

## Written exams of Robotics 2

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All materials are in English, unless indicated (oldies are in Italian)

Year	Date (mm.dd)	Number of exercises	Topics	Notes
2025	09.19	3	Potential terms of a flexible link under gravity with two elastic springs, linear factorization of these terms in the dynamic model, linear approximation, and forced/unforced equilibria Swing-up maneuver for an actuated pendulum along a cubic trajectory and uniform time-scaling for minimum motion time; Pros and cons of a linear factorization of robot dynamics (also, with addition of joint friction models and motor inertias) using standard dynamic parameters or dynamic coefficients in identification and adaptive control problems	---
2025	07.08	4	Step scaling of the Projected Gradient method in the presence of joint velocity limits, with geometric interpretation in the $m=1, n=2$ case; Dynamic model of a planar 4P robot in the vertical plane, with Cartesian inertia matrix and adaptive control; Minimization of inverse inertia-weighted norm of torque, with a reference torque in the cost function and presence of an end-effector force, with a numerical case for a planar PR robot in a 1-dimensional task; Residual design for actuation FDI in a planar RP robot under gravity, and time profiles of residuals for a total failure of joint 1 with robot in motion	solutions

2025	06.11	5	<p>Redundancy resolution schemes with task priority on a 3R planar arm with an end-effector velocity and an obstacle clearance velocity*;</p> <p>Two-mass system with a flexible transmission under gravity: dynamic model, equilibria, inverse dynamics, feedforward and initial conditions for perfect tracking;</p> <p>Energy-driven torque control design (with regions for joint velocity norm);</p> <p>Adaptive tracking control of minimal dimension for an actuated pendulum with partial knowledge of parameters;</p> <p>Two cases of natural and artificial constraints for a wedge in a rectangular groove</p>	solutions, MATLAB code
2025	04.30 (Midterm Test)	3	<p>Redundancy resolution by Task Priority or by Task Augmentation for a 5R planar arm performing 3 tasks (with 5 items);</p> <p>Dynamic model of a PPR spatial robot, with analysis of factorization of velocity terms, linearization in terms on a minimal set of dynamic coefficients, and an inverse dynamics problem;</p> <p>Kinetic energy of a 3R spatial robot using the recursive algorithm with moving frames, followed by a LQ optimal redundancy resolution problem in joint acceleration</p>	solutions
2025	02.07	3	<p>Comparison of Projected Gradient (PG), Reduced Gradient (RG) and Task Priority (TP) methods for addressing a redundant task in a 2P2R planar robot with joint limits;</p> <p>Reduced dynamics of a PPR robot with end effector constrained on a linear surface and a hybrid force-velocity control task (with redundancy);</p> <p>Dynamic modeling of a 1-dof robotic servomechanism with a DC motor and a flexible transmission, and related inverse dynamics for a rest-to-rest motion without residual vibrations</p>	---

2025	01.13	2	<p>2R polar robot:</p> <ul style="list-style-type: none"> <li>- Lagrangian dynamics and linear parametrization.</li> <li>- all known regulators and their properties,</li> <li>- inverse dynamics torque on a specific trajectory,</li> <li>- adaptive tracking control design;</li> </ul> <p>3P planar robot on a 2D task:</p> <ul style="list-style-type: none"> <li>- minimum velocity norm and minimum kinetic energy solutions;</li> <li>- mass distribution for a diagonal Cartesian inertia (if possible)</li> </ul>	---
2024	11.07	3	<p>Dynamic model of a PPR robot in a vertical plane, linear parametrization, and upper bound on the gravity term;</p> <p>Inverse dynamics computation for the PPR planar robot for exact execution of a circular trajectory;</p> <p>Hybrid force-motion task definition: writing with a ball pen on a flat sheet</p>	---
2024	09.19	5	<p>Dynamic model of an RP robot with base offset and under gravity, with linear parametrization;</p> <p>RP robot: integral of generalized momentum on a special joint trajectory;</p> <p>Minimization of (twice) the kinetic energy), under constant momentum components, with application to the RP robot;</p> <p>RP robot in contact with a compliant wall: hybrid force-position control with linear and decoupled dynamics (using modified impedance scheme);</p> <p>RP robot without gravity: inverse dynamics problem and minimum motion time under torque bound</p>	solutions
2024	07.08	4	<p>Dynamic model of a spatial PRR robot and its minimal linear parametrization [was part of 7 Jan 2020, without solution];</p> <p>Redundancy resolution at acceleration level for a 2R planar robot in a one-dimensional task [modified from 24 Mar 2023, without solution];</p> <p>Dynamic model of a two-mass system with a damped elastic joint under gravity, with design of a stabilizing feedback for regulation and of inverse dynamics torque for trajectory execution [modified from 19 Oct 2021, without solution];</p>	solutions

