Robotics II

September 21, 2017

Exercise 1

Consider the RP planar robot in Fig. 1, with the coordinates $\boldsymbol{q} = (q_1, q_2)$, the kinematic parameter L_2, and the dynamic parameters d_{c2}, m_2, I_1 and I_2 defined therein.

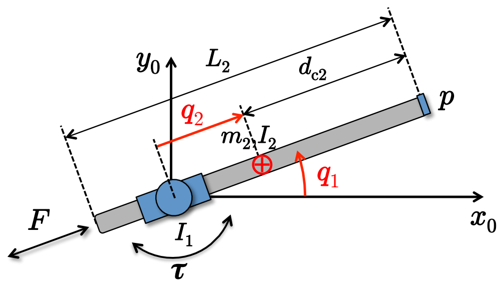

Figure 1: A RP planar robot with the relevant variables and parameters.

- Provide the symbolic expression of the inertia matrix $\boldsymbol{M}(\boldsymbol{q})$, of the Coriolis and centrifugal vector $\boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, and of the gravity vector $\boldsymbol{g}(\boldsymbol{q})$ when the plane $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is inclined with respect to the horizontal plane by an angle $\alpha \in [0, \pi/2]$ around the $\boldsymbol{x}_0$ axis.
- Determine the symbolic expression of $\ddot{\boldsymbol{q}}_0 \in \mathbb{R}^2$, the joint acceleration when the robot starts from rest and the two actuators apply a torque τ and a force F as command inputs.
- Next, assume that

  i) $\alpha = 0$ and the robot is at rest;
  ii) the second link is a uniform thin rod with mass m_2 and inertia $I_2 = (m_2 L_2^2)/12$;
  iii) the torque and the force provided by the motors are bounded: $|\tau| \leq T_{max}$, $|F| \leq F_{max}$;
  iv) the prismatic joint has a limited symmetric range, with $q_2 \in [-L_2, L_2]$.

  In these conditions:

  a. Provide the expression of the squared norm $\|\ddot{\boldsymbol{p}}_0\|^2$, where $\ddot{\boldsymbol{p}}_0 \in \mathbb{R}^2$ is the end-effector acceleration when the robot starts from rest. Verify that this quantity is a function of $\boldsymbol{q}$ and sketch graphically this dependence.
  b. Analyze at least qualitatively how the configurations $\boldsymbol{q}^*_{min}$ and $\boldsymbol{q}^*_{max}$ that provide, respectively, the minimum and maximum of $\|\ddot{\boldsymbol{p}}_0\|^2$ change, when the inertia I_1 of the first link is either much larger or much smaller than I_2 (by 1-2 orders of magnitude).

Exercise 2

A lightweight 6R robot with a spherical wrist operates in a working environment where a human is occasionally present. During normal operation, the robot task is to track accurately a desired smooth trajectory $\boldsymbol{p}_d(t)$ for the end-effector position $\boldsymbol{p} = \boldsymbol{f}_p(\boldsymbol{q}) \in \mathbb{R}^3$ and an associated desired trajectory $\boldsymbol{\phi}_d(t)$ for a minimal representation of the end-effector orientation $\boldsymbol{\phi} = \boldsymbol{f}_\phi(\boldsymbol{q}) \in \mathbb{R}^3$. Assume that:

- The complete dynamic model of the robot in free motion is perfectly known, and is described (with the usual notations) by the following equations

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} - \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}), \tag{1}$$

  where the friction term $\boldsymbol{\tau}_f$ denotes a dissipative action at the joints.
- The direct kinematic functions $\boldsymbol{f}_p$ and $\boldsymbol{f}_\phi$ are known, as well as the 6×6 analytic Jacobian associated to the end-effector task

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{pmatrix} \frac{\partial \boldsymbol{f}_p(\boldsymbol{q})}{\partial \boldsymbol{q}} \\ \frac{\partial \boldsymbol{f}_\phi(\boldsymbol{q})}{\partial \boldsymbol{q}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_p(\boldsymbol{q}) \\ \boldsymbol{J}_\phi(\boldsymbol{q}) \end{pmatrix}, \tag{2}$$

  where the two matrices $\boldsymbol{J}_p$ and $\boldsymbol{J}_\phi$ have dimension 3×6, and matrix $\boldsymbol{J}$ is nonsingular in the region of interest.
- The robot is equipped only with encoders at the joints, and the environment is *not* monitored by any external sensor.

