotics II

October 27, 2017

A robot manipulator having generalized coordinates $\mathbf{q} \in \mathbb{R}^n$ and dynamic model

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \mathbf{u}, \quad (1)$$

with inertia matrix \mathbf{M} , Coriolis and centrifugal terms \mathbf{c} , and gravity vector \mathbf{g} , should perform a task that is described in terms of variables $\mathbf{x} = \mathbf{f}(\mathbf{q}) \in \mathbb{R}^m$, with $m < n$, and is specified by a smooth desired task trajectory $\mathbf{x} = \mathbf{x}_d(t)$, for $t \geq 0$.

Provide the expressions of the feedback laws that solve the following control problems, in which we consider different objectives and different feasible command inputs for the robot system. In the first three cases, possible errors in the execution of the desired task trajectory should be recovered in an exponential way. For all cases, give also some hints on the physical meaning of the pursued solution.

1. Assume the joint velocity $\dot{\mathbf{q}} \in \mathbb{R}^n$ as control input. Execute the desired m -dimensional task trajectory by minimizing instantaneously the objective function

$$H_1 = \frac{1}{2} \|\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} - \mathbf{p}_0\|^2,$$

where $\mathbf{p}_0 \in \mathbb{R}^n$ is a desired vector value for the generalized momentum of the robot.

2. Assume again the joint velocity $\dot{\mathbf{q}} \in \mathbb{R}^n$ as control input. Execute the desired m -dimensional task trajectory by minimizing instantaneously the objective function

$$H_2 = \frac{1}{2} \|\dot{\mathbf{q}} + \alpha \nabla_{\mathbf{q}} H(\mathbf{q})\|^2, \quad \text{with } H = \frac{1}{2} \|\mathbf{g}(\mathbf{q})\|^2,$$

for a scalar $\alpha > 0$.

3. Assume now the joint acceleration $\ddot{\mathbf{q}} \in \mathbb{R}^n$ as control input. Execute the desired m -dimensional task trajectory by minimizing instantaneously the objective function

$$H_3 = \frac{1}{2} \|\mathbf{u}\|^2.$$

4. Assume finally the joint torque $\mathbf{u} \in \mathbb{R}^n$ as control input. Determine \mathbf{u} by minimizing instantaneously the objective function

$$H_4 = \frac{\lambda}{2} \|\mathbf{u}\|^2 + \frac{1}{2} \|\ddot{\mathbf{x}}_d - \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} - \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}}\|^2,$$

for a suitable scalar $\lambda > 0$ and being $\mathbf{J} = (\partial \mathbf{f} / \partial \mathbf{q})$ the $m \times n$ task Jacobian.

Provide next the symbolic expressions of all the terms needed in the control laws for problems 1 and 2 when the robot is the RP planar arm ($n = 2$) of Fig. 1, the task variable is a scalar ($m = 1$) given by $x = \mathbf{f}(\mathbf{q}) = q_2 \cos q_1$, and the desired task trajectory is $x_d(t) = L + L(1 - \cos \omega t)/4$, for some constants $L > 0$ and $\omega > 0$. The robot starts at rest in a generic configuration $q_1(0) = q_{10} > 0$, $q_2(0) = q_{20} > 0$.

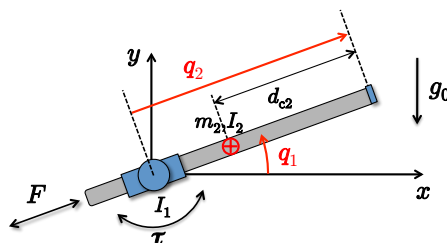


Figure 1: A RP planar robot with the relevant variables and parameters.

[180 minutes, open books but no computer or smartphone]