			Forced equilibria and local asymptotic stabilization of an underactuated 2R planar robot under gravity (Pendubot)	
2024	06.12	4	Kinetic energy and inertia matrix of a 3R robot given only via a DH table with linear factorization of inertial terms; Redundancy resolution for a 4R planar robot in relative pointing task (nominal or with error) with joint range optimization; Proof of global asymptotic stability of a PD motor regulation law for a robot with $n$ elastic joints without gravity; Fault detection and identification of a single actuation fault acting on the first joint of a PR robot under gravity	solutions
2024	04.24 (Midterm Test)	4	3R planar robot: i) Projected Gradient method at velocity level, with maximization of clearance from an obstacle*; ii) Reformulation with Task Priority method (two variants)*; 2R spatial robot under gravity: i) dynamic model and factorization of velocity terms*; ii) linear parametrization with a minimal set of dynamic coefficients*; iii) torque for an inverse dynamics example*; iv) equilibrium configurations; v) mechanical parameters needed to balance the robot under gravity*; vi) end-effector acceleration in response to a tip force*; Two computational uses of the recursive Newton-Euler algorithm; Maximum instantaneous reduction of the total robot energy under bounded torques: i) in a generic state with non-zero joint velocity; ii) in a state with $g(q) \neq 0$ and zero joint velocity	solutions, MATLAB codes
2024	02.16	4	Analysis of torque and acceleration limits related to the inertia matrix, with a skewed 2P planar robot example; Dynamic model of a (skewed) PPR planar robot, with linear parametrization and feedback linearization for trajectory tracking; Sphere-in-hole task description and related hybrid force-velocity control diagram; Redundancy resolution for a (skewed) PPR robot commanded in joint velocity, with minimization of the robot kinetic energy	---

2024	01.24	3	State-space equations from dynamic model, using coordinates $q$ and generalized momentum $p$ as state variables; State-space equations with $x = (q, p)$ for the dynamics of a RP planar robot with friction and under gravity (symbolic and numeric); Questionnaire with 4 questions	---
2023	09.11	1 (6 parts)	4P planar robot in vertical plane: - dynamic model in joint space - Cartesian inertia - joint-space regulation with PD+ and analysis - non-zero joint torque producing no task acceleration - minimum norm joint torque for a desired task acceleration - linear parametrization and adaptive control	---
2023	07.10	3	Two proofs on redundant robot dynamics: dynamically consistent decomposition of joint torques, expression of the task dynamics; For a RPR planar robot: inertia matrix, gravity vector, equilibria, linear parametrization of the gravity vector, bound on the gradient of $g(q)$ ; Sphere-in-Hole with natural and artificial constraints, selection matrices, handling of inconsistent measurements	---
2023	06.12	4	Inertia matrix for a 4R planar robot in absolute coordinates (plus some manipulation and extension); Five redundancy resolution schemes for a 4R planar robot with one or two 2-dimensional tasks, possibly with priority; Analysis of a PD + gravity compensation law for a PRR robot with balanced third link; Reduced dynamics and inversion-based motion and constraining force control of a closed kinematic chain for payload elevation	solutions
2023	04.19 (Midterm Test)	6	SNS redundancy optimization for a 3R planar robot commanded in acceleration with joint velocity and acceleration bounds; Mass and CoM distribution for achieving a structured gravity term in a 3R planar arm;	solutions