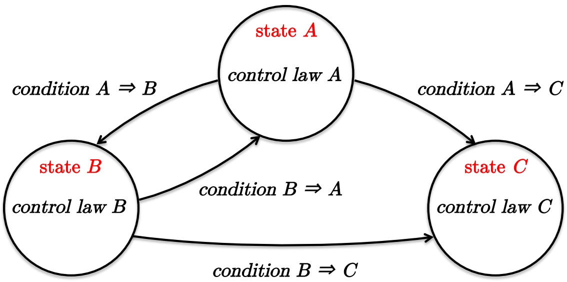

Figure 2: A state diagram of robot control operation in collision-aware tasks.

With reference to the state diagram in Fig. 2, the following collision-aware behavior for safe Human-Robot Interaction (HRI) should be realized through a suitable set of robot control laws and conditions for the transitions:

- During normal operation (state A in the diagram), if a mild contact occurs and is detected, the robot keeps the three-dimensional position task but relaxes the orientation task, trying to accommodate in this way a reflex reaction to the contact (state B).
- Instead, when a severe collision occurs during normal operation, the robot abandons the task completely by bouncing away from the collision area (state C) ad then stops.
- While in state B, the robot may either switch back to normal operation when the contact is no longer present, or abandon also the orientation task and switch to state C in case the interaction forces will increase further.

Specify the control laws and the transition conditions to be used in the state diagram of Fig. 2.

[150 minutes; open books]

Solution

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Exercise 1

Following a Lagrangian approach, we compute first the kinetic energy $T = T_1 + T_2$. We have

$$T_1 = \frac{1}{2} I_1 \dot{q}_1^2, \qquad T_2 = \frac{1}{2} m_2 \left\| \frac{d}{dt} \begin{pmatrix} q_2 \cos q_1 \\ q_2 \sin q_1 \end{pmatrix} \right\|^2 + \frac{1}{2} I_2 \dot{q}_1^2 = \frac{1}{2} \left(I_2 + m_2 q_2^2 \right) \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2,$$

and thus the diagonal inertia matrix

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{pmatrix} I_1 + I_2 + m_2 q_2^2 & 0 \\ 0 & m_2 \end{pmatrix}. \tag{3}$$

Using the Christoffel symbols, the Coriolis and centrifugal terms are easily computed from (3) as

$$\boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{pmatrix} 2 m_2 q_2 \dot{q}_1 \dot{q}_2 \\ -m_2 q_2 \dot{q}_1^2 \end{pmatrix}.$$

For the potential energy due to gravity, $U_g = U_1 + U_2$, we have (up to a constant)

$$U_1 = 0, \qquad U_2 = m_2 \left(g_0 \sin \alpha \right) q_2 \sin q_1,$$

and thus

$$\boldsymbol{g}(\boldsymbol{q}) = \left(\frac{\partial U_g(\boldsymbol{q})}{\partial \boldsymbol{q}} \right)^T = m_2 \, g_0 \sin \alpha \begin{pmatrix} q_2 \cos q_1 \\ \sin q_1 \end{pmatrix}. \tag{4}$$

When the robot is at rest ($\dot{\boldsymbol{q}} = \boldsymbol{0}$), the joint acceleration takes the expression

$$\ddot{\boldsymbol{q}}_0 = \ddot{\boldsymbol{q}} \left|_{\dot{\boldsymbol{q}}=\boldsymbol{0}} \right. = \boldsymbol{M}^{-1}(\boldsymbol{q}) \left(\begin{pmatrix} \tau \\ F \end{pmatrix} - \boldsymbol{g}(\boldsymbol{q}) \right) = \begin{pmatrix} \dfrac{\tau - m_2 g_0 \sin \alpha \, q_2 \cos q_1}{I_1 + I_2 + m_2 q_2^2} \\ \dfrac{F - m_2 g_0 \sin \alpha \sin q_1}{m_2} \end{pmatrix} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix}. \tag{5}$$

The end-effector position and its velocity are

$$\boldsymbol{p} = (q_2 + d_{c2}) \begin{pmatrix} \cos q_1 \\ \sin q_1 \end{pmatrix}, \qquad \dot{\boldsymbol{p}} = \dot{q}_2 \begin{pmatrix} \cos q_1 \\ \sin q_1 \end{pmatrix} + (q_2 + d_{c2}) \, \dot{q}_1 \begin{pmatrix} -\sin q_1 \\ \cos q_1 \end{pmatrix}.$$

Thus, the end-effector acceleration at *zero* joint velocity is

$$\begin{aligned} \ddot{\boldsymbol{p}}_0 = \ddot{\boldsymbol{p}} \left|_{\dot{\boldsymbol{q}}=\boldsymbol{0}} \right. &= \ddot{q}_2 \begin{pmatrix} \cos q_1 \\ \sin q_1 \end{pmatrix} + (q_2 + d_{c2}) \, \ddot{q}_1 \begin{pmatrix} -\sin q_1 \\ \cos q_1 \end{pmatrix} \\ &= \begin{pmatrix} \cos q_1 & -\sin q_1 \\ \sin q_1 & \cos q_1 \end{pmatrix} \begin{pmatrix} \ddot{q}_2 \\ (q_2 + d_{c2}) \, \ddot{q}_1 \end{pmatrix} = \boldsymbol{R}(q_1) \begin{pmatrix} \ddot{q}_2 \\ (q_2 + d_{c2}) \, \ddot{q}_1 \end{pmatrix}, \end{aligned} \tag{6}$$

where $\boldsymbol{R}(\cdot)$ is a 2×2 (planar) rotation matrix. From (6), we have

$$\|\ddot{\boldsymbol{p}}_0\|^2 = \ddot{\boldsymbol{p}}_0^T \, \ddot{\boldsymbol{p}}_0 = \left\| \begin{matrix} \ddot{q}_2 \\ (q_2 + d_{c2}) \, \ddot{q}_1 \end{matrix} \right\|^2 = (q_2 + d_{c2})^2 \, \ddot{q}_1^2 + \ddot{q}_2^2. \tag{7}$$

Using (5) for $\alpha = 0$ in (7), we obtain

$$\left\|\ddot{\boldsymbol{p}}_0\right\|^2 = \frac{1}{m_2^2}\, F^2 + \frac{\left(q_2 + d_{c2}\right)^2}{\left(I_1 + I_2 + m_2 q_2^2\right)^2}\, \tau^2, \tag{8}$$

which shows an actual dependence only on the prismatic joint variable q_2. The two addends in (8) are separately driven by the two motors: the first one is a *radial* contribution due to F, which is scaled just by m_2^2 and is independent from the robot configuration; the second one is the *tangential* contribution (normal to the second link) due to τ, which depends in a nonlinear fashion on q_2, as well as on m_2, d_{c2}, I_1, and I_2 (and their relative values).

It is easy to see that the minimum of $\left\|\ddot{\boldsymbol{p}}_0\right\|^2$ is obtained at $q^*_{2,min} = -d_{c2}$ (with arbitrary q^*_1), namely when the end-effector position is at the origin (on the axis of joint 1). Note also that this value is independent from the dynamic parameters m_2, I_1, and I_2.

From the expression (8), it follows that the maximum value H of the squared norm of the acceleration is given by

$$H = \frac{F^2_{max}}{m_2^2} + \max_{q_2 \in [-L_2, L_2]} \frac{\left(q_2 + d_{c2}\right)^2}{\left(I_1 + I_2 + m_2 q_2^2\right)^2}\, \tau^2_{max},$$

where the maximum bounds on the inputs have been used. Under the given assumption on the mass distribution of link 2, in order to find the absolute maximum of the tangential contribution in $\left\|\ddot{\boldsymbol{p}}_0\right\|^2$ one should study the behavior of the positive function

$$h(q_2) = \left(\frac{q_2 + d_{c2}}{I_1 + I_2 + m_2 q_2^2}\right)^2 = \left(\frac{q_2 + \frac{L_2}{2}}{I_1 + m_2\left(\frac{L_2^2}{12} + q_2^2\right)}\right)^2$$

for q_2 in the closed interval $[-L_2, L_2]$. The stationary points of h satisfy the necessary condition