			Minimization of kinetic energy or of joint velocity norm for a 4P planar robot in a redundant task; Inertia matrix of a RPR spatial robot (with assigned DH frames and coordinates); Symbolic computation of Coriolis and centrifugal terms, three of its factorizations for skew-symmetry (or not), and regressor for a linear parametrization of a 3-dof (RPR) robot with assigned inertia matrix; Minimum-time motion on a rest-to-rest cubic joint trajectory for a vertical PR robot under force/torque input bounds	
2023	03.24	3	Dynamic redundancy resolution for a 2R planar robot in a one-dimensional task (symbolic and numeric solutions); Adaptive trajectory tracking control of a 2R planar robot, with uniform link mass distribution (as thin rods), uncertain but equal link lengths, and unknown but equal link masses; Definition of natural and artificial constraints for a hybrid force-motion task, with choice of suitable references for the controlled variables	---
2023	02.13	3	Three dynamic schemes of redundancy resolution for a planar 3R robot with uniform links; Minimum time swing-up for a 1-DOF pendulum along a cubic rest-to-rest trajectory under torque bounds Study of the position regulation of a system with two masses and an intermediate spring, each controlled by a PD or PD+ffw law	solutions
2023	01.25	2	- Planar 2R robot with balanced second link: Dynamic model of a balanced 2R robot with friction and under gravity and its linear parametrization; Minimum gains for regulation under PD+gravity compensation; Adaptive control for trajectory tracking; Minimum time rest-to-rest transfer under torque bounds keeping the second link fixed (no gravity nor friction) - Hybrid force-velocity control task of a Cartesian robot in contact with a compliant linear surface	solutions

2022	10.21	6	All exercises are related to a PR robot under gravity: Dynamic model; Minimal linear parametrization; Regulation with PD + constant gravity compensation; Computation of the associated initial acceleration; Inverse dynamics command along a desired joint trajectory; Minimum time motion under torque bounds along a special prescribed joint path	solutions
2022	09.09	3	Constrained minimization of joint range function for a planar 3R robot*; Task control for a planar 2R robot at the torque level, with specified transients for the errors along tangential/normal directions to the linear path (with numerical evaluation)*; Dynamic modeling and design of PID and iterative learning control laws for a two-mass system under gravity (plus dynamic modeling with a flexible cable)	solutions; MATLAB codes
2022	07.08	3	Change of coordinates in the dynamic model a generic 3R robot so as to match the work of the actuating torques*; Reduced dynamics and control problem for a generic robot, with the first joint variable being constrained ( $q_1=k$ ); Minimum-time rest-to-rest motion of a Cartesian PP robot under gravity and with bounded input forces*	solutions; MATLAB codes
2022	06.10	4	Adaptive control of a PR robot horizontal plane with regressor $Y$ , given $M(q)$ *; Algorithmic singularities for a planar 4R micro-macro robot performing two tasks, and its task priority solution*; Natural impedance control for elastic joint robots under preliminary joint elastic torque feedback; Minimum-time rest-to-rest motion of an actuated pendulum with bounded torque and bang-bang acceleration profile	solutions; MATLAB codes

2022	04.13 (Midterm Test)	6	Calibration of link lengths with data for a planar 2R robot; Discrete-time redundancy acceleration control minimizing norm of next velocity; Inertia matrix of a 3R spatial robot (polar robot mounted on a rotating base); Projected gradient at velocity level for a planar 3R robot minimizing $H_{\text{range}}$ (with joint limits and task scaling); Minimum and weighted norm of torques for a PR robot on a 1-dimensional task; Gravity term for a vertical PR robot, with bound on the norm of its gradient	solutions
2022	02.03	3	Cartesian regulation of a RPR robot under gravity, with minimal factorization of the gravity term and analysis of singular situation for the control law; Minimization of the acceleration norm with a null-space velocity damping in a 4R planar robot, with a 2-dimensional and then 3-dimensional augmented task; Reduced dynamic model of a constrained planar 2R robot under gravity with only one torque (mimicking an elevator along the y-axis), with inverse dynamics for a static condition and a rest-to-rest motion trajectory	solutions
2022	01.11	3	Minimization of kinetic energy and additional obstacle avoidance for a planar RPR robot; Joint torque control to smoothly stop a robot (at equilibrium) in time $T$ and uniform scaling in presence of an additional acceleration bounds; Reduced dynamic model and control of a planar PR robot with its end-effector constrained on a line segment	solutions