$$\frac{dh(q_2)}{dq_2} = 0 \qquad \Longleftrightarrow \qquad 2\left(\frac{q_2 + \frac{L_2}{2}}{I_1 + m_2\left(\frac{L_2^2}{12} + q_2^2\right)}\right) \frac{I_1 + m_2\left(\frac{L_2^2}{12} + q_2^2\right) - 2 m_2 q_2\left(q_2 + \frac{L_2}{2}\right)}{\left(I_1 + m_2\left(\frac{L_2^2}{12} + q_2^2\right)\right)^2} = 0$$

$$\Longleftrightarrow \qquad \frac{\left(q_2 + \frac{L_2}{2}\right)\left(m_2\, q_2^2 + m_2 L_2\, q_2 - \left(I_1 + m_2 \frac{L_2^2}{12}\right)\right)}{\left(I_1 + m_2\left(\frac{L_2^2}{12} + q_2^2\right)\right)^3} = 0.$$

The zeros of the derivative occur where one of the two polynomial factors (one linear, the other quadratic) at the numerator vanishes. This occurs at[1]

$$q_2 = q^*_{2,min} = -\frac{L_2}{2} \qquad \Rightarrow \quad \text{a minimum of } h(q_2)$$

and at

$$q_2 = q^*_{2,max} = -\frac{L_2}{2} + \sqrt{\left(\frac{L_2}{2}\right)^2 + \frac{I_1}{m_2} + \frac{L_2^2}{12}} \qquad \Rightarrow \quad \text{a maximum —only if} \in [-L_2, L_2].$$

For very large values of the ratio $I_1/I_2 \propto I_1/m_2$, this second expression will be larger than L_2, and thus outside the closed interval of definition for q_2. Therefore, the maximum will occur at the

[1]The second root of the quadratic factor is always strictly lower than $-L_2$, thus outside the interval $[-L_2, L_2]$.

upper limit, i.e., $q^*_{2,max} = L_2$. On the other hand, for very small values of I_1/m_2, neglecting this term and using a Taylor expansion yields $q^*_{2,max} \approx L_2/12$.

As a numerical example, Fig. 3 shows the plots of $h(q_2)$ for various ratios of I_1/I_2, when the second link is a uniform thin rod of mass $m_2 = 1$ [kg] and length $L_2 = 0.5$ [m]. For instance, when $I_1/I_2 = 50$ (red profile on the left), the maximum is at $q^*_{2,max} = L_2 = 0.5$. On the other hand, when $I_1/I_2 = 0.01$ (red profile on the right), the maximum is at $q^*_{2,max} \approx L_2/12 = 0.04$ [m].

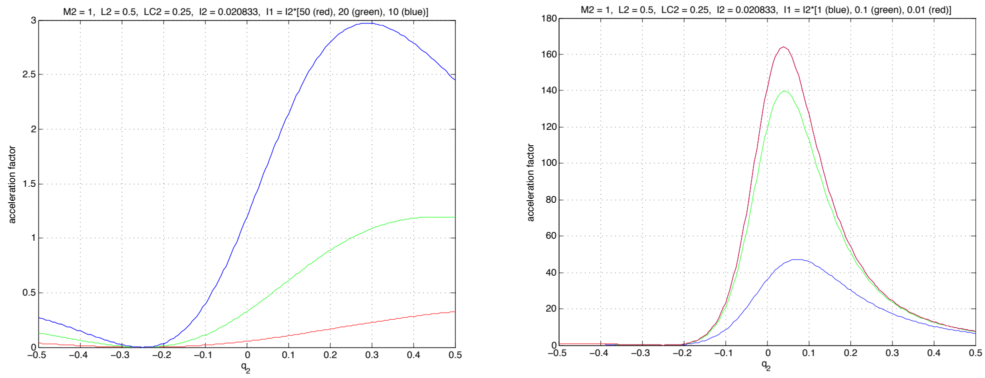

Figure 3: Behavior of the $h(q_2)$ function, for various ratios I_1/I_2: high ratios $50, 20, 10$ (left) and low ratios $= 1, 0.1, 0.01$ (right).

The physical explanation of these behaviors is as follows. When the inertia of the first link is very large, this constant inertia will dominate the effort needed by the first motor to accelerate the robot structure and so the maximum tangential component of the end-effector acceleration will be obtained when the second link is fully stretched. On the other hand, when the first link inertia can be assumed as negligible in the picture, the maximum tangential acceleration of the end-effector will result from the trade-off between two contrasting effects: the amplification of the joint acceleration due to a longer radial extension of the second link and its reduction due to the associated larger inertia seen by the first motor torque. Thus, qualitatively speaking, the peak will be somewhere in between $q_2 = 0$ and $q_2 = L_2$.