2021	10.19	2	Robust redundancy resolution for the two-arm DLR Justin robot, for independent or coordinated tasks of the two end effectors, at the velocity or acceleration level; Lagrangian dynamic model of a visco-elastic robot joint, with the design of a stabilizing feedback for regulation and inverse dynamics for trajectory execution [updated version from 23 March 2007]	---
2021	09.10	3	Deceleration control for a planar 3R redundant robot under input bounds*; Cartesian inertia matrix for the same planar 3R redundant robot*; Impedance control design for a 2-dof Cartesian robot without F/T sensing	solutions; MATLAB codes
2021	07.12	3	Acceleration control problems for a planar 3R robot under input bounds; Modeling and analysis of an incipient blocking actuator fault in a robot; Dynamic modeling and adaptive control of a planar RPR robot, with partially known dynamic parameters	solutions
2021	06.11	5	Weighted pseudoinverse using the pinv function of MATLAB*; Dynamic model of an elastically suspended link and two related dynamic problems; Guessing a robotic structure from inertial and gravity terms (a 3P portal robot); Transformation of generalized forces in two systems of coordinates (planar 3R); Trajectory tracking using feedback linearization along a circle, with transient behaviors specified in a time-varying task frame	solutions; MATLAB code
2021	04.14 (Remote Midterm Test)	3	Dynamics of a planar 2R robot under gravity whose links have a non-symmetric CoM, with its linear parameterization; Comparative methods of redundancy resolution at the velocity level in a planar 3R robot performing 2 tasks, possibly with priority: pseudoinversion of the Extended Jacobian, damped least squares on the Extended Jacobian, task priority method;	solutions

			Minimum time motion under bounded inputs for a PR robot tracing a circular path	
2021	02.04 (Remote)	4	Inertia and gravity terms of a RRPR robot, with their linear parameterization; Minimum-time motion on a linear path for two cooperating Cartesian robots under motor torque bounds; Dynamic self-motion task for a 3R robot with null-space projection or joint space decomposition; 1-dof force regulation with zero error (using PI control) for a rigid robot and a rigid environment	solutions
2021	01.12 (Remote)	5	Kinematic control in a self-motion task for a redundant planar 3R robot so as to minimize the potential energy due to gravity; Torque control for the same planar 3R robot and self-motion task; Dynamic model of a polar RR robot; Two dynamic problems for the same polar RR robot: analysis of a steady-state condition and minimal linear parametrization; Collision detection, isolation and identification for a PPR robot in the horizontal plane	solutions
2020	10.23 (Remote)	4	Regressor for adaptive control of a PR robot moving on a horizontal plane; Minimum weighted torque solutions for 1-dimensional task of the same PR robot; Sufficient conditions for asymptotic stability of a PD plus gravity compensation law on the same PR robot in the vertical plane; Definition of a force-motion hybrid task for a sphere rolling on a plane while following a desired trajectory of the contact point	---

2020	09.11 (Remote)	5	Control formulation for the simultaneous trajectory execution and obstacle avoidance of a planar 3R robot arm; Interaction matrix for the geometric barycenter of three point-features in IBVS, with null space computation; Cartesian inertia matrix of a RP robot, with tip acceleration in response to a force; Reduced dynamics for a planar 3R robot under Cartesian constraints on last link; Comparison of torques between feedback linearization and global Lyapunov-based trajectory tracking control laws	solutions
2020	07.15 (Remote)	5	Inertia matrix of a 4-dof (PRRR) planar robot using absolute coordinates; Task augmented Jacobian for two tasks (on the same 4-dof robot) with minimum norm solutions and analysis of algorithmic singularity; Natural and artificial constraints for a hybrid task of moving a cylinder on a plane; Uniform time scaling to recover torque feasibility for a 2R robot under gravity; Three questions on regulation controllers with specified asymptotic or exponential transients for the errors	solutions
2020	06.05 (Remote)	5	SNS algorithm under velocity bounds for a 3R robot; Analysis of physical conditions for a 2-dof dynamic model; Bound on the gradient of the gravity term for an RP robot under gravity with limited range of the prismatic joint; Dynamic modeling and adaptive control for a 2P robot in the vertical plane with joint friction and payload; Natural and artificial constraints for the hybrid task of closing a door	solutions
2020	04.15 (Remote Midterm Test)	Q 10	Questionnaire with 10 questions, with replies to be elaborated*	solutions; MATLAB codes (for Q2 to Q9)

2020	02.12	3	Dynamic model of a planar RPR robot under gravity, linear parametrization of the gravity vector, bound on its gradient; Two redundancy resolution problems for the same robot above, assuming joint velocity as input; A 1-dof force control problem with combinations of P, I and feedforward laws	---
2020	01.07	2	Dynamic model of a spatial PRR robot, linear parametrization, and adaptive control design; Questionnaire with 4 questions	---
2019	09.11	3	Control law to assign a desired dynamic behavior to the robot kinetic energy in the closed-loop system, with application to a 3R robot moving on the horizontal plane; Minimum-time motion under joint torque/force bounds for a RP planar arm moving its end-effector along a circular path; Impact of two masses, with conservation of kinetic energy and total momentum	solutions
2019	07.11	3	Task priority control of a 3R planar robot for two tasks, with analysis of their compatibility*; Residual vector computation for collision detection and isolation in a RP planar robot; Minimum-time motion under torque bound for an actuated pendulum under gravity on a special class of trajectories with bang-coast-bang acceleration profile*	solutions; MATLAB codes
2019	06.17	6	Gravity term in the dynamic model of the 6-dof Kawasaki S030 robot (with simplified assumptions); Check for a 3x3 matrix to be an inertia matrix; Comparison of Jacobian pseudoinverse and Jacobian transpose velocity commands in redundant robots; Assignment of a linear and decoupled Cartesian error dynamics to a 2R robot in a regulation task;	solutions

			Natural and artificial constraints in a hybrid force-velocity control task formulation	
2019	04.29 (Midterm Test)	5	<p>Check for 2x2 matrices to be inertia matrices;</p> <p>Dynamic model of a PRP planar robot and its linear parametrization;</p> <p>Gravity term in the dynamic model of a nR planar robot, with equilibrium conditions and balancing conditions;</p> <p>Factorizations of the Coriolis/centrifugal terms of a 2-dof robot and a 3-dof robot;</p> <p>Execution of a one-dimensional task with a 2R robot, with minimization of its kinetic energy;</p> <p>Acceleration resolution for a redundant 3R planar robot, under hard joint acceleration constraints</p>	solutions
2018	07.11	4	<p>Inertia matrix and its linear parametrization for a 2R planar robot with elastic joints;</p> <p>Saturated solution for a 4R planar robot executing a Cartesian acceleration under joint acceleration bounds (variant of the SNS algorithm);</p> <p>Interaction matrix for the average position of n point features;</p> <p>Reduced dynamics of a robot under the geometric constraint that a subset of coordinates is constant (<math>\mathbf{q}_b = \mathbf{q}_{bd}</math>)</p>	solutions
2018	06.11	3	<p>Explicit expression of the dynamic terms in the residual vector for collision detection and isolation in a RP robot moving in the vertical plane, and analysis of the possible issues in detection/isolation;</p> <p>Adaptive control law for a 1-dof actuated pendulum, when also the current-to-torque drive gain is unknown;</p> <p>Feedback/feedforward control schemes for force regulation of a mass in contact through a load cell with a stiff environment: equilibria and stability analyses</p>	solutions