Note finally that when the location of the center of mass of the second link (with non-uniformly distributed mass) approaches the tip of the link ($d_{c2} = 0$), the above qualitative behavior remains the same, but the plots in Fig. 3 will become symmetric w.r.t. $q_2 = 0$, with the single minimum at $q^*_{2,min} = 0$ and two maxima in $\pm|q^*_{2,max}| \in [-L_2, L_2]$.

Exercise 2

The problem can be solved by using the residual vector $\boldsymbol{r}$ as a collision monitoring signal, together with a number of ordered positive thresholds on its norm $\|\boldsymbol{r}\|$ to be used in the switching conditions, and suitable control laws for each state.

Based on the known model (1), the residual $\boldsymbol{r} \in \mathbb{R}^6$ can be defined as

$$\boldsymbol{r}(t) = \boldsymbol{K}\left(\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \int_0^t \left(\boldsymbol{\tau} + \boldsymbol{C}^T(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}) + \boldsymbol{r}\right) ds\right), \quad \text{with } \boldsymbol{K} > 0 \text{ (diagonal)}, \tag{9}$$

where $\boldsymbol{\tau}$ is the actual control torque applied in any of the robot states A, B, or C. Using (1), equation (9) implies the dynamic behavior

$$\dot{\boldsymbol{r}} = \boldsymbol{K}\left(\boldsymbol{\tau}_c - \boldsymbol{r}\right), \tag{10}$$

where $\boldsymbol{\tau}_c \in \mathbb{R}^6$ is the joint torque resulting from a collision force/moment occurring anywhere along the robot structure. Indeed, if at some time t the torque τ_c returns to zero, then each component of $\boldsymbol{r}$ will decay exponentially to zero as well. Moreover, for a sufficiently large $\boldsymbol{K}$, from (10) we can use the approximation $\boldsymbol{\tau}_C \simeq \boldsymbol{r}$ and use the residual vector $\boldsymbol{r}$ as a proxy of the unknown joint torque $\boldsymbol{\tau}_c$ due to collision.

With reference to Fig. 2, in the following suitable control laws will be defined for each state.

- **Control in state** A. Define the desired task trajectory as $\boldsymbol{x}_d(t) = \left(\boldsymbol{p}_d^T(t)\ \boldsymbol{\phi}_d^T(t)\right)^T \in \mathbb{R}^6$. In order to accurately follow this smooth trajectory, we use the Cartesian feedback linearization controller

$$\begin{aligned}\boldsymbol{\tau} = {} & \boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q})\left(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_D\left(\dot{\boldsymbol{x}}_d - \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}\right) + \boldsymbol{K}_P\left(\boldsymbol{x}_d - \boldsymbol{f}(\boldsymbol{q})\right) - \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}\right) \\ & + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\dot{\boldsymbol{q}}),\end{aligned} \tag{11}$$

  with 6×6 (typically diagonal) gain matrices $\boldsymbol{K}_P > 0$ and $\boldsymbol{K}_D > 0$. Within this law, the presence of a PD action on the task error allows to recover exponentially transient errors. This is necessary, e.g., when the complete task is partially abandoned and then resumed (in case we are coming back to state A from state B).

- **Control in state** B. In this case, the orientation part of the desired task will be relaxed, while the position task $\boldsymbol{p}_d(t) \in \mathbb{R}^3$ for the robot end-effector should be kept. Therefore, the robot becomes kinematically redundant since the task has dimension $m = 3$ while the robot has $n = 6$ control commands available; the degree of redundancy is thus $n - m = 3$. We continue to achieve Cartesian position tracking, e.g., by using a dynamically consistent redundancy resolution scheme. This control scheme uses the 3×6 Jacobian $\boldsymbol{J}_p$ in a partially feedback linearizing law that is weighted by the inverse of the task inertia matrix $\boldsymbol{\Lambda}(\boldsymbol{q})$ and adds a suitable torque $\boldsymbol{\tau}_0 \in \mathbb{R}^6$ projected in the dynamic null space of the task. We have thus