2018	04.26 (Midterm Test)	4	Dynamic model of a 2P2R planar robot under gravity and its linear parametrization*; Factorizations of the Coriolis/centrifugal terms in a planar 2R robot plus an inverse dynamics problem for the same robot*; General derivation of the Hamiltonian model equations of a robot manipulator, using generalized coordinates $q$ and generalized momentum $p$ as state variables; Minimum norm velocity solution for a (redundant) PPR planar robot with the associated unit inconsistency problem, and use of a weighted pseudoinversion to resolve the issue*	solutions; MATLAB codes
2018	03.27	2	Dynamic model, linear parametrization, approximate linearization and partial feedback linearization control of a 2-dof automated crane (with passive swinging link/payload); PPR planar robot with end-effector in contact with a linear, frictionless surface: nominal commands for a hybrid force/velocity dynamic task addressing also robot redundancy	solutions
2018	02.05	3	Formulation of the task Jacobian and of the related kinematic control problem when the task function is the norm of the task error, and its application to a visual servoing problem with two point-features; Dynamic model of a PRP (cylindrical-like) robot, with a horizontal first prismatic axis, and adaptive trajectory tracking control design; Iterative learning control of an actuated pendulum (single-link under gravity), with convergence analysis of the PD feedback plus (updated) feedforward law	solutions
2018	01.11	3	Redundancy resolution schemes for a planar RP robot in a one-dimensional task; Lagrangian dynamics, analysis, and nonlinear control of a Boulton-Watt centrifugal regulator; Impedance and force control design alternatives for a single mass subject to an external force	solutions

2017	10.27	1 (4 parts)	Definition of four different optimization schemes for redundant robots by local minimization of suitable norms (with application of the first two schemes to a planar RP robot): generalized momentum difference; velocity difference with respect to the (anti)gradient of a function; torque; weighted combination of torque and task acceleration error	---
2017	09.21	2	Dynamics of RP robot in a tilted plane, with computation of the maximum and minimum norm of the tip acceleration as function of the configuration, when starting at rest and under bounds on the command inputs; Definition of control laws and transition conditions among states for a robot operating in the presence of a human, with a Cartesian trajectory task in normal conditions and detection/reaction to mild or severe collisions	solutions
2017	07.11	3	Dynamic terms of an RPP (cylindrical) robot used in a tracking control law; Derivation of a reduced dynamics model and control of a Cartesian 2P robot in a vertical plane, with its end-effector motion being constrained to a linear surface; Steady-state analysis of a planar 3R robot under a Cartesian position regulation law, while in contact with an obstacle: equilibrium control torques, torques at the joints due to contact, momentum-based residual, and estimation of the Cartesian contact force	solutions
2017	06.06	3	Inverse differential solution with minimum norm of the joint jerk for a 3R planar robot using absolute coordinates (nominal case and feedback control to correct initial mismatch with Cartesian trajectory)*; Choice of PD gains in a regulation control law with gravity cancelation for the same 3R planar arm of Ex#1 and under joint torque bounds; Dynamic model of a PRP planar robot, with factorization of the quadratic velocity terms, equilibrium configurations, and linear bound on the Hessian of the potential energy due to gravity*	solutions; MATLAB codes

2017	05.29 (Final Test)	4	<p>Presentation of known regulation control laws in the presence of gravity, with two more specific questions;</p> <p>Interaction matrix for a point feature in an IBVS problem when using polar coordinates in the image plane, and its characteristics*;</p> <p>Gravity term and its parametrization in an adaptive trajectory tracking controller for a PRP planar robot when the other inertial parameters are already known;</p> <p>Definition of task frame and of natural and artificial constraints for a hybrid force/motion control of a robotized surface polishing task</p>	solutions; MATLAB code
2017	03.29 (Midterm Test)	5	<p>Inertia matrix of a 3R spatial robot and its linear parametrization*;</p> <p>Proof of a weighted pseudoinverse in case of rank deficiency of the Jacobian;</p> <p>Saturated solution for a 4R planar robot executing a Cartesian velocity under joint velocity bounds (SNS algorithm)*;</p> <p>Reduced Gradient and Task Augmented solutions for a 3R planar robot*;</p> <p>Calibration equations for a 2R planar robot with uncertain parameters*</p>	solutions; MATLAB codes
2017	01.11	3	<p>Interpretation of the inertia matrix and all possible regulation control laws for a 2R robot, with their design conditions and convergence/stability properties;</p> <p>Use of recursive Newton-Euler algorithm for computing kinematic quantities (similar to Ex#2, 22.09.2014);</p> <p>Redundancy resolution for planar 3R robot at the velocity level in the presence of an obstacle (same as Ex#1, 10.06.2014)</p>	---
2016	10.28	2	<p>Inertia matrix and gravity vector (with equilibrium configurations) of a RP robot in a vertical plane, with second prismatic joint axis skewed;</p> <p>Dynamic modeling, determination of unforced equilibrium states and of steady-state conditions under constant input force, and regulation control for a mechanical system of three masses interconnected by springs with viscous friction</p>	solutions



2016	09.12	2	Inertia matrix and gravity vector (with equilibrium configurations) of a RPR robot in a vertical plane; Dynamic modeling and determination of the equilibrium states for a mechanical system of two masses interconnected by a nonlinear spring	solutions
2016	07.11	3	Dynamic model of a planar 2R robot in absolute coordinates and input torque transformation; Task frame and natural/virtual constraints definition for a square peg-in-hole insertion; Compliance and force control laws in one contact direction, with robustness analysis	solutions
2016	06.06	2	Dynamic modeling of a polar 2R robot and its adaptive trajectory tracking control law; All possible regulation control laws for this robot, with their design conditions and stability properties	only solution to Ex#1
2016	06.01 (Final Test)	1 (4 parts)	Planar 2R robot subject to a single holonomic constraint: reduced dynamic model and its features; equilibrium torques (and associated constraint force); simulation set-up for the reduced model; hybrid force/motion control task	solution
2016	04.13 (Midterm Test)	3	Inertia matrix and Coriolis/centrifugal terms of a planar PRR robot; Gravity vector, its linear parametrization, equilibria and mechanical balance of a planar 4R robot; Singularities of a planar 4R robot for position/orientation tasks, and a joint velocity solution minimizing the distance from the joint range midpoints	solutions
2015	04.15	2	Equilibrium configurations, approximate linearization, and regulation control of RP robot; Motion of a planar 3R robot using redundancy to avoid Cartesian obstacles	modified from 2006.07.13 (in Italian)
2014	10.27	2	Expression of the residual for collision detection and isolation in a planar PRR;	solutions

			Maximum contact force in norm applied at the tip of the planar PRR robot that can be balanced in the presence of hard bounds on the actuator torques	
2014	09.22	2	Dynamic model of a DC motor with elastic transmission, planning of a rest-to-rest trajectory, and its inverse dynamics solution; Use of recursive Newton-Euler algorithm for computing end-effector differential kinematic quantities	---
2014	07.15	1	Analysis of gravity terms and unforced equilibrium configurations for a KUKA LWR with last three joints frozen	---
2014	06.10	2	Redundancy resolution for planar 3R robot at the velocity level in the presence of an obstacle; Dynamic modeling and control of a two-mass system under gravity and with an elastic transmission	---
2014	04.02	2	Definition of kinematic controllers for the two-arm Justin robot; Acceleration analysis for a planar 3R robot subject to an end-effector force	---
2013	09.19	1	Inertia matrix and acceleration analysis for a planar RPPR robot	---
2013	07.15	1	Analysis and estimation of a contact force in three different cases for a planar 3R robot	solution
2013	06.10	1	Kinetic energy using the recursive algorithm with moving frames, inertia matrix and its linear parametrization for a 4-dof robot for which only the DH table is given	---
2013	02.06	1	Reduced dynamic model for a planar RP arm, with its end-effector constrained on a line, and associated hybrid force/motion regulation law	solution
2013	01.09	2	Jacobians for two tasks, algorithmic singularities, and task priority solution for a planar 3R arm; Constrained dynamic model for a planar RP arm, with its end-effector constrained on a line	solutions
2012	07.05	1 (2 parts + bonus)	Dynamics of a 3-dof portal robot for aeronautical industry; Controller for a regulation task; (bonus) Generalized coordinates for the closed kinematic loop	solution