$$\begin{aligned}\boldsymbol{\tau} = {} & \boldsymbol{J}_p^T(\boldsymbol{q})\boldsymbol{\Lambda}(\boldsymbol{q})\Big(\ddot{\boldsymbol{p}}_d + \boldsymbol{K}_{D,p}\left(\dot{\boldsymbol{p}}_d - \boldsymbol{J}_p(\boldsymbol{q})\dot{\boldsymbol{q}}\right) + \boldsymbol{K}_{P,p}\left(\boldsymbol{p}_d - \boldsymbol{f}_p(\boldsymbol{q})\right) - \dot{\boldsymbol{J}}_p(\boldsymbol{q})\dot{\boldsymbol{q}} \\ & \qquad + \boldsymbol{J}_p(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\left(\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_f(\dot{\boldsymbol{q}})\right)\Big), \\ & + \left(\boldsymbol{I} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{\Lambda}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\right)\boldsymbol{\tau}_0,\end{aligned} \tag{12}$$

  with 3×3 (typically diagonal) gain matrices $\boldsymbol{K}_{P,p} > 0$ and $\boldsymbol{K}_{D,p} > 0$, and the 3×3 inertia matrix reduced to the task

$$\boldsymbol{\Lambda}(\boldsymbol{q}) = \left(\boldsymbol{J}_p(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{J}_p^T(\boldsymbol{q})\right)^{-1}.$$

  In (12), the torque $\boldsymbol{\tau}_0 = \boldsymbol{K}_r\boldsymbol{r}$ is used, with $\boldsymbol{K}_r > 0$, so as to obtain a reaction to the collision torque $\boldsymbol{\tau}_c \simeq \boldsymbol{r}$ which is consistent with the remaining Cartesian position task.

- **Control in state** C. In this case, the complete original task is abandoned. The robot reacts to the collision in a stronger or weaker way depending on the intensity (and direction in the joint space) of $\boldsymbol{r}$, which is a proxy of the severity of the collision. Moreover, to avoid bias in the reaction due to the gravity, this term should be cancelled. As a result

$$\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}_r \boldsymbol{r} \tag{13}$$

with $\boldsymbol{K}_r > 0$. Once the contact is lost, $\boldsymbol{r}$ will go to zero. As a result, thanks of the presence of friction, the robot will come to a stop in a zero-gravity condition. Joint velocity damping can be added so as to anticipate the instant when the robot is finally at rest, but this will limit quick reaction to collisions.

Transitions between the states in Fig. 2 will be driven by the actual value of $\|\boldsymbol{r}\| \geq 0$. To this end, define a sequence of positive thresholds for this variable:

$$0 < r_{low} < r_{mild} < r_{severe}.$$

The value r_{low} is the minimum threshold that should be crossed by $\|\boldsymbol{r}\|$ in order to detect reliably contact/collision events (i.e., obtaining few false positives, or eliminating them). The detection instant $t_{detect} \geq 0$ is the first instant at which $\|\boldsymbol{r}(t_{detect})\| \geq r_{low}$. For the choice of this lowest threshold, one takes into account the presence of noise in position sensing and in the generation of an estimate of the velocity $\dot{\boldsymbol{q}}$ by numerical differentiation of the position measures $\boldsymbol{q}$, as well as the remaining model uncertainties. For the two other thresholds, the rationale is that mild collisions will generate small values of the norm of the residual and, conversely, severe collisions will be associated to large values of $\boldsymbol{r}$. The value r_{mild} is chosen only slightly above r_{low}, so that the control system may detect a contact but not yet consider it as a collision, letting thus the robot continue the original motion task. With this in mind, the following switching conditions correctly realize the desired behavior:

- **condition** $A \Rightarrow B$: $r_{mild} \leq \|\boldsymbol{r}\| < r_{severe}$;
- **condition** $A \Rightarrow C$: $\|\boldsymbol{r}\| \geq r_{severe}$;
- **condition** $B \Rightarrow C$: $\|\boldsymbol{r}\| \geq r_{severe}$;
- **condition** $B \Rightarrow A$: $\|\boldsymbol{r}\| < r_{low}$.

Note that the last condition may be replaced also by $\|\boldsymbol{r}\| < r_{mild}$. However, using the more conservative value r_{low} introduces some hysteresis, so that the robot will avoid switching several times between the states A and B when the norm of the residual is oscillating around r_{mild}.

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