2012	06.11	2	Null space of visual interaction matrix for a point feature (MATLAB code included); Regulation task for a planar 3R robot subject to a contact force	solutions
2011	09.12	1	Gravity balance and PD + constant gravity compensation of a 2R robot in the vertical plane with an additional payload	solution
2011	07.04	1	Dynamic modeling and feedback/feedforward force regulation of a two-mass, two-spring system	---
2011	06.17	1	Collision reaction for a 2R planar robot and use of the residual	solution
2010	09.15	1	Inertia matrix and its minimal linear parameterization for a planar RPR robot	solution
2010	07.07	1	Dynamic model of a two-mass/spring/damper system and analysis of a contact force loop	solution
2010	06.15	1	Dynamic model of RP under gravity and minimum torque solution for a one-dimensional acceleration task	solution
2009	09.10	2	Dynamic control of a linear Cartesian trajectory with decoupled error along the tangential and normal directions to the path; Uniform scaling of unfeasible trajectory with respect to torque constraints (without gravity) <b>(in Italian)</b>	solutions <b>(in Italian)</b>
2009	07.10	2	Feasible acceleration region for a 2R planar robot with torque constraints; Definition of natural and artificial constraints for a hybrid task and its realizability with a SCARA robot <b>(in Italian)</b>	solutions <b>(in Italian)</b>
2009	06.10	2	Inertia matrix of a planar PRP robot; Solution with minimum weighted norm of the torque and dynamic control in the task space for redundant robots <b>(in Italian)</b>	solutions <b>(in Italian)</b>
2008	09.11	1	Adaptive control of planar 2R robot with payload and minimal parameterization <b>(in Italian)</b>	sketch of solution <b>(in Italian)</b>
2008	03.20	1	Dynamic model of planar PRR robot under gravity and linear parameterization <b>(in Italian)</b>	---

2007	09.13	2	General structure of the minimal parameterization of the gravity term for planar nR robots; Hybrid task of surface finishing on a sphere with a 3P or 3R robot <b>(in Italian)</b>	---
2007	04.19	2	Dynamic model of PRP robot under gravity; Robot regulation control with assignment of linear error dynamics <b>(in Italian)</b>	solutions <b>(in Italian)</b>
2007	03.23	2	Redundancy resolution for the two-arm DLR Justin robot; Lagrangian dynamic model of a visco-elastic robot joint <b>(in Italian)</b>	---
2006	09.11	1	Planar 3R robot: inertia matrix, optimization of manipulability, minimum norm joint velocity computation <b>(in Italian)</b>	---
2006	07.13	2	Equilibrium configurations and approximate linearization of RP robot; Motion of a planar 3R robot among obstacles with Cartesian artificial potentials <b>(in Italian)</b>	---
2006	06.30	2	End-effector forces and torque at second joint that produce zero acceleration at the joints of a planar 3R robot under gravity; Voronoi diagram <b>(in Italian)</b>	only solution to ex #2 <b>(in Italian)</b>
2005	09.22	1	Kinematic control of a mobile manipulator (unicycle + single link arm) <b>(in Italian)</b>	---
2005	04.14	1 (two parts)	Dynamic model of a polar 2R robot; regulation with PD plus constant gravity compensation law <b>(in Italian)</b>	---
2005	04.05	2	Dynamic model of a planar RP robot, equilibrium configurations under gravity, minimal parameterization and adaptive control; Stabilization of a linear mass in the presence of limited motion range, using artificial potentials (Lyapunov proof) <b>(in Italian)</b>	solutions <b>(in Italian)</b>
2004	04.06	1	Motion planning with approximate cell decomposition <b>(in Italian)</b>	solution <b>(in Italian)</b>
2004	03.25	1	Dynamic model of an offset RP robot under gravity, adaptive control, partial feedback linearization of the first joint dynamics when the second joint is passive (or vice versa) <b>(in Italian)</b>	solution <b>(in Italian)</b>

**Note:** For these\* problems, MATLAB codes for computing solutions and/or for graphics are either embedded in the solution text or available to the students of the course upon request (contact [deluca@diag.uniroma1.it](mailto:deluca@diag.uniroma1.it)).